errata (259)
p. 119, 216 - 224 - 280 - 284 - 301 - 417
above corrections were sent
on April 20 - 1900.

[redacted]

AN

ADVANCED ARITHMETIC

FOR

HIGH SCHOOLS, NORMAL SCHOOLS
AND ACADEMIES

BY

G. A. WENTWORTH, A.M.
AUTHOR OF A SERIES OF TEXT-BOOKS IN MATHEMATICS

TEACHERS' EDITION

BOSTON, U.S.A.
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[redacted]

PREFACE.

THIS edition is intended for teachers, *and for them only.* The publishers will make every effort to keep the book from pupils; and teachers are urged to exercise the utmost care not to lose their copies, or to leave them where pupils can have access to them.

It is hoped that young teachers will derive great advantage from studying the systematic arrangement of the algebraic work, for such attention has been paid to this as the limitation of the page would allow.

It is also expected that many teachers, who are pressed for time, will find great relief by not being obliged to work out every problem in the Algebra.

G. A. WENTWORTH.

EXETER, N.H., September, 1898.

[redacted]

ADVANCED ARITHMETIC.

TEACHERS' EDITION.

Exercise 1. Page 5.

Write in periods, and read:

1. 7000; seven thousand.

2. 7842; seven thousand, eight hundred forty-two.

3. 5043; five thousand, forty-three.

4. 8375; eighty-three hundred seventy-five.

5. 2020; two thousand, twenty.

6. 1753; seventeen hundred fifty-three.

7. 18,757; eighteen thousand, seven hundred fifty-seven.

8. 75,764; seventy-five thousand, seven hundred sixty-four.

9. 22,003; twenty-two thousand, three.

10. 70,856; seventy thousand, eight hundred fifty-six.

11. 234,567; two hundred thirty-four thousand, five hundred sixty-seven.

12. 34,561; thirty-four thousand, five hundred sixty-one.

13. 123,456; one hundred twenty-three thousand, four hundred fifty-six.

14. 654,089; six hundred fifty-four thousand, eighty-nine.

15. 600,897; six hundred thousand, eight hundred ninety-seven.

16. 704,608; seven hundred four thousand, six hundred eight.

17. 350,709; three hundred fifty thousand, seven hundred nine.

18. 240,682; two hundred forty thousand, six hundred eighty-two.

19. 682,000; six hundred eighty-two thousand.

20. 753,110; seven hundred fifty-three thousand, one hundred ten.

21. 703,101; seven hundred three thousand, one hundred one.

22. 870,890; eight hundred seventy thousand, eight hundred ninety.

23. 21,978,564; twenty-one million, nine hundred seventy-eight thousand, five hundred sixty-four.

24. 17,756,423; seventeen million, seven hundred fifty-six thousand, four hundred twenty-three.

25. 300,200,100; three hundred million, two hundred thousand, one hundred.

26. 707,303,202; seven hundred seven million, three hundred three thousand, two hundred two.

27. 3,125,476,890; three billion, one hundred twenty-five million, four hundred seventy-six thousand, eight hundred ninety.

28. 79,501,346,081; seventy-nine billion, five hundred one million, three hundred forty-six thousand, eighty-one.

29. 3,000,872,696; three billion, eight hundred seventy-two thousand, six hundred ninety-six.

30. 72,727,000,000; seventy-two billion, seven hundred twenty-seven million.

Exercise 2. Page 6.

Write in figures, arranged in periods:

1. 600,006.
2. 713,329.
3. 7854.
4. 4,003,330.
5. 110,000,279.
6. 19,000,004,000,309.
7. 7,676,466.
8. 347,651,785.
9. 200,000,207.
10. 400,000,400,004.

Exercise 3. Page 9.

Read:

1. Six million, seven hundred twenty-eight thousand, six hundred forty-two.

2. Three and twenty-four thousand, six hundred fifty-eight hundred-thousandths.

3. Forty-nine thousand, five hundred sixty-eight and four thousand, seven hundred eighty-two ten-thousandths.

4. Thirty-four billion, five hundred ninety-eight million, four hundred ninety-two thousand, two hundred twelve.

5. Four million, two thousand and two hundredths.

6. Eighteen hundred seventy-two and seventeen hundredths.

7. Ninety-four and six hundred fifty-eight thousand, two hundred sixty-five millionths.

8. Three hundred seven ten-thousandths.

9. One hundred and one hundredth.

[redacted]

10. One million, eight hundred seventy-two thousand, five hundred sixty-three and three hundred seventy-two thousandths.

11. Seventeen and eight thousandths.

12. One hundred forty-three and one hundred forty-three hundred-thousandths.

13. Twenty-nine and eighty-one hundred-thousandths.

14. Five million, two hundred sixty-two thousand, eight hundred seventy-three.

15. Eight and seventy-eight hundred fifty-four ten-thousandths.

16. One hundred eighty-two dollars, and twenty-seven cents, five mills.

17. Eight cents, six mills.

18. Seven cents, five mills.

19. Four hundred sixty-three dollars and eighty-seven cents.

20. Twenty thousand, five hundred forty-two dollars and two cents.

21. Seventy-five cents.

22. Four hundred twenty-eight thousand, four hundred twenty-eight and four hundred twenty-eight thousandths.

23. Fifteen hundred forty-two and eighty-seven thousandths.

24. Six hundred forty-two and eight hundred seventy-three thousand, six hundred fifty-four millionths.

25. Four hundred and four hundred-thousandths.

26. Three billion, five hundred forty-three million, three hundred sixty-two thousand, three hundred thirty-eight.

27. Nine ten-millionths.

28. Fifty-two and two hundredths.

29. Fifty-six thousand, four hundred eighty-two and fifty-six hundredths.

30. Eighty-seven million, eight hundred sixty-five thousand, eight hundred forty-two and eighty-seven thousand, eight hundred sixty-six hundred-thousandths.

Exercise 4. Page 9.

Write in figures:

1. 81,000.345.
2. 3741.675.
3. 413.08.
4. 96.096.
5. 9.000043.
6. 154.0032.
7. 0.075.
8. 0.3.
9. 44,044,044.044.
10. 100.000043.

11. 0.000143.
12. 140.000003.
13. 943,000.943.
14. 0.0000722.
15. 13.01468.
16. 4.1009.
17. 101.0101.
18. 17,649,000.
19. 12,000,012,000.
20. 12,000,000,000.012.
21. $8.12.
22. $127.01.
23. $14,278.275.
24. $1000.011.
25. $234.55.
26. $0.25 ; $0.034.
27. 1,489,590.590.
28. 43,677.04006.
29. 3069.0078416.

Exercise 5. Page 11.

ɫead :

Ґhirty-six ; forty ; forty-six ; fifty-eight ; fifty-nine ; eighty-one ; .ety-one ; ninety-three ; one hundred nine ; two hundred nine ;) hundred twenty ; one hundred fifty-nine ; eighteen hundred hty-six ; sixteen hundred sixty-six ; seventeen hundred seventy- ; fourteen hundred fifty-nine ; fifteen hundred eighty-nine.

Express in the Roman system :

ⅭLIII ; LV ; LXXXI ; LXXVII ; XCIX ; CXIII ; CXXVIII ; :IV ; DCCXXIV ; DCXXX ; MXX ; MXL ; MLXXXVIII ; :XXXI ; MCCXVIII ; MCDXCII ; MDCCLXXVI ; MDCCCXCIX ; :CCXIX ; MDLVI ; MDCCCXCVII ; MDCXX ; MDCCLXXXIII ;)CCCXII ; MDCCCLXI ; MDCCCLXXII.

Exercise 6. Page 12.

ŀ. Count to 100 or more by 2's.

), 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 42, 44, 46, 48, 50, 52, 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98, 100.
., 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 67, 69, 71, 73, 75, 77, 81, 83, 85, 87, 89, 91, 93, 95, 97, 99, 101.

ȝ. Count to 100 or more by 3's.

), 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57, 63, 66, 69, 72, 75, 78, 81, 84, 87, 90, 93, 96, 99, 102.

[redacted]

1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40, 43, 46, 49, 52, 55, 58, 61, 64, 67, 70, 73, 76, 79, 82, 85, 88, 91, 94, 97, 100.

2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, 41, 44, 47, 50, 53, 56, 59, 62, 65, 68, 71, 74, 77, 80, 83, 86, 89, 92, 95, 98, 101.

3. Count to 100 or more by 4's.

0, 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 88, 92, 96, 100.

1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61, 65, 69, 73, 77, 81, 85, 89, 93, 97, 101.

2, 6, 10, 14, 18, 22, 26, 30, 34, 38, 42, 46, 50, 54, 58, 62, 66, 70, 74, 78, 82, 86, 90, 94, 98, 102.

3, 7, 11, 15, 19, 23, 27, 31, 35, 39, 43, 47, 51, 55, 59, 63, 67, 71, 75, 79, 83, 87, 91, 95, 99, 103.

4. Count to 100 or more by 5's.

0, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100.

1, 6, 11, 16, 21, 26, 31, 36, 41, 46, 51, 56, 61, 66, 71, 76, 81, 86, 91, 96, 101.

2, 7, 12, 17, 22, 27, 32, 37, 42, 47, 52, 57, 62, 67, 72, 77, 82, 87, 92, 97, 102.

3, 8, 13, 18, 23, 28, 33, 38, 43, 48, 53, 58, 63, 68, 73, 78, 83, 88, 93, 98, 103.

4, 9, 14, 19, 24, 29, 34, 39, 44, 49, 54, 59, 64, 69, 74, 79, 84, 89, 94, 99, 104.

5. Count to 100 or more by 6's.

0, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96, 102.
1, 7, 13, 19, 25, 31, 37, 43, 49, 55, 61, 67, 73, 79, 85, 91, 97, 103.
2, 8, 14, 20, 26, 32, 38, 44, 50, 56, 62, 68, 74, 80, 86, 92, 98, 104.
3, 9, 15, 21, 27, 33, 39, 45, 51, 57, 63, 69, 75, 81, 87, 93, 99, 105.
4, 10, 16, 22, 28, 34, 40, 46, 52, 58, 64, 70, 76, 82, 88, 94, 100.
5, 11, 17, 23, 29, 35, 41, 47, 53, 59, 65, 71, 77, 83, 89, 95, 101.

6. Count to 100 or more by 7's.

0, 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98, 105.
1, 8, 15, 22, 29, 36, 43, 50, 57, 64, 71, 78, 85, 92, 99, 106.
2, 9, 16, 23, 30, 37, 44, 51, 58, 65, 72, 79, 86, 93, 100.
3, 10, 17, 24, 31, 38, 45, 52, 59, 66, 73, 80, 87, 94, 101.

4, 11, 18, 25, 32, 39, 46, 53, 60, 67, 74, 81, 88, 95, 102.
5, 12, 19, 26, 33, 40, 47, 54, 61, 68, 75, 82, 89, 96, 103.
6, 13, 20, 27, 34, 41, 48, 55, 62, 69, 76, 83, 90, 97, 104.

7. Count to 100 or more by 8's.

0, 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, 104.
1, 9, 17, 25, 33, 41, 49, 57, 65, 73, 81, 89, 97, 105.
2, 10, 18, 26, 34, 42, 50, 58, 66, 74, 82, 90, 98, 106.
3, 11, 19, 27, 35, 43, 51, 59, 67, 75, 83, 91, 99, 107.
4, 12, 20, 28, 36, 44, 52, 60, 68, 76, 84, 92, 100.
5, 13, 21, 29, 37, 45, 53, 61, 69, 77, 85, 93, 101.
6, 14, 22, 30, 38, 46, 54, 62, 70, 78, 86, 94, 102.
7, 15, 23, 31, 39, 47, 55, 63, 71, 79, 87, 95, 103.

8. Count to 100 or more by 9's.

0, 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99, 108.
1, 10, 19, 28, 37, 46, 55, 64, 73, 82, 91, 100.
2, 11, 20, 29, 38, 47, 56, 65, 74, 83, 92, 101.
3, 12, 21, 30, 39, 48, 57, 66, 75, 84, 93, 102.
4, 13, 22, 31, 40, 49, 58, 67, 76, 85, 94, 103.
5, 14, 23, 32, 41, 50, 59, 68, 77, 86, 95, 104.
6, 15, 24, 33, 42, 51, 60, 69, 78, 87, 96, 105.
7, 16, 25, 34, 43, 52, 61, 70, 79, 88, 97, 106.
8, 17, 26, 35, 44, 53, 62, 71, 80, 89, 98, 107.

Find the sum of :

9.	**10.**	**11.**	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
3	2	3	5	3	2	5	5	4	5	3	1
5	1	6	6	3	7	3	6	8	5	6	8
7	9	7	7	4	7	2	4	7	3	7	8
6	8	8	8	5	3	1	7	3	6	3	7
21	20	24	26	15	19	11	22	22	19	19	24

21.	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**	**31.**	**32.**
6	9	6	4	4	3	6	7	5	8	2	9
8	5	4	5	4	7	2	5	5	2	9	6
7	4	3	6	3	5	1	8	9	2	9	5
9	3	7	7	7	5	8	3	3	7	4	4
30	21	20	22	18	20	17	23	22	19	24	24

Exercise 7. Page 15.

Find the sum of :

1.	2.	3.	4.	5.	6.
231	341.	430.31	512.87	12.78	1543.1
764	57.8	58.61	36.84	711.56	164.7
995	398.8	488.92	549.71	415.86	1707.8
				1140.20	

7.	8.	9.	10.	11.
1728.	1897.3	475.34	402.56	0.7854
402.56	675.34	6897.65	164.7	3.1416
2130.56	6897.65	1728.	0.5236	2.71828
	9470.29	9100.99	567.7836	6.64528

12.	13.	14.	15.	16.
2.71828	0.7854	2.7113	230.8	32.358
402.56	4.12	27.53	223.	821.9
1897.3	30.103	341.586	2.63	23.04
2302.57828	35.0084	371.8273	373.8	73.7
			830.23	950.998

17.	18.	19.	20.
202.3031	0.0078	653.03	939.303
71.575	7.377	65.303	65.746
65.813	653.03	6.5033	8.2794
339.6911	660.4148	724.8363	1013.3284

21.	22.	23.
2.7182818	0.4342945	1.6093295
3.1415927	0.2098882	15.4323487
0.7853982	4.8104774	3.785
6.6452727	5.4546601	20.8266782

24.	25.	26.
0.4771213	1.6093295	0.6213768
0.2908882	3.2808693	3.785
4.8104774	0.3937043	0.264
2.5399772	0.5235988	15.4323487
0.3937043	0.4342945	1.7320508
8.5121684	6.2417964	21.8347763

27.	28.	29.
0.6213768	0.3937043	1.4142136
1.4142136	0.3047973	1.6093295
3.2808693	1.7320508	0.30103
0.3047973	2.236068	0.381966
4.8104774	0.381966	3.2808693
10.4317344	5.0485864	6.9874084

Exercise 8. Page 17.

Find the sum of :

1.	2.	3.
$45.68	$154.31	$73.86
73.91	296.85	453.71
78.54	736.48	137.64
534.69	345.19	98.87
134.70	782.34	643.48
581.43	78.43	462.71
$1448.95	$2393.60	$1870.27

4.	5.	6.
$498.50	$65.42	$621.65
17.37	638.34	167.32
684.29	763.43	856.96
231.56	809.31	718.83
210.10	798.83	501.49
671.54	835.78	315.72
643.53	356.47	768.44
$2956.89	$4267.58	$3950.41

7.	8.	9.
$791.52	$32.54	$763.89
504.83	254.63	78.23
879.26	63.27	345.61
243.97	131.56	26.73
732.86	506.72	489.56
47.95	283.54	812.35
856.43	345.83	607.28
497.65	643.46	219.07
541.26	708.91	68.72
616.72	463.73	216.78
857.94	67.74	436.74
$6570.39	$3501.93	$4064.96

10.	11.	12.
$8400.07	$1873.33	$2336.29
3212.17	6170.24	336.00
1716.41	4813.25	2456.25
1020.08	662.25	641.25
1452.44	622.64	1174.50
1829.51	692.82	326.03
1929.96	2457.75	1219.87
114.78	2126.76	226.78
89.75	5391.25	276.75
173.67	7349.86	5936.40
17.45	1422.75	1914.78
112.44	9667.50	311.87
1098.75	6000.00	7956.00
6170.24	572.80	1919.66
$27337.72	$49823.20	$27032.43

13.	14.	15.
$1482.40	$773.72	$2406.08
2575.71	442.37	3101.24
3364.27	454.86	1452.09
689.81	358.61	3693.91
1533.61	2003.17	2054.76
735.58	179.56	1231.25
105.69	8493.75	1828.35
261.64	4179.54	1562.50
1516.56	3493.54	6937.50
2197.23	178.17	1987.57
1317.71	727.53	943.27
408.30	2889.42	2312.11
609.53	992.92	1409.28
1679.47	1183.08	2759.94
$18477.51	$26350.24	$33679.85

Exercise 9. Page 19.

1. Subtract by 2's from 20 to 0; from 21 to 1.

   20, 18, 16, 14, 12, 10, 8, 6, 4, 2, 0.
   21, 19, 17, 15, 13, 11, 9, 7, 5, 3, 1.

2. Subtract by 3's from 20 to 2; from 21 to 0.

   20, 17, 14, 11, 8, 5, 2.
   21, 18, 15, 12, 9, 6, 3, 0.

[redacted]

3. Subtract by 4's from 30 to 2; from 31 to 3; from 32 to 0; from 33 to 1.

30, 26, 22, 18, 14, 10, 6, 2.
31, 27, 23, 19, 15, 11, 7, 3.
32, 28, 24, 20, 16, 12, 8, 4, 0.
33, 29, 25, 21, 17, 13, 9, 5, 1.

4. Subtract by 5's from 32 to 2; from 33 to 3; from 34 to 4; from 35 to 0; from 36 to 1.

32, 27, 22, 17, 12, 7, 2.
33, 28, 23, 18, 13, 8, 3.
34, 29, 24, 19, 14, 9, 4.
35, 30, 25, 20, 15, 10, 5, 0.
36, 31, 26, 21, 16, 11, 6, 1.

5. Subtract by 6's from 33 to 3; from 34 to 4; from 35 to 5; from 36 to 0; from 37 to 1; from 38 to 2.

33, 27, 21, 15, 9, 3.
34, 28, 22, 16, 10, 4.
35, 29, 23, 17, 11, 5.
36, 30, 24, 18, 12, 6, 0.
37, 31, 25, 19, 13, 7, 1.
38, 32, 26, 20, 14, 8, 2.

6. Subtract by 7's from 42 to 0; from 43 to 1; from 44 to 2; from 45 to 3; from 46 to 4; from 47 to 5.

42, 35, 28, 21, 14, 7, 0.
43, 36, 29, 22, 15, 8, 1.
44, 37, 30, 23, 16, 9, 2.
45, 38, 31, 24, 17, 10, 3.
46, 39, 32, 25, 18, 11, 4.
47, 40, 33, 26, 19, 12, 5.

7. Subtract by 8's from 42 to 2; from 43 to 3; from 44 to 4; from 45 to 5; from 46 to 6; from 47 to 7.

42, 34, 26, 18, 10, 2.
43, 35, 27, 19, 11, 3.
44, 36, 28, 20, 12, 4.
45, 37, 29, 21, 13, 5.
46, 38, 30, 22, 14, 6.
47, 39, 31, 23, 15, 7.

[redacted]

8. Subtract by 9's from 55 to 1; from 56 to 2; from 57 to 3; from 59 to 5; from 61 to 7; from 62 to 8.

55, 46, 37, 28, 19, 10, 1.
56, 47, 38, 29, 20, 11, 2.
57, 48, 39, 30, 21, 12, 3.
59, 50, 41, 32, 23, 14, 5.
61, 52, 43, 34, 25, 16, 7.
62, 53, 44, 35, 26, 17, 8.

Exercise 10. Page 21.

Find the remainder and prove:

1.	2.	3.	4.	5.	6.	7.	8.
234	343	424	555	676	725	839	999
123	123	123	123	123	123	123	123
111	220	301	432	553	602	716	876
Proof.	Proof.	Proof.	Proof.	Proof.	Proof.	Proof.	Proof.
111	220	301	432	553	602	716	876
123	123	123	123	123	123	123	123
234	343	424	555	676	725	839	999

9.	10.	11.	12.	13.	14.	15.	16.
1000	5120	789	879	978	6378	6855	6853
123	123	456	456	456	456	456	456
877	4997	333	423	522	5922	6399	6397
Proof.	Proof.	Proof.	Proof.	Proof.	Proof.	Proof.	Proof.
877	4997	333	423	522	5922	6399	6397
123	123	456	456	456	456	456	456
1000	5120	789	879	978	6378	6855	6853

17.	18.	19.	20.	21.	22.	23.
7797	7006	3542	4000	974	368	2301
456	456	456	456	779	249	479
7341	6550	3086	3544	195	119	1822
Proof.	Proof.	Proof.	Proof.	Proof.	Proof.	Proof.
7341	6550	3086	3544	195	119	1822
456	456	456	456	779	249	479
7797	7006	3542	4000	974	368	2301

[redacted]

24.	25.	26.	27.	28.	29.	30.
2731	708	1123	891	8103	19,001	2180
929	394	1072	773	5621	3,456	792
1802	314	51	118	2482	15,545	1388
Proof.	Proof.	Proof.	Proof.	Proof.	Proof.	Proof.
1802	314	51	118	2482	15,545	1388
929	394	1072	773	5621	3,456	792
2731	708	1123	891	8103	19,001	2180

31.	32.	33.	34.	35.
$183.45	$716.43	$647.51	$270.04	$125.
76.47	628.74	549.64	128.31	101.50
$106.98	$87.69	$97.87	$141.73	$23.50
Proof.	Proof.	Proof.	Proof.	Proof.
$106.98	$87.69	$97.87	$141.73	$23.50
76.47	628.74	549.64	128.31	101.50
$183.45	$716.43	$647.51	$270.04	$125.00

36.	37.	38.	39.	40.
$247.93	$641.87	$56.27	3.1415927	0.7853982
129.47	333.95	29.89	2.7182818	0.5235988
$118.46	$307.92	$26.38	0.4233109	0.2617994
Proof.	Proof.	Proof.	Proof.	Proof.
$118.46	$307.92	$26.38	0.4233109	0.2617994
129.47	333.95	29.89	2.7182818	0.5235988
$247.93	$641.87	$56.27	3.1415927	0.7853982

41.	42.	43.	44.	45.
4.8104774	2.5399772	0.3937043	3.2808693	3.2808693
0.4342945	0.3937043	0.3047973	0.3047973	1.6093295
4.3761829	2.1462729	0.088907	2.976072	1.6715398
Proof.	Proof.	Proof.	Proof.	Proof.
4.3761829	2.1462729	0.088907	2.976072	1.6715398
0.4342945	0.3937043	0.3047973	0.3047973	1.6093295
4.8104774	2.5399772	0.3937043	3.2808693	3.2808693

[redacted]

46.	47.	48.	49.	50.
3.785	15.4323487	1.7320508	2.236068	2.236068
0.6213768	0.264	1.4142136	1.7320508	0.618034
3.1636232	15.1683487	0.3178372	0.5040172	1.618034
Proof.	Proof.	Proof.	Proof.	Proof.
3.1636232	15.1683487	0.3178372	0.5040172	1.618034
0.6213768	0.264	1.4142136	1.7320508	0.618034
3.785	15.4323487	1.7320508	2.236068	2.236068

51.	52.	53.	54.
0.381966	3.1415927	2.3561945	1.5707963
0.30103	0.7853982	0.7853982	0.7853982
0.080936	2.3561945	1.5707963	0.7853981
Proof.	Proof.	Proof.	Proof.
0.080936	2.3561945	1.5707963	0.7853981
0.30103	0.7853982	0.7853982	0.7853982
0.381966	3.1415927	2.3561945	1.5707963

55.	56.	57.	58.
3.1415927	2.6179939	2.0943951	1.5707963
0.5235988	0.5235988	0.5235988	0.5235988
2.6179939	2.0943951	1.5707963	1.0471975
Proof.	Proof.	Proof.	Proof.
2.6179939	2.0943951	1.5707963	1.0471975
0.5235988	0.5235988	0.5235988	0.5235988
3.1415927	2.6179939	2.0943951	1.5707963

59.	60.	61.	62.
1.0471975	1.	1.4142136	0.618034
0.5235988	0.381966	0.618034	0.381966
0.5235987	0.618034	0.7961796	0.236068
Proof.	Proof.	Proof.	Proof.
0.5235987	0.618034	0.7961796	0.236068
0.5235988	0.381966	0.618034	0.381966
1.0471975	1.	1.4142136	0.618034

[redacted]

63.	**64.**	**65.**	**66.**
9,873,210	8010.101	1,000,000	729,434
8,765,420	4187.94	817,259	613,488
1,107,790	3822.161	182,741	115,946
Proof.	Proof.	Proof.	Proof.
1,107,790	3822.161	182,741	115,946
8,765,420	4187.94	817,259	613,488
9,873,210	8010.101	1,000,000	729,434

67.	**68.**	**69.**	**70.**
6532.18	1718.754	21,205.	42,786.95
1916.47	1389.328	1,787.563	4,278.695
4615.71	329.426	19,417.437	38,508.255
Proof.	Proof.	Proof.	Proof.
4615.71	329.426	19,417.437	38,508.255
1916.47	1389.328	1,787.563	4,278.695
6532.18	1718.754	21,205.	42,786.95

Exercise 11. Page 22.

1. In a till are \$391 in bills, \$67.50 in gold, \$39.75 in silver, and \$2.77 in copper and nickel. How much money is in the till?

\$391.00
67.50
39.75
2.77
\$501.02 *Ans.*

2. Starting out with \$315.75 in one wallet and \$54.37 in another, I pay the grocer \$127.38; the butcher, \$64.17; the shoemaker, \$21.40; the landlord, \$50; the tailor, \$35. What ought I to have left?

\$315.75	\$127.38	\$370.12
54.37	64.17	297.95
\$370.12	21.40	\$72.17 *Ans.*
	50.00	
	35.00	
	\$297.95	

3. On a bill of $753.43 I pay $517.87. How much do I still owe? If I owe $817.87, and have but $637.50, how much do I lack of being able to pay?

$753.43	$817.87
517.87	637.50
$235.56 *Ans.*	$180.37 *Ans.*

4. If a man was born January 1, 1812, how old was he January 1, 1878.

1878	
1812	
66	66 years. *Ans.*

5. America was discovered in 1492. How many years after its discovery was each of the following events?

Settlement of Florida, 1565; of Virginia, 1607; of Massachusetts, 1620; of Quebec, 1608; French and Indian War, 1756; Declaration of Independence, 1776; Inauguration of Washington, 1789; War with England, 1812; Mexican War, 1846; Civil War, 1861.

1565	1607	1620	1608	1756
1492	1492	1492	1492	1492
73	115	128	116	264
1776	1789	1812	1846	1861
1492	1492	1492	1492	1492
284	297	320	354	369

73; 115; 128; 116; 264; 284; 297; 320; 354; 369. *Ans.*

6. The minuend is one hundred million, two hundred fifty-six thousand, three hundred seventy-two, and the subtrahend is nineteen million, nine hundred thousand, nine hundred ninety-nine. Find the remainder.

100,256,372
19,900,999
80,355,373 *Ans.*

7. If the minuend is 9874, and remainder 3185, what is the subtrahend? The subtrahend being 7659, and remainder 675.68, what is the minuend?

9874	675.68
3185	7659.
6689 *Ans.*	8334.68 Ans.

[redacted]

8. The smaller of two numbers is 7.95764328; their difference is 0.00087692. What is the larger number?

$$\begin{array}{l} 7.95764328 \\ 0.00087692 \\ \hline 7.9585202 \quad \textit{Ans.} \end{array}$$

9. The larger of two numbers is 7.95764328, and their difference is 7.153485. What is the smaller number?

$$\begin{array}{l} 7.95764328 \\ 7.153485 \\ \hline 0.80415828 \quad \textit{Ans.} \end{array}$$

10. If the subtrahend is 10,542, and the difference 544.2, what is the minuend?

$$\begin{array}{r} 10{,}542. \\ 544.2 \\ \hline 11{,}086.2 \end{array} \quad \textit{Ans.}$$

11. A man pumps out of a cistern in one hour 243.75 gallons; in the next hour, 227.5 gallons; in 45 minutes more, 137.75 gallons; and the cistern is empty. How many gallons of water were in it?

$$\begin{array}{l} 243.75 \\ 227.5 \\ 137.75 \\ \hline 609. \quad \textit{Ans.} \end{array}$$

12. From what number must I subtract 5 to leave 7? 8 to leave 9? From what number must I subtract 5.1736 to leave 8.1964? 6.231 to leave 9.6648? 74.213 to leave 25.787?

$$\begin{array}{r} 7 \\ 5 \\ \hline 12 \end{array}\ \textit{Ans.} \qquad \begin{array}{r} 9 \\ 8 \\ \hline 17 \end{array}\ \textit{Ans.} \qquad \begin{array}{r} 8.1964 \\ 5.1736 \\ \hline 13.37 \end{array}\ \textit{Ans.}$$

$$\begin{array}{r} 9.6648 \\ 6.231 \\ \hline 15.8958 \end{array}\ \textit{Ans.} \qquad \begin{array}{r} 25.787 \\ 74.213 \\ \hline 100. \end{array}\ \textit{Ans.}$$

13. What must be subtracted from 1 to leave 0.5? to leave 0.53? to leave 0.532? to leave 0.5236? to leave 0.5235988?

1.	1.	1.
0.5	0.53	0.532
0.5 *Ans.*	0.47 *Ans.*	0.468 *Ans.*

1.	1.
0.5236	0.5235988
0.4764 *Ans.*	0.4764012 *Ans.*

14. I start on a journey of 3433 miles. The first day I make 428 miles; the second day, 511 miles; the third, 497 miles; the fourth, 513. How many miles of my journey remained for me at the close of each day? How many miles had I gone at the close of each day?

3433			
428			
3005	after first day.	428	end of first day.
511		511	
2494	after second day.	939	end of second day.
497		497	
1997	after third day.	1436	end of third day.
513		513	
1484	after fourth day.	1949	end of fourth day.

15. Subtract 76,343 from the sum of 61,932, 51,387, 5193, 4674, and 8199; then subtract 23,657 from the remainder.

61,932	
51,387	131,385
5,193	76,343
4,674	55,042
8,199	23,657
131,385	31,385 *Ans.*

16. Jones bought a farm and stock for $7633.90; sold the stock for $305.75; then sold the farm for $7325. How much did he lose?

$305.75	$7633.90
7325.	7630.75
$7630.75	$3.15 *Ans.*

[redacted]

17. If I gave $4375 for my land, and paid for house, barn, sheds, and fences $2789.50, also $973.75 for horses, cattle, tools, etc., what did my farm and stock cost?

$$\begin{array}{r} \$4375. \\ 2789.50 \\ \underline{973.75} \\ \$8138.25 \end{array} \quad \textit{Ans.}$$

18. If I paid $8138.25 for land and cattle, and sold part of the land for $675, and part of the cattle for $217.50, what is the cost of the land and the cattle left?

$$\begin{array}{r} \$675. \\ \underline{217.50} \\ \$892.50 \end{array} \qquad \begin{array}{r} \$8138.25 \\ \underline{892.50} \\ \$7245.75 \end{array} \quad \textit{Ans.}$$

19. John has 158 cents, James has 271 cents; James gives John 56 cents. Which has then more than the other, and how many cents more?

$$\begin{array}{r} 158 \\ \underline{56} \\ 214 \end{array} \text{ John.} \qquad \begin{array}{r} 271 \\ \underline{56} \\ 215 \end{array} \text{ James.}$$

Therefore, James has 1 cent more.

20. A cattle dealer had 228 oxen, 475 sheep, and 49 lambs; he sold 17 oxen, 64 sheep, and 7 lambs. How many animals of each kind did he then have, and how many all together?

$$\begin{array}{rl} 228 & \text{oxen.} \\ \underline{17} & \\ 211 & \text{oxen.} \end{array} \qquad \begin{array}{rl} 475 & \text{sheep.} \\ \underline{64} & \\ 411 & \text{sheep.} \end{array} \qquad \begin{array}{rl} 49 & \text{lambs.} \\ \underline{7} & \\ 42 & \text{lambs.} \end{array} \qquad \begin{array}{r} 211 \\ 411 \\ \underline{42} \\ 664 \end{array}$$

Exercise 12. Page 29.

Find the product of:

1.

$$\begin{array}{r} 0.5235988 \\ \underline{6} \\ 3.1415928 \end{array}$$

2.

$$\begin{array}{r} 0.7853982 \\ \underline{4} \\ 3.1415928 \end{array}$$

3.

$$\begin{array}{r} 3.14159265 \\ \underline{5} \\ 15.70796325 \\ \underline{5} \\ 78.53981625 \end{array}$$

4.

$$\begin{array}{r} 8.75 \\ \underline{30} \\ 262.50 \end{array}$$

[redacted]

5.
6.975
0.07
0.48825

6.
7.81
700
5467.00

7.
65.432
8000
523,456.000

8.
7.85
300
2355.00

9.
10,356.78
0.009
93.21102

10.
0.785398
7.37
5497786
2356194
5497786
5.78838326

11.
0.785398
8.56
4712388
3926990
6283184
6.72300688

12.
0.785398
1001
785398
785398
786.183398

13.
2150.42
0.083
645126
1720336
178.48486

14.
2150.42
0.75
1075210
1505294
1612.8150

15.
2150.42
0.075
1075210
1505294
161.28150

16.
1.4142136
0.7071
14142136
98994952
98994952
0.99999043656

17.
1.41421
1.4142
282842
565684
141421
565684
141421
1.999975782

18.
1.732
1.732
3464
5196
12124
1732
2.999824

19.
2.23607
2.236
1341642
670821
447214
447214
4.99985252

20.
0.618
618
4944
618
3708
381.924

21.
0.618034
0.618035
3090170
1854102
4944272
618034
3708204
0.381966643190

22.
0.12936
12
25872
12936
1.55232

23.
7.92801
0.9
7.135209

24.
58.383
0.39
525447
175149
22.76937

[redacted]

25.

$$\begin{array}{r} 0.28744 \\ 0.08 \\ \hline 0.0229952 \end{array}$$

26.

$$\begin{array}{r} 491.205 \\ 0.065 \\ \hline 2456025 \\ 2947230 \\ \hline 31.928325 \end{array}$$

27.

$$\begin{array}{r} 68.325 \\ 6.25 \\ \hline 341625 \\ 136650 \\ 409950 \\ \hline 427.03125 \end{array}$$

28.

$$\begin{array}{r} 0.732 \\ 1.6 \\ \hline 4392 \\ 732 \\ \hline 1.1712 \end{array}$$

29.

$$\begin{array}{r} 1208.88 \\ 0.438 \\ \hline 967104 \\ 362664 \\ 483552 \\ \hline 529.48944 \end{array}$$

30.

$$\begin{array}{r} 0.0125 \\ 498 \\ \hline 1000 \\ 1125 \\ 500 \\ \hline 6.2250 \end{array}$$

31.

$$\begin{array}{r} 0.007 \\ 7 \\ \hline 0.049 \end{array}$$

32.

$$\begin{array}{r} 0.0001 \\ 1000 \\ \hline 0.1000 \end{array}$$

33.

$$\begin{array}{r} 10.24 \\ 0.235 \\ \hline 5120 \\ 3072 \\ 2048 \\ \hline 2.40640 \end{array}$$

34.

$$\begin{array}{r} 0.00507702 \\ 0.0283 \\ \hline 1523106 \\ 4061616 \\ 1015404 \\ \hline 0.000143679666 \end{array}$$

35.

$$\begin{array}{r} 0.00752 \\ 89.3 \\ \hline 2256 \\ 6768 \\ 6016 \\ \hline 0.671536 \end{array}$$

36.

$$\begin{array}{r} 0.0256 \\ 74.1 \\ \hline 256 \\ 1024 \\ 1792 \\ \hline 1.89696 \end{array}$$

Exercise 13. Page 30.

Express the product of:

1.

$7^5 \times 7^3 = 7^{5+3} = 7^8.$
$8^2 \times 8 = 8^{2+1} = 8^3.$
$2^8 \times 2 = 2^{8+1} = 2^9.$
$5^4 \times 5^2 = 5^{4+2} = 5^6.$

2.

$3.01^2 \times 3.01 = 3.01^{2+1} = 3.01^3.$
$0.67^2 \times 0.67^8 = 0.67^{2+8} = 0.67^{10}.$
$0.208 \times 0.208^3 = 0.208^{1+3} = 0.208^4.$

3.

$2.003^2 \times 2.003^4 = 2.003^{2+4} = 2.003^6.$
$20.03^3 \times 20.03 = 20.03^{3+1} = 20.03^4.$
$20.03 \times 20.03^2 = 20.03^{1+2} = 20.03^3.$

[redacted]

Exercise 14. Page 32.

Find the following products, and test the accuracy by casting out the nines, and by casting out the elevens :

1.

```
        21.3706
         15.243
        -------
         641118
        854824
       427412
     1068530
     213706
    -----------
    325.7520558
         1.8954
    -----------
     13030082232
    16287602790
   29317685022
  26060164464
  3257520558
  ---------------
  617.43044656332
```

The three remainders after the nines are cast out are 1, 6, and 0. $1 \times 6 \times 0 = 0$.

The remainder of the product after the nines are cast out is 0.

The three remainders after the elevens are cast out are 9, 8, and 1. $9 \times 8 \times 1 = 72$, or casting out the elevens, 6.

The remainder of the product after the elevens are cast out is 6.

2.

```
       0.026891
          5.328
       --------
         215128
        53782
       80673
     134455
    -----------
    0.143275248
          29.74
    -----------
      573100992
    1002926736
   1289477232
   286550496
  --------------
  4.26100587552
```

The three remainders after the nines are cast out are 8, 0, 4. $8 \times 0 \times 4 = 0$.

The remainder of the product after the nines are cast out is 0.

The three remainders after the elevens are cast out are 7, 4, and 4. $7 \times 4 \times 4 = 112$, or casting out the elevens, 2.

The remainder of the product after the elevens are cast out is 2.

3.

```
          5.8281
          0.0012
      ----------
          116562
          58281
      ----------
      0.00699372
          0.6827
      ----------
         4895604
        1398744
       5594976
      4196232
  --------------
  0.004774612644
```

[redacted]

The three remainders after the nines are cast out are 6, 3, and 5. $6 \times 3 \times 5 = 90$, or after the nines are cast out, 0.

The remainder of the product after the nines are cast out is 0.

The three remainders after the elevens are cast out are 3, 1, and 7. $3 \times 1 \times 7 = 21$, or after the elevens are cast out, 10.

The remainder of the product after the elevens are cast out is 10.

4.

```
     23.9875
     12.4764
     -------
      959500
    1439250
   1679125
   959500
  479750
 239875
------------
299.27764500
       0.017
------------
2094943515
299277645
------------
5.087719965
```

The three remainders after the nines are cast out are 7, 6, and 8. $7 \times 6 \times 8 = 336$, or after the nines are cast out, 3.

The remainder of the product after the nines are cast out is 3.

The three remainders after the elevens are cast out are 9, 2, and 6. $9 \times 2 \times 6 = 108$, or after the elevens are cast out, 9.

The remainder of the product after the elevens are cast out is 9.

5.

```
        39.801
          1.44
        ------
        159204
       159204
       39801
      --------
      57.31344
       17.9645
      --------
      28656720
     22925376
    34388064
   51582096
  40119408
  5731344
---------------
1029.607292880
```

The three remainders after the nines are cast out are 3, 0, and 5. $3 \times 0 \times 5 = 0$.

The remainder of the product after the nines are cast out is 0.

The three remainders after the elevens are cast out are 3, 1, and 4. $3 \times 1 \times 4 = 12$, or after the elevens are cast out, 1.

The remainder of the product after the elevens are cast out is 1.

6.

```
         5.2817
         0.0165
         ------
         264085
        316902
        52817
     ----------
     0.08714805
         0.8469
     ----------
       78433245
      52288830
     34859220
    69718440
---------------
0.073805683545
```

The three remainders after the nines are cast out are 5, 3, and 0. $5 \times 3 \times 0 = 0$.

The remainder of the product after the nines are cast out is 0.

The three remainders after the elevens are cast out are 6, 0, and 10. $6 \times 0 \times 10 = 0$.

The remainder of the product after the elevens are cast out is 0.

7.

```
   0.54237
        16
   -------
    325422
    54237
   -------
   8.67792
   0.00176
   -------
   5206752
  6074544
  867792
------------
0.0152731392
```

The three remainders after the nines are cast out are 3, 7, and 5. $3 \times 7 \times 5 = 105$, or after the nines are cast out, 6.

The remainder of the product after the nines are cast out is 6.

The three remainders after the elevens are cast out are 7, 5, and 0. $7 \times 5 \times 0 = 0$.

The remainder of the product after the elevens are cast out is 0.

8.

```
        24.271
        3.6485
        121355
       194168
       97084
      145626
      72813
     -----------
     88.5527435
        15.271
     -----------
     885527435
    6198692045
   1771054870
  4427637175
  885527435
 ----------------
 1352.2889459885
```

The three remainders after the nines are cast out are 7, 8, and 7. $7 \times 8 \times 7 = 392$, or after the nines are cast out, 5.

The remainder of the product after the nines are cast out is 5.

The three remainders after the elevens are cast out are 5, 9, and 3. $5 \times 9 \times 3 = 135$, or after the elevens are cast out, 3.

The remainder of the product after the elevens are cast out is 3.

9.

```
      13.256
      14.125
      ------
       66280
      26512
     13256
    53024
   13256
  ----------
  187.241000
```

[redacted]


```
    187.241000
     30.254
    ----------
     748964000
    936205
   374482
  561723
  --------------
  5664.789214000
```

The three remainders after the nines are cast out are 8, 4, and 5.

8 × 4 × 5 = 160, or after the nir are cast out, 7.

The remainder of the prodı after the nines are cast out is 7

The three remainders after t elevens are cast out are 1, 1, and 1 × 1 × 4 = 4.

The remainder of the prodı after the elevens are cast out is

Exercise 15. Page 34.

Find to the fifth decimal the value of:

1.

```
   0.497149 87
 3622185 71
 ----------
   497150
   348004
    24857
     3977
       50
        9
        1
 --------
 0.874048
```

0.87405. *Ans.*

2.

```
  0.79817987
 579924990
 ---------
   718361
    71835
     3192
      160
       71
        6
 --------
 0.793625
```

0.79363. *Ans.*

3.

```
  1.09920986
 494758420
 ---------
   219842
    43968
     8794
      550
       76
        4
        1
 --------
 0.273235
```

0.27324. *Ans.*

4.

```
  0.62208861
 266175610
 ---------
    62209
    37325
     3110
      435
        6
        4
 --------
 0.103089
```

0.10309. *Ans.*

5.

```
  1.75812263
 369519502
 ---------
  3516245
    87906
    15823
      176
       88
       15
        1
 --------
 3.620254
```

3.62025. *Ans.*

6.

```
  0.55630251
 573544330
 ---------
   166891
    16689
     2225
      222
       28
        2
 --------
 0.186057
```

0.18606. *Ans.*

[redacted]

7.	8.	9.
0.75142506	0.05245506	0.33143325
965899899	266175610	317362971
6762825	5246	331433
676283	3147	232003
60124	262	29829
6763	36	663
676	1	199
60	0.008692	10
4	0.00869. *Ans.*	2
7.506735		0.594139
7.50674. *Ans.*		0.59414. *Ans.*

10.	11.	12.
0.90633287	2.84657842	0.546794489
67315545160	245596690	5947967482
543799	2561920	1093589
9063	170794	437435
4532	17079	21871
362	2561	3827
45	142	328
5	14	49
1	1	4
0.557807	2.752581	1.557103
0.55781. *Ans.*	2.75258. *Ans.*	1.55710. *Ans.*

Exercise 16. Page 38.

Find the quotient of:

1.	2.	3.	4.
9)126.409	10)13.31	11)13.31	12)1.728
14.04544	1.331	1.21	0.144

5.	6.	7.	8.
3$\cancel{0}$)3.7632	2$\cancel{0}$)4263.1	4$\cancel{00}$)964.64	6$\cancel{00}$)0.58775
1.2544	2131.55	241.16	0.09796

9.	10.	11.	12.
7$\cancel{00}$)752.30	8$\cancel{0}$)895.6	9$\cancel{00}$)982.54	7$\cancel{000}$)82.610
107.47143	111.95	109.17111	11.80143

[redacted]

13.	14.	15.	16.
5$\cancel{0}\cancel{0}$)836.90	11$\cancel{0}$)9646.4	12$\cancel{0}\cancel{0}$)875.85	2$\cancel{0}\cancel{0}$)274.85
167.38	876.94545	72.9875	137.425

17.	18.	19.	20.
002)1001.	005)004.	004)743.2	0005)31000.
500.5	0.8	185.8	6200.

21.	22.	23.	24.
0012)480000.	00007)9800000.	000009)1098000.	009)1098.
40000.	1400000.	122000.	122.

25.	26.	27.	28.
000009)10980.	009)10.98	11)144100.	11)189.2
1220.	1.22	13100.	17.2

Exercise 17. Page 42.

Find the quotient of:

1.

```
     83
91)7553
   728
    273
    273
```

2.

$62\frac{67}{73}$

```
73)4593
   438
    213
    146
     67
```

3.

$1180\frac{33}{76}$

```
76)89713
   76
   137
    76
    611
    608
     33
```

4.

$610\frac{11}{88}$

```
88)53691
   528
     89
     88
     11
```

5.

$1010\frac{22}{35}$

```
35)35372
   35
     37
     35
      22
```

6.

$2045\frac{201}{408}$

```
408)834561
    816
     1856
     1632
      2241
      2040
       201
```

[redacted]

7.

$1382\frac{232}{247}$
247)341586
247
945
741
2048
1976
726
494
232

8.

$2180\frac{245}{395}$
395)861345
790
713
395
3184
3160
245

9.

$439\frac{329}{843}$
843)370406
3372
3320
2529
7916
7587
329

10.

$1964\frac{145}{498}$
498)978217
498
4802
4482
3201
2988
2137
1992
145

11.

$1523\frac{105}{357}$
357)543816
357
1868
1785
831
714
1176
1071
105

12.

$469\frac{189}{1289}$
1289)604730
5156
8913
7734
11790
11601
189

13.

2.475
132)326.7
264
627
528
990
924
660
660

14.

71.12
1121)79725.52
7847
1255
1121
1345
1121
2242
2242

15.

0.045
906)40.77
3624
4530
4530

[redacted]

16.

9.007
1068)9619.476
9612
7476
7476

17.

70.
$\$38745)\,\$2712150.$
271215
0

18.

3.1416
3937)12368.4792
11811
5574
3937
16377
15748
6299
3937
23622
23622

19.

62.5
1926)120375.
11556
4815
3852
9630
9630

20.

160000.
00016)2560000.
16
96
96
0000

21.

7.58
319)2418.02
2233
1850
1595
2552
2552

22.

640.
03125)2000000.
18750
12500
12500
0

23.

92.8
008302)770425.6
74718
23245
16604
66416
66416

24.

3.35977
0479)1609.3295
1437
1723
1437
2862
2395
4679
4311
3685
3353
3320

25.

1.75499
0917)1609.3295
917
6923
6419
5042
4585
4579
3668
9115
8253
8620
8253
367

[redacted]

26.

94.66644
0017)1609.3295
153
79
68
113
102
112
102
109
102
75
68
7

27.

184.98040
00087)16093.295
87
739
696
433
348
852
783
699
696
350
348
20

28.

1.76471
17)30.
17
130
119
110
102
80
68
120
119
10

29.

1.73410
173)300.
173
1270
1211
590
519
710
692
180
173
70

30.

1.73210
1732)3000.
1732
12680
12124
5560
5196
3640
3464
1760
1732
280

31.

1.73200
17321)30000.
17321
126790
121247
55430
51963
34670
34642
2800

32.

0.00030479
528~~0~~)0.16093295
1584
2532
2112
4209
3696
5135
4752
383
0.0003048. *Ans.*

33.

1.41423
14142)20000.
14142
58580
56568
20120
14142
59780
56568
32120
28284
38360

34.

2.23614
2236)5000.
4472
5280
4472
8080
6708
13720
13416
3040
2236
8040

[redacted]

35.

$213.67
117)$25000.
234
160
117
430
351
790
702
880
819
61
$213.68. *Ans.*

36.

11.00543
1472)16200.
1472
1480
1472
8000
7360
6400
5888
5120
4416
704

37.

0.000064
198~~00~~)0.01270
1188
820
792
28

38.

0.34379
16382)05632.
49146
71740
65528
62120
49146
129740
114674
150660
147438
3222

39.

44.21057
4236~~0~~)187276.0
16944
17836
16944
8920
8472
4480
4236
24400
21180
32200
29652
2548
44.21058. *Ans.*

[redacted]

40.

```
              9.83258
19293)189700.
      173637
       160630
       154344
         62860
         57879
          49810
          38586
          112240
           96465
           157750
           154344
             3406
```

41.

```
           0.00114
00872)00001.
          872
          1280
           872
           4080
           3488
            592
```

0.00115. *Ans.*

42.

```
          7.76378
18246)141658.
      127722
       139360
       127722
        116380
        109476
          69040
          54738
          143020
          127722
           152980
           145968
             7012
```

43.

```
                  33.41590
$30377)$1015075.
         91131
         103765
          91131
          126340
          121508
            48320
            30377
            179430
            151885
             275450
             273393
               20570
```

33.41591. *Ans.*

44.

```
        $12.62
2473)$31212.
      2473
       6482
       4946
       15360
       14838
         5220
         4946
          274
```

45.

```
        $17.70
176)$3115.20
      176
      1355
      1232
       1232
       1232
          0
```

46.

```
        $553.06
5135)$2840000.
     25675
      27250
      25675
       15750
       15405
         34500
         30810
          3690
      $553.07.  Ans.
```

47.

```
         354.2
3542)1254576.4
     10626
      19197
      17710
       14876
       14168
         7084
         7084
```

48.

```
       0.86605
0866)0750.
     6928
      5720
      5196
       5240
       5196
         4400
         4330
           70
```

49.

```
          311.12396
31113)9680000.
      93339
       34610
       31113
        34970
        31113
         38570
         31113
          74570
          62226
          123440
           93339
           301010
           280017
            209930
            186678
             23252
            311.12397.  Ans.
```

50.

```
          395708.73088
64037)25340000000.
      192111
       612890
       576333
        365570
        320185
         453850
         448259
          559100
          512296
           468040
           448259
            197810
            192111
             569900
             512296
              576040
              512296
               63744
         395,708.73089.  Ans.
```

51.

0.0000025

2534000) 0.0064037

5068

13357

12670

687

52.

0.13457

5504) 0740.7

5504

19030

16512

25180

22016

31640

27520

41200

38528

2672

53.

31.36125

17359) 544400.

52077

23630

17359

62710

52077

106330

104154

21760

17359

44010

34718

92920

86795

6125

54.

0.15753

2322) 0365.8

2322

13360

11610

17500

16254

12460

11610

8500

6966

1534

0.15754. *Ans.*

55.

```
       $17.566
143) $2512
      143
      1082
      1001
        810
        715
         950
         858
          92

$17.57. Ans.
```

56.

```
        4[illegible]
[illegible]) [illegible]
       144
        2[illegible]0
        232
          80
          72
           80
           72
            80
            72
             80
             72
              80
              72
               80
               72
                8
```

57.

```
              $79.92
1789.2) $143000.00
         125244
          177560
          161028
           165320
           161028
             42920
             35784
              7136
```

58.

```
        0.15454
121) 18.7
     121
      660
      605
       550
       484
        660
        605
         550
         484
          66
0.15455. Ans.
```

59.

```
          286.96306
1728) 495872.1765
      3456
      15027
      13824
       12032
       10368
        16641
        15552
         10897
         10368
           5296
           5184
            11250
            10368
              882
286.96307. Ans.
```

60.

```
         35.32532
528) 18651.7725
     1584
      2811
      2640
       1717
       1584
        1337
        1056
         2812
         2640
          1725
          1584
           1410
           1056
            354
35.32533. Ans.
```

[redacted]

61.

243.66937
231) 56287.625
462
1008
924
847
693
1546
1386
1602
1386
2165
2079
860
693
1670
1617
53

62.

17.97170
4356Ø) 78284.7375
4356
34724
30492
42327
39204
31233
30492
7417
4356
30615
30492
1230

63.

68.02571
27225) 1852000.
163350
218500
217800
70000
54450
155500
136125
193750
190575
31750
27225
4525

64.

17.01117
215042) 3658117.
215042
1507697
1505294
240300
215042
252580
215042
375380
215042
1603380
1505294
98086

65.

51.02040
196) 10000.
980
200
196
400
392
800
784
160
51.02041. *Ans.*

66.

$213.77
1025) $219120.
2050
1412
1025
3870
3075
7950
7175
7750
7175
575
$213.78. *Ans.*

[redacted]

67.

```
             3.89699
5645376)22000000.
        16936128
         50638720
         45163008
          54757120
          50808384
           39487360
           33872256
            56151040
            50808384
             53426560
             50808384
              2618176
```

68.

```
      0.01239
1331)0016.5
      1331
       3190
       2662
        5280
        3993
        12870
        11979
          891
```

0.01240. *Ans.*

69.

```
       65.58593
1152)75555.
     6912
      6435
      5760
       6750
       5760
        9900
        9216
         6840
         5760
         10800
         10368
           4320
           3456
            864
```

65.58594. *Ans.*

70.

```
          2.21592
55056)122000.
      110112
       118880
       110112
         87680
         55056
         326240
         275280
          509600
          495504
           140960
           110112
            30848
```

2.21593. *Ans.*

[redacted]

71.

7.14842
107716)770000.
754012
159880
107716
521640
430864
907760
861728
460320
430864
294560
215432
79128

7.14843. *Ans.*

72.

9.11274
72426)660000.
651834
81660
72426
92340
72426
199140
144852
542880
506982
358980
289704
69276

9.11275. *Ans.*

73.

0.03156
1728)54.55
5184
2710
1728
9820
8640
11800
10368
1432

0.03157. *Ans*

74.

1.04823
44723)46880.
44723
215700
178892
368080
357784
102960
89446
135140
134169
971

75.

0.00196
444)0.874
444
4300
3996
3040
2664
376

0.00197. *Ans.*

76.

10.36515
528~~0~~)5472.8
528
1928
1584
3440
3168
2720
2640
800
528
2720
2640
80

77.

1130.
00018)20340.
18
23
18
54
54
0

78.

0.0081
108)0.8748
864
108
108

79.

1200900.
00037)44433300.
37
74
74
333
333
00

80.

0.0016
2003)3.2048
2003
12018
12018

[redacted]

Exercise 18. Page 44.

Reduce to a single expression:

1. $(16 - 11 + 2) \times 5 = 7 \times 5 = 35.$
2. $(4 \times 15) \div (2 \times 3) = 60 \div 6 = 10.$
3. $(84 \div 7) + (4 + 5 - 6) = 12 + 3 = 15.$
4. $(44 - 31) \times (14 - 11) = 13 \times 3 = 39.$
5. $(96 \div 6 + 5) - (6 \times 8 \div 16) = 21 - 3 = 18.$
6. $(52 - 5 \times 7) + (4 \times 5) - 16 \div 2 = 17 + 20 - 8 = 29.$
7. $52 - 5 \times 7 + 4 \times 5 - 16 \div 2 = 52 - 35 + 20 - 8 = 29.$
8. $(62 + 3 - 15) \div 10 + (6 \times 7 - 30) \div 3 = 50 \div 10 + 12 \div 3$
   $= 5 + 4 = 9.$

Exercise 19. Page 45.

By the use of reciprocals, find the value of:

1. $8 \times 0.25 = 8 \div 4 = 2.$
2. $171 \div 0.25 = 171 \times 4 = 684.$
3. $876 \times 1.25 = 876 \div 0.8 = 8760 \div 8 = 1095.$
4. $132 \times 2.5 = 132 \div 0.4 = 1320 \div 4 = 330.$
5. $591 \div 2.5 = 591 \times 0.4 = 236.4.$
6. $756 \div 0.125 = 756 \times 8 = 6048.$
7. $268 \times 25 = 268 \div 0.04 = 26{,}800 \div 4 = 6700.$
8. $753 \div 25 = 753 \times 0.04 = 30.12.$
9. $567 \div 625 = (567 \div 5) \times 0.008 = 113.4 \times 0.008 = 0.9072.$
10. $1764 \times 0.025 = 1764 \div 40 = 44.1.$
11. $5381 \div 0.025 = 5381 \times 40 = 215{,}240.$
12. $7452 \div 0.875 = 7452 \times 8 \div 7 = 59{,}616 \div 7 = 8516.6.$
13. $651 \times 0.33333 = 651 \div 3 = 217.$
14. $456 \times 6.66667 = 456 \div 0.15 = 45{,}600 \div 15 = 3040.$
15. $1554 \times 0.16667 = 1554 \div 6 = 259.$
16. $432 \div 1.33333 = 432 \times 0.75 = 324.$
17. $375 \div 16.66667 = 375 \times 0.06 = 22.5.$
18. $225 \div 6.66667 = 225 \times 0.15 = 33.75.$

[redacted]

Exercise 20. Page 47.

Divide by the contracted method:

1. 11.4285285 by 3.1415927 to six decimal places.

3.637813

31~~415927~~) 114285285.
94247781
20037504
18849556
1187948
942478
245470
219911
25559
25132
427
314
113
94

2. 0.004239239 by 3.2783278 to five decimal places.

0.00129

327~~83278~~) 42392.39
32783
9609
6557
3052
2950

3. 437 by 215.253 to five decimal places.

2.03017

21~~5253~~) 437000.
430506
64940
64576
364
215
149

4. 0.0053 by 72.654 to eight decimal places.

0.00007294

72~~654~~) 5.30000
508578
21422
14531
6891
6539
352
300

5. 6 by 0.1573 to three decimal places.

38.143

15~~73~~) 60000.
4719
12810
12584
2260
1573
687
629
58
47

6. 0.11 by 1937.43 to eight decimal places.

0.00005677

19~~3743~~) 11.0000
96872
13128
11624
1504
1356
148
135

[redacted]

7. 44.2 by 0.768547 to five decimal places.

```
            57.51112
768547) 44200000.
        3842735
         5772650
         5379829
          3928210
          3842735
            85475
            76855
             8620
             7685
              935
              769
              166
              154
```

8. 0.6587465 by 0.5475869 five decimal places.

```
              1.20298
5475869) 6587465.
         5475869
         1111596
         1095174
           16422
           10952
            5470
            4928
             442
             438
```

9. 46 by 0.00751515151 to thr decimal places.

```
                    6120.967
751515151) 4600000000000.
           450909091
             9090909
             7515152
             1575757
             1503030
               72727
               67636
                5091
                4509
                 582
                 526
```

Exercise 21. Page 48.

Express the value of:

1. $10^1 = 10.$

$10^2 = 10 \times 10 = 100.$

$10^3 = 10 \times 10 \times 10 = 1000.$

$10^4 = 10 \times 10 \times 10 \times 10 = 10{,}000.$

$10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100{,}000.$

$10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1{,}000{,}000.$

$10^7 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10{,}000{,}000.$

$10^8 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 100{,}000{,}000.$

[redacted] [redacted]

2. $10^3 \div 10^2 = 10^{3-2} = 10.$

$10^8 \div 10^5 = 10^{8-5} = 10^3.$

$10^5 \div 10^8 = \frac{1}{10^{8-5}} = \frac{1}{10^3}.$

$10^9 \div 10^4 = 10^{9-4} = 10^5.$

3. $9.99^4 \div 9.99^2 = 9.99^{4-2} = 9.99^2.$

$9.99^{108} \div 9.99^{110} = \frac{1}{9.99^{110-108}} = \frac{1}{9.99^2}.$

$9.99^{16} \div 9.99^{18} = \frac{1}{9.99^{18-16}} = \frac{1}{9.99^2}.$

4. $1.01^{25} \div 1.01^{22} = 1.01^{25-22} = 1.01^3.$

$1.01^{12} \div 1.01^{15} = \frac{1}{1.01^{15-12}} = \frac{1}{1.01^3}.$

$1.01^{19} \div 1.01^{16} = 1.01^{19-16} = 1.01^3.$

Exercise 22. Page 49.

Find the following quotients and test the accuracy of the work by casting out the nines:

1.

```
               73.03522
215042)15705641.692
        1505294
         652701
         645126
          757569
          645126
          1124432
          1075210
            492220
            430084
             621360
             430084
             191276
```

The remainder after the nines are cast out from the divisor is 5; from the quotient, 4; from the remainder, 8; from the dividend, 1.

$5 \times 4 + 8 = 28.$

$28 \div 9 = 3$ with remainder 1.

2.

```
                    8.79530
5645376)49652789.6
         45163008
          44897816
          39517632
           53801840
           50808384
            29934560
            28226880
             17076800
             16936128
              1406720
```

The remainder after the nines are cast out from the divisor is 0; from the quotient, 5; from the remainder, 2; from the dividend, 2.

$0 \times 5 + 2 = 2.$

[redacted]

3\.

636.61828
31416)20000000.
188496
115040
94248
207920
188496
194240
188496
57440
31416
260240
251328
89120
62832
262880
251328
11552

The remainder after the nines are cast out from the divisor is 6; from the quotient, 4; from the remainder, 5; from the dividend, 2.

$$6 \times 4 + 5 = 29.$$

$$29 \div 9 = 3 \text{ with remainder } 2.$$

4\.

24.16166
7854)189765.7
15708
32685
31416
12697
7854
48430
47124
13060
7854
52060
47124
49360
47124
2236

The remainder after the nines are cast out from the divisor is 6; from the quotient, 8; from the remainder, 4; from the dividend, 7.

$$6 \times 8 + 4 = 52.$$

$$52 \div 9 = 5 \text{ with remainder } 7.$$

5\.

1997.58881
1439874)2876276200.
1439874
14364022
12958866
14051560
12958866
10926940
10079118
8478220
7199370
12788500
11518992
12695080
11518992
11760880
11518992
2418880
1439874
979006

The remainder after the nines are cast out from the divisor is 0; from the quotient, 2; from the remainder, 4; from the dividend, 4.

$$0 \times 2 + 4 = 4.$$

[redacted]

6.

```
         1328.74761
658298)874711900.
       658298
       2164139
       1974894
        1892450
        1316596
         5758540
         5266384
          4921560
          4608086
           3134740
           2633192
            5015480
            4608086
             4073940
             3949788
              1241520
               658298
               583222
```

The remainder after the nines are cast out from the divisor is 2; from the quotient, 3; from the remainder, 4; from the dividend, 1.

$2 \times 3 + 4 = 10.$

$10 \div 9 = 1$ with remainder 1.

7.

```
         191.94170
149796)28752100.
       149796
       1377250
       1348164
         290860
         149796
         1410640
         1348164
           624760
           599184
            255760
            149796
            1059640
            1048572
              110680
```

The remainder after the nines are cast out from the divisor is 0; from the quotient, 5; from the remainder, 7; from the dividend, 7.

$0 \times 5 + 7 = 7.$

Exercise 23. Page 50.

Express in words:

1. 327.244.

Three hundred twenty-seven and two hundred forty-four thousandths.

2. 80.9056.

Eighty and nine thousand fifty-six ten-thousandths.

3. 0.390012.

Three hundred ninety thousand twelve millionths.

4. 20,000.002.

Twenty thousand and *two thousandths*.

5. 0.0000008.

Eight ten-millionths.

6. 41.27105.

Forty-one and twenty-seven thousand one hundred five hundred-thousandths.

Write in figures :

7. Two hundred thirty-five and eight hundred thirty-five thousandths.

235.835.

8. Seventy-four and two hundred three thousand six millionths.

74.203006.

9. Twelve hundred and eight thousand three ten-millionths.

1200.0008003.

10. Five thousand sixty-four millionths.

0.005064.

11. One million and four tenths.

1,000,000.4.

12. Six hundred-millionths.

0.00000006.

13. Multiply and divide 789.365 by 10 ; by 100 ; by 100,000.

7893.65 ; 78.9365 ; 78,936.5 ; 7.89365 ; 78,936,500 ; 0.00789365.

14. Multiply and divide 0.004 by 100 ; by 10,000 ; by 1000.

0.4 ; 0.00004 ; 40 ; 0.0000004 ; 4 ; 0.000004.

15. Multiply and divide 436 by 1,000,000 ; by 1000 ; by 10.

436,000,000 ; 0.000436 ; 436,000 ; 0.436 ; 4360 ; 43.6.

16. Multiply and divide 0.1 by ten ; by ten millions.

1 ; 0.01 ; 1,000,000 ; 0.00000001.

Find the value of :

17. 21.3706 + 15.243 + 1.8954 + 0.026891 + 5.328 + 29.74.

$$\begin{array}{r}
21.3706 \\
15.243 \\
1.8954 \\
0.026891 \\
5.328 \\
29.74 \\
\hline
73.603891
\end{array}$$

[redacted]

18. 57 + 0.0057 + 6.8 + 1200 + 0.847 + 159.2 + 3.

$$\begin{array}{r} 57. \\ 0.0057 \\ 6.8 \\ 1200. \\ 0.847 \\ 159.2 \\ 3. \\ \hline 1426.8527 \end{array}$$

19. 0.0012 + 10 + 5.8281 + 5 + 39.43 + 0.6827 + 1.

$$\begin{array}{r} 0.0012 \\ 10. \\ 5.8281 \\ 5. \\ 39.43 \\ 0.6827 \\ 1. \\ \hline 61.942 \end{array}$$

20. 23.9875 — 12.4764; 35.14732 — 27.62815.

$$\begin{array}{r} 23.9875 \\ 12.4764 \\ \hline 11.5111 \end{array} \qquad \begin{array}{r} 35.14732 \\ 27.62815 \\ \hline 7.51917 \end{array}$$

21. 102.1274 — 83.072; 39.801 — 17.9645.

$$\begin{array}{r} 102.1274 \\ 83.072 \\ \hline 19.0554 \end{array} \qquad \begin{array}{r} 39.801 \\ 17.9645 \\ \hline 21.8365 \end{array}$$

22. 30 — 5.2817; 1.7 — 0.8469.

$$\begin{array}{r} 30. \\ 5.2817 \\ \hline 24.7183 \end{array} \qquad \begin{array}{r} 1.7 \\ 0.8469 \\ \hline 0.8531 \end{array}$$

23. 1 — 0.54237; 100 — 0.00176.

$$\begin{array}{r} 1. \\ 0.54237 \\ \hline 0.45763 \end{array} \qquad \begin{array}{r} 100. \\ 0.00176 \\ \hline 99.99824 \end{array}$$

[redacted]

24. 24.271 − 3.6485 + 15.271 − 13.256 − 14.125.

	3.6485	
24.271	13.256	39.542
15.271	14.125	31.0295
39.542	31.0295	8.5125

25. 52 + 0.52 − 17.8946 − 30.254 − 0.5 + 21.12.

52.	17.8946	
0.52	30.254	73.64
21.12	0.5	48.6486
73.64	48.6486	24.9914

26. 41.289 × 0.5; 0.268 × 0.9; 0.112 × 0.2.

41.289	0.268	0.112
0.5	0.9	0.2
20.6445	0.2412	0.0224

27. 2.435 × 4.23; 71.651 × 3.37; 0.251 × 0.04.

2.435	71.651	0.251
4.23	3.37	0.04
7305	501557	0.01004
4870	214953	
9740	214953	
10.30005	241.46387	

28. 0.0012 × 0.005; 2.26823 × 200; 5.6125 × 0.0768.

0.0012	2.26823	5.6125
0.005	200	0.0768
0.000006	453.646	449000
		336750
		392875
		0.43104

29. 0.7 × 7 × 0.07; 0.15625 × 23.7 × 0.00192 × 5.

0.7	0.15625	3.703125
7	23.7	0.00192
4.9	109375	7406250
0.07	46875	33328125
0.343	31250	3703125
	3.703125	0.00711
		5
		0.03555

[redacted]

30. $(2.465 + 1.21) \times (3.2 - 2.89)$.

$$\begin{aligned}&(2.465 + 1.21) \times (3.2 - 2.89)\\ &= 3.675 \times 0.31\\ &= 1.13925.\end{aligned}$$

31. $(3.01)^2$; $(0.045)^2$; $(0.0081)^2$; $(5.1004)^3$; $(0.76)^3$.

```
   3.01        0.045         0.0081
   3.01        0.045         0.0081
    301          225             81
   903          180             648
 9.0601     0.002025     0.00006561

        5.1004             0.76
        5.1004             0.76
        204016              456
      51004                532
    255020               0.5776
   26.01408016             0.76
        5.1004            34656
    10405632064          40432
   2601408016          0.438976
 13007040080
 132.682214448064
```

32. $(0.125)^2 \times (0.32)^3$.

```
  0.125        0.32        0.032768
  0.125        0.32        0.015625
    625          64          163840
   250          96          65536
  125         0.1024       196608
0.015625        0.32      163840
               2048       32768
              3072      0.000512
           0.032768
```

33. Divide 291.84 by 6; 0.12936 by 12; 7.92801 by 0.9.

```
6)291.84     12)0.12936     9)79.2801
  48.64         0.01078        8.8089
```

[redacted]

34. Divide 58.383 by 0.39; 0.28744 by 0.08; 491.205 by 0.065.

```
     149.7
39)5838.3
   39
   193
   156
    378
    351
     273
     273
```

```
8)28.744
   3.593
```

```
      7557
65)491205
   455
    362
    325
     370
     325
      455
      455
```

35. Divide 68.325 by 6.25; 0.732 by 1.6; 1208.88 by 0.438.

```
      10.932
625)6832.5
    625
     5825
     5625
      2000
      1875
       1250
       1250
```

```
   0.4575
16)7.32
   64
    92
    80
    120
    112
      80
      80
```

```
        2760
438)1208880
     876
     3328
     3066
      2628
      2628
         0
```

36. Divide 498 by 0.0125; 7 by 0.007; 1000 by 0.0001.

The reciprocal of 0.0125 is 80.

```
  498
   80
39840
```

```
7)7000
  1000
```

```
1)10000000
  10000000
```

37. Divide 0.235 by 10.24; 27 by 12; 0.00507702 by 0.0283.

```
     0.02294
1024)23.5
     2048
      3020
      2048
       9720
       9216
        5040
        4096
         944
```

0.02295. *Ans.*

```
12)27.
   2.25
```

```
     0.1794
283)50.7702
    283
    2247
    1981
     2660
     2547
      1132
      1132
```

[redacted]

38. Divide 89.3 by 0.00752; 74.1 by 0.0256; 1 by 0.128.

```
      11875
752)8930000
     752
     1410
      752
      6580
      6016
       5640
       5264
        3760
        3760
```

```
       2894.53125
256)741000.
    512
    2290
    2048
     2420
     2304
      1160
      1024
       1360
       1280
        800
        768
         320
         256
          640
          512
          1280
          1280
```

```
       7.8125
128)1000.
     896
     1040
     1024
      160
      128
       320
       256
        640
        640
```

39. Divide 0.39842 by 3.7164 ; 281.5 by 13.789; 0.0005 by 0.0028.

```
          0.10720
37164)3984.2
      37164
       267800
       260148
         76520
         74328
          21920
```

0.10721. *Ans.*

```
             20.41482
13789)281500.
      27578
        57200
        55156
         20440
         13789
          66510
          55156
          113540
          110312
            32280
            27578
             4702
```

```
     0.17857
28)5.
   28
   220
   196
    240
    224
     160
     140
      200
      196
        4
```

[redacted]

40. Divide 63.04128 by 912.85; 287.209 by 0.00493; 2000 by 0.0059.

```
       0.06905
91285) 6304.128
       547710
        827028
        821565
         546300
         456425
          89875
0.06906. Ans.
```

```
    58257.40365
493) 28720900.
     2465
      4070
      3944
       1269
        986
       2830
       2465
        3650
        3451
         1990
         1972
           1800
           1479
            3210
            2958
             2520
             2465
               55
```

```
   338983.05084
59) 20000000.
    177
     230
     177
      530
      472
       580
       531
        490
        472
         180
         177
          300
          295
           500
           472
            280
            236
             44
```

338,983.05085. *Ans.*

Exercise 24. Page 51.

Find the value of:

1. 1.4 + 2.08 + 3.895.

```
1.4
2.08
3.895
7.375
```

2. 2.8 + 2.08 + 0.28 + 0.028 + 0.812.

```
2.8
2.08
0.28
0.028
0.812
6.
```

3. 1.667 + 0.4 + 0.286 + 6.08 + 0.636 + 0.931.

```
1.667
0.4
0.286
6.08
0.636
0.931
10.
```

4. 6.125 — 0.57.

```
6.125
0.57
5.555
```

5. (4.625 + 1.146) — (1.2 + 3.571)
= 5.771 — 4.771
= 1.

[redacted]

6. $6.913 - (2.85 - 0.937)$
$= 6.913 - 1.913$
$= 5.$

7. $24 - 2.4 + (5 - 3.508) - 3.092$
$= 24 - 2.4 + 1.492 - 3.092$
$= 25.492 - 5.492$
$= 20.$

8. $10 - (4.25 - 2.5 + 2 - 0.625 - 0.4 - 2.02) - 0.295$
$= 10 - (6.25 - 5.545) - 0.295$
$= 10 - 0.705 - 0.295$
$= 10 - 1$
$= 9.$

9. $1.5 \times 0.08 \times 0.5.$

$$\begin{array}{r} 1.5 \\ 0.08 \\ \hline 0.12 \\ 0.5 \\ \hline 0.06 \end{array}$$

10. $0.1204 \times 0.0168 \times 100.$

$$\begin{array}{r} 0.1204 \\ 0.0168 \\ \hline 9632 \\ 7224 \\ 1204 \\ \hline 0.00202272 \\ 100 \\ \hline 0.202272 \end{array}$$

11. $0.04 \times 3.25 \times 0.06.$

$$\begin{array}{r} 3.25 \\ 0.04 \\ \hline 0.13 \\ 0.06 \\ \hline 0.0078 \end{array}$$

12. $36 \times 0.002 \times 2.05 \times 0.00765.$

$$\begin{array}{r} 36 \\ 0.002 \\ \hline 0.072 \\ 2.05 \\ \hline 360 \\ 144 \\ \hline 0.1476 \end{array} \qquad \begin{array}{r} 0.1476 \\ 0.00765 \\ \hline 7380 \\ 8856 \\ 10332 \\ \hline 0.00112914 \end{array}$$

13. $0.139 \times 28 + 42 \times 0.002 + 6 \times 0.004 - 0.05 \times 20$
$= 3.892 + 0.084 + 0.024 - 1$
$= 4 - 1$
$= 3.$

14. $(10 - 1.25) \times 0.2 + 0.02 \times 2.8 + (80.3 \times 0.1 - 5.3) \times 10 - 805.3 \times 0.02$
$= 8.75 \times 0.2 + 0.02 \times 2.8 + (8.03 - 5.3) \times 10 - 805.3 \times 0.02$
$= 1.75 + 0.056 + 27.3 - 16.106$
$= 29.106 - 16.106$
$= 13.$

[redacted]

15. 28.3696 ÷ 1.49.

```
        19.04
149)2836.96
    149
    1346
    1341
       596
       596
```

16. 0.27 ÷ 0.00225.

```
        120
225)27000
    225
     450
     450
       0
```

17. 8.8779 ÷ 175.8.

```
         0.0505
1758)88.779
     8790
       8790
       8790
```

18. 0.0427 ÷ 92.3.

```
       0.00046
923)0.427
     3692
      5780
      5538
       242
```

19. 0.28744 ÷ 800.

```
8)0.0028744
  0.0003593
```

20. 491.205 ÷ 650.

```
     0.7557
65)49.1205
   455
    362
    325
     370
     325
      455
      455
```

21. 68.325 ÷ 6250.

```
      0.010932
625)6.8325
    625
     5825
     5625
      2000
      1875
       1250
       1250
```

22. 0.732 ÷ 16,000.

```
     0.00004575
16)0.000732
        64
         92
         80
         120
         112
           80
           80
```

23. 1208.88 ÷ 0.438.

```
        2760
438)1208880
    876
    3328
    3066
     2628
     2628
        0
```

[redacted]

24. $2 \div 0.01 - (0.2 \div 0.02 + 0.8 \div 10) + 36.48 \div 8 - (4 \div 0.05 - 2 + 0.6 \div 1.25)$
$$= 200 - (10 + 0.08) + 4.56 - (80 - 2 + 0.48)$$
$$= 200 - 10.08 + 4.56 - 78.48$$
$$= 204.56 - 88.56$$
$$= 116.$$

25. $72.2 \div 10 - 2 \div (0.5 \div 1.60) + 2.125 \div (1.75 - 0.5)$
$$= 72.2 \div 10 - 2 \div 0.3125 + 2.125 \div 1.25$$
$$= 7.22 - 6.4 + 1.7$$
$$= 8.92 - 6.4$$
$$= 2.52.$$

Exercise 25. Page 52.

1. What number subtracted 88 times from 80,005 will leave 13 as a remainder?

$$\begin{array}{r} 80005 \\ 13 \\ \hline 79992 \end{array} \qquad \begin{array}{r} 909 \\ 88\overline{)79992} \\ 792 \\ \hline 792 \\ 792 \\ \hline \end{array}$$

2. If 7 men can build a wall in 16 days, how many men will it take to build a wall three times as long in half the time?

$$\begin{array}{r} 7 \\ 3 \\ \hline 21 \\ 2 \\ \hline 42 \end{array}$$

3. How many minutes are there between 25 minutes past 8 in the morning and midnight?

$$\begin{array}{r} 35 \\ 180 \\ 720 \\ \hline 935 \end{array}$$

4. If the velocity of sound is 1090 feet per second, at what distance is a gun fired, the report of which I hear 11 seconds after seeing the flash? (5280 feet make a mile.)

1090
11
1090
1090
11990

2.27083
5280)11990.
10560
14300
10560
37400
36960
44000
42240
17600
15840

2.27083 miles. *Ans.*

5. How long will it take to travel 30.2375 miles at the rate of 8.85 miles per hour?

3.4166
885)3023.75
2655
3687
3540
1475
885
5900
5310
5900
5310
590

3.4167 hours. *Ans.*

6. If the circumference of a circle is 3.1416 times the diameter, find the circumference of a circle whose diameter is 6.8 feet; also, find the diameter of a circle whose circumference is 20 inches.

3.1416
6.8
251328
188496
21.36288

21.363 feet. *Ans.*

6.366
31416)200000.
188496
115040
94248
207920
188496
194240
188496

6.366 inches. *Ans.*

7. How much wire will be required to make a hoop 30 inches in diameter, allowing 2 inches for the joining?

```
 3.1416
     30
 94.248
     2.
 96.248
```

96.248 inches. *Ans.*

8. How many times would the hoop of Ex. 7 turn in going half a mile?

```
2)5280                          336.
  2640              94248)31680000.
                          282744
  2640                     340560
    12                     282744
  5280                      578160
 2640                       565488
 31680
```

9. Cork, whose weight is 0.24 of the weight of water, weighs 15 pounds per cubic foot. What is the weight of 6 cubic feet of oak, if the weight of oak is 0.934 of the weight of water?

```
      62.5
 24)1500.                 62.5
    144                  0.934
     60                   2500
     48                  1875
    120                 5625
    120                 58.375
                             6
                        350.25
```

350.25 pounds. *Ans.*

10. From what number can 847 be subtracted 307 times, and leave a remainder of 49?

```
    847
    307
   5929
  2541
 260029
     49
 260078
```

11. What is the 235th part of 141,235?

```
         601
 235)141235
     1410
        235
        235
```

12. What will 343 barrels of flour cost at $6.37 a barrel?

$$\begin{array}{r} \$6.37 \\ 343 \\ \hline 1911 \\ 2548 \\ 1911 \\ \hline \$2184.91 \end{array}$$

13. Twelve makes a dozen, and 12 dozen makes a gross. How many steel pens in 28 gross? What will a gross of eggs cost at 27 cents a dozen?

$12 \times 12 = 144.$

$$\begin{array}{r} 144 \\ 28 \\ \hline 1152 \\ 288 \\ \hline 4032 \end{array} \qquad \begin{array}{r} \$0.27 \\ 12 \\ \hline 54 \\ 27 \\ \hline \$3.24 \end{array}$$

14. How much must be added to $4429 to make the sum equal to 43 × $241?

$$\begin{array}{r} \$241 \\ 43 \\ \hline 723 \\ 964 \\ \hline \$10363 \\ 4429 \\ \hline \$5934 \end{array}$$

15. What number deducted from the 26th part of 2262 will leave the 87th part of the same number?

$$\begin{array}{r} 87 \\ 26\overline{)2262} \\ 208 \\ \hline 182 \\ 182 \\ \hline \end{array} \qquad \begin{array}{r} 26 \\ 87\overline{)2262} \\ 174 \\ \hline 522 \\ 522 \\ \hline \end{array} \qquad \begin{array}{r} 87 \\ 26 \\ \hline 61 \end{array}$$

16. At the ordinary rate, 123 words a minute, how long will it take a man to deliver a speech of 15 pages, each of 28 lines, each line containing 11 words? How long would it have taken Daniel Webster to deliver the same speech, whose rate was 93 words a minute?

$$\begin{array}{r} 15 \\ 28 \\ \hline 120 \\ 30 \\ \hline 420 \\ 11 \\ \hline 420 \\ 420 \\ \hline 4620 \end{array} \qquad \begin{array}{r} 37.5 \\ 123\overline{)4620.} \\ 369 \\ \hline 930 \\ 861 \\ \hline 690 \\ 615 \\ \hline 75 \end{array} \qquad \begin{array}{r} 49.6 \\ 93\overline{)4620.} \\ 372 \\ \hline 900 \\ 837 \\ \hline 630 \\ 558 \\ \hline 72 \end{array}$$

37.6 minutes; 49.7 minutes. *Ans.*

[redacted]

17. How long will it take a railway train to go from New York to San Francisco, 3310 miles, at the rate of 1973 feet a minute?

```
  3310                      8858
  5280            1973)17476800
264800                 15784
 6620                   16928
16550                   15784
17476800                 11440
                          9865
                         15750
```

8858 minutes = 147 hours 38 minutes. *Ans.*

18. How many hours will it take to count a million, at the rate of 67 a minute?

```
  67            248.75
  60    4020)100000.
4020          804
              1960
              1608
               3520
               3216
                3040
                2814
                 2260
                 2010
                  250
```

248.76 hours. *Ans.*

19. If you put into a box 17 cents a day, including Sundays, beginning January 1 and ending July 4, how much money will there be in the box?

```
 31         185
 28        0.17
 31        1295
 30        185
 31       31.45
 30
  4       $31.45. Ans.
185
```

20. If a man's income is $3000 a year, and his daily expenses average $7.68, what does he save in a year?

```
  $7.68        $3000.
    365       2803.20
   3840       $196.80
  4608
 2304
$2803.20
```

21. In a question of division the quotient was 87.83, the divisor, 759. What was the dividend?

```
   87.83
     759
   79047
  43915
 61481
66662.97
```

22. What is the nearest number to 7196 that will contain 372 without a remainder?

```
       19
372)7196        7196
    372          128
    3476        7068
    3348
     128
```

[redacted]

23. It is 3.1416 times as far round a wheel as across it. How many times will a wheel 4.5 feet across turn in going 23 miles of 5280 feet each ?

```
                                                  8590  Ans.
  3.1416            5280          141372)1214400000
     4.5              23                 1130976
  157080           15840                  834240
 125664            10560                  706860
 14.1372          121440                 1273800
                                         1272348
                                           14520
```

24. How many gallons of 231 cubic inches are contained in a cubic foot of 1728 cubic inches ? in a bushel of 2150.42 cubic inches ? How many cubic feet in a bushel ? How many bushels in 31.5 gallons ?

```
        7.48                       9.309
   231)1728.                 231)2150.42
       1617                      2079
        1110                       714
         924                       693
         1860                       2120
         1848                       2079
```

```
        1.244                                3.38
  1728)2150.42        31.5       215042)727650.
       1728            231              645126
        4224           315               825240
        3456          945                645126
         7682        630                 1801140
         6912        7276.5              1720336
          7700
          6912
```

25. Seven children had left to them $7186 apiece ; one died, and his share was divided among the surviving six. How much had each then ?

```
6)$7186.
  $1197.67
   7186.
  $8383.67
```

26. How long will it take 2 men to do what 1 man can do in 6 days ? what 4 men can do in 3 days ? what 3 men can do in 4 days ?

6 days ÷ 2 = 3 days.
2 × 3 days = 6 days.
(3 × 4 days) ÷ 2 = 6 days.

[redacted]

27. Divide \$1.80 among Thomas, Richard, and Henry in such a way that Henry shall receive 3 cents for every 5 cents that Thomas gets, and Richard shall receive 2 cents for every 3 cents that Henry gets.

2	10) \$1.80
3	\$0.18
5	2
10	\$0.36, R.'s.
\$0.18	\$0.18
3	5
\$0.54, H.'s.	\$0.90, T.'s.

28. Divide \$87.84 between B and C so that C shall get \$19 as often as B gets \$17.

	2.44
19	36) 87.84
17	72
36	158
	144
	144
	144
\$2.44	\$2.44
17	19
1708	2196
244	244
\$41.48, B's.	\$46.36, C's.

29. Three partners received for goods: one, \$371.63; the second, \$285.40; the third, \$411.91. They paid for the goods \$879.34, and divided the profit equally among them. How much did each receive?

\$371.63	\$1068.94
285.40	879.34
411.91	3) \$189.60
\$1068.94	\$63.20

30. If there are 12 inches in a foot, how many inches long is a wall 35 feet in length? If a brick and its share of mortar is 8.4 inches long, how many bricks in length is the wall?

35	50
12	84) 4200
70	420
35	0
420	

31. If a brick and its mortar is 2.4 inches high, how many bricks are required to build a wall 12 feet high, 35 feet long, if the width of the wall is the width of two bricks?

12	60	60
12	24) 1440	50
144	144	3000
	0	2
		6000

32. What is the total weight of the wall of Ex. 31, if a brick with its share of the mortar weighs 4.13 pounds? What is the weight after a long rain, when the weight is increased to 4.27 pounds for each brick?

4.13	4.27
6000	6000
24780	25620

24,780 pounds;
25,620 pounds. Ans.

33. How many pounds does each foot in length of the wall of Ex. 31 weigh?

```
     708           732
35)24780      35)25620
   245           245
    280           112
    280           105
                   70
                   70
```

34. If 60.98 cubic inches of brick weigh 4 pounds, how many cubic inches of brick weigh 1 pound? How many pounds will a cubic foot (1728 cubic inches) weigh?

```
 4)60.98
   15.245

          113.34
15245)1728000.
      15245
       20350
       15245
        51050
        45735
         53150
         45735
          74150
          60980
          13170
113.35 pounds. Ans.
```

35. If a cubic foot of water weighs 62.5 pounds, how many times as heavy as water is brick?

```
       1.8
625)1133.5
    625
    5085
    5000
```

36. Light moves through the air at the rate of 186,500 miles a second. How many times can it go around the earth in a second, if the distance round the earth is 24,897.714 miles?

```
                  7.4
24897714)186500000.
         174283998
          122160020
           99590856
           22569164
7.5. Ans.
```

37. Light moves through the air at the rate of 300,190 kilometers a second. How many times can it go around the earth in a second, if the distance round the earth is 40,007.5 kilometers?

```
             7.5
400075)3001900.
       2800525
        2013750
        2000375
```

38. A minute is 60 seconds. How many miles and how many kilometers can light travel through air in a minute?

```
  186500
      60
11190000
  300190
      60
18011400
```

11,190,000 miles;
18,011,400 kilometers. *Ans.*

[redacted]

39. An hour is 60 minutes. How many miles and how many kilometers can light travel in an hour?

```
 11190000
       60
671400000
 18011400
       60
1080684000
```

671,400,000 miles;
1,080,684,000 kilometers. *Ans.*

40. The distance round the earth, given in Ex. 37, is measured on a north and south line. Around the equator the distance is 40,075.45 kilometers. How many times could light move round the equator in one minute?

```
               7.49
4007545)30019000.
        28052815
         19661850
         16030180
          36316700
          36067905
```

```
 7.49
   60
449.4
```

41. Find the reciprocal of the difference between 31.24 and 31.23768.

```
31.24
31.23768
 0.00232
```

```
     431.034
232)100000.
    928
     720
     696
      240
      232
        800
        696
        1040
         928
         112
```

42. The Hanoverian mile is 25,400 Hanoverian feet long, and each foot is 0.9542 of an English foot. Find to four places of decimals the fraction that an English mile of 5280 English feet is of a Hanoverian mile.

```
   0.9542
    25400
  3816800
 47710
19084
24236.6800
```

```
                  0.2178
2423668)258000.
        4847336
         4326640
         2423668
         19029720
         16965676
          20640440
          19389344
           1251096
```

[redacted]

43. Express in inches the length of a meter, given that a meter is one ten-millionth of a quarter of the earth's circumference, that the circumference is 3.14159 times the diameter, that the diameter of the earth is 7911.7 miles, and that a mile is 5280 × 12 inches.

5280
12
63360
7911.7
443520
63360
63360
570240
443520
501285312.
3.14159
4511567808
2506426560
501285312
2005141248
501285312
1503855936
4)1574832923.32608
393708230.83152
0.0000001
39.370823083152

39.3708 inches. *Ans.*

44. How must a number be altered that its reciprocal may be doubled?

Divided by 2.

45. What effect is produced on the sum of two numbers, if the same number is added to each of them? What effect on the difference?

It is increased by twice the number; no effect.

46. What effect is produced on the product of two numbers, if both numbers are multiplied by the same number? What effect on the quotient?

It is multiplied by the square of the number; no effect.

[redacted]

47. What effect is produced on the *remainder*, if both divisor and dividend are multiplied by the same number? If both are divided by the same number?

It is multiplied by the number; it is divided by the number.

48. In going from one planet to another, light probably moves faster than in air. Suppose it moves at the rate of 309,800 kilometers a second, how many seconds would it take light to perform each of the following journeys:

Moon to Earth	375,500	kilometers.
Sun to Earth	147,250,000	"
Sun to Mercury	56,900,000	"
Sun to Venus	106,400,000	"
Sun to Mars	224,100,000	"
Sun to the Asteroids	400,000,000	"
Sun to Jupiter	765,400,000	"
Sun to Saturn	1,403,000,000	"
Sun to Uranus	2,817,000,000	"
Sun to Neptune	4,421,000,000	"
Sun to the nearest star . .	24,000,000,000,000	"

1.21
3098~~00~~)3755.
3098
6570
6196
3740
3098

475.3
3098~~00~~)1472500.
12392
23330
21686
16440
15490
9500
9294

183.7
3098~~00~~)569000.
3098
25920
24784
11360
9294
20660

343.4
3098~~00~~)1064000.0
9294
13460
12392
10680
9294
13860
12392

723.4
3098~~00~~)2241000.
21686
7240
6196
10440
9294
11460

1291.2
3098~~00~~)4000000.
3098
9020
6196
28240
27882
3580
3098
4820

2470.6
309800)7654000.
6196
14580
12392
21880
21686
19400
18588

4528.7
309800)14030000.
12392
16380
15490
8900
6196
27040
24784
22560
21686

9092.9
309800)28170000.
27882
28800
27882
9180
6196
29840
27882

14270.5
309800)44210000.
3098
13230
12392
8380
6196
21840
21686
15400

77469335
309800)240000000000
21686
23140
21686
14540
12392
21480
18588
28920
27882
10380
9294
10860
9294
15660
15490

49. A kilometer is about 0.6214 of a mile. How many miles is each of the planets from the sun?

14725
6214
58900
14725
29450
88350
Earth, 91501150

5690
6214
22760
5690
11380
34140
Mercury, 35357660

10640
6214
42560
10640
21280
63840
Venus, 66116960

22410
6214
89640
22410
44820
134460
Mars, 139255740

6214
40000
Asteroids, 248560000

76540
6214
306160
76540
153080
459240
Jupiter, 475619560

140300	281700	442100
6214	6214	6214
561200	1126800	1768400
140300	281700	442100
280600	563400	884200
841800	1690200	2652600
Saturn, 871824200	Uranus, 1750483800	Neptune, 2747209400

50. If 11.75 tons of coal cost $82.25, what will 21.4 tons cost?

```
         $7          21.4
1175)$8225              7       $149.80. Ans.
      8225          149.8
```

51. Find the number of hours it will take a locomotive running at the rate of 27 miles an hour to make the distance passed over in 13.25 hours by another locomotive that has a velocity of 43.5 miles an hour.

```
 13.25            21.34722  Ans.
  43.5         27)576.375
  6625            54
 3975              36
5300               27
576.375             93
                    81
                    127
                    108
                     195
                     189
                       60
                       54
                       60
                       54
                        6
```

Exercise 26. Page 60.

1. Change 5427^{m} to kilometers; to millimeters; to centimeters.

$5427^{m} = 5.427^{km} = 5{,}427{,}000^{mm} = 542{,}700^{cm}$.

2. How many meters in 6853^{mm}? how many centimeters? what part of a kilometer?

$6853^{mm} = 6.853^{m} = 685.3^{cm} = 0.006853^{km}$.

[redacted]

3. Write 49.7^{m} as centimeters; as millimeters; as the decimal of a kilometer.

$$49.7^{m} = 4970^{cm} = 49{,}700^{mm} = 0.0497^{km}.$$

4. How many centimeters in 12.4^{km}? how many millimeters?

$$12.4^{km} = 1{,}240{,}000^{cm} = 12{,}400{,}000^{mm}.$$

5. Change 1230^{m} to kilometers; to centimeters.

$$1230^{m} = 1.23^{km} = 123{,}000^{cm}.$$

6. Write 1230^{cm} as meters; as millimeters.

$$1230^{cm} = 12.3^{m} = 12{,}300^{mm}.$$

7. Find in meters the value of $0.435^{m} + 852^{cm} + 4263^{mm} + 0.1595^{km}$.

$$\begin{array}{r} 0.435^{m} \\ 8.52 \\ 4.263 \\ \underline{159.5} \\ 172.718^{m} \end{array}$$

8. Find in meters the value of $0.927^{km} - 6495^{cm}$; $4.37^{cm} - 42.87^{mm}$.

$$\begin{array}{r} 927.^{m} \\ \underline{64.95} \\ 862.05^{m} \end{array} \qquad \begin{array}{r} 0.0437^{m} \\ \underline{0.04287} \\ 0.00083^{m} \end{array}$$

9. Find in meters the value of 8×0.0457^{km}; 3.04×60.93^{cm}; 5.43×67.2^{mm}.

$$\begin{array}{r} 45.7^{m} \\ \underline{8} \\ 365.6^{m} \end{array} \qquad \begin{array}{r} 0.6093^{m} \\ \underline{3.04} \\ 24372 \\ \underline{18279} \\ 1.852272^{m} \end{array} \qquad \begin{array}{r} 0.0672^{m} \\ \underline{5.43} \\ 2016 \\ 2688 \\ \underline{3360} \\ 0.364896^{m} \end{array}$$

10. Find in meters the value of $38{,}019^{mm} \div 0.097$; $0.41^{km} \div 25.625$.

$$\begin{array}{r} 391.948^{m} \\ 97\overline{)\,38019.^{m}} \\ \underline{291} \\ 891 \\ \underline{873} \\ 189 \\ \underline{97} \\ 920 \\ \underline{873} \\ 470 \\ \underline{388} \\ 820 \\ \underline{776} \end{array} \qquad \begin{array}{r} 16^{m} \\ 25625\overline{)\,410000^{m}} \\ \underline{25625} \\ 153750 \\ \underline{153750} \end{array}$$

11. At \$1.87 a meter, what is the cost of 6.20^{m} of cloth?

$$\begin{array}{r} \$1.87 \\ \underline{6.2} \\ 374 \\ \underline{1122} \\ \$11.594 \end{array}$$

\$11.59. *Ans.*

12. At \$0.75 a meter, what is the cost of 60^{m} of cloth?

$$\begin{array}{r} \$0.75 \\ \underline{60} \\ \$45.00 \end{array}$$

[redacted]

13. From a piece of cloth containing 47.60^{m} a tailor cuts off three pieces: the first of 3.80^{m}, the second of 1.30^{m}, and the third of 45^{cm}. How many meters of the cloth are left?

3.8^{m}
1.3
0.45
5.55^{m}

47.6^{m}
5.55
42.05^{m}

14. What is the value of 60^{cm} of cloth at \$5.20 a meter?

\$5.20
0.6
\$3.12

15. If \$6.00 is paid for a railroad ticket to travel 440^{km}, what is the fare per kilometer?

\$0.0136
440) \$6.000
440
1600
1320
2800
2640
160

16. If a train goes 288^{km} in 9 hours, how many meters does it go in a minute? (1 hour = 60 minutes.)

60
9
540

533.33^{m}
54) $28800.^{m}$
270
180
162
180
162
180
162
180
162

17. If a man walks at the rate of 6^{km} an hour, what part of an hour will it take him to walk 420^{m}?

$6^{km} = 6000^{m}$

0.07
6000) 420.00
42000

18. A railroad carried 412 passengers 18^{km} for \$88.992; at the same rate, what will it receive for carrying 350 passengers 35^{km}?

412
18
3296
412
7416

\$0.012
7416) \$88.992
7416
14832
14832

350
35
1750
1050
12250

12250
0.012
24500
1225
147.000 \$147. *Ans.*

Exercise 27. Page 62.

1. Change $1{,}854{,}276^{qm}$ to hektars; to square kilometers.

$1{,}854{,}276^{qm} = 185.4276^{ha}$
$= 1.854276^{qkm}$.

2. How many hektars in 2.7856^{qkm}?

$2.7856^{qkm} = 278.56^{ha}$.

3. Write 1.7431^{qm} as square centimeters; as square millimeters.

$$1.7431^{qm} = 17{,}431^{qcm}$$
$$= 1{,}743{,}100^{qmm}.$$

4. How many square kilometers in 17,467.5^{ha}?

$$17{,}467.5^{ha} = 174.675^{qkm}.$$

5. How many square meters in 1.3614^{qkm}?

$$1.3614^{qkm} = 1{,}361{,}400^{qm}.$$

6. How many square meters in 2.25^{ha}?

$$2.25^{ha} = 22{,}500^{qm}.$$

7. How many square centimeters in 0.0137^{qm}?

$$0.0137^{qm} = 137^{qcm}.$$

8. Write 3.571^{qcm} as square millimeters.

$$3.571^{qcm} = 357.1^{qmm}.$$

9. If a field contains 7500^{ca}, how many ars does it contain? What part of a hektar?

$$7500^{ca} = 75^{a} = 0.75^{ha}.$$

10. How many square meters must be added to 22,612^{qm} to make 4^{ha} 62^{a} 17^{ca}?

$$4^{ha}\,62^{a}\,17^{ca} = 46{,}217^{ca} = 46{,}217^{qm}.$$

$$\begin{array}{r} 46{,}217^{qm} \\ \underline{22{,}612} \\ 23{,}605^{qm} \end{array} \quad \textit{Ans.}$$

11. A field containing 72.4^{a} is sold at 15 cents a square meter. What is received for the field?

$$72.4^{a} = 7240^{qm}.$$

$$7240 \times \$0.15 = 0.15 \times \$7240.$$

$$\begin{array}{r} \$7240 \\ \underline{0.15} \\ 36200 \\ \underline{7240} \\ \$1086.00 \end{array} \qquad \$1086. \; \textit{Ans.}$$

12. If 62^{a} 12^{ca} of land is sold for $1366.64, what is the price per square meter?

$$62^{a}\,12^{ca} = 6212^{qm}.$$

$$\begin{array}{r} \$0.22 \quad \textit{Ans.} \\ 6212\overline{)\$1366.64} \\ \underline{12424} \\ 12424 \\ \underline{12424} \end{array}$$

13. How many square centimeters must be taken from 12,473^{qcm} to leave 1^{qm} 14^{qdm} 53^{qcm}?

$$1^{qm}\,14^{qdm}\,53^{qcm} = 11{,}453^{qcm}.$$

$$\begin{array}{r} 12473^{qcm} \\ \underline{11453} \\ 1020^{qcm} \end{array} \quad \textit{Ans.}$$

Exercise 28. Page 64.

1. Write 2.25^{cbm} as cubic centimeters.

$$2.25^{cbm} = 2{,}250{,}000^{ccm}. \quad \textit{Ans.}$$

2. Change 2,162,875^{ccm} to cubic meters.

$$2{,}162{,}875^{ccm} = 2.162875^{cbm}. \quad \textit{Ans.}$$

3. Change 0.0175^{cbm} to cubic millimeters.

$$0.0175^{cbm} = 17{,}500{,}000^{cmm}. \quad \textit{Ans.}$$

4. Change 46,164^{ccm} to cubic decimeters.

$$46{,}164^{ccm} = 46.164^{cdm}. \quad \textit{Ans.}$$

[redacted]

5. What is the equivalent of 0.875^{dkst} in cubic meters? in cubic centimeters?

$0.875^{dkst} = 8.75^{st} = 8.75^{cbm}$
$= 8,750,000^{ccm}$.

6. How many sters are there in 14.75^{dkst} of wood? how many decisters?

$14.75^{dkst} = 147.5^{st}$
$= 1475^{dst}$.

7. What is the cost of 28.25^{dkst} of wood at \$1.25 a ster?

$28.25^{dkst} = 282.5^{st}$.

$$\begin{array}{r} 282.5 \\ 1.25 \\ \hline 14125 \\ 5650 \\ 2825 \\ \hline 353.125 \end{array}$$

\$353.13. *Ans.*

8. Find the cost of an oak beam containing 1250^{cdm} at \$25 a cubic meter.

$1250^{cdm} = 1.25^{cbm}$.

$$\begin{array}{r} 1.25 \\ 25 \\ \hline 625 \\ 250 \\ \hline 31.25 \end{array}$$

\$31.25. *Ans.*

9. How many cubic centimeters must be added to $1,262,376^{ccm}$ to make $2^{cbm}\ 2^{cdm}\ 2^{ccm}$?

$2^{cbm}\ 2^{cdm}\ 2^{ccm} = 2,002,002^{ccm}$.

$$\begin{array}{r} 2,002,002^{ccm} \\ 1,262,376 \\ \hline 739,626^{ccm} \end{array}$$ *Ans.*

10. How many cubic millimeters must be taken from $22,350,000,000^{cmm}$ to leave 20^{cbm} $22^{cdm}\ 222^{ccm}$?

$20^{cbm}\ 22^{cdm}\ 222^{ccm}$
$= 20,022,222,000^{cmm}$.

$$\begin{array}{r} 22,350,000,000^{cmm} \\ 20,022,222,000 \\ \hline 2,327,778,000^{cmm} \end{array}$$ *Ans.*

Exercise 29. Page 65.

1. How many liters in 1.7^{cbm}? in $157,854^{ccm}$?

$1.7^{cbm} = 1700^{l}$
$157,854^{ccm} = 157.854^{l}$.

2. How many cubic centimeters in 9.5^{l}? in 0.015^{l}?

$9.5^{l} = 9500^{ccm}$.
$0.015^{l} = 15^{ccm}$.

3. Change 1.25^{hl} to cubic centimeters; to the fraction of a cubic meter.

$1.25^{hl} = 125^{l} = 125,000^{ccm}$
$= 0.125^{cbm}$.

4. Change 431.88^{l} to hektoliters; to the fraction of a cubic meter.

$431.88^{l} = 4.3188^{hl}$
$= 0.43188^{cbm}$.

5. Write 0.375^{cbm} as liters; as cubic centimeters.

$0.375^{cbm} = 375^{l}$
$= 375,000^{ccm}$.

6. Write $734{,}159.651^{ccm}$ as liters; as hektoliters; as cubic meters.

$734{,}159.651^{ccm}$
$= 734.159651^{l}$
$= 7.34159651^{hl}$
$= 0.734159651^{cbm}$.

7. How many cubic meters in $8{,}573{,}412.867^{ccm}$?

$8{,}573{,}412.867^{ccm}$
$= 8.573412867^{cbm}$.

8. Change 0.734578912^{cbm} to cubic centimeters; to liters.

0.734578912^{cbm}
$= 734{,}578.912^{ccm}$
$= 734.578912^{l}$.

9. Change 1731.5^{l} to cubic meters; to cubic centimeters.

$1731.5^{l} = 1.7315^{cbm}$
$= 1{,}731{,}500^{ccm}$.

Exercise 30. Page 66.

1. How many kilos in 1.73^{t}? in 0.341 of a ton?

$1.73^{t} = 1730^{kg}$.
$0.341^{t} = 341^{kg}$.

2. How many kilos will a hektoliter of water weigh?

100^{kg}. *Ans.*

3. Change $13{,}756^{mg}$ to grams; to the fraction of a kilo.

$13{,}756^{mg} = 13.756^{g}$
$= 0.013756^{kg}$.

4. What is the weight in grams of 346.1^{ccm} of water?

346.1^{g}. *Ans.*

5. Find the weight in kilograms of 0.37615^{cbm} of water.

376.15^{kg}. *Ans.*

6. Change 0.6778^{kg} to milligrams.

$0.6778^{kg} = 677{,}800^{mg}$.

7. How many milligrams in the third part of 17.4^{g}?

$\frac{1}{3}$ of $17.4^{g} = 5.8^{g}$
$= 5800^{mg}$. *Ans.*

Exercise 31. Page 67.

1. Add 17.3^{m}, 87.41^{m}, 271^{cm}, 380^{mm}, and 1.79^{m}.

$$\begin{array}{r} 17.3^{m} \\ 87.41 \\ 2.71 \\ 0.38 \\ 1.79 \\ \hline 109.59^{m} \end{array}$$

2. Add 15.87^{m}, 394.6^{dm}, 47.52^{m}, 7538^{cm}, and 75.89^{m}.

$$\begin{array}{r} 15.87^{m} \\ 39.46 \\ 47.52 \\ 75.38 \\ 75.89 \\ \hline 254.12^{m} \end{array}$$

[redacted]

3. Add 187^{cm}, 49.3^{m}, 317^{mm}, and 6.138^{m}.

$$\begin{array}{r} 1.87^{m} \\ 49.3 \\ 0.317 \\ 6.138 \\ \hline 57.625^{m} \end{array}$$

4. In a room the doorsill is 3^{cm} high; the door, 2.34^{m}; the finish over the door, 13.7^{cm}; and the distance from the finish to the ceiling is 93^{cm}. What is the height of the room?

$$\begin{array}{l} 0.03^{m} \\ 2.34 \\ 0.137 \\ 0.93 \\ \hline 3.437^{m} \end{array}$$

5. The distance to the post-office is 3.31^{km}; thence to the mill, 1.711^{km}; thence to the store, 3.718^{km}; thence home, 2.543^{km}. How long is the circuit?

$$\begin{array}{l} 3.31^{km} \\ 1.711 \\ 3.718 \\ 2.543 \\ \hline 11.282^{km} \end{array}$$

6. The distance from Portland, Me., to Boston is 174^{km}; Boston to Albany, 317^{km}; Albany to Buffalo, 478^{km}; Buffalo to Chicago, 863^{km}; Chicago to Omaha, 789^{km}; Omaha to Cheyenne, 830^{km}. How far is it from Cheyenne to Portland? from Cheyenne to Albany? from Boston to Chicago? from Boston to Cheyenne?

(1)

$$\begin{array}{r} 830^{km} \\ 789 \\ 863 \\ 478 \\ 317 \\ 174 \\ \hline 3451^{km} \end{array}$$

(2)

$$\begin{array}{r} 830^{km} \\ 789 \\ 863 \\ 478 \\ \hline 2960^{km} \end{array}$$

(3)

$$\begin{array}{r} 317^{km} \\ 478 \\ 863 \\ \hline 1658^{km} \end{array}$$

(4)

$$\begin{array}{r} 317^{km} \\ 478 \\ 863 \\ 789 \\ 830 \\ \hline 3277^{km} \end{array}$$

7. If I travel 789.7^{km} a day, how far shall I go in 7 days? in 8.5? in 19.6? in 27.8? in 365?

$$\begin{array}{r} 789.7^{km} \\ 7 \\ \hline 5527.9^{km} \end{array}$$

$$\begin{array}{r} 789.7^{km} \\ 8.5 \\ \hline 39485 \\ 63176 \\ \hline 6712.45^{km} \end{array}$$

$$\begin{array}{r} 789.7^{km} \\ 19.6 \\ \hline 47382 \\ 71073 \\ 7897 \\ \hline 15478.12^{km} \end{array}$$

$$\begin{array}{r} 789.7^{km} \\ 27.8 \\ \hline 63176 \\ 55279 \\ 15794 \\ \hline 21953.66^{km} \end{array}$$

$$\begin{array}{r} 789.7^{km} \\ 365 \\ \hline 39485 \\ 47382 \\ 23691 \\ \hline 288240.5^{km} \end{array}$$

8. How much will 3^{m} of cloth cost at $1.37 a meter? How much will 5.38^{m} cost at $2.63 a meter?

$1.37	$2.63
3	5.38
$4.11 *Ans.*	2104
	789
	1315
	$14.1494
	$14.15. *Ans.*

9. How much will 13.4^{kg} of opium be worth at $8.48 a kilo? 28.79^{kg}, at $7.96 a kilo?

$8.48	28.79
13.4	7.96
3392	17274
2544	25911
848	20153
$113.632	229.1684
$113.63. *Ans.*	$229.17. *Ans.*

10. If one barrel of flour weighs 88.9^{kg}, how many barrels can be filled from 444.5^{t} of flour?

5000 *Ans.*

889) 4445000
4445
000

11. How many steps 80^{cm} long will a man take in walking a kilometer?

$1^{km} = 100{,}000^{cm}$.

80) 100000
1250 *Ans.*

12. At 16 cents a liter, what is the cost of 52.4^{hl} of olive oil?

$52.4^{hl} = 5240^{l}$.

5240
0.16
31440
5240
838.40 $838.40 *Ans.*

13. What is the cost of 6^{dkst} 4^{st} of oak wood at $1.75 per ster?

$6^{dkst}\ 4^{st} = 64^{st}$.

$1.75
64
700
1050
$112.00 *Ans.*

14. If a pasture contains $22{,}408^{ca}$, how many ars does it contain? how many hektars?

$22{,}408^{ca} = 224.08^{a}$
$= 2.2408^{ha}$.

15. Find the circumference of a circle 1^{m} in diameter.

3.1416^{m}. *Ans.*

16. Find to the nearest tenth of a millimeter the circumferences of circles whose diameters are, respectively, 83^{m}; 3.71^{m}; 32.8^{m}; 10.4^{cm}; 11.8^{cm}; 167.1^{mm}; 39.3^{mm}.

3.1416	3.1416	3.1416
83000	3710	32800
94248000	314160	25132800
251328	219912	62832
260752.8	94248	94248
$260{,}752.8^{mm}$. *Ans.*	11655.3360	103044.4800
	$11{,}655.3^{mm}$. *Ans.*	$103{,}044.5^{mm}$. *Ans.*

3.1416	3.1416	3.1416	3.1416
104	118	167.1	39.3
125664	251328	31416	94248
31416	31416	219912	282744
326.7264	31416	188496	94248
326.7^{mm}. *Ans.*	370.7088	31416	123.46488
	370.7^{mm}. *Ans.*	524.96136	123.5^{mm}. *Ans.*
		525^{mm}. *Ans.*	

17. What is the length of the earth's orbit, to the nearest meter, if the diameter of the orbit is 294,481,217^{km}?

```
      294481217km
         3.1416
      1766887302
      294481217
    1177924868
    294481217
   883443651
  925,142,191.3272km
```

925,142,191,327^{m}. *Ans.*

18. What is the circumference of a carriage wheel 1.31^{m} in diameter? How far will it go in turning once? 17 times?

(1)

```
  3.1416
    1.31
   31416
  94248
 31416
 4.115496
```

4.115^{m}. *Ans.*

(2)

4.115^{m}. *Ans.*

(3)

```
4.115m
    17
 28805
 4115
```

69.955^{m}. *Ans.*

19. How many times must the wheel of Ex. 18 turn in going 69.429^{m}? 73.513^{m}? 17.27^{km}?

```
             17 nearly
 4115)69429
      4115
      28279

             18 nearly
 4115)73513
      4115
      32363

             4197 nearly
 4115)17270000
      16460
        8100
        4115
        39850
        37035
         28150
```

20. Find the reciprocal of 3.1416 to the fifth place.

```
              0.31830
 31416)10000.0
        94248
         57520
         31416
         261040
         251328
           97120
           94248
            28720
```

0.31831. *Ans.*

21. How thick through is a tree whose girth is 2.97^{m}?

```
   0.31831
      2.97
    222817
   286479
   63662
 0.9453807
```

0.945^{m}. *Ans.*

22. What is the diameter of a wheel that turns 19.5 times in going 107.25^{m}?

```
         5.5m
195) 1072.5m
      975
       975
       975
```

```
  0.31831
      5.5
   159155
  159155
 1.750705
```

1.75^{m}. *Ans.*

23. What is the diameter of a rope of which the circumference is 20^{cm}?

```
 0.31831
      20
  6.3662
```

6.3662^{cm}. *Ans.*

Exercise 32. Page 69.

1. Find the area of a rectangle 17^{cm} by 19^{cm}.

```
 19
 17
133
19
323
```

323^{qcm}. *Ans.*

2. In a rectangular township 16^{km} by 7^{km}, how many hektars? If there are in it 47.3^{km} of highway, averaging 11.7^{m} wide, how much land is left for other uses?

```
 47300
  11.7
331100
 473
473
553410
```

553,410^{qm} = 55.341^{ha}.

```
 16
  7
112
```

112^{qkm} = 11,200^{ha}. *Ans.*

11200^{ha}
55.341
11,144.659^{ha} *Ans.*

3. In a rectangular field 751.3^{m} long and 189.3^{m} wide is a rectangular garden 31.4^{m} by 17.8^{m}. How many hektars in the field? How many exclusive of the garden?

[redacted]


```
   751.3             31.4
   189.3             17.8
   22539             2512
  67617             2198
 60104              314
 7513               558.92
142221.09
```

$558^{qm} = 0.056^{ha}$.

$142{,}221^{qm} = 14.222^{ha}$. *Ans.*

```
 14.222ha
  0.056
 14.166ha
```

14.166^{ha} *Ans.*

4. If my garden contains 941.65^{qm} and my neighbor's 747.37^{qm}, what is the area in hektars of both taken together?

$941.65^{qm} = 0.094165^{ha}$
$747.37^{qm} = 0.074737^{ha}$
0.168902^{ha}

0.1689^{ha}. *Ans.*

5. If a painter can cover 8.786^{qm} in an hour, how many square meters can he cover in 1.78 hours? in 3.86 hours? in 4.57 hours?

```
   8.786qm          8.786qm          8.786qm
    1.78             3.86             4.57
   70288            52716            61502
  61502            70288            43930
  8786            26358            35144
  15.63908qm       33.91396qm       40.15202qm
```

15.639^{qm}. *Ans.* 33.914^{qm}. *Ans.* 40.152^{qm}. *Ans.*

6. How many hektars in each of three rectangular fields: one measuring 315.71^{m} by 78.91^{m}; a second, 293.6^{m} by 84.84^{m}; the third, 346.8^{m} by 71.82^{m}? How many in the three together?

```
   315.71          293.6           346.8
    78.91          84.84           71.82
   31571          11744            6936
  284139         23488           27744          2.4913ha
 252568         11744           3468           2.4909
220997         23488           24276           2.4907
24912.6761     24909.024       24907.176       7.4729ha  Ans.
```

2.4913^{ha}. *Ans.* 2.4909^{ha}. *Ans.* 2.4907^{ha}. *Ans.*

7. Find the price of a rectangular field, 346.8^{m} by 71.82^{m}, at \$67.50 a hektar; at \$384 a hektar; and at \$2.375 a square meter.

From Example 6, the field contains 2.4907^{ha}.

2.4907	2.4907
67.50	384
1245350	99628
174349	199256
149442	74721
168.122250	956.4288
\$168.12. *Ans.*	\$956.43. *Ans.*

\$2.375 per square meter = \$23,750 per hektar.

\$23750
2.4907
166250
213750
95000
47500
\$59154.1250

\$59,154.13. *Ans.*

8. Find the length of a rectangle 17^{cm} wide that contains 306^{qcm}. What length of carpet 75^{cm} wide is required to make 27^{qm}?

18	36
17)306	75)2700
17	225
136	450
136	450
18^{cm}. *Ans.*	36^{m}. *Ans.*

9. A room is 16^{m} long, 8^{m} wide, and 8^{m} high; another room is 7^{m} long, 7^{m} wide, and 3^{m} high. How many square meters of painting on the walls of both rooms, if no allowance is made for doors and windows? How many more square meters of painting on the walls of the larger room than on those of the smaller?

7^{m}	14	16^{m}	24	384^{qm}	384^{qm}
7	2	8	2	84	84
14^{m}	28	24^{m}	48	468^{qm} *Ans.*	300^{qm} *Ans.*
	3		8		
	84		384		

[redacted]

10. What is the area of a circle 27^{cm} in diameter? of a circle 1^{m} in diameter?

27
27
189
54
729

0.7854
729
70686
15708
54978
572.5566

572.5566^{qcm}. *Ans.*

$1 \times 1 \times 0.7854 = 0.7854.$

0.7854^{qm}. *Ans.*

11. What is the area in hektars of a circular field 784^{m} in diameter?

784
784
3136
6272
5488
614656

614656^{qm}
0.7854
2458624
3073280
4917248
4302592
482750.8224^{qm}

$= 48.275^{ha}$. *Ans.*

12. Find the area of a circle 31^{cm} in diameter.

31
31
31
93
961

0.7854
961
7854
47124
70686
754.7694

754.7694^{qcm}. *Ans.*

13. Find the area of a circle whose radius is 24^{m}.

24
24
96
48
576

3.1416
576
188496
219912
157080
1809.5616

1809.5616^{qm}. *Ans.*

14. If a circle has a radius of 7^{cm}, how many square centimeters does it contain?

7
7
49

3.1416
49
282744
125664
153.9384

153.9384^{qcm}. *Ans.*

[redacted]

15. In a rectangular sheet of zinc 1.76^{m} long and 89^{cm} wide are two circular openings, one of which has a radius of 10.5^{cm}, the other a radius of 9.2^{cm}. What is the area of the zinc left?

```
  10.5             3.1416           9.2
  10.5             110.25           9.2
  ----             ------           ---
   525             157080           184
 105                62832          828
 ------            31416           -----
 110.25           31416            84.64
                  ----------
                  346.361400
```

$346.3614^{qcm} = 0.03464^{qm}$.

```
   3.1416          0.02659qm        1.76
   84.64           0.03464qm        0.89
  -------          ---------        ----
  125664           0.06123qm        1584
 188496                            1408
125664                             ------
251328                             1.5664
----------
265.905024                         1.5664qm
                                   1.06123
                                   ---------
                                   1.50517qm  Ans.
```

$265.905024^{qcm} = 0.02659^{qm}$.

1.50517^{qm} *Ans.*

16. A piece of land in the form of a circle has a radius of 40^{m}; in the middle of it is a pond forming a circle of 15^{m} radius. What is the total surface? the surface of the pond? the surface of the land to cultivate?

```
 3.1416        40        3.1416
    225        40          1600
 ------      ----      --------
 157080      1600      18849600
 62832                 31416
62832                  -------
------                 5026.56
706.86                  706.86
                       -------
                       4219.7
```

5026.56^{qm}; 706.86^{qm}; 4219.7^{qm}. *Ans.*

17. How deep is a well, if the wheel whose diameter is 75^{cm} makes 26 revolutions in raising the bucket?

$26 \times 3.1416 \times 75^{cm} = 6126.12^{cm} = 61.2612^{m}$. *Ans.*

```
   75            3.1416
   26              1950
  ----          -------
  450           1570800
 150           282744
 ----          31416
 1950          ---------
               6126.1200
```

[redacted]

18. How many square centimeters of surface on a ball 7^{cm} in diameter?

```
 7          3.1416
 7              49
49          282744
           125664
           153.9384
```

153.9384^{qcm}. *Ans.*

19. How many square centimeters of surface on a ball 18^{cm} in diameter?

```
 18          3.1416
 18             324
144          125664
 18           62832
324          94248
            1017.8784
```

1017.8784^{qcm}. *Ans.*

20. How many square meters of surface on a hemispherical dome 11.27^{m} in diameter?

```
  11.27          127.0129qm
  11.27            3.1416
   7889          7620774
  2254           1270129
 1127           5080516
1127            1270129
127.0129       3810387
            2)399.023726664qm
              199.511863332qm
```

199.5119^{qm}. *Ans.*

21. What is the interior surface of a hemispherical basin 12^{cm} in diameter?

```
 12        3.1416
 12           144
144        125664
          125664
          31416
       2)452.3904
         226.1952
```

226.1952^{qcm}. *Ans.*

22. What is the interior surface of a hemispherical vase 70^{cm} in diameter?

```
  70        3.1416
  70          4900
4900      28274400
          125664
      2)15393.8400
         7696.92
```

7696.92^{qcm}. *Ans.*

23. How many meters of carpet 60^{cm} wide will be required for a room 6^{m} long and 5.4^{m} wide, the strips running lengthwise? how many meters would be required if the carpet were 80^{cm} wide?

$\cancel{6}\cancel{0})\underline{54\cancel{0}}.$ Hence, 9 strips will
$\quad 9$
be required.

$9 \times 6^{m} = 54^{m}$. *Ans.*

$8\cancel{0})\underline{54\cancel{0}}.$ Hence, 7 strips would
$\quad 6.7$
be required.

$7 \times 6^{m} = 42^{m}$. *Ans.*

[redacted]

24. How many meters of carpet 56^{cm} wide will be required for a room 8.32^{m} long and 6.6^{m} wide, strips running lengthwise?

$$\begin{array}{r} 11 \\ 56\overline{)660} \\ 56 \\ \hline 100 \\ 56 \\ \hline 44 \end{array}$$

Hence, 12 strips will be required.

$$\begin{array}{r} 8.32^{m} \\ 12 \\ \hline 1664 \\ 832 \\ \hline 99.84^{m} \end{array}$$ *Ans.*

25. How many meters of carpet 70^{cm} wide will be required for a room 7^{m} long and 5.4^{m} wide, strips running across the room?

$$7\cancel{0}\overline{)70\cancel{0}} \quad 10$$

$$\begin{array}{r} 5.4^{m} \\ 10 \\ \hline 54^{m} \end{array}$$ *Ans.*

26. How many meters of carpet 80^{cm} wide will be required for a room 6^{m} long and 5.47^{m} wide, strips running across the room?

$$\begin{array}{r} 7.5 \\ 80\overline{)600} \\ 560 \\ \hline 400 \\ 400 \\ \hline \end{array}$$

Hence, 8 strips will be required.

$$\begin{array}{r} 5.47^{m} \\ 8 \\ \hline 43.76^{m} \end{array}$$ *Ans.*

27. How many meters of carpet 90^{cm} wide will be required for a room 5^{m} long and 4.5^{m} wide, strips running lengthwise? How much will it cost, at $1.875 a meter?

$$9\cancel{0}\overline{)45\cancel{0}} \quad 5$$

$$\begin{array}{r} 5^{m} \\ 5 \\ \hline 25^{m} \end{array}$$ *Ans.*

$$\begin{array}{r} \$1.875 \\ 25 \\ \hline 9375 \\ 3750 \\ \hline \$46.875 \end{array}$$

$46.88. *Ans.*

28. How many meters of carpet 75^{cm} wide will be required for a room 5.25^{m} long and 4.75^{m} wide, strips running across the room? Find the cost, at $2.125 a meter.

$$\begin{array}{r} 7 \\ 75\overline{)525} \\ 525 \\ \hline \end{array}$$

$$\begin{array}{r} 4.75^{m} \\ 7 \\ \hline 33.25^{m} \end{array}$$ *Ans.*

$$\begin{array}{r} \$2.125 \\ 33.25 \\ \hline 10625 \\ 4250 \\ 6375 \\ 6375 \\ \hline \$70.65625 \end{array}$$

$70.66. *Ans.*

29. How many meters of carpet 75^{cm} wide will be required for a room 5.6^{m} square? How wide a strip will have to be turned under? How much will the carpet cost, at $1.25 a meter?

$$\begin{array}{r} 7 \\ 75\overline{)560} \\ 525 \\ \hline 35 \end{array}$$

$$\begin{array}{r} 5.6^{m} \\ 8 \\ \hline 44.8^{m} \end{array}$$ *Ans.*

Hence, 8 strips will be required.

[redacted]

$$\begin{array}{r} 75^{cm} \\ 35 \\ \hline 40^{cm} \end{array}$$ to turn under. *Ans.*

$$\begin{array}{r} \$1.25 \\ 44.8 \\ \hline 1000 \\ 500 \\ 500 \\ \hline \$56.000 \end{array}$$

$56. *Ans.*

30. Find the area of the walls of a room whose length is 6.12^m, breadth 5.05^m, and height 3.5^m.

Perimeter $= 2 \times (6.12^m + 5.05^m)$

$= 22.34^m$.

$$\begin{array}{r} 22.34 \\ 3.5 \\ \hline 11170 \\ 6702 \\ \hline 78.19 \end{array}$$

78.19^{qm}. *Ans.*

31. How many rolls of paper 45^{cm} wide and 8^m long, allowing 11.19^{qm} for doors and windows, will be required to paper the room of Ex. 30?

$$\begin{array}{r} 78.19^{qm} \\ 11.19 \\ \hline 67^{qm} \end{array} \qquad \begin{array}{r} 0.45 \\ 8 \\ \hline 3.60 \end{array}$$

$$\begin{array}{r} 18 \\ 36\overline{)670} \\ 36 \\ \hline 310 \\ 288 \\ \hline 22 \end{array}$$

19 rolls. *Ans.*

32. Find the cost of papering a room 8^m long, 5.5^m wide, and 4.5^m high, with paper 50^{cm} wide and 7.5^m in a roll, at $1.25 a roll, put on; if there is a baseboard 25^{cm} wide running round the room, and an allowance of 11^{qm} is made for doors and windows.

$$\begin{array}{r} 8 \\ 5.5 \\ \hline 13.5 \\ 2 \\ \hline 27 \\ 4.5 \\ \hline 135 \\ 108 \\ \hline 121.5 \\ 17.75 \\ \hline 103.75 \end{array} \qquad \begin{array}{r} 27 \\ 0.25 \\ \hline 135 \\ 54 \\ \hline 6.75 \\ 11. \\ \hline 17.75 \end{array} \qquad \begin{array}{r} 7.5 \\ 0.5 \\ \hline 3.75 \end{array}$$

$$\begin{array}{r} 27 \\ 375\overline{)10375} \\ 750 \\ \hline 2875 \\ 2625 \\ \hline 250 \end{array}$$

Hence, 28 rolls will be required.

$$\begin{array}{r} \$1.25 \\ 28 \\ \hline 1000 \\ 250 \\ \hline \$35.00 \end{array}$$ *Ans.*

33. Find the cost of plastering the room of Ex. 32, at $0.50 a square meter.

$$\begin{array}{r} 5.5 \\ 8 \\ \hline 44. \end{array} \qquad \begin{array}{r} 103.75^{qm} \\ 44. \\ \hline 147.75^{qm} \end{array}$$

$147.75 \times \$0.50 = \$73.88$. *Ans.*

34. Find the cost of papering a room 5.5m long, 4.8m wide, and 3.2m high, with paper 45cm wide, 7.5m in a roll, at $0.875 a roll, put on, allowing 12m² for bases, doors, etc.

Hence, 16 rolls will be required.

5.5	7.5	3.375)53.920(15	$0.875
4.8	0.45	3375	16
10.3	375	20170	5250
2	300	16875	875
20.6	3.375	3295	$14.00 *Ans.*
3.2			
412			
618			
65.92			
12			
53.92			

35. Find the cost of plastering the room of Ex. 34, at $0.45 a square meter.

5.5	26.40m² ceiling.	80.32
4.8	53.92 walls.	0.45
440	80.32m²	40160
220		32128
26.40		36.1440 $36.14. *Ans.*

36. Find the cost of papering a room 6m square and 3.5m high, with paper 45cm wide and 7.5m in a roll, at $0.75 a roll, put on; and of putting on a border, at 5 cents per running meter.

Hence, 25 rolls will be required.

6		24		
6	7.5	3375)84000		
12	0.45	6750		
2	375	16500	$0.75	
24	300	13500	25	
3.5	3.375	3000	375	24
120			150	0.05
72			$18.75 *Ans.*	1.20
84				$1.20. *Ans.*

[redacted]

37. Find the cost of plastering the room of Ex. 36, at $0.36 a square meter.

$$\begin{array}{r} 6 \\ \underline{6} \\ 36 \\ \underline{84} \\ 120 \end{array} \qquad \begin{array}{r} \$0.36 \\ \underline{120} \\ 720 \\ \underline{36} \\ \$43.20 \end{array}$$ *Ans.*

38. Find the cost of papering a room 13^m long, 12^m wide, and 7^m high, with paper 45^{cm} wide and 7.5^m in a roll, at $1.50 a roll, put on; and of putting on a border, at $0.30 a running meter, allowing 115^{qm} for baseboard, doors, etc.

$$\begin{array}{r} 13 \\ \underline{12} \\ 25 \\ \underline{2} \\ 50 \\ \underline{7} \\ 350 \\ \underline{115} \\ 235 \end{array} \qquad \begin{array}{r} 7.5 \\ \underline{0.45} \\ 375 \\ \underline{300} \\ 3.375 \end{array} \qquad \begin{array}{r} 69 \\ 3375\overline{)235000} \\ \underline{20250} \\ 32500 \\ \underline{30375} \\ 2125 \end{array}$$

Hence, 70 rolls will be required.

$$\begin{array}{r} \$1.50 \\ \underline{70} \\ \$105.00 \end{array}$$ *Ans.*

$$\begin{array}{r} \$0.30 \\ \underline{50} \\ \$15.00 \end{array}$$ *Ans.*

39. Find the cost of plastering the room of Ex. 36, at $0.60 a square meter.

$$\begin{array}{r} 13 \\ \underline{12} \\ 156 \\ \underline{235} \\ 391 \end{array}$$

$391 \times \$0.60 = \$234.60$. *Ans.*

40. How many meters, board measure, in a board 8^m long, 20^{cm} wide, and 20^{mm} thick?

$$\begin{array}{r} 8 \\ \underline{0.2} \\ 1.6 \end{array}$$ 1.6^m. *Ans.*

41. How many meters, board measure, in a joist 5^m long, 25^{cm} wide, and 75^{mm} thick?

$$\frac{5 \times 0.25 \times \overset{3}{\cancel{75}}}{\cancel{25}} = 3.75.$$

3.75^m. *Ans.*

42. How many meters, board measure, in a stick of timber 15^m long and 40^{cm} square?

$$\frac{15 \times 0.40 \times \overset{16}{\cancel{400}}}{\cancel{25}} = 96.$$

96^m. *Ans.*

43. [illegible]

[illegible] $= [illegible]$ *Ans.*

44. [illegible]

[illegible] *Ans.*

[illegible] $\times$ $[illegible] = [illegible]$ *Ans.*

45. [illegible] at $14 a hundred meters?

[illegible]	$14
4	0.42
[illegible]	28
[illegible]	56
[illegible]	$5.88 *Ans.*

[illegible] *Ans.*

46. Find the cost of ten joists [illegible] wide, and 7.5cm thick, at $11 a hundred meters.

$$\frac{[illegible] \times 0.1 \times \overset{3}{75}}{25} = 13.5$$

13.5
0.11
[illegible]

47. Find the cost of thirty-six planks, each 4m long, 27.8cm wide, and 75mm thick, at $16 a hundred meters.

$$\frac{36 \times 4 \times 0.278 \times \overset{3}{75}}{25} = 120.096.$$

0.278	120.096
4	0.16
1.112	720576
3	120096
3.336	19.21536
36	$19.22. *Ans.*
20016	
10008	
120.096	

48. Find the cost of three sticks of timber, each 8m long, 22.5cm wide, and 20cm thick, at $17.50 a hundred meters.

$$\frac{3 \times 8 \times 0.225 \times \overset{8}{\cancel{200}}}{\cancel{25}} = 43.2.$$

$0.175
43.2
350
525
700
$7.56 *Ans.*

49. Find the cost of a board [illegible]m long, 28cm wide at one end [illegible] 35cm at the other, and 31.25mm [illegible]k, at $0.30 a meter.

[illegible] of (28cm + 35cm) = 31.5cm

[redacted]

$$\begin{array}{r} 8.25 \\ 0.315 \\ \hline 4125 \\ 825 \\ 2475 \\ \hline 2.59875 \end{array} \qquad \begin{array}{r} 2.59875 \\ 1.25 \\ \hline 1299375 \\ 519750 \\ 259875 \\ \hline 3.2484375 \end{array}$$

$$\begin{array}{r} 3.248 \\ 0.30 \\ \hline 0.97440 \end{array} \quad \$0.97. \textit{ Ans.}$$

50. Find the cost of a stick of timber 10^m long, 25^{cm} thick, 30^{cm} wide at one end and 25^{cm} wide at the other, at \$14 a hundred meters.

$\frac{1}{2}$ of $(30^{cm} + 25^{cm}) = 27.5^{cm}$.

$$\frac{10 \times 0.275 \times \overset{10}{\cancel{250}}}{\cancel{25}} = 27.5.$$

$$\begin{array}{r} 27.5 \\ 0.14 \\ \hline 1100 \\ 275 \\ \hline 3.850 \end{array} \quad \$3.85. \textit{ Ans.}$$

51. Find the cost of the floor boards, 32^{mm} thick, for a two-story building 16^m by 10.5^m at \$30 a hundred meters.

$$\frac{2 \times 16 \times \overset{2.1}{\cancel{10.5}} \times 32}{\underset{5}{\cancel{25}}} = 430.08.$$

$$\begin{array}{r} 4.3008 \\ 30 \\ \hline 129.0240 \end{array} \quad \$129.02. \textit{ Ans.}$$

52. Find the cost of the floor timbers, 25^{cm} by 50^{mm}, for the building of Ex. 51, if the timbers run lengthwise and are placed on edge 30^{cm} apart, and are worth \$11.50 a hundred meters.

$50^{mm} = 5^{cm}$.

$\therefore$ each timber with its space occupies $30^{cm} + 5^{cm} = 35^{cm}$ of space.

The width of the house is 10.5^m or 1050^{cm}.

$\therefore$ each floor requires $\frac{1050}{35} = 30$ timbers, and both floors require 60 timbers.

$$\frac{60 \times 16 \times 0.25 \times \overset{2}{\cancel{50}}}{\cancel{25}} = 480.$$

$$\begin{array}{r} \$0.1150 \\ 480 \\ \hline 92000 \\ 4600 \\ \hline \$55.2000 \end{array} \quad \$55.20. \textit{ Ans.}$$

53. Find the cost of the fencing to inclose a field 150^m long and 75^m wide; the posts are set 2.5^m apart, and cost \$0.25 apiece; the fence is 5 boards high; the bottom board is 30^{cm}, the top board 25^{cm}, and the other three each 22.5^{cm} wide, and the boards cost \$13.25 a hundred meters.

Perimeter $= 2 \times (150^m + 75^m)$
$= 450^m$.

$450 \div 2.5 = 180$, number of posts.

$180 \times \$0.25 = \$45$.

Total width of the boards

$= 30^{cm} + 25^{cm} + 3 \times 22.5^{cm}$

$= 30^{cm} + 25^{cm} + 67.5^{cm} = 122.5^{cm}$.

$$\begin{array}{r} 1.225 \\ 450 \\ \hline 61250 \\ 4900 \\ \hline 551.250 \end{array} \qquad \begin{array}{r} 551.25 \\ 0.1325 \\ \hline 275625 \\ 110250 \\ 165375 \\ 55125 \\ \hline 73.040625 \end{array} \qquad \begin{array}{r} \$73.04 \\ 45. \\ \hline \$118.04 \end{array} \textit{ Ans.}$$

Exercise 33. Page 77.

1. How many cubic centimeters in a block 9^{cm} long, 7^{cm} wide, and 6^{cm} deep?

9	63
7	6
63	378 *Ans.*

2. If wood is cut into 120^{cm} lengths, and a pile is 43.7^{m} long and 1.4^{m} high, how many steres of wood are there in the pile?

43.7	52.44
1.2	1.4
874	20976
437	5244
52.44	73.416 *Ans.*

3. How many hektoliters of grain will a bin hold, 11.2^{m} long, 4.34^{m} wide, and 2.83^{m} deep?

11.2	48.608
4.34	2.83
448	145824
336	388864
448	97216
48.608	137.56064

$137.56064^{cbm} = 1375.6064^{hl}$. *Ans.*

4. If a liter of grain weighs 0.81 of the weight of a liter of water, find the weight of the grain in the bin of Ex. 3.

1375.6064^{hl} of water weighs $137,560.64^{kg}$.

137560.64^{kg}
0.81
13756064
110048512
111424.1184^{kg} *Ans.*

5. A bin 16^{m} by 9.7^{m}, and 2.8^{m} deep, is full of oats, worth \$0.98 a hektoliter. What is the whole worth?

16	4345.6
9.7	0.98
112	347648
144	391104
155.2	4258.688
2.8	\$4258.69. *Ans.*
12416	
3104	
434.56	

$434.56^{cbm} = 4345.6^{hl}$.

6. How many liters does a vat 197^{cm} long, 87^{cm} wide, and 63^{cm} deep hold? What weight of water will be required to fill it?

197	17139
87	63
1379	51417
1576	102834
17139	1079757

$1,079,757^{ccm} = 1079.757^{l}$. *Ans.*
1079.757^{kg}. *Ans.*

7. Add 1341^{ccm}, 231^{l}, and 2.13^{hl}, and give the sum in terms of each of the three units.

$1,341^{ccm}$
231,000
213,000
$445,341^{ccm}$. *Ans.*
445.341^{l}. *Ans.*
4.45341^{hl}. *Ans.*

[redacted]

8. If a spring delivers 467.8^{l} each minute, how many hektoliters will it deliver in 60 minutes? in 37 minutes? in 78 minutes?

$$\begin{array}{r} 4.678^{hl} \\ 60 \\ \hline 280.68^{hl} \end{array} \quad \textit{Ans.}$$

$$\begin{array}{r} 4.678^{hl} \\ 37 \\ \hline 32746 \\ 14034 \\ \hline 173.086^{hl} \end{array} \quad \textit{Ans.}$$

$$\begin{array}{r} 4.678^{hl} \\ 78 \\ \hline 37424 \\ 32746 \\ \hline 364.884^{hl} \end{array} \quad \textit{Ans.}$$

9. If 67.3^{l} of oil in a vat with perpendicular sides fills it to a depth of 173^{mm}, how deep will 13.7 times that quantity fill it? How many hektoliters will there be?

$$\begin{array}{r} 173^{mm} \\ 13.7 \\ \hline 1211 \\ 519 \\ 173 \\ \hline 2370.1^{mm} \end{array} \qquad \begin{array}{r} 0.673^{hl} \\ 13.7 \\ \hline 4711 \\ 2019 \\ 673 \\ \hline 9.2201^{hl} \end{array} \quad \textit{Ans.}$$

$=2.3701^{m}$. *Ans.*

10. One cask contains 171.4^{l} of oil; another, 209.3^{l}; a third, 73.8^{l}; while a square vat, 137^{cm} each way, is filled to a depth of 69^{cm}. Find in liters and in hektoliters the amount of oil in the four vessels together.

$$\begin{array}{r} 137 \\ 137 \\ \hline 959 \\ 411 \\ 137 \\ \hline 18769 \end{array} \qquad \begin{array}{r} 18769 \\ 69 \\ \hline 168921 \\ 112614 \\ \hline 1295061 \end{array}$$

$1{,}295{,}061^{ccm} = 1295.061^{l}$.

$$\begin{array}{r} 171.4^{l} \\ 209.3 \\ 73.8 \\ 1295.061 \\ \hline 1749.561^{l} \end{array} \quad \textit{Ans.}$$

17.49561^{hl}. *Ans.*

11. How many liters of air in a room 7.8^{m} long, 6.23^{m} wide, and 3^{m} high?

$$\begin{array}{r} 6.23 \\ 7.8 \\ \hline 4984 \\ 4361 \\ \hline 48.594 \\ 3 \\ \hline 145.782 \end{array}$$

$145.782^{cbm} = 145{,}782^{l}$. *Ans.*

12. If a person's breathing spoils the air at the rate of 0.2175^{cbm} a minute, how long will it take three persons sitting in the closed room of Ex. 11 to spoil the air?

$$\begin{array}{r} 0.2175^{cbm} \\ 3 \\ \hline 0.6525^{cbm} \end{array}$$

$$\begin{array}{r|l} & 223.42 \\ \hline 6525 & 1457820. \\ & 13050 \\ \hline & 15282 \\ & 13050 \\ \hline & 22320 \\ & 19575 \\ \hline & 27450 \\ & 26100 \\ \hline & 13500 \\ & 13050 \\ \hline & 450 \end{array}$$

223.42 minutes. *Ans.*

13. How long, at the same rate as in Ex. 11, will the air in a hall 22^{m} long, 16^{m} wide, and 7^{m} high last 280 persons?

```
  22        0.2175cbm
  16             280
 132        174000
 22         4350
 352        60.9000cbm
   7
2464
```

```
          40.4
609)24640.
      2436
       2800   40.5 minutes.
       2436            Ans.
        364
```

14. How many cubic centimeters in a ball 10^{ccm} in diameter?

$0.5236 \times (10^{3})^{ccm} = 523.6^{ccm}$. *Ans.*

15. Into a cubical box 20^{cm} on an edge, and full of water, an iron ball 20^{cm} in diameter is gently lowered until it touches the bottom. Find in liters and in cubic centimeters the volume of the water left in the box.

```
 20      0.5236
 20        8000
400      4188.8
 20
8000     8000ccm
         4188.8
         3811.2ccm. Ans.
         3.8112l.   Ans.
```

16. If cast iron weighs 7.207 times as much as water, what is the weight of a cast iron ball 5^{cm} in diameter?

$0.5236 \times (5^{3})^{ccm} = 0.5236 \times 125^{ccm}$.

```
 0.5236        65.45g
    125        7.207
  26180        45815
 10472        13090
 5236        45815
 65.4500     471.69815g Ans.
```

17. A rubber ball is 6.2^{cm} in diameter. What is the amount of rubber in the ball?

```
   6.2           238.328ccm
   6.2             0.5236
   124           1429968
  372           714984
 38.44         476656
   6.2        1191640
  7688        124.7885408ccm
23064                   Ans.
238.328
```

18. If the circumference of a cannon ball is 52^{cm}, find the volume of the ball.

```
0.31831        16.55212
     52        16.55212
  63662         3310424
 59155         1655212
16.55212      3310424
             8276060
            8276060
           9931272
          1655212
          273.9726764944
```

[redacted]


```
    273.97268
     16.55212
     54794536
    27397268
   54794536
  136986340
 136986340
164383608
27397268
4534.8286760816
```

```
    4534.82868
      0.5236
   2720897208
  1360448604
  906965736
 2267414340
2374.436296848
```

2374.436^{ccm}. *Ans.*

19. How many cubic centimeters of oil are there in a cylindrical cup 10^{cm} across when the oil is 38^{mm} deep?

$38^{mm} = 3.8^{cm}$.

```
  78.54
   3.8
  62832
 23562
208.452
```

298.452^{ccm}. *Ans.*

20. What is the capacity of a cylindrical cup 95^{mm} across and 11.08^{cm} deep?

$95^{mm} = 9.5^{cm}$.

```
  9.5
  9.5
  475
 855
90.25
```

```
    0.7854
     90.25
     39270
    15708
  70686
  70.88235
     11.08
   56705880
  7088235
 7088235
785.3764380
```

$785.3^{ccm} = 0.785^{l}$. *Ans.*

21. What is the capacity of a cylindrical vessel 16.24^{cm} across and 19.95^{cm} deep? 75.4^{mm} across and 87.9^{mm} deep?

```
      16.24
      16.24
       6496
      3248
     9744
    1624
    263.7376
      0.7854
    10549504
   13186880
  21099008
 18461632
 207.1395
   19.95
 10356975
18642555
18642555
2071395
4132.433025
```

$4132.433025^{ccm} = 4.132^{l}$. *Ans.*

```
      75.4
      75.4
      3016
     3770
    5278
    5685.16
     0.7854
    2274064
   2842580
  4548128
 3979612
 4465.124664
 4465.125
     87.9
 40186125
31255875
35721000
392484.4875
```

$392,484.4875^{ccm} = 0.392^{l}$. *Ans.*

[redacted]

22. How many cubic meters of wood in a round stick of equal size throughout, 37^{cm} in diameter, and 8.4^{m} long?

37	0.7854	0.10752126
37	1369	8.4
259	70686	43008504
111	47124	86017008
1369	23562	0.903178584
	7854	0.9032^{cbm}. *Ans.*
	1075.2126	

1075.2126^{qcm} = 0.10752126^{qm}.

23. A cylindrical stand-pipe whose diameter is 12^{m} and whose height is 22^{m} is filled with water. Find the weight of the water.

$0.7854 \times (12 \times 12 \times 22)^{cbm}$
$= 2488.1472^{cbm}$.

12	0.7854
12	3168
144	62832
22	47124
288	7854
288	23562
3168	2488.1472

2488.1472^{cbm} of water weighs 2488.1472^{t}. *Ans.*

24. Find the number of liters of water in a well, if its diameter is 1.2^{m} and the depth of the water is 2^{m}.

$0.7854 \times (1.2 \times 1.2 \times 2)^{cbm}$
$= 2.261952^{cbm}$
$= 2261.952^{l}$. *Ans.*

1.2	0.7854
1.2	2.88
1.44	62832
2	62832
2.88	15708
	2.261952

25. A cylindrical cup 90^{mm} in diameter is partly filled with water. Into the cup is dropped a piece of iron, and the water rises 63^{mm}. What is the volume of the piece of iron?

$0.7854 \times (90 \times 90 \times 63)^{qmm}$
$= 400{,}789.62^{cmm}$
$= 400.78962^{ccm}$. *Ans.*

90	0.7854
90	510300
8100	2356200
63	7854
24300	39270
486	400789.6200
510300	

Exercise 34. Page 79.

1. What is the weight, in kilograms, of a hektoliter of water? of 73.8^{l} of water? of a cubic meter of water? of a cubic centimeter of water?

1^{hl} of water weighs 100^{kg}. *Ans.*

73.8^{l} of water weighs 73.8^{kg}. *Ans.*

1^{cbm} of water weighs 1000^{kg}. *Ans.*

1^{ccm} of water weighs 0.001^{kg}. *Ans.*

2. If a man buys half a ton of potatoes for \$20, and retails them all, without waste, at 5 cents a kilogram, what profit does he make on the whole?

$$\begin{array}{r} \$0.05 \\ 500 \\ \hline \$25.00 \\ 20. \\ \hline \$5 \text{ } \textit{Ans.} \end{array}$$

3. What is the weight of water required to fill a vat 98^{cm} long, 71^{cm} wide, and 38^{cm} deep?

$$\begin{array}{r} 98 \\ 71 \\ \hline 98 \\ 686 \\ \hline 6958 \end{array} \qquad \begin{array}{r} 6958 \\ 38 \\ \hline 55664 \\ 20874 \\ \hline 264.404 \end{array}$$

264.404^{kg}. *Ans.*

4. If the vat of the last example is filled with brine weighing 1.04^{kg} to the liter, what is the weight of the brine?

$$\begin{array}{r} 264.404^{kg} \\ 1.04 \\ \hline 1057616 \\ 264404 \\ \hline 274.98^{kg} \end{array}$$

Ans.

5. If the vat of Ex. 3 is filled with wine weighing 0.981^{kg} to the liter, what is the weight of the wine?

$$\begin{array}{r} 264.404^{kg} \\ 0.981 \\ \hline 264404 \\ 2115232 \\ 2379636 \\ \hline 259.38^{kg} \end{array}$$

Ans.

6. What is the total weight of 13 men averaging 73.48^{kg} each?

$$\begin{array}{r} 73.48^{kg} \\ 13 \\ \hline 22044 \\ 7348 \\ \hline 955.24^{kg} \end{array}$$

Ans.

7. How many kilograms, and how many tons, will 3.6175^{cbm} of brick weigh, at 2 tons to a cubic meter? at 2.34 tons?

3.6175×2^{t}
$= 7.235^{t}$
$= 7235^{kg}$.

3.6175×2.34^{t}
$= 8.46495^{t}$
$= 8464.95^{kg}$.

$$\begin{array}{r} 3.6175 \\ 2.34 \\ \hline 144700 \\ 108525 \\ 72350 \\ \hline 8.46495 \end{array}$$

[redacted]

8. From a barrel containing 67^{kg} of granulated sugar there are taken three parcels of 2.75^{kg} each, and four parcels of 7.50^{kg} each. How much is left in the barrel?

$$\begin{array}{r} 2.75^{kg} \\ 3 \\ \hline 8.25^{kg} \end{array} \qquad \begin{array}{r} 7.5^{kg} \\ 4 \\ \hline 30.0^{kg} \end{array}$$

$$\begin{array}{r} 30.^{kg} \\ 8.25 \\ \hline 38.25^{kg} \end{array}$$

$$\begin{array}{r} 67.^{kg} \\ 38.25 \\ \hline 28.75^{kg} \end{array} \textit{ Ans.}$$

9. Into how many pills of 325^{mg} each can a mass of 7.8^{g} be divided?

$$\begin{array}{r} 24 \\ 325\overline{)7800} \\ 650 \\ \hline 1300 \\ 1300 \\ \hline \end{array} \textit{ Ans.}$$

10. A mass of 21.8^{g} is divided into 60 pills. What is the weight of each pill?

$$6\cancel{0})\overline{2180.^{mg}}$$
$$363.333^{mg} \textit{ Ans.}$$

11. A bag, when empty, weighs 213^{g}; when full of silver five-franc pieces, $20^{kg}\ 5^{hg}\ 13^{g}$. A five-franc piece weighs 25^{g}. How many five-franc pieces will the bag hold?

$$20^{kg}\ 5^{hg}\ 13^{g} = 20{,}513^{g}.$$
$$20{,}513^{g} - 213^{g} = 20{,}300^{g}.$$

$$\begin{array}{r} 812 \\ 25\overline{)20300} \\ 200 \\ \hline 30 \\ 25 \\ \hline 50 \\ 50 \\ \hline \end{array} \textit{ Ans.}$$

12. A vessel, when empty, weighs 2.7^{kg}; and when full of water 4235^{dkg}. What would it weigh if filled with milk which is 1.03 times as heavy as water?

$$4235^{dkg} = 42.35^{kg}.$$

$$\begin{array}{r} 42.35^{kg} \\ 2.7 \\ \hline 39.65^{kg} \end{array}\text{, weight of water.}$$

$$\begin{array}{r} 39.65 \\ 1.03 \\ \hline 11895 \\ 3965 \\ \hline 40.8395^{kg} \end{array}\text{, weight of milk.}$$

$$\begin{array}{r} 40.8395 \\ 2.7 \\ \hline 43.5395^{kg} \end{array} \textit{ Ans.}$$

Exercise 35. Page 81.

1. If a stone weighs 1.3^{kg} in air and 0.68^{kg} in water, and the stone and a block of wood together weigh 1.55^{kg} in air and 0.63^{kg} in water, what is the specific gravity of the block of wood?

$1.55^{kg} - 1.3^{kg} = 0.25^{kg}$, the weight of the wood in the air.

$1.55^{kg} - 0.63^{kg} = 0.92^{kg}$, the weight of the water displaced by the stone and the wood.

$1.3^{kg} - 0.68^{kg} = 0.62^{kg}$, the weight of the water displaced by the stone alone.

Therefore, $0.92^{kg} - 0.62^{kg} = 0.3^{kg}$, the weight of the water displaced by the wood.

$0.25 \div 0.3 = 0.833$, the specific gravity of the wood.

2. What is the weight of 8.17^{hl} of alcohol, specific gravity 0.83 ?

$$
\begin{array}{r}
817^{kg} \\
0.83 \\
\hline
2451 \\
6536 \\
\hline
678.11^{kg}\ \textit{Ans.}
\end{array}
$$

3. What will 97^{l} of alcohol weigh, of specific gravity 0.817 ? of specific gravity 0.819 ? of specific gravity 0.823 ? 0.838 ? 0.847 ?

0.817^{kg}	0.819^{kg}	0.823^{kg}	0.838^{kg}	0.847^{kg}
97	97	97	97	97
5719	5733	5761	5866	5929
7353	7371	7407	7542	7623
79.249^{kg}	79.443^{kg}	79.831^{kg}	81.286^{kg}	82.159^{kg}

4. A bar of aluminum 113^{mm} long, 17^{mm} wide, and 13^{mm} thick, is said to be of specific gravity 2.57. What does it weigh ? If it really is of specific gravity 2.67, what does it weigh ?

113	1921	24.973^{g}	24.973^{g}
17	13	2.57	2.67
791	5763	174811	174811
113	1921	124865	149838
1921	24973	49946	49946
	$24,973^{cmm} = 24.973^{ccm}$.	64.18^{g} *Ans.*	66.67791^{g}
			66.68^{g}. *Ans.*

5. What would be the specific gravity of the aluminum in Ex. 4 if the bar weighed 65.137^{g}?

```
           2.608 Ans.
24973)65137.
      49946
      151910
      149838
        207200
        199784
```

6. What is the weight of a bar of aluminum 371^{mm} by 63^{mm} by 84^{mm}, specific gravity being 2.63?

```
   371
    63
  1113
 2226
 23373
    84
 93492
186984
1963332
```

```
   1.963332kg
       2.63
    5889996
  11779992
  3926664
  5.16356316kg
  5.1636kg. Ans.
```

7. An irregular mass of copper, gently lowered into a pail brimful of water, caused 1.374^{l} to run over. What did it weigh if of specific gravity 8.91? if 8.89?

```
  1.374kg
  8.91
  1374
 12366
10992
12.242kg Ans.
```

```
  1.374kg
   8.89
  12366
 10992
10992
12.21486kg
12.215kg. Ans.
```

8. What would be the specific gravity of the copper in Ex. 7 if the mass weighed 12.3016^{kg}?

```
          8.953 Ans.
1374)12301.6
     10992
      13096
      12366
        7300
        6870
         4300
         4122
```

9. A plate of iron 137^{cm} long, 64.3^{cm} wide, and 4.31^{cm} thick weighs 277.54^{kg}. What is its specific gravity? What would the same mass weigh at specific gravity 7.47? at 7.79?

```
 137
 64.3
 411
548
822
8809.1
```

```
   8809.1
     4.31
   88091
 264273
352364
37967.221
```

$37,967.221^{ccm} = 37.97^{l}$.

```
          7.309 Ans.
3797)27754.
     26579
      11750
      11391
        35900
        34173
```

```
37.967221kg
      7.47
 265770547
151868884
265770547
283.615kg Ans.
```

```
 37.967221kg
      7.79
 ---------
 341704989
265770547
265770547
-----------
295.76465159kg
```

295.765kg. *Ans.*

10. What is the specific gravity of sea water when a hektoliter weighs 102.58kg? when 3l weighs 3077g?

```
100)102.58
    1.0258 Ans.
  3)3.077
    1.0257 Ans.
```

11. What is the specific gravity of a substance of which 7.3ccm weighs 31.5g?

```
     4.315 Ans.
73)315.
   292
    230
    219
     110
      73
     370
     365
```

12. If a cubic meter of sand weighs 1723kg, what is its specific gravity? If 3.4cbm of gravel weighs 134 tons, what is its specific gravity?

```
1000)1723.
     1.723 Ans.
```

```
   2.098 Ans.
34)71.34
   68
    334
    306
     280
     272
```

13. If a cubic centimeter of metal weighs 7.3g, what is its specific gravity?

7.3. *Ans.*

14. What is the specific gravity of a fluid weighing 2.317kg to a liter?

2.317. *Ans.*

15. If a body weighs 3.71kg in air and 2.38kg in water, what is its specific gravity?

```
3.71kg          2.789 Ans.
2.38       133)371.
1.33kg          266
                1050
                 931
                 1190
                 1064
                  1260
                  1197
```

16. A piece of ore weighing 3.77kg weighs in water only 2.53kg. What is its specific gravity?

```
3.77kg          3.04 Ans.
2.53       124)377.
1.24kg          372
                  500
                  496
```

[redacted]

17. How many cubic centimeters in a stone which loses 17.8^{g} of its weight when weighed in water? What is its specific gravity if it weighs 33.7^{g} in air?

17.8^{ccm}. *Ans.*

```
        1.893  Ans.
178)337.
    178
    1590
    1424
     1660
     1602
       580
       534
```

18. In a wrought-iron bottle I find 2.63^{l} of quicksilver, weighing 35.81^{kg}; in another 2.59^{l}, weighing 35.193^{kg}; in a third, 2.617^{l}, weighing 35.571^{kg}. What is the specific gravity of each? What would be the specific gravity of the mixture if the three were emptied into one vessel?

```
        13.616  Ans.
263)3581.
    263
    951
    789
    1620
    1578
      420
      263
      1570
```

```
        13.588  Ans.
259)3519.3
    259
     929
     777
     1523
     1295
      2280
      2072
       2080
       2072
```

```
          13.592  Ans.
2617)35571.
     2617
      9401
      7851
      15500
      13085
       24150
       23553
         5970
         5234
```

2.63^{l}	35.81^{kg}
2.59	35.193
2.617	35.571
7.837^{l}	106.574^{kg}

$106.574^{kg} \div 7.837^{kg} = 13.599$. *Ans.*

19. A plate of iron 89^{cm} by 17^{cm} by 7^{cm} weighs 79.43^{kg}. What is its specific gravity?

```
  89
  17
 623
 89
1513
   7
10591
```

```
            7.5  Ans.
10591)79430.
      74137
       52930
```

[redacted]

20. What is the specific gravity of a rectangular block of wood 1.6^{m} long, 0.3^{m} wide, and 0.15^{m} thick, if, floating in water on its face 0.3^{m} wide, it sinks to a depth of 0.12^{m}?

Volume of the block is $(1.6 \times 0.3 \times 0.15)^{cbm}$.

Volume of the water displaced is $(1.6 \times 0.3 \times 0.12)^{cbm}$.

Weight of the water displaced is $(1.6 \times 0.3 \times 0.12)^{t}$.

Weight of the block is $(1.6 \times 0.3 \times 0.12)^{t}$.

Therefore, the specific gravity of the wood

$$= \frac{\cancel{1.6} \times \cancel{0.3} \times \overset{4}{\cancel{0.12}}}{\cancel{1.6} \times \cancel{0.3} \times \underset{5}{\cancel{0.15}}} = \frac{4}{5} = 0.8. \quad \textit{Ans.}$$

Exercise 36. Page 83.

1. If 3 men eat 8^{kg} of bread a week, how much will 1 man eat at the same rate? How much will 7 men? How much will 3 men eat in 1 day? How much will 1 man eat in 1 day? How much will 7 men eat in 1 day? in 1 week? in 5 weeks?

$$3\overline{)8.00^{kg}} \quad 2.67^{kg}$$

$$2.67^{kg} \times 7 = 18.67^{kg}$$

$$7\overline{)8.00^{kg}} \quad 1.14^{kg}$$

$$3\overline{)1.14^{kg}} \quad 0.38^{kg}$$

$$0.38^{kg} \times 7 = 2.67^{kg}$$

$$2.67^{kg} \times 7 = 18.67^{kg}$$

$$18.67^{kg} \times 5 = 93.33^{kg}$$

2. At the same rate as in Ex. 1, how much will 17 men eat in 3 weeks and 4 days?

3 weeks 4 days = 25 days.

$$\begin{array}{r} 25 \\ \underline{17} \\ 175 \\ \underline{25} \\ 425 \\ \underline{0.38} \\ 3400 \\ \underline{1275} \\ 161.50 \end{array}$$

161.5^{kg}. *Ans.*

3. If 1^{hl} of oats is enough for 5 horses 1 week, how much is enough for 1 horse 1 week? for 1 horse 7 weeks? for 11 horses 17 weeks?

$$5\overline{)1.0^{hl}} \quad 0.2^{hl} = 20^{l} \quad \textit{Ans.}$$

$$20^{l} \times 7 = 140^{l} \quad \textit{Ans.}$$

$$\begin{array}{r} 17 \\ \underline{11} \\ 17 \\ \underline{17} \\ 187 \end{array}$$

$187 \times 20^{l} = 3740^{l}$. *Ans.*

4. If 2^{hl} of grain is enough for 3 horses 5 days, how much is enough for 3 horses 1 day? for 1 horse 1 day? for 7 horses 6 days?

```
  3
  5
3)15
  5
```

$5)\underline{2.0^{hl}}$
$0.4^{hl} = 40^{l}$ *Ans.*

$3)\underline{40.^{l}}$
13.33^{l} *Ans.*

```
   13.33
      42
   -----
    2666
   5332
  ------
  559.86
```

$559.86^{l} = 560^{l}$ *Ans.*

5. Mix 17^{l} of vinegar, costing 6 cents a liter, with 39^{l} at 5 cents, 21^{l} at 7 cents, and 13^{l} of water costing nothing. Find the number of liters, and the cost.

```
  17      39      21
0.06    0.05    0.07
----    ----    ----
1.02    1.95    1.47
```

```
17^l        $1.02
39           1.95
21           1.47
13          -----
--          $4.44 Ans.
90^l Ans.
```

6. For how much a liter must I sell the mixture of Ex. 5 to gain 96 cents? to gain $1.41?

```
$4.44         $0.06
 0.96      90)$5.40
-----          540
$5.40          ---
```

```
$4.44         $0.065
 1.41      90)$5.85
-----          540
$5.85          ---
               450
               450
               ---
```

7. A grocer sold 421 kegs of butter for $4995.25; 56 kegs brought $12.50 a keg, 91 brought $11.75 a keg, and 100 kegs brought $12.25 a keg. For how much a keg were the other kegs sold?

```
 $12.50          $11.75
     56              91
 ------          ------
   7500            1175
  6250            10575
 -------         --------
$700.00          $1069.25
                   700.00
 $12.25           1225.00
    100          --------
--------         $2994.25
$1225.00
```

```
$4995.25
 2994.25
--------
$2001.00
```

```
 56          421
 91          247
100          ---
---          174
247
```

```
         $11.50
   174)$2001.
        174
        ---
         261
         174
         ---
          870
          870
          ---
```

8. If 3 tons of coal cost $15.75, how many tons will $36.75 buy?

```
3)$15.75              7 Ans.
  $5.25        525)3675
                   3675
                   ----
```

[redacted]

9. If 5^m of cloth cost \$18.75, what will 7^m cost?

$$\begin{array}{r} 5)\$18.75 \\ \hline \$3.75 \\ 7 \\ \hline \$26.25 \end{array}$$

\$26.25 *Ans.*

10. If a tap running 3.5^l a minute fills a tub in 16 minutes, how long will a tap delivering 5^l a minute be in filling the same tub?

$$\begin{array}{r} 3.5 \\ 16 \\ \hline 210 \\ 35 \\ \hline 5)56.0 \\ \hline 11.2 \end{array}$$

11.2 minutes. *Ans.*

11. If both taps of the last example are opened at once, how soon will they fill the tub?

$$\begin{array}{r} 3.5 \\ 5. \\ \hline 8.5 \end{array} \qquad \begin{array}{r} 6.6 \\ 85\overline{)560.} \\ 510 \\ \hline 500 \end{array}$$

6.6 minutes. *Ans.*

12. If 3 men can dig 378^m of ditch in 2 days, how long will it take 5 men, at the same rate, to dig 787^m?

$$\begin{array}{r|l} 2 & 378^m \\ \hline 3 & 189^m \\ \hline & 63^m \\ & 5 \\ \hline & 315^m \end{array} \qquad \begin{array}{r} 2.5 \\ 315\overline{)787.} \\ 630 \\ \hline 1570 \end{array}$$

2.5 days. *Ans.*

13. Into a tub that will hold 48^l, one tap is delivering water at the rate of 3.7^l a minute; while out of it, by another tap, the water is running at 2.5^l a minute. How long will it take to fill the tub, beginning with it empty?

$$\begin{array}{r} 3.7^l \\ 2.5 \\ \hline 1.2^l \end{array} \qquad \begin{array}{r} 12)480 \\ \hline 40 \end{array}$$

40 minutes. *Ans.*

14. A tap discharges into a tub 4.2^l a minute; from the tub water is also running, by a second tap; the water in the tub gains 30^l in 18 minutes. How fast is the second tap discharging?

$$\begin{array}{r} 4.2^l \\ 18 \\ \hline 336 \\ 42 \\ \hline 75.6^l \\ 30. \\ \hline 45.6^l \end{array} \qquad \begin{array}{r} 2.5^l \\ 18\overline{)45.6^l} \\ 36 \\ \hline 96 \\ 90 \\ \hline \end{array}$$

2.5^l a minute. *Ans.*

15. If a wheel is 1.2^m across, how many times will it turn in going one kilometer?

$$\begin{array}{r} 3.1416 \\ 1.2 \\ \hline 62832 \\ 31416 \\ \hline 3.76992 \end{array} \qquad \begin{array}{r} 265 \\ 377\overline{)100000} \\ 754 \\ \hline 2460 \\ 2262 \\ \hline 1980 \\ 1885 \\ \hline \end{array}$$

$3.76992^m = 0.00377^{km}$.

265. *Ans.*

16. How many times in a minute does the wheel of the last example turn, when the carriage is driven at the rate of 14^{km} an hour?

$$14 \div 60 = 0.23.$$

```
 265
0.23
----
 795
530
-----
60.95
```

61 times. *Ans.*

17. What is the weight of the water in a tank if it takes 1 hour and 38 minutes, at the rate of 8.7^{l} a minute, to empty the tank?

```
60        98
38       8.7
--       ---
98       686
        784
        -----
        852.6
```

852.6^{kg}. *Ans.*

18. If we replace the water of Ex. 17 with oil worth \$18.75 a hektoliter, what will the contents of the tank be worth?

8.526^{hl} of water are required to weigh 852.6^{kg}.

```
       8.526
       18.75
       -----
       42630
      59682
     68208
     8526
   ---------
   159.86250
```

\$159.86. *Ans.*

Exercise 37. Page 85.

1. A train leaves Paris at 11 o'clock A.M., and reaches Lyons at 10 o'clock P.M. How many meters does it travel in an hour, the distance from Paris to Lyons being 512.7^{km}?

There are 11 hours between 11 A.M. and 10 P.M.

$$512.7^{km} \div 11 = 46.609^{km} = 46{,}609^{m}. \text{ Ans.}$$

2. A railroad has a single track 11.450^{km} long. How many rails 4.569^{m} in length did it require to lay the track?

There are two lines of rails. Therefore the length of the rails is

$$2 \times 11.450^{km} = 22.900^{km} = 22{,}900^{m}.$$

```
         5012
     --------
4569)22900000
     22845
     -----
        5500
        4569
        ----
         9310
         9138
```

The number of rails required was 5013. *Ans.*

[redacted]

3. A book is 2.1^{cm} in thickness; each leaf is 0.05^{mm} thick. Find the number of pages in the book.

The number of leaves is $21 \div 0.05 = 420$.

The number of pages is $2 \times 420 = 840$. *Ans.*

4. The cost of opening a canal amounts to $25,400 a kilometer. How much will a canal cost which is 113.253^{km} in length?

$$\begin{array}{r} 113.253 \\ 25400 \\ \hline 45301200 \\ 566265 \\ 226506 \\ \hline 2876626.200 \end{array}$$

$2,876,626.20. *Ans.*

5. The expense of laying out a paved road is $12,500 a kilometer. How much will a road cost which is 72.053^{km} long?

$$\begin{array}{r} 72.053 \\ 12500 \\ \hline 36026500 \\ 144106 \\ 72053 \\ \hline 900662.50 \end{array}$$

$900,662.50. *Ans.*

6. The cost of building a railroad is about $78,000 a kilometer in France, and only $25,000 in the United States. How much will it cost in each country to make a road 295.671^{km} long?

$$\begin{array}{r} 295.671 \\ 78000 \\ \hline 2365368000 \\ 2069697 \\ \hline 23062338.000 \end{array}$$

$$\begin{array}{r} 4)\underline{29567100} \\ 7391775 \end{array}$$

$23,062,338, France;
$7,391,775, U.S. *Ans.*

[redacted]

7. If you must go up 211 steps to reach the top of a tower, and each step is 195^{mm} high, what is the height of the tower?

$195^{mm} = 0.195^{m}$.

$$\begin{array}{r} 0.195^{m} \\ \underline{211} \\ 195 \\ 195 \\ \underline{390} \\ 41.145^{m} \end{array}\ \textit{Ans.}$$

8. A house has 5 stories, each story has 19 stairs, each stair is 16^{cm} in height. Find the height of the floor of the fifth story from the ground.

$16^{cm} = 0.16^{m}$.

$$\begin{array}{r} 0.16^{m} \\ \underline{19} \\ 144 \\ \underline{16} \\ 3.04^{m} \\ \underline{4} \\ 12.16^{m} \end{array}\ \textit{Ans.}$$

9. A ream of paper contains 20 quires, each quire has 24 sheets; the ream is 13.5^{cm} in thickness. Find the thickness of each sheet.

In one ream there are 20×24 sheets $= 480$ sheets. If 480 sheets are 13.5^{cm} thick, the thickness of one sheet $= 13.5^{cm} \div 480 = 0.028^{cm}$. *Ans.*

$$\begin{array}{r} 0.028 \\ 48\overline{)1.35} \\ \underline{96} \\ 390 \\ \underline{384} \end{array}$$

10. The equator on a terrestrial globe measures 0.80^{m} in circumference. By the aid of a tape measure we find that the distance between two cities on this globe is 0.046^{m}. What is really the distance in kilometers between the two cities? (The earth's equator is $40{,}075.45^{km}$.)

The ratio of the distance on the globe between the two cities to the equator is $0.046^{m} \div 0.80^{m} = 0.0575$. Therefore the actual distance between the two cities is $0.0575 \times 40{,}075.45^{km} = 2304.338^{km}$. *Ans.*

$$\begin{array}{r} 8\underline{)0.46} \\ 0.0575 \end{array} \qquad \begin{array}{r} 40075.45^{km} \\ \underline{0.0575} \\ 20037725 \\ 28052815 \\ \underline{20037725} \\ 2304.338375^{km} \end{array}$$

[redacted]

11. Upon a military map we find that the distance from Paris to St. Denis is 78^{mm}. What is the distance in kilometers from Paris to St. Denis? The map is made on the scale of 1 to 80,000; that is, 1^m on the map represents $80,000^m$ of actual measurement upon the ground.

The actual distance is 80,000 times the distance on the map; that is, $80,000 \times 78^{mm} = 6,240,000^{mm} = 6.24^{km}$. *Ans.*

12. Find the number of revolutions made by the wheels of a carriage in traveling 82^{km}. The wheels are 1354^{mm} in diameter.

$82^{km} = 82,000,000^{mm}$.

The circumference of the wheels is $3.1416 \times 1354^{mm} = 4253.7264^{mm}$. The number of revolutions is the total distance divided by the circumference of the wheel, or $82,000,000^{mm} \div 4253.7264^{mm} = 19,277$. *Ans.*

```
     3.1416
       1354
    -------
     125664
    157080
    94248
   31416
  ---------
  4253.7264
```

```
                  19277
  42537264)820000000000
           42537264
           ---------
           394627360
           382835376
           ----------
            117919840
             85074528
            ----------
             328453120
             297760848
             ----------
              306922720
              297760848
              ---------
```

13. How many hektars in a square kilometer? how many ars? how many square meters?

$$1^{qkm} = 100^{ha},$$
$$= 10,000^{a},$$
$$= 1,000,000^{qm}.$$

14. France has about $542,000^{qkm}$. How many hektars does it measure?

$$542,000^{qkm} = 542,000 \times 100^{ha}$$
$$= 54,200,000^{ha}. \textit{ Ans.}$$

[redacted]

15. A piece of land 1224.5^{m} square is sold at \$ 140 a hektar. How much does the land bring?

1224.5
1224.5
61225
48980
24490
24490
12245
1499400.25

149.94
140
599760
14994
20991.60

\$ 20,991.60. *Ans.*

$1,499,400.25^{qm} = 149.94^{ha}$.

16. The total surface measurement of the glass in the windows of a house is 182^{qm}. How many panes of 53^{cm} by 48^{cm} will it take to supply the windows?

$182^{qm} = 1,820,000^{qcm}$.

53
48
424
212
2544

715.4
2544)1820000.
17808
3920
2544
13760
12720
10400
10176

716 panes. *Ans.*

17. How many square slabs of marble 150^{qcm} on the surface will it require to pave a court whose area is 25.35^{qm}?

$25.35^{qm} = 253,500^{qcm}$.

The number of slabs required is $253,500^{qcm} \div 150^{qcm} = 1690$. *Ans.*

1690
15)25350
15
103
90
135
135
0

18. A speculator bought 31.0728^{ha} of land for \$1296 a hektar. For how much a square meter must he sell it to realize a profit of \$1937 ?

```
   31.0728
      1296
  --------
   1864368
  2796552
  621456
 310728
 ---------
 40270.3488

$40,270.35  cost.
  1,937.    profit.
-----------
$42,207.35  selling price.
```

```
               $0.136  Ans.
310728)$42207.35
        310728
        -------
        1113455
         932184
        -------
        1812710
```

19. A man is offered \$6000 for 2.5^a of land. He declines to sell; and soon after, the town gives him \$25.20 a square meter. How much did he make by refusing the first offer ?

$2.5^a = 250^{qm}$.

```
 $25.20
    250
 ------
 126000
 5040
 -------
$6300.00
 6000.
 -------
$300.  Ans.
```

20. A man surveys a piece of land and finds that it measures 14.0715^{ha}. He afterwards discovers that his chain was too short by 0.03^m. How can he calculate the real superficial measurement of the land without surveying it again ? (A surveyor's chain is 10^m long.)

$10.00 - 0.03 = 9.97.$

$9.97 \div 10 = 0.997.$

```
  0.997
  0.997
 ------
   6979
  8973
 8973
--------
0.994009
```

```
    14.0715ha
    0.994009
   ---------
    1266435
   562860
  1266435
 1266435
 ----------
 13.987ha  Ans.
```

[redacted]

21. A pile of wood is 4.25^{m} long, 1.33^{m} thick, and 2.60^{m} high. How many sters are there in it?

$$\begin{array}{r} 4.25 \\ \underline{1.33} \\ 1275 \\ 1275 \\ \underline{425} \\ 5.6525 \end{array} \qquad \begin{array}{r} 5.6525 \\ \underline{2.6} \\ 339150 \\ \underline{113050} \\ 14.69650 \end{array}$$

14.6965^{st}. *Ans.*

22. The railroad from Paris to Orleans has a double track; each rail is 4^{m} long, and the distance from Paris to Orleans is 121^{km}. What is the number of rails used in laying the track? If the width of the road is 15^{m}, how many hektars of land does the road include?

There are four lines of rails. $4 \times 121^{km} = 484^{km} = 484{,}000^{m}$ of rails. If one rail is 4^{m} long, in $484{,}000^{m}$ there are $484{,}000 \div 4 = 121{,}000$ rails. $15^{m} = 0.015^{km}$. The area of the road is

$$(121 \times 0.015)^{qkm} = 1.815^{qkm} = 181.5^{ha}. \text{ Ans.}$$

$$\begin{array}{r} 121^{km} \\ \underline{4\phantom{^{km}}} \\ 484^{km} \end{array} \qquad \begin{array}{r} 4\overline{)484000} \\ 121000 \end{array} \qquad \begin{array}{r} 121 \\ \underline{0.015} \\ 605 \\ \underline{121} \\ 1.815 \end{array}$$

23. Find the number of ars in a surface which a ream of paper (480 sheets) will cover. The sheets are 30.3^{cm} long and 195^{mm} wide.

$195^{mm} = 19.5^{cm}$.

$$\begin{array}{r} 19.5 \\ \underline{30.3} \\ 585 \\ \underline{585} \\ 590.85 \end{array} \qquad \begin{array}{r} 590.85^{qcm} \\ \underline{480} \\ 4726800 \\ \underline{236340} \\ 283608.00^{qcm} \end{array}$$

$283{,}608^{qcm} = 28.36^{qm} = 0.2836^{a}$. *Ans.*

24. A beam is 7.070^{m} long; its two other dimensions are 0.258^{m} and 87^{mm}. Find its volume.

$87^{mm} = 0.087^{m}$

$$\begin{array}{r} 0.258 \\ \underline{0.087} \\ 1806 \\ \underline{2064} \\ 0.022446 \end{array} \qquad \begin{array}{r} 0.022446 \\ \underline{7.07} \\ 157122 \\ \underline{157122} \\ 0.15869322 \end{array}$$

0.15869^{cbm}. *Ans.*

[redacted]

25. A bar of iron 3^{m} long measures 45^{mm} square on the end where it has been evenly cut. The bar is heated and drawn out to a greater length by being passed through an orifice 24^{mm} square. What is the length of the bar after the operation?

$$45^{mm} = 0.045^{m}. \quad 24^{mm} = 0.024^{m}.$$

The volume of the bar is $(0.045 \times 0.045 \times 3)^{cbm} = 0.006075^{cbm}$. The area of the end, after the bar has been heated, is

$$(0.024 \times 0.024)^{qm} = 0.000576^{qm}.$$

Therefore the length of the bar is $(0.006075 \div 0.000576)^{m} = 10.547^{m}$. *Ans.*

0.045
0.045
225
180
0.002025
3
0.006075

0.024
0.024
96
48
0.000576

10.547
576)6075.
576
3150
2880
2700
2304
3960

26. A reservoir is 1.50^{m} wide, 2.80^{m} long, and 1.25^{m} deep. Find how many liters it contains when full, and to what height it would be necessary to raise it that it might contain 10^{cbm}.

The volume of the reservoir is $(1.5 \times 2.8 \times 1.25)^{cbm} = 5.25^{cbm} = 5250^{l}$. *Ans.*

The area of the bottom is $(1.5 \times 2.8)^{qm} = 4.2^{qm}$; therefore, in order to contain 10^{cbm}, the height must be $(10 \div 4.2)^{m} = 2.38^{m}$. *Ans.*

1.5
2.8
120
30
4.20
1.25
2100
840
420
5.2500

2.38
42)100.
84
160
126
340
336

[redacted]

27. Suppose a box to be 3.75^{m} long, 3.50^{m} wide, and 0.50^{m} high. How much lime would it take to fill it with mortar, reckoning that 1^{cbm} of lime after being slaked becomes 1.80^{cbm} of mortar?

The volume of the box is $(3.75 \times 3.50 \times 0.50)^{cbm} = 6.5625^{cbm}$. Since 1^{cbm} of mortar when slaked becomes 1.8^{cbm}, the box will hold 6.5625^{cbm} of slaked mortar, which is the same as $6.5625^{cbm} \div 1.8 = 3.646^{cbm}$ of dry mortar.

3.75
3.5
1875
1125
13.125
0.5
6.5625

3.646
18)65.625
54
116
108
82
72
105

3.646^{cbm}. *Ans.*

28. A chest has the following dimensions: 1.17^{m}, 0.90^{m}, 1.04^{m}. If 0.12 of the volume of the chest is deducted for packing, how many cakes of soap 13^{cm} square on the bottom and 29^{cm} thick could be put in it?

The volume of a cake of soap is $(13 \times 13 \times 29)^{ccm} = 4901^{ccm}$. The volume of the chest, deducting waste of room in packing, is

$$0.88 \times (1.17 \times 0.90 \times 1.04)^{cbm} = 0.9637056^{cbm} = 963{,}705.6^{ccm}.$$

Therefore, the chest will hold $(963{,}705.6 \div 4901)$ cakes of soap.

13
13
39
13
169
29
1521
338
4901

1.17
1.04
468
117
1.2168
0.9
1.09512
0.88
876096
876096
0.9637056

196
4901)963705.6
4901
47360
44109
32515
29406

196. *Ans.*

[redacted]

29. A cubic meter of dry plaster makes 1.18^{cbm} when tempered; tempered plaster increases 1 in every 100, twenty-four hours after it is mixed. What volume of tempered plaster would be obtained from 55 sacks of 25^{l} each of dry plaster?

$25^{l} = 0.025^{cbm}$. The volume of the plaster is $55 \times 0.025^{cbm} = 1.375^{cbm}$. As 1^{cbm} makes 1.18^{cbm} when tempered, 1.375^{cbm} will make $1.375 \times 1.18^{cbm} = 1.6225^{cbm}$. In twenty-four hours its volume will be $1.01 \times 1.6225^{cbm} = 1.6387^{cbm}$. *Ans.*

0.025	1.375	1.6225
55	1.18	1.01
125	11000	16225
125	1375	16225
1.375	1375	1.638725
	1.62250	

30. A reservoir is 2.80^{m} long, 1.50^{m} wide, and 1.25^{m} deep. How many liters will be required to fill 0.80 of it?

1.5	4.20	5250^{l}
2.8	1.25	0.8
120	2100	$4200.^{l}$ *Ans.*
30	840	
4.20	420	
	5.2500	
	$5.25^{cbm} = 5250^{l}$.	

31. A man buys 1415^{hl} of wheat for \$3.50 a hektoliter; but the measure used proves too small, the mistake amounting to 3^{l} in every hektoliter. What was the quantity of wheat delivered to the purchaser, the cost, and the reduction which ought to be made to him on account of the error?

The mistake was 3^{l} in 100^{l}, or he received only 0.97 of $1415^{h} = 1372.55^{hl}$. If 1^{hl} of wheat cost \$3.50, 1415^{hl} cost $1415 \times$ \$3.50 = \$4952.50. A reduction of 0.03 of \$4952.50 = \$148.58 ought to be made.

1415^{hl}	1415	\$4952.50
0.97	3.50	0.03
9905	70750	\$148.5750
12735	4245	
1372.55^{hl}	4952.50	

[redacted]

32. The dimensions of a tile are as follows: length 22^{cm}, width 11^{cm}, thickness 55^{mm}. Find the volume of the tile, and the number of tiles in a pile of 25^{cbm}.

$55^{mm} = 5.5^{cm}$. The volume of a tile is $(22 \times 11 \times 5.5)^{ccm} = 1331^{ccm}$. $25^{cbm} = 25{,}000{,}000^{ccm}$. In the pile there will be $25{,}000{,}000 \div 1331 = 18{,}782$ tiles.

```
   22
   11
   --
   22
  22
  ---
  242
  5.5
 ----
 1210
1210
------
1331.0
```

```
        18782
1331)25000000
     1331
     ----
     11690
     10648
     -----
      10420
       9317
      -----
      11030
      10648
      -----
        3820
        2662
        ----
```

33. The measurement of a pile of wood shows that a ster could be filled from it 25.68 times. Find the volume of the pile in cubic meters, reckoning the length of the logs to be 1.15^{m}.

The volume of the pile is $25.68 \times (1 \times 1 \times 1.15)^{cbm} = 29.532^{cbm}$. *Ans.*

```
  25.68
   1.15
  -----
  12840
 2568
2568
-------
29.5320
```

34. A liter of air weighs 1.273^{g}. How much does a cubic meter of air weigh? How many times as heavy as air is water?

$1^{cbm} = 1000^{l}$. Therefore 1^{cbm} of air weighs $1000 \times 1.273^{g} = 1273^{g} = 1.273^{kg}$. *Ans.*

1^{cbm} of water weighs 1000^{kg}.

Therefore, water is $1000 \div 1.273 = 785.55$ times as heavy as air.

```
            785.54
1273)1000000.
      8911
      -----
      10890
      10184
      -----
        7060
        6365
        ----
         6950
         6365
         ----
          5850
          5092
          ----
           758
```

35. A package of candles that weighs 465^g is sold for 28 cents. At the same rate what is the price of a kilogram of candles?

1^g of candles costs \$0.28 ÷ 465 = \$0.000602. Therefore 1^{kg} costs 1000 × \$0.000602 = \$0.602. \$0.60. *Ans.*

36. How many times will 3.243^t of water fill a liter measure?

As 1^t of water will fill a cubic meter, 3.243^t will fill $3.243^{cbm} = 3243^l$.
3243 times. *Ans.*

37. Express in kilograms the weight of 43.4578^{ccm} of pure water.

43.4578^{ccm} of water weighs $43.4578^g = 0.0434578^{kg}$. *Ans.*

38. The volume of the axle of an engine is 0.245^{cbm}. Find its weight, if the specific gravity of the iron is 7.8.

0.245^{cbm} of water weighs 0.245^t, and 0.245^{cbm} of iron weighs

$$7.8 \times 0.245^t = 1.911^t. \textit{ Ans.}$$

```
 0.245
   7.8
 -----
  1960
 1715
 ------
 1.9110
```

39. Find the volume of a gram of the following substances: proof spirit, specific gravity 0.865; tin, specific gravity 7.291; lead, specific gravity 11.35; copper, specific gravity 8.85; silver, specific gravity 10.47; cork, specific gravity 0.240.

[redacted]

1^{ccm} of water weighs 1^{g}. Hence, the volume of a substance equals 1^{ccm} divided by its specific gravity.

(i.)

```
         1.16
865)1000.
     865
     1350
      865
      4850
```

1.16^{ccm}. *Ans.*

(ii.)

```
          0.14
7291)1000.0
      7291
      27090
```

0.14^{ccm}. *Ans.*

(iii.)

```
         0.088
1135)100.00
       9080
        9200
        9080
```

0.088^{ccm}. *Ans.*

(iv.)

```
        0.113
885)100.0
     885
     1150
      885
      2650
```

0.113^{ccm}. *Ans.*

(v.)

```
         0.095
1047)100.00
       9423
        5770
        5235
```

0.095^{ccm}. *Ans.*

(vi.)

```
      4.167
24)100.
    96
     40
     24
     160
     144
```

4.167^{ccm}. *Ans.*

40. Olive oil costs 60 cents a kilogram. What is the price of a liter? The specific gravity of olive oil is 0.914.

1^{l} of olive oil weighs 0.914^{kg}. As 1^{kg} costs \$0.60, 1^{l} costs $0.914 \times \$0.60 = \$0.548$. *Ans.*

```
 0.914
  0.60
0.54840
```

41. Pure alcohol costs \$1.87 a kilogram. What is the price of a liter? The specific gravity of alcohol is 0.792.

1^{l} of alcohol weighs 0.792^{kg}. As 1^{kg} costs \$1.87, 1^{l} costs $0.792 \times \$1.87 = \$1.48$. *Ans.*

```
 $1.87
 0.792
   374
  1683
 1309
$1.48104
```

42. A man wishes to build a shed large enough to hold 135^{st} of wood; if the shed is to be 3^{m} high and 5^{m} wide, how long must it be?

$135^{st} = 135^{cbm}$. The area of one end is $(3 \times 5)^{qm} = 15^{qm}$. Therefore, the length must be $(135 \div 15)^{m} = 9^{m}$.

43. In a country where firewood is cut 1.16^{m} long, what must be the height of the ster that it may hold a cubic meter?

The height must be

$(1 \div 1.16)^{m} = 0.86207^{m}$. *Ans.*

```
      0.86207
116)100.0
     928
     ---
      720
      696
      ---
       240
       232
       ---
        800
```

44. If a ster of cork costs \$20.00, how much would 100^{kg} cost, the cork weighing 0.25 as much as water?

1^{st} of cork weighs 250^{kg}, and costs \$20.00. 100^{kg} will cost $\frac{100}{250}$ of \$20.00 = 0.4 of \$20.00 = \$8.00. *Ans.*

45. A liter of powder weighs 825^{g}. What will be the volume in cubic centimeters of a charge for a gun if the charge weighs 5^{g}?

The specific gravity of powder is 0.825. It takes $(1 \div 0.825)^{ccm}$ of powder to weigh 1^{g}; therefore to weigh 5^{g} it takes $(5 \div 0.825)^{ccm} = 6.06^{ccm}$. *Ans.*

```
      6.06
825)5000.
    4950
    ----
     5000
     4950
     ----
```

46. Out of gold which weighs 19.362 times as much as water, sheets of gold foil are made which are 0.010^{mm} in thickness. What surface will 3^{g} of gold cover?

$0.010^{mm} = 0.001^{cm}$. The volume of the gold is $3^{ccm} \div 19.362 = 0.154943^{ccm}$. Therefore the surface is $(0.154943 \div 0.001)^{qcm} = 154.943^{qcm}$. *Ans.*

```
         0.154942
19362)3000.0
      19362
      ------
      106380
       96810
      ------
        95700
        77448
       ------
        182520
        174258
        ------
          82620
          77448
          -----
           51720
           38724
           -----
           12996
```

47. Find the weight of an oak board 3.25^{m} long, 0.31^{m} wide, and 0.04^{m} thick, if the specific gravity of the oak is 0.808.

[redacted]

The volume of the board is $(3.25 \times 0.31 \times 0.04)^{\text{cbm}} = 0.0403^{\text{cbm}}$. 1^{cbm} of oak weighs 0.808^{t}; therefore 0.0403^{cbm} weighs $0.0403 \times 0.808^{\text{t}} = 0.0325624^{\text{t}} = 32.5624^{\text{kg}}$. *Ans.*

```
     3.25
     0.31
     ----
      325
     975
   ------
   1.0075
     0.04
 --------
 0.040300

   0.0403
    0.808
   ------
     3224
   3224
---------
0.0325624
```

48. Find the weight of a bar of iron having the following dimensions: length 3.6^{m}, width 6^{cm}, thickness 2^{cm}, if the specific gravity of the iron is 7.8.

$3.6^{\text{m}} = 360^{\text{cm}}$.

```
    360
      6
   ----
   2160
      2
   ----
   4320

   4320
    7.8
  -----
  34560
 30240
-------
33696.0
```

$33{,}696^{\text{g}} = 33.696^{\text{kg}}$. *Ans.*

49. How many lead balls each weighing 27^{g} can be obtained by melting a cubic mass of lead 0.356^{m} on an edge, if the specific gravity of the lead is 11.35?

$0.355^{\text{m}} = 35.6^{\text{cm}}$.

```
     35.6
     35.6
     ----
     2136
    1780
   1068
  -------
  1267.36
     35.6
  -------
   760416
  633680
 380208
---------
45118.016
```

```
    45118.016
        11.35
 ------------
    225590080
   135354048
   45118016
  45118016
 ------------
 512089.48160g
```

```
      18966
 27)512089
    27
    ---
    242
    216
    ---
     260
     243
     ---
      178
      162
      ---
       169
       162
       ---
```

18,966. *Ans.*

50. Marble costs \$30.95 a cubic meter, and the specific gravity of marble is 2.73. If a block of marble weighs 1260^{kg}, what is its volume and what is it worth?

1^{cbm} of marble weighs 2.73^{t}. $1260^{kg} = 1.26^{t}$.

```
      0.4615              0.4615
 273)126.0                 30.95
     1092                 23075
      1680               41535
      1638              13845
       420              14.283425
       273
       1470
       1365
```

Volume $= 0.4615^{cbm}$;
cost $=$ \$14.28. *Ans.*

51. Sea water contains 28 parts, by weight, of salt in 1000. A liter of sea water weighs 1.025^{kg}. How many kilograms of salt can be obtained from 126.276842^{cbm} of sea water?

1^{kg} of sea water contains 0.028^{kg} of salt.

```
    126.276842          129433.763
      1.025                 0.028
    631384210           1035470104
   252553684            258867526
  126276842             3624.145364
  129.433763050
```

3624.145^{kg}. *Ans.*

52. An empty cask weighs 17.06^{kg}; when filled with water it weighs 275.8^{kg}. How many liters does it hold? How many casks of this size will it take for the wine from a vat containing 3.008^{cbm}?

The cask will hold $275.8^{kg} - 17.06^{kg} = 258.74^{kg}$ of water. It takes 258.74^{l} of water to weigh 258.74^{kg}. Therefore the cask will hold 258.74^{l}. *Ans.*

$3.008^{cbm} = 3008^{l}$. If one cask holds 258.74^{l}, to hold 3008^{l} it will take $3008 \div 258.74 = 12$ casks. *Ans.*

```
                              12
  275.80kg            25874)300800
   17.06                    25874
  258.74kg                  42060
```

[redacted]

53. It takes about 2.048^{hl} of wheat to sow a hektar. How many cubic meters will it take to sow a square kilometer?

$1^{qkm} = 100^{ha}$. 1^{ha} will require $100 \times 204.8^{l} = 20{,}480^{l} = 20.48^{cbm}$. *Ans.*

54. A piece of road 1^{km} long and 7^{m} wide is to be macadamized to the depth of 33^{cm}. What will the work cost at 43 cents a cubic meter?

$1^{km} = 1000^{m}$; $33^{cm} = 0.33^{m}$.

$$\begin{array}{r} 0.33 \\ \underline{7} \\ 2.31 \\ \underline{1000} \\ 2310. \end{array} \qquad\qquad \begin{array}{r} 2310 \\ \underline{0.43} \\ 6930 \\ \underline{9240} \\ 993.30 \end{array} \quad \$993.30. \textit{ Ans.}$$

55. A gasometer holds $28{,}000^{cbm}$ of gas. How many jets will this gasometer feed for an evening, when each jet burns 125^{l} an hour, and is used 4 hours?

Each jet will burn $4 \times 125^{l} = 500^{l}$ each evening. $28{,}000^{cbm} = 28{,}000{,}000^{l}$. The gasometer will feed $28{,}000{,}000 \div 500 = 56{,}000$ jets.

56. The city of Venice is situated in the midst of a great lake of salt water, communicating with the sea, and all the rain water is caught for the cisterns. Ordinary years the fall of rain in Venice is 82^{cm}; the surface of the city, after the canals have been deducted, is 520^{ha}. Reckoning the population at 115,530, how many liters a day of rain water can each inhabitant have?

$520^{ha} = 5{,}200{,}000^{qm}$; $82^{cm} = 0.82^{m}$.

The average amount of rain water is $(5{,}200{,}000 \times 0.82)^{cbm} = 4{,}264{,}000^{cbm} = 4{,}264{,}000{,}000^{l}$.

Each person can use per year $4{,}264{,}000{,}000^{l} \div 115{,}530$, or, per day, $4{,}264{,}000{,}000^{l} \div (115{,}530 \times 365) = 101.118^{l}$. *Ans.*

$$\begin{array}{r} 0.82 \\ \underline{5200000} \\ 16400000 \\ \underline{410} \\ 4264000.00 \end{array} \qquad \begin{array}{r} 115530 \\ \underline{365} \\ 577650 \\ 693180 \\ \underline{346590} \\ 42168450 \end{array} \qquad \begin{array}{r} 101.118 \\ 4216845)\overline{426400000.} \\ \underline{4216845} \\ 4715500 \\ \underline{4216845} \\ 4986550 \\ \underline{4216845} \\ 7697050 \\ \underline{4216845} \\ 34802050 \\ 33734760 \end{array}$$

[redacted]

57. Find the weight of a bar of iron 5.35^m long, 4.56^{cm} thick, and 3.54^{cm} wide. Find, also, the width of an oak beam 4.30^m long, 9.12^{cm} thick, which has the same weight. The specific gravity of the oak to be reckoned at 1.026, that of the iron at 7.788.

$5.35^m = 535^{cm}$. $4.30^m = 430^{cm}$. $535 \times 4.56 \times 3.54 \times 7.788 = 67{,}258.6$. Therefore the weight of the iron is $67{,}259.6^{kg}$. *Ans.*

The volume of the oak beam is $67{,}258.596992^{ccm} \div 1.026 = 65{,}554.2^{ccm}$. The area of one side of the oak beam is $(430 \times 9.12)^{qcm} = 3921.6^{qcm}$; therefore the thickness is $(65{,}554.2 \div 3921.6)^{cm} = 16.72^{cm}$.

$$\begin{array}{r} 4.56 \\ \underline{3.54} \\ 1824 \\ 2280 \\ \underline{1368} \\ 16.1424 \end{array} \qquad \begin{array}{r} 16.1424 \\ \underline{535} \\ 807120 \\ 484272 \\ \underline{807120} \\ 8636.1840 \end{array} \qquad \begin{array}{r} 8636.184 \\ \underline{7.788} \\ 69089472 \\ 69089472 \\ 60453288 \\ \underline{60453288} \\ 67258.600992 \end{array}$$

$$\begin{array}{l} 65554.2 \\ 1026\overline{)67258600.992} \\ \underline{6156} \\ 5698 \\ \underline{5130} \\ 5686 \\ \underline{5130} \\ 5560 \\ \underline{5130} \\ 4300 \\ \underline{4104} \\ 1969 \end{array} \qquad \begin{array}{l} 16.72 \\ 39216\overline{)655542.} \\ \underline{39216} \\ 263382 \\ \underline{235296} \\ 280860 \\ \underline{274512} \\ 63480 \end{array}$$

16.72^{cm}. *Ans.*

58. Find the specific gravity and volume of a body weighing 35^{kg} in air and 30^{kg} in water.

The weight of the water displaced by the body is 5^{kg}.

The weight of the body in air is 35^{kg}.

Therefore the specific gravity is $35 \div 5 = 7$. 5^{kg} of water occupies 5^l of space. 5^l. *Ans.*

[redacted]

59. A ster of piled oak wood weighs 425^{kg}; the specific gravity of the wood is 0.74. What is the volume occupied by the spaces between the logs? For how much must 100^{kg} of separate sticks be sold to bring the same amount as when sold at $2.20 a ster?

If there were no spaces between the logs, the ster of wood would weigh 740^{kg}. Therefore the spaces, if filled with wood, would weigh $740^{kg} - 425^{kg} = 315^{kg}$. Therefore, the volume of the spaces is $(315 \div 740)^{cbm} = 0.42568^{cbm}$. 100^{kg} ought to be sold for $\frac{100}{425}$ of \$2.20 $=$ \$220 $\div$ 425 $=$ \$0.518.

```
     0.42568                  $0.518
74)31.5                  425)$220.0
   296                        2125
   ---                        ----
    190                        750
    148                        425
    ---                        ----
     420                       3250
     370
     ---
      500
      444
      ---
       560
```

0.42568cbm;
\$0.518. *Ans.*

60. Wrought iron sells for \$7.00 per 100^{kg}. A bar of iron 4.5^{cm} wide, 3.3^{cm} thick costs \$5.08; what is its length, reckoning the specific gravity of the iron at 7.4?

\$7.00 per 100^{kg} is the same as \$0.07 per kilogram. An iron bar that costs \$5.08 must weigh $(5.08 \div 0.07)^{kg} = 72.57143^{kg}$, and its volume is $(72.57143 \div 7.4)^{l} = 9.8069^{l} = 9806.9^{ccm}$. The area of an end of the bar is $(4.5 \times 3.3)^{qcm} = 14.85^{qcm}$. Therefore the length is $(9806.9 \div 14.85)^{cm} = 660.4^{cm} = 6.604^{m}$. *Ans.*

```
      9.8069                    660.4
74)725.7143             1485)980690.0
   666                        8910
   ---                        ----
    597                        8969
    592                        8910
    ---                        ----
     514                        5900
     444
     ---
      703
      666
```

[redacted]

61. Experiment shows that water weighs 770 times as much as air; and the specific gravity of mercury is 13.6. How many liters of air will it take to weigh as much as a liter of mercury?

Water is 770 times as heavy as air, and mercury is 13.6 times as heavy as water. Therefore mercury is 13.6 × 770 times as heavy as air.

$$\begin{array}{r} 13.6 \\ \underline{770} \\ 9520 \\ \underline{9520} \\ 10472.0 \end{array}$$

$10{,}4\cancel{6}2^{l}$. *Ans.*
7

62. A mass of lead weighing 753^{kg} is made into sheets 0.1^{mm} thick. Find in square meters the surface which can be covered by the sheets thus obtained. The specific gravity of the lead is 11.3.

The volume of the lead is $(753 \div 11.3)^{l} = 66.637^{l} = 0.066637^{cbm}$.

$0.1^{mm} = 0.0001^{m}$. The surface of the lead is

$(0.066637 \div 0.0001)^{qm} = 666.37^{qm}$. *Ans.*

$$\begin{array}{r} 66.637 \\ 113\overline{)7530.} \\ \underline{678} \\ 750 \\ \underline{678} \\ 720 \\ \underline{678} \\ 420 \\ \underline{339} \\ 810 \\ \underline{791} \end{array}$$

63. A rectangular sheet of tin of uniform thickness is 85^{cm} wide, 1.35^{m} long, and weighs 268^{g}. What is its thickness, if the specific gravity of tin is 7.3?

The volume of the tin is $(268 \div 7.3)^{ccm} = 33.7109^{ccm}$; $1.35^{m} = 135^{cm}$. The area of the tin is $(135 \times 85)^{qcm} = 11{,}475^{qcm}$; therefore its thickness is $(36.7109 \div 11475)^{cm} = 0.0032^{cm}$. *Ans.*

$$\begin{array}{r} 36.7109 \\ 73\overline{)2680.} \\ \underline{219} \\ 490 \\ \underline{438} \\ 520 \\ \underline{511} \\ 90 \\ \underline{73} \\ 700 \\ \underline{657} \end{array} \qquad \begin{array}{r} 135 \\ \underline{85} \\ 675 \\ \underline{1080} \\ 11475 \end{array} \qquad \begin{array}{r} 0.0032 \\ 11475\overline{)36.7109} \\ \underline{34325} \\ 23859 \\ \underline{22950} \end{array}$$

64. The fine coal which collects about the shafts of the mines and in the coal yards, was for a long time wasted, because it could not be burned in stoves and grates. Now this dust is mixed with tar in proportion of 92^{kg} of dust and 8^{kg} of tar; the mixture is heated, and afterwards pressed in rectangular moulds 14.75^{cm}, by 18.5^{cm}, by 29^{cm}; each one of these blocks weighs 10^{kg}. They are sold at \$3.00 a ton, and make excellent fuel for heating steam boilers. Find the specific gravity of this fuel; also, the sum which would be realized in thus utilizing $800{,}000^{t}$ of coal dust, the cost of tar, mixing, etc., being \$0.50 a ton.

Volume of a block is $(14.75 \times 18.5 \times 29)^{ccm} = 7913.375^{ccm} = 7.913375^{l}$. Specific gravity is $10 \div 7.913375 = 1.264$. $800{,}000^{t}$ of coal dust will make $800{,}000^{t} \div 0.92 = 869{,}565.217^{t}$ of the mixture. $869{,}565.217^{t}$ at \$2.50 per ton $= 869{,}565.217 \times \$2.50 = \$2{,}173{,}913.04$. *Ans.*

```
     14.75
     18.5
    ------
     7375
    11800
    1475
   -------
   272.875
        29
   -------
   2455875
   545750
   --------
   7913.375
```

```
              1.264
7913375)10000000.
         7913375
        --------
         20866250
         15826750
        ---------
          50395000
          47480250
         ---------
           29147500
```

```
     869565.217
92)80000000.
   736
   ---
    640
    552
    ---
     880
     828
     ---
      520
      460
      ---
       600
       552
       ---
        480
        460
        ---
         200
         184
         ---
          160
           92
          ---
          680
          644
          ---
```

```
   869565.217
         2.50
  -----------
  43478260850
 1739130434
 ------------
 2173913.04250
```

[redacted]

65. A bar of iron a millimeter square on the end will break under a tension of 30^{kg}. Find the length at which a suspended bar of iron will break from its own weight, if the specific gravity of the iron is 7.8.

$30^{kg} = 0.03^{l}$.

The volume of the iron bar is $(0.03 \div 7.8)^{cbm} = 0.00384615^{cbm}$. The area of an end of the bar is $1^{qmm} = 0.000001^{qm}$. Therefore the length of the bar is

$$(0.00384615 \div 0.000001)^{m} = 3846.15^{m}.$$

```
      0.00384615
78)0.300
   234
    660
    624
     360
     312
      480
      468
       120
        78
        420
        390
```

66. Fifty-three kilograms of starch are obtained from 100^{kg} of wheat. A hektar of land produces 1363 of wheat; a hektoliter of wheat weighs 78^{kg}. If the wheat harvested from a field measuring 2^{ha} and 33^{qm} is taken to a starch factory, how much starch will be made from it?

0.53^{kg} of starch is obtained from 1^{kg} of wheat. 1^{l} of wheat weighs 0.78^{kg}. 1^{ha} produces 1363×0.78^{kg} of wheat $= 1063.14^{kg}$. $2^{ha}\,33^{qm} = 2.0033^{ha}$. 2.0033^{ha} produces $2.0033 \times 1063.14^{kg} = 2129.788362^{kg}$ of wheat. The amount of starch is

$0.53 \times 2129.788362^{kg} = 1128.7878^{kg}$.

```
      1363
      0.78
     10904
     9541
    1063.14
     2.0033
    318942
   318942
  212628
  2129.788362

   2129.788362
          0.53
   6389365086
  10648941810
  1128.78783186
```

67. A gardener wishes to provide glass for his hotbeds. The beds cover 2.65^{a}; the panes will cover 0.75 of the whole surface, the rest being taken up by the frames and alleys. First, find how many panes measuring 45^{cm} by 37^{cm} it will take to cover the beds; then find the price of the glass, at a cost of 95 cents a square meter.

$45^{cm} = 0.45^{m}$; $37^{cm} = 0.37^{m}$; $2.65^{a} = 265^{qm}$.

Total area of the glass is 0.75 of $265^{qm} = 198.75^{qm}$. The area of one pane is $(0.45 \times 0.37)^{qm} = 0.1665^{qm}$. Therefore the number of panes needed is $198.75 \div 0.1665 = 1194$. At \$0.95 per square meter, 198.75^{qm} will cost $198.75 \times \$0.95 = \$188.81$.

$$\begin{array}{r} 0.45 \\ 0.37 \\ \hline 315 \\ 135 \\ \hline 0.1665 \end{array} \qquad \begin{array}{r} 1194 \\ 1665\overline{)1987500} \\ 1665 \\ \hline 3225 \\ 1665 \\ \hline 15600 \\ 14985 \\ \hline 6150 \end{array} \qquad \begin{array}{r} 198.75 \\ 0.95 \\ \hline 99375 \\ 178875 \\ \hline 188.8125 \end{array}$$

1194 panes ; \$188.81. *Ans.*

68. A jar full of water weighs 1.325^{kg}; filled with mercury it weighs 12.540^{kg}. Find the capacity and the weight of the jar, if the specific gravity of the mercury is 13.59.

The weight of the jar and the jar full of mercury is 12.540^{kg}. The weight of the jar and the jar full of water is 1.325^{kg}. Therefore the difference in weight between the mercury and the water is $12.540^{kg} - 1.325^{kg} = 11.215^{kg}$. $13.59 - 1 = 12.59$, the specific gravity of a liquid of which the jar full without the jar weighs 11.215^{kg}. Hence the capacity of the jar is $(11.215 \div 12.59)^{l} = 0.89078^{l}$. 0.89078^{l} of water weighs 0.89078^{kg}. Hence, the weight of jar is $1.325^{kg} - 0.89078^{kg} = 0.43422^{kg} = 434.22^{g}$.

$$\begin{array}{r} 12.540 \\ 1.325 \\ \hline 11.215 \end{array} \qquad \begin{array}{r} 0.89078 \\ 1259\overline{)1121.5} \\ 10072 \\ \hline 11430 \\ 11331 \\ \hline 9900 \\ 8813 \\ \hline 10870 \\ 10072 \\ \hline \end{array} \qquad \begin{array}{r} 1.325 \\ 0.89078 \\ \hline 0.43422 \end{array}$$

$\left.\begin{array}{l} \text{Capacity} = 0.89078^{l}; \\ \text{weight} \ \ = 434.22^{g}. \end{array}\right\}$ *Ans.*

[redacted]

69. A hektoliter of rape seed weighs 63^{kg}, and 32^{l} of oil can be extracted from it. How many kilograms of the seed will it take to make a hektoliter of oil?

$1^{hl} = 100^{l}$. If 32^{l} of oil can be extracted from 63^{kg} of seed, 1^{l} of oil can be extracted from $63^{kg} \div 32 = 1.96875^{kg}$ of seed, and 100^{l} of oil can be extracted from $100 \times 1.96875^{kg} = 196.875^{kg}$ of seed.

```
      1.96875
  32)63.
     32
     310
     288
      220
      192
       280
       256
        240
        224
         160
         160
```

196.875^{kg}. *Ans.*

70. Common burning gas is 0.97 of the weight of air, and a liter of air weighs 1.293^{g}. In a shop there are 65 jets, each one of which burns 123^{l} an hour, and is used 5 hours in the winter evenings. Find the weight of the gas used in a month of 26 days, and the expense of lighting the shop, when gas costs 6 cents a cubic meter.

1^{l} of gas weighs $0.97 \times 1.293^{g} = 1.25421^{g}$. 65 jets, each burning 123^{l} an hour, and used 5 hours an evening for 26 days, will use $65 \times 5 \times 26 \times 123^{l} = 1{,}039{,}350^{l}$, the weight of which is $1{,}039{,}350 \times 1.25421^{g} = 1{,}303{,}563.16^{g} = 1303.563^{kg}$. $1{,}039{,}350^{l} = 1039.35^{cbm}$. The expense at \$0.06 per cubic meter is $1039.35 \times \$0.06 = \$62.36$.

```
  1.293g        123l         1.25421        1039.35
  0.97           65          1039350           0.06
  9051          615          6271050        62.3610
 11637          738           376263        $62.36.  Ans.
1.25421g       7995l        1128789
                  5         376263
              39975l        125421
                 26         1303563.16350
             239850
             79950
            1039350
```

[redacted]

71. A merchant buys one kind of wine at 30 cents a liter, another kind at 21 cents a liter; he mixes the two kinds by putting 5^l of the first with 8^l of the second. For how much a liter must he sell the mixture in order to gain \$3.75 a hektoliter?

5^l at \$0.30 per liter costs \$1.50.

8^l at \$0.21 per liter costs \$1.68.

Therefore 13^l of the mixture costs \$1.50 + \$1.68 = \$3.18, and 1^l costs \$3.18 ÷ 13 = \$0.2446. Again, if \$3.75 per hektoliter is equivalent to a gain of \$0.0375 per liter, to make \$3.75 per hektoliter the merchant must sell the wine for \$0.0375 + \$0.2446 = \$0.2821 per liter. *Ans.*

$$\begin{array}{r} \$0.30 \\ 5 \\ \hline \$1.50 \end{array} \qquad \begin{array}{r} \$0.21 \\ 8 \\ \hline \$1.68 \\ 1.50 \\ \hline \$3.18 \end{array} \qquad \begin{array}{r} 0.2446 \\ 13)\overline{3.18} \\ 26 \\ \hline 58 \\ 52 \\ \hline 60 \\ 52 \\ \hline 80 \\ 78 \\ \hline \end{array}$$

72. If it requires 360 tiles to drain an ar of land, what will it cost to drain 17.784^{ha}, when the tiles cost \$20 a thousand, and the expense of laying is the same as the cost of the tiles?

The expense of laying the tiles and their cost is \$40 per thousand. $17.784^{ha} = 1778.4^{a}$. To drain 1778.4^{a} of land 1778.4×360 tiles $= 640{,}224$ tiles $= 640.224$ thousand are needed. 640.224 thousand at \$40 per thousand cost 640.224 × \$40 = \$25,608.96. *Ans.*

$$\begin{array}{r} 1778.4 \\ 360 \\ \hline 1067040 \\ 53352 \\ \hline 640224.0 \end{array} \qquad \begin{array}{r} 640.224 \\ 40 \\ \hline 25608.960 \end{array}$$

73. Hewn stone of medium durability ought not to support, as a permanent weight, more than 0.07 of the weight that is required to crush it. A certain kind of stone used for building will be crushed

under a weight of 250^{kg} a square centimeter. What is the greatest height to which a wall constructed of this material can be safely carried, if the specific gravity of the stone is 2.1?

250^{kg} per square centimeter is equivalent to $250,000^{g}$ per square centimeter. 0.07 of $250,000^{g} = 17,500^{g}$ ought to be the pressure on a square centimeter. Therefore the volume of the imaginary prism ought to be $(17,500 \div 2.1)^{ccm} = 8333.33^{ccm}$, or the height ought to be $8333.33^{cm} = 83.333^{m}$.

$$\begin{array}{r} 8333.33 \\ 21\overline{)175000.00} \\ 168 \\ \hline 70 \\ 63 \\ \hline 70 \end{array}$$

83.333^{m}. *Ans.*

74. Several different kinds of wines are mixed as follows: 245^{l} at 20 cents a liter, 547^{l} at 15 cents a liter, 344^{l} at 25 cents a liter. How much does the mixture cost a liter?

245^{l} at \$0.20 per liter costs	\$49.00
547^{l} at \$0.15 per liter costs	\$82.05
344^{l} at \$0.25 per liter costs	\$86.00
1136^{l} of the mixture costs	\$217.05

Therefore 1^{l} costs \$217.05 ÷ 1136 = \$0.191. *Ans.*

$$\begin{array}{r} 245 \\ 0.20 \\ \hline 49.00 \end{array} \qquad \begin{array}{r} 547 \\ 0.15 \\ \hline 2735 \\ 547 \\ \hline 82.05 \end{array} \qquad \begin{array}{r} 344 \\ 0.25 \\ \hline 1720 \\ 688 \\ \hline 86.00 \end{array} \qquad \begin{array}{r} \$0.191 \\ 1136\overline{)\$217.05} \\ 1136 \\ \hline 10345 \\ 10224 \\ \hline 1210 \\ 1136 \\ \hline \end{array}$$

75. A farmer wishes to drain a field of 8.75^{ha}. Each hektar requires 750^{m} of ditches. The opening of these ditches costs 10 cents a running meter; the tiles are 30^{cm} long, and cost \$15 a thousand. He pays 2 cents a meter for laying the tiles, and 4 cents a meter for filling the ditches. What is the cost of draining the field?

There are required $8.75 \times 750^m = 6562.5^m$ of ditches. The expense of opening the ditches, laying the tiles, and filling the ditches is \$0.10 + \$0.02 + \$0.04 = \$0.16 per meter. 6562.5^m will cost 6562.5 × \$0.16 = \$1050.00. $30^{cm} = 0.3^m$. For 6562.5^m, $6562.5 \div 0.3 = 21{,}875$ tiles are necessary. The tiles cost \$15 per thousand. Therefore 21.875 thousand cost 21.875 × \$15 = \$328.13. Hence cost of draining the field is \$1050.00 + \$328.13 = \$1378.13.

8.75	6562.5	21.875	\$1050.
750	0.16	15	328.13
43750	393750	109375	\$1378.13 *Ans.*
6125	65625	21875	
6562.50	1050.000	328.125	

76. A silver five-franc piece weighs 25^g, and is composed of 9 parts of pure silver and 1 part of pure copper. A silver two-franc piece weighs 10^g, and is composed of 835 parts of pure silver and 165 parts of pure copper. A silver twenty-centime piece weighs 1^g, and has the same composition as the two-franc piece. Find the total weight of pure silver and of pure copper contained in 272 five-franc pieces, 145 two-franc pieces, and 179 twenty-centime pieces.

$$272 \times 25^g = 6800^g.$$
$$0.1 \times 6800^g = 680^g, \text{ copper.}$$
$$6800^g - 680^g = 6120^g, \text{ silver.}$$
$$145 \times 10^g = 1450^g.$$
$$179 \times 1^g = 179^g;\quad 1450^g + 179^g = 1629^g.$$

1629^g	1629^g.
0.165	268.785
8145	1360.215^g, silver.
9774	
1629	
268.785^g, copper.	

1360.215^g	268.785^g
6120.	680.
7480.215^g *Ans.*	948.785^g *Ans.*

[redacted]

77. The dimensions of the bottom of a rectangular box are 70^{cm} by 50^{cm}. If the box contains exactly a hektoliter of wheat when full, what is the height of the box?

$$1^{hl} = 100^{l} = 100{,}000^{ccm}.$$

$$\frac{\overset{200}{1\cancel{00000}}^{ccm}}{7\cancel{0} \times \cancel{50}} = \frac{200}{7}^{cm} = 28.571^{cm}. \quad \textit{Ans.}$$

78. If a stick of oak timber 54 centimeters wide and 65 centimeters thick costs \$25 at \$16 a cubic meter, what is the length of the stick?

The volume of the stick of timber $= \frac{25}{16}^{cbm} = \frac{25000000}{16}^{ccm}$
$= 1{,}562{,}500^{ccm}.$

$$\begin{array}{r} 54 \\ \underline{65} \\ 270 \\ \underline{324} \\ 3510 \end{array} \qquad \begin{array}{r} 445.156 \\ 351\cancel{0})\overline{156250.} \\ \underline{1404} \\ 1585 \\ \underline{1404} \\ 1810 \\ \underline{1755} \\ 550 \\ \underline{351} \\ 1990 \\ \underline{1755} \\ 2350 \\ \underline{2106} \\ 244 \end{array}$$

$445.157^{cm} = 4.45157^{m}$. *Ans.*

79. A rectangular box whose bottom is a square 28^{cm} on a side, and whose height is 19.2^{cm}, is exactly filled with gold twenty-franc pieces, in piles touching each other. If a twenty-franc piece is 35^{mm} in diameter, and 1.28^{mm} thick, what is the value of the gold in the box?

$28^{cm} = 280^{mm}$. $\qquad 19.2^{cm} = 192^{mm}$.

$$\begin{array}{r} 8 \\ 35)\overline{280} \\ \underline{280} \end{array} \qquad \begin{array}{r} 150 \\ 128)\overline{19200} \\ \underline{128} \\ 640 \\ \underline{640} \\ 0 \end{array}$$

Hence, the number of piles of pieces is 8 × 8, or 64, and the number in a pile is 150.

Therefore, the number of pieces = 64 × 150 = 9600.

9600 × 20 francs = 192,000 francs. *Ans.*

80. If 1^{hl} of coal yields 1854^{cbm} of gas, and one burner consumes 140^{l} of gas in an hour, how many hektoliters of coal are required to supply 2800 burners for 144 hours?

$$\begin{array}{r} 144 \\ 2800 \\ \hline 115200 \\ 288 \\ \hline 403200 \end{array} \qquad \begin{array}{r} 140^{l} = 0.14^{cbm}. \\ 403200 \\ 0.14 \\ \hline 1612800 \\ 4032 \\ \hline 56448.00 \end{array} \qquad \begin{array}{r} 30.44 \\ 1854)\overline{56448.} \\ 5562 \\ \hline 8280 \\ 7416 \\ \hline 8640 \\ 7416 \\ \hline 1224 \end{array}$$

30.45^{hl}. *Ans.*

81. How many liters of water in a cylindrical well 1.96^{m} in diameter, if the water is 2.84^{m} deep?

$$\begin{array}{r} 1.96 \\ 1.96 \\ \hline 1176 \\ 1764 \\ 196 \\ \hline 3.8416 \end{array} \qquad \begin{array}{r} 3.8416 \\ 0.7854 \\ \hline 153664 \\ 192080 \\ 307328 \\ 268912 \\ \hline 3.01719264 \\ 2.84 \\ \hline 1206877056 \\ 2413754112 \\ 603438528 \\ \hline 8.5688270976 \end{array}$$

$8.568827^{cbm} = 8568.827^{l}$. *Ans.*

[redacted]

Exercise 38. Page 98.

Find the prime factors of:

1.

$$\begin{array}{r|l} 2^2 & 148 \\ \hline & 37 \end{array}$$

$2^2\times37$. *Ans.*

2.

$$\begin{array}{r|l} 2^3 & 264 \\ \hline 3 & 33 \\ \hline & 11 \end{array}$$

$2^3\times3\times11$. *Ans.*

3.

$$\begin{array}{r|l} 2 & 178 \\ \hline & 89 \end{array}$$

2×89. *Ans.*

4.

$$\begin{array}{r|l} 3 & 183 \\ \hline & 61 \end{array}$$

3×61. *Ans.*

5.

$$\begin{array}{r|l} 1 & 173 \\ \hline & 173 \end{array}$$

1×173. *Ans.*

6.

$$\begin{array}{r|l} 11 & 187 \\ \hline & 17 \end{array}$$

11×17. *Ans.*

7.

$$\begin{array}{r|l} 2 & 346 \\ \hline & 173 \end{array}$$

2×173. *Ans.*

8.

$$\begin{array}{r|l} 7^3 & 343 \\ \hline & 1 \end{array}$$

7^3. *Ans.*

9.

$$\begin{array}{r|l} 2 & 210 \\ \hline 3 & 105 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

$2\times3\times5\times7$. *Ans.*

10.

$$\begin{array}{r|l} 1 & 353 \\ \hline & 353 \end{array}$$

1×353. *Ans.*

11.

$$\begin{array}{r|l} 2^5 & 5280 \\ \hline 3 & 165 \\ \hline 5 & 55 \\ \hline & 11 \end{array}$$

$2^5\times3\times5\times11$. *Ans.*

12.

$$\begin{array}{r|l} 3 & 231 \\ \hline 7 & 77 \\ \hline & 11 \end{array}$$

$3\times7\times11$. *Ans.*

13.

$$\begin{array}{r|l} 2^3 & 31416 \\ \hline 3 & 3927 \\ \hline 7 & 1309 \\ \hline 11 & 187 \\ \hline & 17 \end{array}$$

$2^3\times3\times7\times11\times17$. *Ans.*

14.

$$\begin{array}{r|l} 37 & 1369 \\ \hline & 37 \end{array}$$

37×37. *Ans.*

15.

$$\begin{array}{r|l} 2^3 & 1368 \\ \hline 3^2 & 171 \\ \hline & 19 \end{array}$$

$2^3\times3^2\times19$. *Ans.*

16.

$$\begin{array}{r|l} 13 & 247 \\ \hline & 19 \end{array}$$

13×19. *Ans.*

17.

$$\begin{array}{r|l} 3 & 327 \\ \hline & 109 \end{array}$$

3×109. *Ans.*

18.

$$\begin{array}{r|l} 1 & 179 \\ \hline & 179 \end{array}$$

1×179. *Ans.*

19.

$$\begin{array}{r|l} 1 & 83 \\ \hline & 83 \end{array}$$

1×83. *Ans.*

20.

$$\begin{array}{r|l} 5^3 & 2125 \\ \hline & 17 \end{array}$$

$5^3\times17$. *Ans.*

21.

$$\begin{array}{r|l} 13 & 2353 \\ \hline & 181 \end{array}$$

13×181. *Ans.*

22.

$$\begin{array}{r|l} 1 & 2333 \\ \hline & 2333 \end{array}$$

1×2333. *Ans.*

[redacted]

23.

$$\begin{array}{r|l} 3 & 165 \\ \hline 5 & 55 \\ \hline & 11 \end{array}$$

$3 \times 5 \times 11$. *Ans.*

24.

$$\begin{array}{r|l} 2^3 & 168 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

$2^3 \times 3 \times 7$. *Ans.*

25.

$$\begin{array}{r|l} 2^2 & 2148 \\ \hline 3 & 537 \\ \hline & 179 \end{array}$$

$2^2 \times 3 \times 179$. *Ans.*

26.

$$\begin{array}{r|l} 2 & 16662 \\ \hline 3 & 8331 \\ \hline & 2777 \end{array}$$

$2 \times 3 \times 2777$. *Ans.*

27.

$$\begin{array}{r|l} 3 & 321 \\ \hline & 107 \end{array}$$

3×107. *Ans.*

28.

$$\begin{array}{r|l} 3 & 1551 \\ \hline 11 & 517 \\ \hline & 47 \end{array}$$

$3 \times 11 \times 47$. *Ans.*

29.

$$\begin{array}{r|l} 2 & 38 \\ \hline & 19 \end{array}$$

2×19. *Ans.*

30.

$$\begin{array}{r|l} 2 & 82 \\ \hline & 41 \end{array}$$

2×41. *Ans.*

31.

$$\begin{array}{r|l} 3 & 129 \\ \hline & 43 \end{array}$$

3×43. *Ans.*

32.

$$\begin{array}{r|l} 2^3 & 72 \\ \hline 3^2 & 9 \\ \hline & 1 \end{array}$$

$2^3 \times 3^2$. *Ans.*

33.

$$\begin{array}{r|l} 2 & 66 \\ \hline 3 & 33 \\ \hline & 11 \end{array}$$

$2 \times 3 \times 11$. *Ans.*

34.

$$\begin{array}{r|l} 2^2 & 68 \\ \hline & 17 \end{array}$$

$2^2 \times 17$. *Ans.*

35.

$$\begin{array}{r|l} 5 & 65 \\ \hline & 13 \end{array}$$

5×13. *Ans.*

36.

$$\begin{array}{r|l} 2^2 & 76 \\ \hline & 19 \end{array}$$

$2^2 \times 19$. *Ans.*

37.

$$\begin{array}{r|l} 2 & 86 \\ \hline & 43 \end{array}$$

2×43. *Ans.*

38.

$$\begin{array}{r|l} 2^3 & 88 \\ \hline & 11 \end{array}$$

$2^3 \times 11$. *Ans.*

39.

$$\begin{array}{r|l} 2 & 142 \\ \hline & 71 \end{array}$$

2×71. *Ans.*

40.

$$\begin{array}{r|l} 2 & 326 \\ \hline & 163 \end{array}$$

2×163. *Ans.*

41.

$$\begin{array}{r|l} 2^4 & 368 \\ \hline & 23 \end{array}$$

$2^4 \times 23$. *Ans.*

42.

$$\begin{array}{r|l} 2^4 & 464 \\ \hline & 29 \end{array}$$

$2^4 \times 29$. *Ans.*

43.

$$\begin{array}{r|l} 2^2 & 292 \\ \hline & 73 \end{array}$$

$2^2 \times 73$. *Ans.*

44.

$$\begin{array}{r|l} 2 & 362 \\ \hline & 181 \end{array}$$

2×181. *Ans.*

45.

$$\begin{array}{r|l} 5 & 365 \\ \hline & 73 \end{array}$$

5×73. *Ans.*

46.

$$\begin{array}{r|l} 2 & 730 \\ \hline 5 & 365 \\ \hline & 73 \end{array}$$

$2 \times 5 \times 73$. *Ans.*

47.

$$\begin{array}{r|l} 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

$2 \times 3 \times 7$. *Ans.*

48.

$$\begin{array}{r|l} 2^2 & 868 \\ \hline 7 & 217 \\ \hline & 31 \end{array}$$

$2^2 \times 7 \times 31$. *Ans.*

49.

$$\begin{array}{r|l} 3^3 & 999 \\ \hline & 37 \end{array}$$

$3^3 \times 37$. *Ans.*

50.

$$\begin{array}{r|l} 2 & 822 \\ \hline 3 & 411 \\ \hline & 137 \end{array}$$

$2 \times 3 \times 137$. *Ans.*

51.

$$\begin{array}{r|l} 2 & 1346 \\ \hline & 673 \end{array}$$

2×673. *Ans.*

[redacted]

52.

$$\begin{array}{r|l} 3^3 & 7641 \\ \hline & 283 \end{array}$$

$3^3 \times 283$. *Ans.*

53.

$$\begin{array}{r|r} 2 & 6234 \\ \hline 3 & 3117 \\ \hline & 1039 \end{array}$$

$2 \times 3 \times 1039$. *Ans.*

54.

$$\begin{array}{r|r} 2 & 234 \\ \hline 3^2 & 117 \\ \hline & 13 \end{array}$$

$2 \times 3^2 \times 13$. *Ans.*

55.

$$\begin{array}{r|l} 3 & 579 \\ \hline & 193 \end{array}$$

3×193. *Ans.*

56.

$$\begin{array}{r|l} 1 & 577 \\ \hline & 577 \end{array}$$

1×577. *Ans.*

57.

$$\begin{array}{r|r} 2^2 & 212 \\ \hline & 53 \end{array}$$

$2^2 \times 53$. *Ans.*

58.

$$\begin{array}{r|r} 2 & 126 \\ \hline 3^2 & 63 \\ \hline & 7 \end{array}$$

$2 \times 3^2 \times 7$. *Ans.*

59.

$$\begin{array}{r|r} 2^7 & 128 \\ \hline & 1 \end{array}$$

2^7. *Ans.*

60.

$$\begin{array}{r|r} 2^{13} & 8192 \\ \hline & 1 \end{array}$$

2^{13}. *Ans.*

61.

$$\begin{array}{r|r} 2 & 8190 \\ \hline 3^2 & 4095 \\ \hline 5 & 455 \\ \hline 7 & 91 \\ \hline & 13 \end{array}$$

$2 \times 3^2 \times 5 \times 7 \times 13$. *Ans.*

62.

$$\begin{array}{r|l} 7 & 8197 \\ \hline & 1171 \end{array}$$

7×1171. *Ans.*

63.

$$\begin{array}{r|r} 5^5 & 3125 \\ \hline & 1 \end{array}$$

5^5. *Ans.*

64.

$$\begin{array}{r|r} 7^4 & 2401 \\ \hline & 1 \end{array}$$

7^4. *Ans.*

65.

$$\begin{array}{r|r} 11^3 & 1331 \\ \hline & 1 \end{array}$$

11^3. *Ans.*

66.

$$\begin{array}{r|r} 3^2 & 78309 \\ \hline 7 & 8701 \\ \hline 11 & 1243 \\ \hline & 113 \end{array}$$

$3^2 \times 7 \times 11 \times 113$. *Ans.*

67.

$$\begin{array}{r|r} 3 & 25179 \\ \hline 7 & 8393 \\ \hline 11 & 1199 \\ \hline & 109 \end{array}$$

$3 \times 7 \times 11 \times 109$. *Ans.*

68.

$$\begin{array}{r|r} 2^3 & 61600 \\ \hline 2^2 & 7700 \\ \hline 5^2 & 1925 \\ \hline 7 & 77 \\ \hline & 11 \end{array}$$

$2^5 \times 5^2 \times 7 \times 11$. *Ans.*

69.

$$\begin{array}{r|r} 2^3 & 48048 \\ \hline 2 & 6006 \\ \hline 3 & 3003 \\ \hline 7 & 1001 \\ \hline 11 & 143 \\ \hline & 13 \end{array}$$

$2^4 \times 3 \times 7 \times 11$. *Ans.*

70.

$$\begin{array}{r|r} 2 & 401478 \\ \hline 3 & 200739 \\ \hline 7 & 66913 \\ \hline 11 & 9559 \\ \hline & 869 \end{array}$$

$2 \times 3 \times 7 \times 11 \times 869$. *Ans.*

71.

$$\begin{array}{r|r} 2^3 & 278208 \\ \hline 2^3 & 34776 \\ \hline 3^2 & 4347 \\ \hline 3 & 483 \\ \hline 7 & 161 \\ \hline & 23 \end{array}$$

$2^6 \times 3^3 \times 7 \times 23$. *Ans.*

72.

$$\begin{array}{r|r} 3 & 493185 \\ \hline 5 & 164395 \\ \hline 7 & 32879 \\ \hline 7 & 4697 \\ \hline 11 & 671 \\ \hline & 61 \end{array}$$

$3 \times 5 \times 7^2 \times 11 \times 61$. *Ans.*

Exercise 39. Page 99.

Find the prime factors of :

1.

$8.4 = 84 \times 0.1$.

$$\begin{array}{r|r} 2^2 & 84 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

$2^2 \times 3 \times 7 \times 0.1$. *Ans.*

2.

$7.6 = 76 \times 0.1$.

$$\begin{array}{r|r} 2^2 & 76 \\ \hline & 19 \end{array}$$

$2^2 \times 19 \times 0.1$. *Ans.*

3.

$1.08 = 108 \times 0.01$.

$$\begin{array}{r|r} 2^2 & 108 \\ \hline 3^3 & 27 \\ \hline & 1 \end{array}$$

$2^2 \times 3^3 \times 0.01$. *Ans.*

4.

$0.144 = 144 \times 0.001$.

$$\begin{array}{r|r} 2^4 & 144 \\ \hline 3^2 & 9 \\ \hline & 1 \end{array}$$

$2^4 \times 3^2 \times 0.001$. *Ans.*

5.

$0.036 = 36 \times 0.001$.

$$\begin{array}{r|r} 2^2 & 36 \\ \hline 3^2 & 9 \\ \hline & 1 \end{array}$$

$2^2 \times 3^2 \times 0.001$. *Ans.*

6.

$0.037 = 37 \times 0.001$.

$$\begin{array}{r|r} 1 & 37 \\ \hline & 37 \end{array}$$

$1 \times 37 \times 0.001$. *Ans.*

7.

$21.45 = 2145 \times 0.01$.

$$\begin{array}{r|r} 3 & 2145 \\ \hline 5 & 715 \\ \hline 11 & 143 \\ \hline & 13 \end{array}$$

$3 \times 5 \times 11 \times 13 \times 0.01$. *Ans.*

8.

$14.6 = 146 \times 0.1$.

$$\begin{array}{r|r} 2 & 146 \\ \hline & 73 \end{array}$$

$2 \times 73 \times 0.1$. *Ans.*

9.

$2.61 = 261 \times 0.01$.

$$\begin{array}{r|r} 3^2 & 261 \\ \hline & 29 \end{array}$$

$3^2 \times 29 \times 0.01$. *Ans.*

10.

$21.2 = 212 \times 0.1$.

$$\begin{array}{r|r} 2^2 & 212 \\ \hline & 53 \end{array}$$

$2^2 \times 53 \times 0.1$. *Ans.*

11.

$78.54 = 7854 \times 0.01$.

$$\begin{array}{r|r} 2 & 7854 \\ \hline 3 & 3927 \\ \hline 7 & 1309 \\ \hline 11 & 187 \\ \hline & 17 \end{array}$$

$2 \times 3 \times 7 \times 11 \times 17 \times 0.01$. *Ans.*

12.

$0.5236 = 5236 \times 0.0001$.

$$\begin{array}{r|r} 2^2 & 5236 \\ \hline 7 & 1309 \\ \hline 11 & 187 \\ \hline & 17 \end{array}$$

$2^2 \times 7 \times 11 \times 17 \times 0.0001$. *Ans.*

[redacted]

13.

$0.00052 = 52 \times 0.00001.$

$$\begin{array}{r|l} 2^2 & 52 \\ \hline & 13 \end{array}$$

$2^2 \times 13 \times 0.00001.$ *Ans.*

14.

$8.67 = 867 \times 0.01.$

$$\begin{array}{r|l} 3 & 867 \\ \hline 17^2 & 289 \\ \hline & 1 \end{array}$$

$3 \times 17^2 \times 0.01.$ *Ans.*

15.

$48.3 = 483 \times 0.1.$

$$\begin{array}{r|l} 3 & 483 \\ \hline 7 & 161 \\ \hline & 23 \end{array}$$

$3 \times 7 \times 23 \times 0.1.$ *Ans.*

16.

$99.99 = 9999 \times 0.01.$

$$\begin{array}{r|l} 3^2 & 9999 \\ \hline 11 & 1111 \\ \hline & 101 \end{array}$$

$3^2 \times 11 \times 101 \times 0.01.$ *Ans.*

17.

$5.04 = 504 \times 0.01.$

$$\begin{array}{r|l} 2^3 & 504 \\ \hline 3^2 & 63 \\ \hline & 7 \end{array}$$

$2^3 \times 3^2 \times 7 \times 0.01.$ *Ans.*

18.

$1.485 = 1485 \times 0.001.$

$$\begin{array}{r|l} 3^3 & 1485 \\ \hline 5 & 55 \\ \hline & 11 \end{array}$$

$3^3 \times 5 \times 11 \times 0.001.$ *Ans.*

19.

$0.216 = 216 \times 0.001.$

$$\begin{array}{r|l} 2^3 & 216 \\ \hline 3^3 & 27 \\ \hline & 1 \end{array}$$

$2^3 \times 3^3 \times 0.001.$ *Ans.*

20.

$34.87 = 3487 \times 0.01.$

$$\begin{array}{r|l} 11 & 3487 \\ \hline & 317 \end{array}$$

$11 \times 317 \times 0.01.$ *Ans.*

21.

$32.4 = 324 \times 0.1.$

$$\begin{array}{r|l} 2^2 & 324 \\ \hline 3^4 & 81 \\ \hline & 1 \end{array}$$

$2^2 \times 3^4 \times 0.1.$ *Ans.*

22.

$5.115 = 5115 \times 0.001.$

$$\begin{array}{r|l} 3 & 5115 \\ \hline 5 & 1705 \\ \hline 11 & 341 \\ \hline & 31 \end{array}$$

$3 \times 5 \times 11 \times 31 \times 0.001.$ *Ans.*

23.

$71.2 = 712 \times 0.1.$

$$\begin{array}{r|l} 2^3 & 712 \\ \hline & 89 \end{array}$$

$2^3 \times 89 \times 0.1.$ *Ans.*

24.

$2.993 = 2993 \times 0.001.$

$$\begin{array}{r|l} 41 & 2993 \\ \hline & 73 \end{array}$$

$41 \times 73 \times 0.001.$ *Ans.*

Exercise 40. Page 102.

1. Find the G. C. M. of 27 and 33.

$$\begin{array}{r|ll} 3 & 27 & 33 \\ \hline & 9 & 11 \end{array}$$

3. *Ans.*

2. Find the G. C. M. of 13 and 39.

$$\begin{array}{r|ll} 13 & 13 & 39 \\ \hline & 1 & 3 \end{array}$$

13. *Ans.*

[redacted]

3. Find the G. C. M. of 8 and 28.

$$\begin{array}{r|rr} 2^2 & 8 & 28 \\ \hline & 2 & 7 \end{array}$$

$2^2 = 4$. *Ans.*

4. Find the G. C. M. of 27 and 45.

$$\begin{array}{r|rr} 3^2 & 27 & 45 \\ \hline & 3 & 5 \end{array}$$

$3^2 = 9$. *Ans.*

5. Find the G. C. M. of 81 and 108.

$$\begin{array}{r|rr} 3^3 & 81 & 108 \\ \hline & 3 & 4 \end{array}$$

$3^3 = 27$. *Ans.*

6. Find the G. C. M. of 4, 10, 12.

$$\begin{array}{r|rrr} 2 & 4 & 10 & 12 \\ \hline & 2 & 5 & 6 \end{array}$$

2. *Ans.*

7. Find the G. C. M. of 4, 6, 10.

$$\begin{array}{r|rrr} 2 & 4 & 6 & 10 \\ \hline & 2 & 3 & 5 \end{array}$$

2. *Ans.*

8. Find the G. C. M. of 9, 12, 21.

$$\begin{array}{r|rrr} 3 & 9 & 12 & 21 \\ \hline & 3 & 4 & 7 \end{array}$$

3. *Ans.*

9. Find the G. C. M. of 10, 15, 25.

$$\begin{array}{r|rrr} 5 & 10 & 15 & 25 \\ \hline & 2 & 3 & 5 \end{array}$$

5. *Ans.*

10. Find the G. C. M. of 14, 98, 42.

$$\begin{array}{r|rrr} 2 & 14 & 98 & 42 \\ \hline 7 & 7 & 49 & 21 \\ \hline & 1 & 7 & 3 \end{array}$$

$2 \times 7 = 14$. *Ans.*

11. Find the G. C. M. of 30, 18, 54.

$$\begin{array}{r|rrr} 2 & 30 & 18 & 54 \\ \hline 3 & 15 & 9 & 27 \\ \hline & 5 & 3 & 9 \end{array}$$

$2 \times 3 = 6$. *Ans.*

12. Find the G. C. M. of 14, 56, 42.

$$\begin{array}{r|rrr} 2 & 14 & 56 & 42 \\ \hline 7 & 7 & 28 & 21 \\ \hline & 1 & 4 & 3 \end{array}$$

$2 \times 7 = 14$. *Ans.*

13. Find the G. C. M. of 96, 36, 48.

$$\begin{array}{r|rrr} 2^2 & 96 & 36 & 48 \\ \hline 3 & 24 & 9 & 12 \\ \hline & 8 & 3 & 4 \end{array}$$

$2^2 \times 3 = 12$. *Ans.*

14. Find the G.C.M. of 84, 105, 63.

$$\begin{array}{r|rrr} 3 & 84 & 105 & 63 \\ \hline 7 & 28 & 35 & 21 \\ \hline & 4 & 5 & 3 \end{array}$$

$3 \times 7 = 21$. *Ans.*

15. Find the G. C. M. of 24, 60, 84, 128.

$$\begin{array}{r|rrrr} 2^2 & 24 & 60 & 84 & 128 \\ \hline & 6 & 15 & 21 & 32 \end{array}$$

$2^2 = 4$. *Ans.*

16. Find the G. C. M. of 45, 81, 27, 90.

$$\begin{array}{r|rrrr} 3^2 & 45 & 81 & 27 & 90 \\ \hline & 5 & 9 & 3 & 10 \end{array}$$

$3^2 = 9$. *Ans.*

17. Find the G. C. M. of 78, 18, 54, 42.

$$\begin{array}{r|rrrr} 2 & 78 & 18 & 54 & 42 \\ \hline 3 & 39 & 9 & 27 & 21 \\ \hline & 13 & 3 & 9 & 7 \end{array}$$

$2 \times 3 = 6$. *Ans.*

[redacted]

18. Find the G. C. M. of 98, 28, 70, 42.

2	98	28	70	42
7	49	14	35	21
	7	2	5	3

$2 \times 7 = 14$. *Ans.*

19. Find the G. C. M. of 96, 112, 80, 32.

2^4	96	112	80	32
	6	7	5	2

$2^4 = 16$. *Ans.*

20. Find the G. C. M. of 24, 96, 48, 120.

2^3	24	96	48	120
3	3	12	6	15
	1	4	2	5

$2^3 \times 3 = 24$. *Ans.*

21. Find the G. C. M. of 84, 252, 168, 210.

2	84	252	168	210
3	42	126	84	105
7	14	42	28	35
	2	6	4	5

$2 \times 3 \times 7 = 42$. *Ans.*

22. Find the G. C. M. of 33, 88, 77, 55.

11	33	88	77	55
	3	8	7	5

11. *Ans.*

23. Find the G. C. M. of 252, 315, 420, 504.

3	252	315	420	504
7	84	105	140	168
	12	15	20	24

$3 \times 7 = 21$. *Ans.*

24. Find the G. C. M. of 128, 192, 320, 368, 432.

2^4	128	192	320	368	432
	8	12	20	23	27

$2^4 = 16$. *Ans.*

25. Find the G. C. M. of 136, 204, 357, 459.

17	136	204	357	459
	8	12	21	27

17. *Ans.*

26. Find the G. C. M. of 909, 1414, 2323, 4242.

101	909	1414	2323	4242
	9	14	23	42

101. *Ans.*

Exercise 41. Page 104.

1. Find the G. C. M. of 2479 and 3589.

```
  2479)3589(1
       2479
    10|1110
     3| 111
        37)2479(67
           222
            259
            259
```

37. *Ans.*

2. Find the G. C. M. of 3045 and 6195.

```
5|3045    6195
3| 609    1239
   203 )  413(2
          406
           7)203(29
             14
              63
              63
```

$5 \times 3 \times 7 = 105$. *Ans.*

3. Find the G. C. M. of 568 and 712.

```
2³|568    712
   71  )  89(1
          71
        2|18
       3²| 9
           1
```

$2^3 = 8$. *Ans.*

4. Find the G. C. M. of 11,023 and 6493.

```
6493)11023(1
      6493
   10|4530
    3| 453
     151)6493(43
         604
          453
          453
```

151. *Ans.*

5. Find the G. C. M. of 1485 and 2160.

```
5 |1485   2160
3³| 297    432
     11     16
```

$5 \times 3^3 = 135$. *Ans.*

6. Find the G. C. M. of 7040 and 7392.

```
2⁵|7040    7392
10| 220     231
 2|  22
    11)231(21
       22
        11
        11
```

$2^5 \times 11 = 352$. *Ans*

7. Find the G. C. M. of 2760 and 4485.

```
3 |2760   4485
5 | 920   1495
2³| 184    299
    23)299(13
       23
        69
        69
```

$3 \times 5 \times 23 = 345$. *Ans.*

8. Find the G. C. M. of 1177 and 2675.

```
11|1177
   107)2675(25
       214
        535
        535
```

107. *Ans.*

9. Find the G. C. M. of 78,473 and 94,653.

```
78473)94653(1
      78473
   10|16180
    2| 1618
      809)78473(97
          7281
           5663
           5663
```

809. *Ans.*

10. Find the G. C. M. of 35,143 and 10,283.

```
10283)35143(3
      30849
    2| 4294
   19| 2147
       113)10283(91
            1017
             113
113. Ans.    113
```

11. Find the G. C. M. of 44,323 and 61,087.

```
44323)61087(1
      44323
  2²| 16764
  3 |  4191
  11|  1397
      127)44323(349
           381
            622
            508
            1143
127. Ans.   1143
```

12. Find the G. C. M. of 232,353 and 39,699.

```
11 | 39699    232353
3² |  3609     21123
      401   )  2347(5
               2005
            2 | 342
            3²| 171
                19)401(21
                    38
                     21
                     19
                      2)19(9
                        18
11 × 3² = 99. Ans.       1
```

13. Find the G. C. M. of 33,853 and 35,017.

```
33853)35017(1
      33853
  2²|  1164
  3 |   291
        97)33853(349
            291
             475
             388
              873
97. Ans.      873
```

14. Find the G. C. M. of 5115 and 7254.

```
 3| 5115    7254
 5| 1705    2418
11|  341
     31)2418(78
         217
          248
          248
 3 × 31 = 93. Ans.
```

15. Find the G. C. M. of 2268 and 3348.

```
2²| 2268    3348
3²|  567     837
3 |   63      93
      21      31
 2² × 3³ = 108. Ans.
```

16. Find the G. C. M. of 1003 and 2419.

```
1003)2419(2
     2006
   7| 413
      59)1003(17
          59
          413
59. Ans.  413
```

17. Find the G. C. M. of 419 and 52,301.

```
419)52301(124
    419
    1040
     838
     2021
     1676
   3 | 345
   5 | 115
  23 |  23
         1
```

1. *Ans.*

18. Find the G. C. M. of 30,072 and 133,784.

```
2³ | 30072    133784
 7 |  3759     16723
 3 |   537      2389
      179)2389(13
          179
           599
           537
        2 | 62
       31 | 31
        1 | 179
            179
```

$2^3 \times 7 = 56$. *Ans.*

19. Find the G. C. M. of 4257 and 10,836.

```
3² | 4257       10836
11 |  473     7 | 1204
       43  )      172(4
                  172
```

$3^2 \times 43 = 387$. *Ans.*

20. Find the G. C. M. of 17,104 and 27,794.

```
2  | 17104    27794
2³ |  8552    13897
     1069)13897(13
          1069
           3207
           3207
```

$2 \times 1069 = 2138$. *Ans.*

Exercise 42. Page 104.

1. Find the G. C. M. of 855, 1197, 1596.

```
3 | 855       1197       1596
3 | 285        399   4 |  532
5 |  95    7 | 133   7 |  133
     19         19         19
```

$3 \times 19 = 57$. *Ans.*

2. Find the G. C. M. of 3864, 3404, 3657.

```
4 | 3864    3404    3)3657
6 |  966     851      1219
7 |  161
     23)     851      1219
              37        53
```

23. *Ans.*

3. Find the G. C. M. of 15,561, 11,115, 13,585.

```
13 | 11115       13585        15561
 9 |   855   5 |  1045    7 |  1197
 5 |    95  11 |   209    9 |   171
        19          19           19
```

$13 \times 19 = 247$. *Ans.*

4. Find the G. C. M. of 2943, 2616, 4578.

```
3 | 2943      2616        4578
9 |  981   8 | 872   2 |  1526
     109       109   7 |   763
                           109
```

$3 \times 109 = 327$. *Ans.*

[redacted]

5. Find the G. C. M. of 1177, 1391, 1819.

$$\begin{array}{r|l} 11 & 1177 \\ \hline & 107 \end{array}$$

$$\begin{array}{r} 107)1819(17 \\ \underline{107} \\ 749 \\ \underline{749} \end{array}$$

$$\begin{array}{r} 107)1391(13 \\ \underline{107} \\ 321 \\ \underline{321} \end{array}$$

107. *Ans.*

6. Find the G. C. M. of 4939, 1347, 3143.

$$\begin{array}{r|l} 11 & 4939 \\ \hline & 449 \end{array}$$

$$\begin{array}{r} 449)1347(3 \\ \underline{1347} \end{array}$$

$$\begin{array}{r} 449)3143(7 \\ \underline{3143} \end{array}$$

449. *Ans.*

7. Find the G. C. M. of 740, 333, 296.

$$\begin{array}{r|l} 2 & 740 \\ \hline 10 & 370 \\ \hline & 37 \end{array} \qquad \begin{array}{r|l} 9 & 333 \\ \hline & 37 \end{array} \qquad \begin{array}{r|l} 8 & 296 \\ \hline & 37 \end{array}$$

37. *Ans.*

8. Find the G. C. M. of 833, 1785, 1309.

$$\begin{array}{r|l} 7 & 833 \\ \hline & 119 \end{array} \qquad \begin{array}{r|r} 3 & 1785 \\ \hline 5 & 595 \\ \hline & 119 \end{array} \qquad \begin{array}{r|l} 11 & 1309 \\ \hline & 119 \end{array}$$

119. *Ans.*

9. Find the G. C. M. of 4994, 7491, 9988, 12,485, 16,571.

$$\begin{array}{r|l} 2 & 4994 \\ \hline 11 & 2497 \\ \hline & 227 \end{array} \qquad \begin{array}{r|l} 3 & 7491 \\ \hline 11 & 2497 \\ \hline & 227 \end{array} \qquad \begin{array}{r|l} 4 & 9988 \\ \hline 11 & 2497 \\ \hline & 227 \end{array} \qquad \begin{array}{r|r} 5 & 12485 \\ \hline 11 & 2497 \\ \hline & 227 \end{array} \qquad \begin{array}{r} 73 \\ 227)\overline{16571} \\ \underline{1589} \\ 681 \\ \underline{681} \end{array}$$

227. *Ans.*

Exercise 43. Page 107.

1. Find the L. C. M. of 6, 14, 21.

$$\begin{array}{r|ccc} 2 & 6 & 14 & 21 \\ \hline & \cancel{3} & \cancel{7} & 21 \end{array}$$

$2 \times 21 = 42$. *Ans.*

2. Find the L. C. M. of 8, 12, 3, 24.

$\cancel{8} \quad \cancel{12} \quad \cancel{3} \quad 24$

24. *Ans.*

3. Find the L. C. M. of 6, 10, 15.

$$\begin{array}{r|ccc} 2 & 6 & 10 & 15 \\ \hline & \cancel{3} & \cancel{5} & 15 \end{array}$$

$2 \times 15 = 30$. *Ans.*

4. Find the L. C. M. of 9, 12, 18, 4.

$$\begin{array}{r|cccc} 2 & \cancel{9} & 12 & 18 & \cancel{4} \\ \hline 3 & & 6 & 9 & \\ \hline & & 2 & 3 & \end{array}$$

$2^2 \times 3^2 = 36$. *Ans.*

5. Find the L. C. M. of 15, 21, 35.

$$\begin{array}{r|rrr} 3 & 15 & 21 & 35 \\ \hline & \cancel{5} & \cancel{7} & 35 \end{array}$$

$3 \times 35 = 105$. *Ans.*

6. Find the L. C. M. of 12, 20, 24.

$$\begin{array}{r|rrr} 2 & \cancel{12} & 20 & 24 \\ \hline 2 & & 10 & 12 \\ \hline & & 5 & 6 \end{array}$$

$2^2 \times 5 \times 6 = 120$. *Ans.*

7. Find the L. C. M. of 14, 24, 28.

$$\begin{array}{r|rrr} 2^2 & \cancel{14} & 24 & 28 \\ \hline & & 6 & 7 \end{array}$$

$2^2 \times 6 \times 7 = 168$. *Ans.*

8. Find the L. C. M. of 12, 15, 20.

$$\begin{array}{r|rrr} 3 & 12 & 15 & 20 \\ \hline & \cancel{4} & \cancel{5} & 20 \end{array}$$

$3 \times 20 = 60$. *Ans.*

9. Find the L. C. M. of 16, 24, 32.

$$\begin{array}{r|rrr} 2^3 & \cancel{16} & 24 & 32 \\ \hline & & 3 & 4 \end{array}$$

$2^3 \times 3 \times 4 = 96$. *Ans.*

10. Find the L. C. M. of 21, 33, 77.

$$\begin{array}{r|rrr} 3 & 21 & 33 & 77 \\ \hline & \cancel{7} & \cancel{11} & 77 \end{array}$$

$3 \times 77 = 231$. *Ans.*

11. Find the L. C. M. of 27, 33, 99.

$$\begin{array}{r|rrr} 3^2 & 27 & \cancel{33} & 99 \\ \hline & 3 & & 11 \end{array}$$

$3^3 \times 11 = 297$. *Ans.*

12. Find the L.C.M. of 7, 11, 13.

$$\begin{array}{|rrr} 7 & 11 & 13 \\ \hline \end{array}$$

$7 \times 11 \times 13 = 1001$. *Ans.*

13. Find the L. C. M. of 77, 55, 35.

$$\begin{array}{r|rrr} 5 & 77 & 55 & 35 \\ \hline & 77 & \cancel{11} & \cancel{7} \end{array}$$

$5 \times 77 = 385$. *Ans.*

14. Find the L. C. M. of 16, 18, 27, 72.

$$\begin{array}{r|rrrr} 2^3 & 16 & \cancel{18} & 27 & 72 \\ \hline & 2 & & 27 & \cancel{9} \end{array}$$

$2^4 \times 27 = 432$. *Ans.*

15. Find the L. C. M. of 10, 12; 22, 33, 60.

$$\begin{array}{r|rrrrr} 2 & \cancel{10} & \cancel{12} & 22 & 33 & 60 \\ \hline 3 & & & \cancel{11} & 33 & 30 \\ \hline & & & & 11 & 10 \end{array}$$

$2 \times 3 \times 11 \times 10 = 660$. *Ans.*

16. Find the L. C. M. of 15, 16, 18, 20, 22, 24.

$$\begin{array}{r|rrrrrr} 2 & 15 & 16 & 18 & 20 & 22 & 24 \\ \hline 2 & 15 & 8 & 9 & 10 & 11 & 12 \\ \hline 2 & 15 & 4 & 9 & \cancel{5} & 11 & 6 \\ \hline 3 & 15 & 2 & 9 & & 11 & \cancel{3} \\ \hline & 5 & 2 & 3 & & 11 & \end{array}$$

$2^4 \times 3^2 \times 5 \times 11 = 7920$. *Ans.*

17. Find the L. C. M. of 56, 64, 70, 84, 112.

$$\begin{array}{r|rrrrr} 2 & \cancel{56} & 64 & 70 & 84 & 112 \\ \hline 2 & & 32 & 35 & 42 & 56 \\ \hline 2^2 & & 16 & 35 & 21 & 28 \\ \hline 7 & & 4 & 35 & 21 & \cancel{7} \\ \hline & & 4 & 5 & 3 & \end{array}$$

$2^6 \times 3 \times 5 \times 7 = 6720$. *Ans.*

[redacted]

18. Find the L. C. M. of 48, 54, 81, 144, 162.

$$\begin{array}{r|rrrrr} 2 & \cancel{48} & \cancel{54} & \cancel{81} & 144 & 162 \\ \hline 3^2 & & & & 72 & 81 \\ \hline & & & & 8 & 9 \end{array}$$

$2^4 \times 3^4 = 1296$. *Ans.*

19. Find the L. C. M. of 75, 100, 120, 150, 180.

$$\begin{array}{r|rrrrr} 10 & \cancel{75} & 100 & 120 & 150 & 180 \\ \hline 2 & & 10 & 12 & 15 & 18 \\ \hline 3 & & \cancel{5} & 6 & 15 & 9 \\ \hline & & & 2 & 5 & 3 \end{array}$$

$2^3 \times 3^2 \times 5^2 = 1800$. *Ans.*

20. Find the L. C. M. of 112, 168, 196, 224.

$$\begin{array}{r|rrrr} 2^2 & \cancel{112} & 168 & 196 & 224 \\ \hline 2 & & 42 & 49 & 56 \\ \hline 7 & & 21 & 49 & 28 \\ \hline & & 3 & 7 & 4 \end{array}$$

$2^5 \times 3 \times 7^2 = 4704$. *Ans.*

21. Find the L. C. M. of 7, 14, 15, 21, 45.

$$\begin{array}{r|rrrrr} 3 & \cancel{7} & 14 & \cancel{15} & 21 & 45 \\ \hline & & 14 & & \cancel{7} & 15 \end{array}$$

$3 \times 14 \times 15 = 630$. *Ans.*

22. Find the L. C. M. of 16, 25, 81.

$$\begin{array}{|rrr} 16 & 25 & 81 \\ \hline \end{array}$$

$16 \times 25 \times 81 = 32{,}400$. *Ans.*

23. Find the L. C. M. of 26, 39, 52, 65.

$$\begin{array}{r|rrrr} 13 & \cancel{26} & 39 & 52 & 65 \\ \hline & & 3 & 4 & 5 \end{array}$$

$13 \times 3 \times 4 \times 5 = 780$. *Ans.*

24. Find the L. C. M. of 80, 72, 225, 48.

$$\begin{array}{r|rrrr} 2^3 & 80 & 72 & 225 & 48 \\ \hline 2 & 10 & 9 & 225 & 6 \\ \hline & \cancel{5} & \cancel{9} & 225 & \cancel{3} \end{array}$$

$2^4 \times 225 = 3600$. *Ans.*

25. Find the L. C. M. of 10, 20, 30, 40, 50, 60.

$$\begin{array}{r|rrrrrr} 2 & \cancel{10} & \cancel{20} & \cancel{30} & 40 & 50 & 60 \\ \hline 2 & & & & 20 & 25 & 30 \\ \hline 5 & & & & 10 & 25 & 15 \\ \hline & & & & 2 & 5 & 3 \end{array}$$

$2^3 \times 3 \times 5^2 = 600$. *Ans.*

26. Find the L. C. M. of 30, 42, 105, 70.

$$\begin{array}{r|rrrr} 2 & 30 & 42 & 105 & 70 \\ \hline & \cancel{15} & \cancel{21} & 105 & \cancel{35} \end{array}$$

$2 \times 105 = 210$. *Ans.*

27. Find the L. C. M. of 36, 24, 35, 20.

$$\begin{array}{r|rrrr} 2^2 & 36 & 24 & 35 & 20 \\ \hline 3 & 9 & 6 & 35 & \cancel{5} \\ \hline & 3 & 2 & 35 & \end{array}$$

$2^3 \times 3^2 \times 35 = 2520$. *Ans.*

28. Find the L. C. M. of 7, 11, 14, 15.

$$\begin{array}{|rrrr} \cancel{7} & 11 & 14 & 15 \\ \hline \end{array}$$

$11 \times 14 \times 15 = 2310$. *Ans.*

29. Find the L. C. M. of 12, 18, 27, 63, 28.

$$\begin{array}{r|rrrrr} 2 & 12 & 18 & 27 & 63 & 28 \\ \hline 2 & 6 & \cancel{9} & 27 & 63 & 14 \\ \hline 3^2 & \cancel{3} & & 27 & 63 & \cancel{7} \\ \hline & & & 3 & 7 & \end{array}$$

$2^2 \times 3^3 \times 7 = 756$. *Ans.*

30. Find the L. C. M. of 34, 26, 65, 85, 51, 39.

2	34	26	65	85	51	39
5	~~17~~	~~13~~	65	85	51	39
3			~~13~~	~~17~~	51	39
					17	13

$2 \times 3 \times 5 \times 13 \times 17 = 6630$. *Ans.*

31. Find the L. C. M. of 12, 18, 96, 144.

2^3	~~12~~	~~18~~	96	144
2			12	18
3			6	9
			2	3

$2^5 \times 3^2 = 288$. *Ans.*

32. Find the L. C. M. of 84, 156, 63, 99.

2^2	84	156	63	99
3	~~21~~	39	63	99
3		13	21	33
		13	7	11

$2^2 \times 3^2 \times 7 \times 11 \times 13 = 36,036$. *Ans.*

33. Find the L. C. M. of 17, 51, 119, 210.

17	~~17~~	51	119	210
		~~3~~	~~7~~	210

$17 \times 210 = 3570$. *Ans.*

34. Find the L. C. M. of 16, 30, 48, 56, 72.

2	~~16~~	30	48	56	72
2^2		15	24	28	36
3		15	6	7	9
		5	2	7	3

$2^4 \times 3^2 \times 5 \times 7 = 5040$. *Ans.*

35. Find the L. C. M. of 27, 33, 54, 69, 132.

2	27	33	54	69	132
3			27	69	66
			9	23	22

$2 \times 3^3 \times 23 \times 22 = 27,324$. *Ans.*

36. Find the L. C. M. of 15, 26, 39, 65, 180.

2	~~15~~	26	39	65	180
3		~~13~~	39	65	90
5			~~13~~	65	30
				13	6

$2^2 \times 3^2 \times 5 \times 13 = 2340$. *Ans.*

37. Find the L. C. M. of 44, 126, 198, 280, 330.

2	44	126	198	280	330
2	22	63	99	140	165
3	~~11~~	63	99	70	165
7		21	33	70	55
5		~~3~~	33	10	55
			33	2	~~11~~

$2^3 \times 3^2 \times 5 \times 7 \times 11 = 27,720$. *Ans.*

38. Find the L. C. M. of 50, 338, 675, 975.

5	50	338	675	975
5	10	338	135	195
3	2	338	27	39
		338	9	~~13~~

$5^2 \times 3^3 \times 338 = 228,150$. *Ans.*

39. Find the L. C. M. of 552, 575, 920.

2^3	552	575	920
23	69	575	~~115~~
	3	25	

$2^3 \times 23 \times 3 \times 25 = 13,800$. *Ans.*

[redacted]

40. Find the L. C. M. of 228, 304, 342.

$$\begin{array}{r|rrr} 2 & 228 & 304 & 342 \\ \hline 2 & 114 & 152 & 171 \\ \hline 19 & 57 & 76 & 171 \\ \hline & & 4 & 9 \end{array}$$

$2^4 \times 3^2 \times 19 = 2736$. *Ans.*

41. Find the L. C. M. of 1080 and 1260.

$$\begin{array}{r|rr} 10 & 1080 & 1260 \\ \hline 2 & 108 & 126 \\ \hline 3^2 & 54 & 63 \\ \hline & 6 & 7 \end{array}$$

$2^3 \times 3^3 \times 5 \times 7 = 7560$. *Ans.*

42. Find the L. C. M. of 600 and 480.

$$\begin{array}{r|rr} 2^3 & 600 & 480 \\ \hline 3 & 75 & 60 \\ \hline 5 & 25 & 20 \\ \hline & 5 & 4 \end{array}$$

$2^5 \times 3 \times 5^2 = 2400$. *Ans.*

43. Find the L. C. M. of 1564 and 1932.

$$\begin{array}{r|rr} 2^2 & 1564 & 1932 \\ \hline 23 & 391 & 483 \\ \hline & 17 & 21 \end{array}$$

$2^2 \times 23 \times 17 \times 21 = 32{,}844$. *Ans.*

44. Find the L. C. M. of 2530 and 1760.

$$\begin{array}{r|rr} 2 & 2530 & 1760 \\ \hline 5 & 1265 & 880 \\ \hline 11 & 253 & 176 \\ \hline & 23 & 16 \end{array}$$

$2^5 \times 5 \times 11 \times 23 = 40{,}480$. *Ans.*

45. Find the L. C. M. of 936 and 2925.

$$\begin{array}{r|rr} 3^2 & 936 & 2925 \\ \hline 13 & 104 & 325 \\ \hline & 8 & 25 \end{array}$$

$2^3 \times 3^2 \times 5^2 \times 13 = 23{,}400$. *Ans.*

46. Find the L. C. M. of 3432 and 4032.

$$\begin{array}{r|rr} 2^3 & 3432 & 4032 \\ \hline 3 & 429 & 504 \\ \hline & 143 & 168 \end{array}$$

$2^3 \times 3 \times 143 \times 168 = 576{,}576$. *Ans.*

47. Find the L. C. M. of 1875 and 2425.

$$\begin{array}{r|rr} 5^2 & 1875 & 2425 \\ \hline & 75 & 97 \end{array}$$

$5^2 \times 75 \times 97 = 181{,}875$. *Ans.*

48. Find the L. C. M. of 1632 and 2976.

$$\begin{array}{r|rr} 2^3 & 1632 & 2976 \\ \hline 2^2 & 204 & 372 \\ \hline 3 & 51 & 93 \\ \hline & 17 & 31 \end{array}$$

$2^5 \times 3 \times 17 \times 31 = 50{,}592$. *Ans.*

49. Find the L. C. M. of 1001 and 2233.

$$\begin{array}{r|rr} 11 & 1001 & 2233 \\ \hline 7 & 91 & 203 \\ \hline & 13 & 29 \end{array}$$

$7 \times 11 \times 13 \times 29 = 29{,}029$. *Ans.*

50. Find the L. C. M. of 539 and 1463.

$$\begin{array}{r|rr} 7 & 539 & 1463 \\ \hline 11 & 77 & 209 \\ \hline & 7 & 19 \end{array}$$

$7^2 \times 11 \times 19 = 10{,}241$. *Ans.*

Exercise 44. Page 108.

1. Find the L. C. M. of 424 and 583.

```
8 |424
   53)583(11
      53
       53
       53
```

L. C. M. $= 11 \times 424 = 4664$. *Ans.*

2. Find the L. C. M. of 319 and 407.

```
11 |319    407
    29     37
```

L. C. M. $= 29 \times 407 = 11,803$. *Ans.*

3. Find the L. C. M. of 1679 and 1932.

```
4 |1932
3 | 483
7 | 161
     23)1679(73
          161
           69
           69
```

L. C. M. $= 73 \times 1932 = 141,036$. *Ans.*

4. Find the L. C. M. of 1003 and 2419.

```
1003)2419(2
     2006
   7| 413
      59)1003(17
          59
          413
          413
```

L. C. M. $= 17 \times 2419 = 41,123$. *Ans.*

5. Find the L. C. M. of 1003 and 1357.

```
1003)1357(1
     1003
   6| 354
      59)1003(17
          59
          413
          413
```

L. C. M. $= 17 \times 1357 = 23,069$. *Ans.*

6. Find the L. C. M. of 899 and 961.

```
899)961(1
    899
  2| 62
    31)899(29
        62
        279
        279
```

L. C. M. $= 29 \times 961 = 27,869$. *Ans.*

7. Find the L. C. M. of 407, 703, 444.

```
11 |407
    37)703(19
        37
        333
        333
```

L. C. M. $= 11 \times 19 \times 444 = 92,796$. *Ans.*

8. Find the L. C. M. of 411, 959, 2055.

```
~~411~~  959  2055
959)2055(2
    1918
     137)959(7
         959
```

L. C. M. $= 7 \times 2055 = 14,385$. *Ans.*

[redacted]

9. Find the L. C. M. of 221 and 351.

221)351(1
221
10|130
13)221(17
13
91
91

L. C. M. = 17 × 351 = 5967. *Ans.*

10. Find the L. C. M. of 1426 and 989.

2|1426
713)989(1
713
4|276
3|69
23)713(31
69
23
23

L.C.M.=2×31×989=61,318. *Ans.*

11. Find the L. C. M. of 3864, 3404, 3657.

2^3	3864	3404	3657
3	966	851	3657
23	322	851	1219
	14	37	53

$2^3 \times 3 \times 7 \times 23 \times 37 \times 53$
= 7,577,304. *Ans.*

12. Find the L. C. M. of 539 and 253.

11	253	539
	23	49

L. C. M. =23×539=12,397. *Ans.*

13. Find the L.C.M. of 2943, 2616, 4578.

8|2616
327)2943(9
2943

2	2616	2943	4578
327	1308	2943	2289
	4	9	7

2×4×7×9×327=164,808. *Ans.*

14. Find the L. C. M. of 2863 and 1151.

L. C. M. = 1151 × 2863
= 3,295,313. *Ans.*

15. Find the L. C. M. of 1177, 1391, 1819.

107	1177	1391	1819
	11	13	17

11|1177
107)1391(13
107
321
321

L. C. M. = 13 × 17 × 1177
= 260,117. *Ans.*

16. Find the L. C. M. of 5317 and 2863.

7|2863
409)5317(13
409
1227
1227

L. C. M. = 13 × 2863 = 37,219. *Ans.*

17. Find the L. C. M. of 12,703 and 12,879.

L. C. M. = 12,703 × 12,879
= 163,601,937. *Ans.*

18. Find the L. C. M. of 23,309 and 10,753.

L. C. M. $= 10{,}753 \times 23{,}309$
$= 250{,}641{,}677$. *Ans.*

19. Find the L. C. M. of 4939 and 3143.

$$7\,\underline{|3143}$$
$$449)4939(11$$
$$\underline{449}$$
$$449$$
$$\underline{449}$$

L. C. M. $= 11 \times 3143 = 34{,}573$. *Ans.*

20. Find the L. C. M. of 4199 and 6137.

$$4199)6137(1$$
$$4199$$
$$6\,\boxed{1938}$$
$$323)4199(13$$
$$\underline{323}$$
$$969$$
$$\underline{969}$$

L.C.M. $= 13 \times 6137 = 79{,}891$. *Ans.*

Exercise 45. Page 112.

Reduce to a whole or a mixed number:

1. $\frac{13}{7} = 1\frac{6}{7}$.
2. $\frac{21}{8} = 2\frac{5}{8}$.
3. $\frac{25}{4} = 6\frac{1}{4}$.
4. $\frac{107}{11} = 9\frac{8}{11}$.
5. $\frac{91}{9} = 10\frac{1}{9}$.
6. $\frac{63}{7} = 9$.
7. $\frac{72}{8} = 9$.
8. $\frac{44}{5} = 8\frac{4}{5}$.
9. $\frac{9}{2} = 4\frac{1}{2}$.
10. $\frac{19}{3} = 6\frac{1}{3}$.
11. $\frac{50}{10} = 5$.
12. $\frac{51}{16} = 3\frac{3}{16}$.
13. $\frac{45}{15} = 3$.
14. $\frac{45}{13} = 3\frac{6}{13}$.
15. $\frac{62}{5} = 12\frac{2}{5}$.
16. $\frac{55}{6} = 9\frac{1}{6}$.
17. $\frac{44}{4} = 11$.
18. $\frac{112}{19} = 5\frac{17}{19}$.
19. $\frac{213}{16} = 13\frac{5}{16}$.
20. $\frac{242}{22} = 11$.
21. $\frac{374}{25} = 14\frac{24}{25}$.
22. $\frac{481}{13} = 37$.
23. $\frac{299}{23} = 13$.
24. $\frac{529}{29} = 18\frac{7}{29}$.
25. $\frac{787}{43} = 18\frac{13}{43}$.
26. $\frac{8975}{25} = 359$.
27. $\frac{3823}{75} = 50\frac{73}{75}$.
28. $\frac{4684}{177} = 26\frac{82}{177}$.

Exercise 46. Page 113.

Reduce to an improper fraction:

1. $4 = \frac{8}{2}$.
2. $5 = \frac{5}{1}$.
3. $6 = \frac{30}{5}$.
4. $8 = \frac{48}{6}$.
5. $11 = \frac{33}{3}$.
6. $7 = \frac{49}{7}$.
7. $3 = \frac{27}{9}$.
8. $14 = \frac{168}{12}$.
9. $9 = \frac{126}{14}$.
10. $18 = \frac{198}{11}$.
11. $12 = \frac{144}{12}$.
12. $16 = \frac{256}{16}$.

Exercise 47. Page 113.

Reduce to an improper fraction:

1. $3\frac{4}{9} = \frac{31}{9}$.
2. $5\frac{9}{10} = \frac{59}{10}$.
3. $12\frac{4}{11} = \frac{136}{11}$.
4. $8\frac{3}{7} = \frac{59}{7}$.
5. $25\frac{2}{5} = \frac{127}{5}$.
6. $17\frac{1}{3} = \frac{52}{3}$.

[redacted]

7. $8\frac{5}{12} = \frac{101}{12}$.

8. $9\frac{9}{14} = \frac{135}{14}$.

9. $162\frac{8}{11} = \frac{1790}{11}$.

10. $44\frac{6}{7} = \frac{314}{7}$.

11. $2\frac{108}{109} = \frac{326}{109}$.

12. $3\frac{44}{45} = \frac{179}{45}$.

13. $10\frac{7}{19} = \frac{197}{19}$.

14. $12\frac{13}{15} = \frac{193}{15}$.

15. $84\frac{17}{20} = \frac{1697}{20}$.

16. $16\frac{18}{23} = \frac{386}{23}$.

17. $17\frac{7}{12} = \frac{211}{12}$.

18. $19\frac{5}{9} = \frac{176}{9}$.

19. $14\frac{17}{19} = \frac{283}{19}$.

20. $21\frac{17}{100} = \frac{2117}{100}$.

21. $6\frac{41}{50} = \frac{341}{50}$.

22. $16\frac{8}{25} = \frac{408}{25}$.

23. $11\frac{19}{24} = \frac{283}{24}$.

24. $8\frac{9}{16} = \frac{137}{16}$.

25. $12\frac{3}{4} = \frac{51}{4}$.

26. $27\frac{1}{2} = \frac{55}{2}$.

27. $111\frac{2}{3} = \frac{335}{3}$.

28. $36\frac{88}{85} = \frac{3148}{85}$.

29. $11\frac{109}{1000} = \frac{11109}{1000}$.

30. $3\frac{71}{75} = \frac{296}{75}$.

31. $16\frac{11}{12} = \frac{203}{12}$.

32. $15\frac{1}{42} = \frac{631}{42}$.

33. $108\frac{6}{25} = \frac{2706}{25}$.

34. $51\frac{3}{17} = \frac{870}{17}$.

35. $40\frac{16}{19} = \frac{776}{19}$.

36. $864\frac{13}{97} = \frac{83821}{97}$.

Exercise 48. Page 115.

Reduce to lowest terms :

1. $\frac{120}{192} = \frac{15}{24} = \frac{5}{8}$. *Ans.*

2. $\frac{105}{135} = \frac{21}{27} = \frac{7}{9}$. *Ans.*

3. $\frac{928}{1520} = \frac{116}{165}$. *Ans.*

4. $\frac{1728}{2448} = \frac{216}{306} = \frac{108}{153} = \frac{12}{17}$. *Ans.*

5. $\frac{1296}{6561} = \frac{144}{729} = \frac{16}{81}$. *Ans.*

6. $\frac{2310}{3080} = \frac{231}{308} = \frac{21}{28} = \frac{3}{4}$. *Ans.*

7. $\frac{1848}{2352} = \frac{231}{294} = \frac{77}{98} = \frac{11}{14}$. *Ans.*

8. $\frac{3960}{12672} = \frac{495}{1584} = \frac{55}{176} = \frac{5}{16}$. *Ans.*

9. $\frac{1848}{3003} = \frac{616}{1001} = \frac{88}{143} = \frac{8}{13}$. *Ans.*

10. $\frac{924}{1092} = \frac{231}{273} = \frac{77}{91} = \frac{11}{13}$. *Ans.*

11. $\frac{2640}{2970} = \frac{88}{99} = \frac{8}{9}$. *Ans.*

12. $\frac{324}{1092} = \frac{81}{273} = \frac{27}{91}$. *Ans.*

13. $\frac{6732}{9108} = \frac{1683}{2277} = \frac{153}{207} = \frac{17}{23}$. *Ans.*

14. $\frac{6840}{27360} = \frac{171}{684} = \frac{19}{76} = \frac{1}{4}$. *Ans.*

15. $\frac{5760}{7000} = \frac{576}{700} = \frac{144}{175}$. *Ans.*

16. $\frac{875}{10000} = \frac{7}{80}$. *Ans.*

17. $\frac{2208}{4140} = \frac{552}{1035} = \frac{184}{345} = \frac{8}{15}$. *Ans.*

18. $\frac{1015}{1566} = \frac{35}{54}$. *Ans.*

19. $\frac{516}{2107} = \frac{12}{49}$. *Ans.*

20. $\frac{3872}{92807} = \frac{352}{8437} = \frac{32}{767}$. *Ans.*

21. Reduce to lowest terms $\frac{78473}{94653}$.

```
78473)94653(1
      78473
   10|16180
    2| 1618
       809)78473(97
           7281
            5663
            5663
```

G. C. M. = 809.

$\frac{78473}{94653} = \frac{97}{117}$. *Ans.*

22. Reduce to lowest terms $\frac{17596}{26145}$.

```
4|17596
  4399)26145(5
       21995
    10|4150
     5| 415
        83)4399(53
           415
            249
            249
```

G. C. M. = 83.

$\frac{17596}{26145} = \frac{212}{315}$. *Ans.*

23. Reduce to lowest terms $\frac{44323}{61087}$.

$$
\begin{array}{l}
44323)61087(1 \\
\quad 44323 \\
4\,|\,16764 \\
3\,|\,4191 \\
11\,|\,1397 \\
\quad 127)44323(349 \\
\qquad 381 \\
\qquad \overline{622} \\
\qquad 508 \\
\qquad \overline{1143} \\
\qquad 1143
\end{array}
$$

G. C. M. = 127.

$\frac{44323}{61087} = \frac{349}{481}$. *Ans.*

24. Reduce to lowest terms $\frac{339}{1243}$.

$$
\begin{array}{l}
3\,|\,339 \\
\quad 113)1243(11 \\
\qquad 1243
\end{array}
$$

G. C. M. = 113.

$\frac{339}{1243} = \frac{3}{11}$. *Ans.*

25. Reduce to lowest terms $\frac{1177}{2675}$.

$$
\begin{array}{l}
11\,|\,1177 \\
\quad 107)2675(25 \\
\qquad 214 \\
\qquad \overline{535} \\
\qquad 535
\end{array}
$$

G. C. M. = 107.

$\frac{1177}{2675} = \frac{11}{25}$. *Ans.*

26. Reduce to lowest terms $\frac{19075}{25615}$.

$\frac{19075}{25615} = \frac{3815}{5123} = \frac{35}{47}$. *Ans.*

$$
\begin{array}{l}
5\,|\,3815 \\
7\,|\,763 \\
\quad 109)5123(47 \\
\qquad 436 \\
\qquad \overline{763} \\
\qquad 763
\end{array}
$$

G.C.M. = 109.

27. Reduce to lowest terms $\frac{14141}{16289}$.

$$
\begin{array}{l}
14141)16289(1 \\
\quad 14141 \\
12\,|\,2148 \\
\quad 179)14141(79 \\
\qquad 1253 \\
\qquad \overline{1611} \\
\qquad 1611
\end{array}
$$

G. C. M. = 179.

$\frac{14141}{16289} = \frac{79}{91}$. *Ans.*

28. Reduce to lowest terms $\frac{428571}{999999}$.

$\frac{428571}{999999} = \frac{47619}{111111} = \frac{15873}{37037} = \frac{1443}{3367}$
$= \frac{111}{259} = \frac{3}{7}$. *Ans.*

Exercise 49. Page 115.

1. Reduce $\frac{3}{4}$ to 20ths.

$$\frac{3}{4} = \frac{5 \times 3}{5 \times 4} = \frac{15}{20}.$$

2. Reduce $\frac{2}{3}$ to 24ths.

$$\frac{2}{3} = \frac{8 \times 2}{8 \times 3} = \frac{16}{24}.$$

3. Reduce $\frac{3}{5}$ to 50ths.

$$\frac{3}{5} = \frac{10 \times 3}{10 \times 5} = \frac{30}{50}.$$

4. Reduce $\frac{7}{13}$ to 39ths.

$$\frac{7}{13} = \frac{3 \times 7}{3 \times 13} = \frac{21}{39}.$$

5. Reduce $\frac{5}{18}$ to 90ths.

$$\frac{5}{18} = \frac{5 \times 5}{5 \times 18} = \frac{25}{90}.$$

6. Reduce $\frac{2}{9}$ to 108ths.

$$\frac{2}{9} = \frac{12 \times 2}{12 \times 9} = \frac{24}{108}.$$

[redacted]

7. Reduce $\frac{3}{16}$ to 144ths.

$$\frac{3}{16}=\frac{9\times3}{9\times16}=\frac{27}{144}.$$

8. Reduce $\frac{7}{18}$ to 144ths.

$$\frac{7}{18}=\frac{8\times7}{8\times18}=\frac{56}{144}.$$

9. Reduce $\frac{7}{12}$ to 156ths.

$$\frac{7}{12}=\frac{13\times7}{13\times12}=\frac{91}{156}.$$

Exercise 50. Page 116.

1. Find the product of $\frac{3}{4}\times2$.

$$\frac{3}{\underset{2}{\cancel{4}}}\times\cancel{2}=\frac{3}{2}=1\tfrac{1}{2}.\ \textit{Ans.}$$

2. Find the product of $\frac{3}{4}\times9$.

$$\frac{3}{4}\times9=\frac{27}{4}=6\tfrac{3}{4}.\ \textit{Ans.}$$

3. Find the product of $10\times\frac{2}{5}$.

$$\overset{2}{\cancel{10}}\times\frac{2}{\cancel{5}}=4.\ \textit{Ans.}$$

4. Find the product of $15\times\frac{3}{3}$.

$$15\times\frac{\cancel{3}}{\cancel{3}}=15.\ \textit{Ans.}$$

5. Find the product of $\frac{9}{21}\times7$.

$$\frac{\overset{3}{\cancel{9}}}{\underset{3}{\cancel{21}}}\times\cancel{7}=3.\ \textit{Ans.}$$

6. Find the product of $16\times\frac{5}{8}$.

$$\overset{2}{\cancel{16}}\times\frac{5}{\cancel{8}}=10.\ \textit{Ans.}$$

7. Find the product of $\frac{5}{8}\times2$.

$$\frac{5}{\underset{4}{\cancel{8}}}\times\cancel{2}=\frac{5}{4}=1\tfrac{1}{4}.\ \textit{Ans.}$$

8. Find the product of $\frac{2}{15}\times5$.

$$\frac{2}{\underset{3}{\cancel{15}}}\times\cancel{5}=\tfrac{2}{3}.\ \textit{Ans.}$$

9. Find the product of $27\times\frac{5}{9}$.

$$\overset{3}{\cancel{27}}\times\frac{5}{\cancel{9}}=15.\ \textit{Ans.}$$

10. Find the product of $\frac{13}{20}\times2$.

$$\frac{13}{\underset{10}{\cancel{20}}}\times\cancel{2}=\frac{13}{10}=1\tfrac{3}{10}.\ \textit{Ans.}$$

11. Find the product of $\frac{13}{20}\times3$.

$$\frac{13}{20}\times3=\frac{39}{20}=1\tfrac{19}{20}.\ \textit{Ans.}$$

12. Find the product of $\frac{13}{20}\times4$.

$$\frac{13}{\underset{5}{\cancel{20}}}\times\cancel{4}=\frac{13}{5}=2\tfrac{3}{5}.\ \textit{Ans.}$$

13. Find the product of $5\times\frac{13}{20}$.

$$\cancel{5}\times\frac{13}{\underset{4}{\cancel{20}}}=\frac{13}{4}=3\tfrac{1}{4}.\ \textit{Ans.}$$

14. Find the product of $6\times\frac{13}{20}$.

$$\overset{3}{\cancel{6}}\times\frac{13}{\underset{10}{\cancel{20}}}=\frac{39}{10}=3\tfrac{9}{10}.\ \textit{Ans.}$$

[redacted]

15. Find the product of $7 \times \frac{13}{20}$.

$7 \times \frac{13}{20} = \frac{91}{20} = 4\frac{11}{20}$. *Ans.*

16. Find the product of $8 \times \frac{13}{20}$.

$\overset{2}{\cancel{8}} \times \frac{13}{\underset{5}{\cancel{20}}} = \frac{26}{5} = 5\frac{1}{5}$. *Ans.*

17. Find the product of $\frac{13}{20} \times 10$.

$\frac{13}{\underset{2}{\cancel{20}}} \times \cancel{10} = \frac{13}{2} = 6\frac{1}{2}$. *Ans.*

18. Find the product of $\frac{13}{20} \times 12$.

$\frac{13}{\underset{5}{\cancel{20}}} \times \overset{3}{\cancel{12}} = \frac{39}{5} = 7\frac{4}{5}$. *Ans.*

19. Find the product of $\frac{13}{20} \times 15$.

$\frac{13}{\underset{4}{\cancel{20}}} \times \overset{3}{\cancel{15}} = \frac{39}{4} = 9\frac{3}{4}$. *Ans.*

20. Find the product of $\frac{13}{20} \times 20$.

$\frac{13}{\cancel{20}} \times \cancel{20} = 13$. *Ans.*

21. Find the product of $\frac{5}{36}$ of 324.

$\frac{5}{\cancel{36}}$ of $\overset{9}{\cancel{324}} = 45$. *Ans.*

22. Find the product of $\frac{7}{13}$ of 273.

$\frac{7}{\cancel{13}}$ of $\overset{21}{\cancel{273}} = 147$. *Ans.*

23. Find the product of $\frac{10}{11}$ of 242.

$\frac{10}{\cancel{11}}$ of $\overset{22}{\cancel{242}} = 220$. *Ans.*

24. Find the product of $340 \times \frac{8}{17}$.

$\overset{20}{\cancel{340}} \times \frac{8}{\cancel{17}} = 160$. *Ans.*

25. Find the product of $450 \times \frac{7}{10}$.

$\overset{45}{\cancel{450}} \times \frac{7}{\cancel{10}} = 315$. *Ans.*

26. Find the product of $\frac{6}{100} \times 1000$.

$\frac{6}{\cancel{100}} \times \overset{10}{\cancel{1000}} = 60$. *Ans.*

27. Find the product of $\frac{9}{50} \times 210$.

$\frac{9}{\underset{5}{\cancel{50}}} \times \overset{21}{\cancel{210}} = \frac{189}{5} = 37\frac{4}{5}$. *Ans.*

28. Find the product of $\frac{12}{25} \times 90$.

$\frac{12}{\underset{5}{\cancel{25}}} \times \overset{18}{\cancel{90}} = \frac{216}{5} = 43\frac{1}{5}$. *Ans.*

29. Find the product of $\frac{5}{7}$ of 434.

$\frac{5}{\cancel{7}}$ of $\overset{62}{\cancel{434}} = 310$. *Ans.*

30. Find the product of $468 \times \frac{11}{9}$.

$\overset{52}{\cancel{468}} \times \frac{11}{\cancel{9}} = 572$. *Ans.*

31. Find the product of $30 \times \frac{12}{11}$.

$30 \times \frac{12}{11} = \frac{360}{11} = 32\frac{8}{11}$. *Ans.*

32. Find the product of $100 \times \frac{16}{15}$.

$\overset{20}{\cancel{100}} \times \frac{16}{\underset{3}{\cancel{15}}} = \frac{320}{3} = 106\frac{2}{3}$. *Ans.*

[redacted]

33. Find the product of $\frac{25}{12} \times 54$.

$$\frac{25}{\underset{2}{\cancel{12}}} \times \overset{9}{\cancel{54}} = \frac{225}{2} = 112\tfrac{1}{2}. \text{ Ans.}$$

34. Find the product of $\frac{21}{32} \times 48$.

$$\frac{21}{\underset{2}{\cancel{32}}} \times \overset{3}{\cancel{48}} = \frac{63}{2} = 31\tfrac{1}{2}. \text{ Ans.}$$

35. Find the product of $72 \times \frac{19}{16}$.

$$\overset{9}{\cancel{72}} \times \frac{19}{\underset{2}{\cancel{16}}} = \frac{171}{2} = 85\tfrac{1}{2}. \text{ Ans.}$$

36. Find the product of $\frac{15}{32}$ of 128.

$$\frac{15}{\cancel{32}} \text{ of } \overset{4}{\cancel{128}} = 60. \text{ Ans.}$$

Exercise 51. Page 118.

Find the product of:

1. $\frac{2}{3}$ of $\frac{7}{11}$.

$$\tfrac{2}{3} \text{ of } \tfrac{7}{11} = \tfrac{14}{33}. \text{ Ans.}$$

2. $\frac{3}{7}$ of $2\frac{1}{10}$.

$$\frac{3}{7} \text{ of } 2\tfrac{1}{10} = \frac{3}{\cancel{7}} \times \frac{\overset{3}{\cancel{21}}}{10} = \frac{9}{10}. \text{ Ans.}$$

3. $\frac{3}{7}$ of $\frac{5}{9}$.

$$\frac{\cancel{3}}{7} \text{ of } \frac{5}{\underset{3}{\cancel{9}}} = \frac{5}{21}. \text{ Ans.}$$

4. $2\frac{2}{5} \times 2\frac{1}{2}$.

$$2\tfrac{2}{5} \times 2\tfrac{1}{2} = \frac{\overset{6}{\cancel{12}}}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{2}} = 6. \text{ Ans.}$$

5. $4\frac{4}{5} \times 2\frac{1}{7}$.

$$4\tfrac{4}{5} \times 2\tfrac{1}{7} = \frac{24}{\cancel{5}} \times \frac{\overset{3}{\cancel{15}}}{7} = \frac{72}{7} = 10\tfrac{2}{7}. \text{ Ans.}$$

6. $4\frac{5}{6} \times 9\frac{1}{3}$.

$$4\tfrac{5}{6} \times 9\tfrac{1}{3} = \frac{29}{\underset{3}{\cancel{6}}} \times \frac{\overset{14}{\cancel{28}}}{3} = \frac{406}{9} = 45\tfrac{1}{9}. \text{ Ans.}$$

7. $\frac{1}{3}$ of $\frac{3}{5}$ of 10.

$$\frac{1}{\cancel{3}} \text{ of } \frac{\cancel{3}}{\cancel{5}} \text{ of } \overset{2}{\cancel{10}} = 2. \text{ Ans.}$$

8. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{2}{3}$.

$$\frac{\cancel{2}}{\cancel{3}} \text{ of } \frac{\cancel{3}}{\underset{\cancel{2}}{\cancel{4}}} \text{ of } \frac{2}{3} = \frac{1}{3}. \text{ Ans.}$$

9. $\frac{4}{5} \times \frac{5}{6} \times \frac{3}{7} \times 4\frac{1}{5}$.

$$\frac{4}{5} \times \frac{5}{6} \times \frac{3}{7} \times 4\tfrac{1}{5} = \frac{\overset{2}{\cancel{4}}}{\cancel{5}} \times \frac{\cancel{5}}{\underset{\cancel{2}}{\cancel{6}}} \times \frac{\cancel{3}}{\cancel{7}} \times \frac{\overset{3}{\cancel{21}}}{5} = \frac{6}{5} = 1\tfrac{1}{5}. \text{ Ans.}$$

10. $\frac{5}{6} \times 4\frac{1}{2}$.

$$\frac{5}{6}\times 4\frac{1}{2} = \frac{5}{\underset{2}{\cancel{6}}}\times\frac{\overset{3}{\cancel{9}}}{2} = \frac{15}{4} = 3\frac{3}{4}.\ \textit{Ans.}$$

11. $\frac{8}{9}$ of $\frac{9}{10}$ of $\frac{5}{7}$ of $\frac{3}{4}$ of $\frac{1}{5}$ of $15\frac{3}{4}$.

$$\frac{8}{9} \text{ of } \frac{9}{10} \text{ of } \frac{5}{7} \text{ of } \frac{3}{4} \text{ of } \frac{1}{5} \text{ of } 15\frac{3}{4} = \frac{\overset{2}{\cancel{8}}}{\cancel{9}}\times\frac{\cancel{9}}{\underset{5}{\cancel{10}}}\times\frac{\cancel{5}}{\cancel{7}}\times\frac{3}{\cancel{4}}\times\frac{1}{\cancel{5}}\times\frac{\overset{9}{\cancel{63}}}{4} = \frac{27}{20} = 1\frac{7}{20}.\ \textit{Ans.}$$

12. $5\frac{3}{4} \times 8\frac{2}{5}$.

$$5\frac{3}{4}\times 8\frac{2}{5} = \frac{23}{\underset{2}{\cancel{4}}}\times\frac{\overset{21}{\cancel{42}}}{5} = \frac{483}{10} = 48\frac{3}{10}.\ \textit{Ans.}$$

13. $\frac{2}{3} \times \frac{4}{7} \times \frac{7}{15} \times 7\frac{1}{2}$.

$$\frac{2}{3}\times\frac{4}{7}\times\frac{7}{15}\times 7\frac{1}{2} = \frac{\cancel{2}}{3}\times\frac{4}{\cancel{7}}\times\frac{\cancel{7}}{\cancel{15}}\times\frac{\cancel{15}}{\cancel{2}} = \frac{4}{3} = 1\frac{1}{3}.\ \textit{Ans.}$$

14. $\frac{3}{5}$ of $\frac{10}{27}$ of $\frac{9}{20}$ of $8\frac{1}{3}$.

$$\frac{3}{5} \text{ of } \frac{10}{27} \text{ of } \frac{9}{20} \text{ of } 8\frac{1}{3} = \frac{\cancel{3}}{\cancel{5}}\times\frac{\cancel{10}}{\underset{3}{\cancel{27}}}\times\frac{\cancel{9}}{\underset{2}{\cancel{20}}}\times\frac{\overset{5}{\cancel{25}}}{\cancel{3}} = \frac{5}{6}.\ \textit{Ans.}$$

15. $\frac{8}{11} \times \frac{20}{21} \times \frac{35}{48} \times 2\frac{10}{19}$.

$$\frac{8}{11}\times\frac{20}{21}\times\frac{35}{48}\times 2\frac{10}{19} = \frac{8}{11}\times\frac{20}{\underset{3}{\cancel{21}}}\times\frac{\overset{5}{\cancel{35}}}{\cancel{48}}\times\frac{\cancel{48}}{19} = \frac{800}{627} = 1\frac{173}{627}.\ \textit{Ans.}$$

16. $\frac{42}{43} \times \frac{13}{105} \times 1\frac{7}{308}$.

$$\frac{42}{43}\times\frac{13}{105}\times 1\frac{7}{308} = \frac{\overset{\cancel{21}}{\cancel{42}}}{43}\times\frac{13}{\underset{\cancel{21}}{\cancel{105}}}\times\frac{\overset{\overset{9}{\cancel{63}}}{\cancel{315}}}{\underset{\underset{22}{\cancel{154}}}{\cancel{308}}} = \frac{117}{946}.\ \textit{Ans.}$$

17. $\frac{5}{8} \times \frac{120}{121} \times \frac{66}{85} \times 17$.

$$\frac{\cancel{5}}{\cancel{8}}\times\frac{\overset{15}{\cancel{120}}}{\underset{11}{\cancel{121}}}\times\frac{\overset{6}{\cancel{66}}}{\underset{\cancel{17}}{\cancel{85}}}\times\cancel{17} = \frac{90}{11} = 8\frac{2}{11}.\ \textit{Ans.}$$

18. $\frac{38}{39} \times \frac{52}{57} \times \frac{69}{86} \times 1\frac{20}{23}$.

$$\frac{38}{39}\times\frac{52}{57}\times\frac{69}{86}\times 1\frac{20}{23} = \frac{\overset{\cancel{19}}{\cancel{38}}}{\underset{\cancel{13}}{\cancel{39}}}\times\frac{\overset{4}{\cancel{52}}}{\underset{3}{\cancel{57}}}\times\frac{\overset{\cancel{3}}{\cancel{69}}}{\underset{\cancel{2}}{\cancel{86}}}\times\frac{\cancel{43}}{\cancel{23}} = \frac{4}{3} = 1\frac{1}{3}.\ \textit{Ans.}$$

[redacted]

19. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$ of $\frac{9}{10}$ of 10.

$$\frac{1}{\cancel{2}} \text{ of } \frac{\cancel{2}}{\cancel{3}} \text{ of } \frac{\cancel{3}}{4} \text{ of } \frac{5}{\cancel{6}} \text{ of } \frac{\cancel{6}}{\cancel{7}} \text{ of } \frac{\cancel{7}}{\cancel{8}} \text{ of } \frac{\cancel{8}}{\cancel{9}} \text{ of } \frac{\cancel{9}}{\cancel{10}} \text{ of } \cancel{10} = \frac{5}{4} = 1\tfrac{1}{4}. \textit{ Ans.}$$

20. $\frac{7}{25}$ of $\frac{8}{11}$ of 30.

$$\frac{7}{\underset{5}{\cancel{25}}} \text{ of } \frac{8}{11} \text{ of } \overset{6}{\cancel{30}} = \frac{336}{55} = 6\tfrac{6}{55}. \textit{ Ans.}$$

21. $\frac{113}{355} \times \frac{85}{226} \times \frac{12}{35} \times 1\frac{3}{4}$.

$$\frac{113}{355} \times \frac{85}{226} \times \frac{12}{35} \times 1\tfrac{3}{4} = \frac{\cancel{113}}{\underset{71}{\cancel{355}}} \times \frac{\overset{17}{\cancel{85}}}{\underset{2}{\cancel{226}}} \times \frac{\overset{\overset{3}{\cancel{6}}}{\cancel{12}}}{\underset{5}{\cancel{35}}} \times \frac{\cancel{7}}{\underset{2}{\cancel{4}}} = \frac{51}{710}. \textit{ Ans.}$$

22. $\frac{7}{8} \times \frac{3}{4} \times \frac{8}{11} \times \frac{4}{5}$ of $\frac{5}{8}$ of $\frac{3}{4}$ of 8.

$$\frac{7}{\cancel{8}} \times \frac{3}{\cancel{4}} \times \frac{\cancel{8}}{11} \times \frac{\cancel{4}}{\cancel{5}} \text{ of } \frac{\cancel{5}}{\cancel{8}} \text{ of } \frac{3}{4} \text{ of } \cancel{8} = \frac{63}{44} = 1\tfrac{19}{44}. \textit{ Ans.}$$

23. $\frac{2}{13}$ of $\frac{39}{40}$ of $\frac{52}{117}$.

$$\frac{\cancel{2}}{\cancel{13}} \text{ of } \frac{\overset{\cancel{3}}{\cancel{39}}}{\underset{\underset{5}{\cancel{20}}}{\cancel{40}}} \text{ of } \frac{\overset{\cancel{13}}{\cancel{52}}}{\underset{\underset{3}{\cancel{9}}}{\cancel{117}}} = \frac{1}{15}. \textit{ Ans.}$$

24. $\frac{9}{11} \times \frac{7}{12} \times \frac{22}{63} \times 48$.

$$\frac{\cancel{9}}{\cancel{11}} \times \frac{\cancel{7}}{\underset{\cancel{6}}{\cancel{12}}} \times \frac{\overset{\cancel{2}}{\cancel{22}}}{\underset{\cancel{7}}{\cancel{63}}} \times \overset{8}{\cancel{48}} = 8. \textit{ Ans.}$$

25. $\frac{39}{40}$ of $\frac{7}{65}$ of $\frac{25}{42}$ of 12.

$$\frac{\overset{\cancel{3}}{\cancel{39}}}{\underset{\underset{4}{\cancel{8}}}{\cancel{40}}} \text{ of } \frac{\cancel{7}}{\underset{\cancel{5}}{\cancel{65}}} \text{ of } \frac{\overset{\cancel{5}}{\cancel{25}}}{\underset{\underset{\cancel{2}}{\cancel{14}}}{\cancel{42}}} \text{ of } \overset{\overset{3}{\cancel{6}}}{\cancel{12}} = \frac{3}{4}. \textit{ Ans.}$$

26. $1\frac{2}{3} \times 4\frac{1}{2} \times \frac{3}{8}$.

$$1\tfrac{2}{3} \times 4\tfrac{1}{2} \times \frac{3}{8} = \frac{5}{\cancel{3}} \times \frac{9}{2} \times \frac{\cancel{3}}{8} = \frac{45}{16} = 2\tfrac{13}{16}. \textit{ Ans.}$$

27. $2\frac{4}{9} \times 1\frac{3}{7} \times 1\frac{13}{15} \times 8$.

$$2\tfrac{4}{9} \times 1\tfrac{3}{7} \times 1\tfrac{13}{15} \times 8 = \frac{22}{9} \times \frac{\overset{2}{\cancel{10}}}{\cancel{7}} \times \frac{\overset{4}{\cancel{28}}}{\underset{3}{\cancel{15}}} \times 8 = \frac{1408}{27} = 52\tfrac{4}{27}. \textit{ Ans.}$$

28. $3\frac{5}{7} \times 2\frac{1}{2}$ of $1\frac{8}{13} \times 1\frac{4}{11}$.

$$3\tfrac{5}{7} \times 2\tfrac{1}{2} \text{ of } 1\tfrac{8}{13} \times 1\tfrac{4}{11} = \frac{\overset{\cancel{2}}{\cancel{26}}}{\cancel{7}} \times \frac{5}{\cancel{2}} \times \frac{\overset{3}{\cancel{21}}}{\cancel{13}} \times \frac{15}{11} = \frac{225}{11} = 20\tfrac{5}{11}. \quad \textit{Ans.}$$

29. $\frac{11}{12} \times 5\frac{1}{3} \times 4\frac{1}{8} \times \frac{7}{22} \times 5$.

$$\frac{11}{12} \times 5\tfrac{1}{3} \times 4\tfrac{1}{8} \times \frac{7}{22} \times 5 = \frac{\cancel{11}}{12} \times \frac{\overset{\cancel{2}}{\cancel{16}}}{\cancel{3}} \times \frac{\overset{11}{\cancel{33}}}{\cancel{8}} \times \frac{7}{\underset{\cancel{2}}{\cancel{22}}} \times 5 = \frac{385}{12} = 32\tfrac{1}{12}. \quad \textit{Ans.}$$

30. $\frac{2}{5}$ of $\frac{7}{15} \times 8\frac{2}{7} \times \frac{6}{29}$ of $1\frac{17}{18}$.

$$\frac{2}{5} \text{ of } \frac{7}{15} \times 8\tfrac{2}{7} \times \frac{6}{29} \text{ of } 1\tfrac{17}{18} = \frac{\cancel{2}}{\cancel{5}} \times \frac{\cancel{7}}{\underset{5}{\cancel{15}}} \times \frac{\overset{2}{\cancel{58}}}{\cancel{7}} \times \frac{\overset{2}{\cancel{6}}}{\cancel{29}} \times \frac{\overset{7}{\cancel{35}}}{\underset{9}{\cancel{18}}} = \frac{28}{45}. \quad \textit{Ans.}$$

31. $\frac{17}{82} \times \frac{27}{38} \times \frac{123}{153}$.

$$\frac{\cancel{17}}{\underset{2}{\cancel{82}}} \times \frac{\overset{9}{\cancel{27}}}{38} \times \frac{\overset{\cancel{41}}{\cancel{123}}}{\underset{\underset{\cancel{17}}{\cancel{51}}}{\cancel{153}}} = \frac{9}{76}. \quad \textit{Ans.}$$

32. $\frac{292}{465} \times \frac{573}{657} \times \frac{248}{1719}$.

$$\frac{\overset{4}{\cancel{292}}}{\underset{\underset{5}{\cancel{155}}}{\cancel{465}}} \times \frac{\overset{\cancel{191}}{\cancel{573}}}{\underset{9}{\cancel{657}}} \times \frac{\overset{8}{\cancel{248}}}{\underset{9}{\cancel{1719}}} = \frac{32}{405}. \quad \textit{Ans.}$$

33. $\frac{1205}{2763}$ of $\frac{712}{2169}$ of $\frac{1535}{1068}$.

$$\frac{\overset{5}{\cancel{1205}}}{\underset{9}{\cancel{2763}}} \text{ of } \frac{\overset{\overset{2}{\cancel{8}}}{\cancel{712}}}{\underset{9}{\cancel{2169}}} \text{ of } \frac{\overset{5}{\cancel{1535}}}{\underset{\underset{3}{\cancel{12}}}{\cancel{1068}}} = \frac{50}{243}. \quad \textit{Ans.}$$

34. $\frac{4}{25} \times 7\frac{1}{7} \times 6\frac{2}{3} \times \frac{21}{80}$.

$$\frac{4}{25} \times 7\tfrac{1}{7} \times 6\tfrac{2}{3} \times \frac{21}{80} = \frac{\cancel{4}}{\cancel{25}} \times \frac{\overset{2}{\cancel{50}}}{\cancel{7}} \times \frac{\cancel{20}}{\cancel{3}} \times \frac{\overset{\cancel{3}}{\cancel{21}}}{\underset{\cancel{4}}{\cancel{80}}} = 2. \quad \textit{Ans.}$$

35. $12\frac{1}{2} \times \frac{8}{15} \times 16\frac{2}{3} \times \frac{9}{50}$.

$$12\tfrac{1}{2} \times \frac{8}{15} \times 16\tfrac{2}{3} \times \frac{9}{50} = \frac{\overset{5}{\cancel{25}}}{\cancel{2}} \times \frac{\overset{4}{\cancel{8}}}{\underset{\cancel{5}}{\cancel{15}}} \times \frac{\cancel{50}}{\cancel{3}} \times \frac{\overset{\cancel{3}}{\cancel{9}}}{\cancel{50}} = 20. \quad \textit{Ans.}$$

36. $37\frac{1}{2} \times \frac{12}{25} \times \frac{10}{11} \times \frac{11}{40}$.

$$37\tfrac{1}{2} \times \frac{12}{25} \times \frac{10}{11} \times \frac{11}{40} = \frac{\overset{3}{\cancel{75}}}{2} \times \frac{\overset{3}{\cancel{12}}}{\cancel{25}} \times \frac{\cancel{10}}{\cancel{11}} \times \frac{\cancel{11}}{\underset{\cancel{4}}{\cancel{40}}} = \frac{9}{2} = 4\tfrac{1}{2}. \quad \textit{Ans.}$$

[redacted]

37. $\frac{5}{16}\times\frac{9}{20}\times\frac{4}{21}\times 2\frac{1}{3}$.

$$\frac{5}{16}\times\frac{9}{20}\times\frac{4}{21}\times2\frac{1}{3}=\frac{\cancel{5}}{16}\times\frac{\overset{\cancel{3}}{\cancel{9}}}{\underset{\cancel{4}}{\cancel{20}}}\times\frac{\cancel{4}}{\underset{\cancel{3}}{\cancel{21}}}\times\frac{\cancel{7}}{\cancel{3}}=\frac{1}{16}.\quad \textit{Ans.}$$

38. $8\frac{1}{2}\times\frac{5}{12}\times1\frac{1}{17}\times\frac{4}{5}$.

$$8\frac{1}{2}\times\frac{5}{12}\times1\frac{1}{17}\times\frac{4}{5}=\frac{\cancel{17}}{\cancel{2}}\times\frac{\cancel{5}}{\underset{\cancel{2}}{\cancel{12}}}\times\frac{\overset{3}{\cancel{18}}}{\cancel{17}}\times\frac{\overset{\cancel{2}}{\cancel{4}}}{\cancel{5}}=3.\quad \textit{Ans.}$$

39. $62\frac{1}{2}\times\frac{9}{50}\times\frac{2}{3}\times15$.

$$62\frac{1}{2}\times\frac{9}{50}\times\frac{2}{3}\times15=\frac{\overset{5}{\cancel{125}}}{2}\times\frac{\overset{3}{\cancel{9}}}{\underset{\cancel{25}}{\cancel{50}}}\times\frac{\cancel{2}}{\cancel{3}}\times15=\frac{225}{2}=112\frac{1}{2}.\quad \textit{Ans.}$$

40. $\frac{8}{75}\times87\frac{1}{2}\times\frac{3}{10}\times\frac{1}{7}$.

$$\frac{8}{75}\times87\frac{1}{2}\times\frac{3}{10}\times\frac{1}{7}=\frac{\overset{2}{\overset{\cancel{4}}{\cancel{8}}}}{\underset{\cancel{3}}{\cancel{75}}}\times\frac{\overset{\cancel{7}}{\cancel{175}}}{\cancel{2}}\times\frac{\cancel{3}}{\underset{5}{\cancel{10}}}\times\frac{1}{\cancel{7}}=\frac{2}{5}.\quad \textit{Ans.}$$

41. $1\frac{1}{9}\times1\frac{1}{10}\times3\frac{3}{11}\times\frac{1}{16}$.

$$1\frac{1}{9}\times1\frac{1}{10}\times3\frac{3}{11}\times\frac{1}{16}=\frac{\cancel{10}}{\cancel{9}}\times\frac{\cancel{11}}{\cancel{10}}\times\frac{\overset{\cancel{4}}{\cancel{36}}}{\cancel{11}}\times\frac{1}{\underset{4}{\cancel{16}}}=\frac{1}{4}.\quad \textit{Ans.}$$

42. $6\frac{3}{5}\times\frac{15}{22}\times\frac{8}{9}\times\frac{3}{8}$.

$$6\frac{3}{5}\times\frac{15}{22}\times\frac{8}{9}\times\frac{3}{8}=\frac{\overset{3}{\cancel{33}}}{\cancel{5}}\times\frac{\overset{\cancel{3}}{\cancel{15}}}{\underset{2}{\cancel{22}}}\times\frac{\cancel{8}}{\underset{\cancel{3}}{\cancel{9}}}\times\frac{\cancel{3}}{\cancel{8}}=\frac{3}{2}=1\frac{1}{2}.\quad \textit{Ans.}$$

43. $\frac{7}{18}$ of $\frac{9}{14}$ of $\frac{35}{36}$ of $10\frac{2}{7}$.

$$\frac{7}{18}\text{ of }\frac{9}{14}\text{ of }\frac{35}{36}\text{ of }10\frac{2}{7}=\frac{\cancel{7}}{\underset{2}{\cancel{18}}}\times\frac{\cancel{9}}{\underset{\cancel{7}}{\cancel{14}}}\times\frac{\overset{5}{\cancel{35}}}{\cancel{36}}\times\frac{\overset{\cancel{2}}{\cancel{72}}}{\cancel{7}}=\frac{5}{2}=2\frac{1}{2}.\quad \textit{Ans.}$$

44. $\frac{11}{68}\times2\frac{19}{33}\times1\frac{5}{9}\times27$.

$$\frac{11}{68}\times2\frac{19}{33}\times1\frac{5}{9}\times27=\frac{\cancel{11}}{\underset{2}{\underset{\cancel{4}}{\cancel{68}}}}\times\frac{\overset{5}{\cancel{85}}}{\underset{\cancel{3}}{\cancel{33}}}\times\frac{\overset{7}{\cancel{14}}}{\cancel{9}}\times\overset{\cancel{3}}{\cancel{27}}=\frac{35}{2}=17\frac{1}{2}.\quad \textit{Ans.}$$

[redacted]

45. $2\frac{1}{28} \times 1\frac{32}{63} \times \frac{7}{180} \times 2\frac{4}{19}$.

$$2\tfrac{1}{28} \times 1\tfrac{32}{63} \times \frac{7}{180} \times 2\tfrac{4}{19} = \frac{\overset{19}{\cancel{57}}}{\underset{2}{\cancel{28}}} \times \frac{\overset{\cancel{5}}{\cancel{95}}}{\underset{\underset{\cancel{3}}{\cancel{9}}}{\cancel{63}}} \times \frac{\cancel{7}}{\underset{36}{\cancel{180}}} \times \frac{\overset{\cancel{3}}{\cancel{42}}}{\cancel{19}} = \frac{19}{72}. \quad \textit{Ans.}$$

46. $\frac{27}{44} \times 1\frac{29}{120} \times \frac{13}{60} \times 12\frac{2}{9}$.

$$\frac{27}{44} \times 1\tfrac{29}{120} \times \frac{13}{60} \times 12\tfrac{2}{9} = \frac{\overset{\cancel{9}}{\cancel{27}}}{\underset{4}{\cancel{44}}} \times \frac{149}{\underset{\underset{4}{\cancel{12}}}{\cancel{120}}} \times \frac{13}{60} \times \frac{\overset{\cancel{11}}{\cancel{110}}}{\cancel{9}} = \frac{1937}{960} = 2\tfrac{17}{960}. \quad \textit{Ans.}$$

47. $\frac{112}{119} \times 1\frac{3}{25} \times \frac{85}{98} \times \frac{7}{32}$.

$$\frac{112}{119} \times 1\tfrac{3}{25} \times \frac{85}{98} \times \frac{7}{32} = \frac{\overset{\cancel{7}}{\cancel{112}}}{\underset{\cancel{17}}{\cancel{119}}} \times \frac{\overset{\cancel{2}}{\cancel{28}}}{\underset{5}{\cancel{25}}} \times \frac{\overset{\cancel{5}}{\cancel{85}}}{\underset{\cancel{14}}{\cancel{98}}} \times \frac{\cancel{7}}{\underset{\cancel{2}}{\cancel{32}}} = \frac{1}{5}. \quad \textit{Ans.}$$

48. $3\frac{1}{9} \times 2\frac{5}{36} \times 1\frac{5}{22} \times \frac{18}{35}$.

$$3\tfrac{1}{9} \times 2\tfrac{5}{36} \times 1\tfrac{5}{22} \times \frac{18}{35} = \frac{\overset{\overset{7}{\cancel{14}}}{\cancel{28}}}{\cancel{9}} \times \frac{\overset{\cancel{7}}{\cancel{77}}}{\underset{\cancel{2}}{\cancel{36}}} \times \frac{\overset{3}{\cancel{27}}}{\underset{\cancel{2}}{\cancel{22}}} \times \frac{\cancel{18}}{\underset{5}{\cancel{35}}} = \frac{21}{5} = 4\tfrac{1}{5}. \quad \textit{Ans.}$$

49. $1\frac{11}{25} \times \frac{22}{63} \times \frac{7}{18} \times 1\frac{1}{44}$.

$$1\tfrac{11}{25} \times \frac{22}{63} \times \frac{7}{18} \times 1\tfrac{1}{44} = \frac{\overset{\cancel{2}}{\cancel{36}}}{\underset{5}{\cancel{25}}} \times \frac{22}{\underset{\cancel{9}}{\cancel{63}}} \times \frac{\cancel{7}}{\cancel{18}} \times \frac{\overset{\cancel{5}}{\cancel{45}}}{\underset{\cancel{2}}{\cancel{44}}} = \frac{1}{5}. \quad \textit{Ans.}$$

50. $15\frac{3}{7} \times \frac{7}{36} \times \frac{19}{63} \times \frac{42}{57}$.

$$15\tfrac{3}{7} \times \frac{7}{36} \times \frac{19}{63} \times \frac{42}{57} = \frac{\overset{\cancel{3}}{\cancel{108}}}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{36}} \times \frac{\cancel{19}}{\underset{3}{\cancel{63}}} \times \frac{\overset{2}{\cancel{42}}}{\underset{\cancel{3}}{\cancel{57}}} = \frac{2}{3}. \quad \textit{Ans.}$$

51. $\frac{102}{637} \times \frac{91}{9} \times \frac{1}{25} \times \frac{35}{54}$.

$$\frac{\overset{\cancel{3}}{\cancel{102}}}{\underset{\cancel{7}}{\cancel{637}}} \times \frac{\cancel{91}}{\underset{3}{\cancel{9}}} \times \frac{1}{\underset{5}{\cancel{25}}} \times \frac{\overset{\cancel{7}}{\cancel{35}}}{\cancel{54}} = \frac{1}{15}. \quad \textit{Ans.}$$

52. $1\frac{9}{247} \times 1\frac{28}{53} \times \frac{323}{432} \times \frac{117}{272}$.

$$1\tfrac{9}{247} \times 1\tfrac{28}{53} \times \frac{323}{432} \times \frac{117}{272} = \frac{\overset{\cancel{16}}{\cancel{256}}}{\underset{\cancel{13}}{\cancel{247}}} \times \frac{\overset{3}{\cancel{81}}}{53} \times \frac{\overset{\cancel{19}}{\cancel{323}}}{\underset{\cancel{16}}{\cancel{432}}} \times \frac{\overset{9}{\cancel{117}}}{\underset{\cancel{17}}{\cancel{272}}} = \frac{27}{53}. \quad \textit{Ans.}$$

[redacted]

53. $\frac{4}{9} \times \frac{3}{11} \times 6\frac{2}{7} \times 9\frac{3}{5} \times 2\frac{1}{2} \times 63 \times \frac{13}{144}$.

$$\frac{4}{9}\times\frac{3}{11}\times6\tfrac{2}{7}\times9\tfrac{3}{5}\times2\tfrac{1}{2}\times63\times\frac{13}{144}=\frac{\overset{2}{\cancel{4}}}{\cancel{9}}\times\frac{\cancel{3}}{\cancel{11}}\times\frac{\overset{4}{\cancel{44}}}{\cancel{7}}\times\frac{\cancel{48}}{\cancel{5}}\times\frac{\cancel{5}}{\cancel{2}}\times\overset{9}{\cancel{63}}\times\frac{13}{\underset{3}{\cancel{144}}}=104.\ \textit{Ans.}$$

54. $6\frac{1}{2} \times 11\frac{3}{7} \times 16\frac{4}{11} \times \frac{2}{13}$ of $\frac{7}{80}$ of $\frac{1}{90}$.

$$6\tfrac{1}{2}\times11\tfrac{3}{7}\times16\tfrac{4}{11}\times\frac{2}{13}\text{ of }\frac{7}{80}\text{ of }\frac{1}{90}=\frac{\cancel{13}}{\cancel{2}}\times\frac{\cancel{80}}{\cancel{7}}\times\frac{\overset{2}{\cancel{180}}}{11}\times\frac{2}{\cancel{13}}\times\frac{\cancel{7}}{\cancel{80}}\times\frac{1}{\cancel{90}}=\frac{2}{11}.\ \textit{Ans.}$$

55. $2\frac{3}{4} \times 7\frac{7}{11} \times 2 \times 1\frac{1}{3} \times \frac{3}{56} \times \frac{7}{27} \times \frac{27}{49}$.

$$2\tfrac{3}{4}\times7\tfrac{7}{11}\times2\times1\tfrac{1}{3}\times\frac{3}{56}\times\frac{7}{27}\times\frac{27}{49}=\frac{\cancel{11}}{\cancel{4}}\times\frac{\overset{3}{\cancel{84}}}{\cancel{11}}\times\cancel{2}\times\frac{\cancel{4}}{\cancel{3}}\times\frac{\cancel{3}}{\underset{\cancel{28}}{\cancel{56}}}\times\frac{\cancel{7}}{\cancel{27}}\times\frac{\cancel{27}}{\underset{7}{\cancel{49}}}=\frac{3}{7}.\ \textit{Ans.}$$

Exercise 52. Page 119.

Find the product of:

1. $9\times6\frac{5}{6}=61\frac{1}{2}$.

2. $8\times17\frac{1}{5}=137\frac{3}{5}$.

3. $19\times5\frac{1}{4}=99\frac{3}{4}$.

4. $7\times12\frac{1}{3}=86\frac{1}{3}$.

5. $10\times15\frac{1}{2}=155$.

6. $6\times1\frac{7}{8}=11\frac{1}{4}$.

7. $12\times2\frac{3}{4}=33$.

8. $17\times6\frac{1}{8}=104\frac{1}{8}$.

9. $19\times1\frac{1}{19}=20$.

10. $24\times16\frac{5}{6}=404$.

11. $32\times22\frac{3}{8}=716$.

12. $40\times8\frac{1}{5}=328$.

13. $41\times9\frac{1}{6}=375\frac{5}{6}$.

14. $18\times7\frac{4}{5}=140\frac{2}{5}$.

15. $19\times6\frac{4}{13}=119\frac{11}{13}$.

16. $20\times5\frac{1}{3}=106\frac{2}{3}$.

17. $15\times\$9\frac{1}{2}=\$142\frac{1}{2}$.

18. $6\times8\frac{3}{5}=51\frac{3}{5}$.

19. $11\times8\frac{4}{5}=96\frac{4}{5}$.

20. $100\times6\frac{2}{3}=666\frac{2}{3}$.

21. $5\times3\frac{1}{8}=15\frac{5}{8}$.

22. $6\times17\frac{1}{7}=102\frac{6}{7}$.

23. $32\times6\frac{2}{5}=204\frac{4}{5}$.

24. $13\times3\frac{3}{7}=44\frac{4}{7}$.

25. $12\times6\frac{3}{5}=79\frac{1}{5}$.

26. $8\frac{2}{11}\times12=98\frac{2}{11}$.

27. $20\frac{1}{4}\times5=101\frac{1}{4}$.

28. $6\frac{2}{3}\times18=120$.

29. $11\times11\frac{1}{9}=122\frac{2}{9}$.

30. $18\times12\frac{4}{5}=230\frac{2}{5}$.

31. $36\times4\frac{1}{6}=150$.

32. $12\times20\frac{2}{7}=243\frac{3}{7}$.

33. $12\times48\frac{24}{25}=587\frac{13}{25}$.

34. $11\times24\frac{4}{9}=268\frac{8}{9}$.

35. $7\times19\frac{3}{5}=137\frac{1}{5}$.

36. $8\times16\frac{1}{4}=130$.

37. $5\times29\frac{1}{6}=145\frac{5}{6}$.

38. $16\times3\frac{3}{7}=54\frac{6}{7}$.

39. $19\times12\frac{1}{17}=229\frac{2}{17}$.

40. $23\times42\frac{5}{6}=985\frac{1}{6}$.

41. $18\times12\frac{7}{8}=231\frac{3}{4}$.

42. $22\times22\frac{1}{22}=485$.

43. $12\times161\frac{17}{19}=1942\frac{14}{19}$.

44. $9\times144\frac{1}{6}=1297\frac{1}{2}$.

45. $10\times112\frac{7}{9}=1127\frac{7}{9}$.

46. $14\times42\frac{5}{9}=595\frac{7}{9}$.

47. $161\times4\frac{2}{3}=751\frac{1}{3}$.

48. $140\times5\frac{7}{12}=781\frac{2}{3}$.

Exercise 53. Page 120.

1. Divide $\frac{24}{35}$ by 6.

$$\frac{24}{35} \div 6 = \frac{1}{\cancel{6}} \times \frac{\overset{4}{\cancel{24}}}{35} = \frac{4}{35}. \textit{ Ans.}$$

2. Divide $\frac{10}{11}$ by 5.

$$\frac{10}{11} \div 5 = \frac{1}{\cancel{5}} \times \frac{\overset{2}{\cancel{10}}}{11} = \frac{2}{11}. \textit{ Ans.}$$

3. Divide $\frac{3}{7}$ by 8.

$$\frac{3}{7} \div 8 = \frac{1}{8} \times \frac{3}{7} = \frac{3}{56}. \textit{ Ans.}$$

4. Divide $18\frac{2}{3}$ by 7.

$$18\tfrac{2}{3} \div 7 = \frac{1}{\cancel{7}} \times \frac{\overset{8}{\cancel{56}}}{3} = 2\tfrac{2}{3}. \textit{ Ans.}$$

5. Divide $\frac{5}{8}$ by $\frac{3}{4}$.

$$\frac{5}{8} \div \frac{3}{4} = \frac{\cancel{4}}{3} \times \frac{5}{\underset{2}{\cancel{8}}} = \frac{5}{6}. \textit{ Ans.}$$

6. Divide $\frac{12}{16}$ by $\frac{3}{8}$.

$$\frac{12}{16} \div \frac{3}{8} = \frac{\overset{2}{\cancel{8}}}{\cancel{3}} \times \frac{\overset{\cancel{3}}{\cancel{12}}}{\underset{\cancel{4}}{\cancel{16}}} = 2. \textit{ Ans.}$$

7. Divide $1\frac{3}{4}$ by $3\frac{1}{3}$.

$$1\tfrac{3}{4} \div 3\tfrac{1}{3} = \frac{7}{4} \div \frac{10}{3} = \frac{3}{10} \times \frac{7}{4} = \frac{21}{40}. \textit{ Ans.}$$

8. Divide $5\frac{1}{5}$ by $4\frac{2}{3}$.

$$5\tfrac{1}{5} \div 4\tfrac{2}{3} = \frac{26}{5} \div \frac{14}{3} = \frac{3}{\underset{7}{\cancel{14}}} \times \frac{\overset{13}{\cancel{26}}}{5} = \frac{39}{35} = 1\tfrac{4}{35}. \textit{ Ans.}$$

9. Divide $8\frac{2}{9}$ by $4\frac{1}{9}$.

$$8\tfrac{2}{9} \div 4\tfrac{1}{9} = \frac{74}{9} \div \frac{37}{9} = \frac{\cancel{9}}{\cancel{37}} \times \frac{\overset{2}{\cancel{74}}}{\cancel{9}} = 2. \textit{ Ans.}$$

10. Divide $7\frac{1}{5}$ by $4\frac{2}{7}$.

$$7\tfrac{1}{5} \div 4\tfrac{2}{7} = \frac{36}{5} \div \frac{30}{7} = \frac{7}{\underset{5}{\cancel{30}}} \times \frac{\overset{6}{\cancel{36}}}{5} = \frac{42}{25} = 1\tfrac{17}{25}. \textit{ Ans.}$$

11. Divide $6\frac{3}{4}$ by $9\frac{1}{2}$.

$$6\tfrac{3}{4} \div 9\tfrac{1}{2} = \frac{27}{4} \div \frac{19}{2} = \frac{\cancel{2}}{19} \times \frac{27}{\underset{2}{\cancel{4}}} = \frac{27}{38}. \textit{ Ans.}$$

12. Divide $8\frac{2}{3}$ by $4\frac{2}{3}$.

$$8\tfrac{2}{3} \div 4\tfrac{2}{3} = \frac{26}{3} \div \frac{14}{3} = \frac{\cancel{3}}{\underset{7}{\cancel{14}}} \times \frac{\overset{13}{\cancel{26}}}{\cancel{3}} = \frac{13}{7} = 1\tfrac{6}{7}. \textit{ Ans.}$$

[redacted]

13. Divide $3\frac{8}{9}$ by $\frac{14}{27}$.

$$3\tfrac{8}{9} \div \tfrac{14}{27} = \frac{35}{9} \div \frac{14}{27} = \frac{\overset{3}{\cancel{27}}}{\underset{2}{\cancel{14}}} \times \frac{\overset{5}{\cancel{35}}}{\cancel{9}} = \frac{15}{2} = 7\tfrac{1}{2}. \text{ } \textit{Ans.}$$

14. Divide $4\frac{3}{7}$ by $6\frac{8}{9}$.

$$4\tfrac{3}{7} \div 6\tfrac{8}{9} = \frac{31}{7} \div \frac{62}{9} = \frac{9}{\underset{2}{\cancel{62}}} \times \frac{\cancel{31}}{7} = \frac{9}{14}. \text{ } \textit{Ans.}$$

15. Divide 5 by $4\frac{2}{7}$.

$$5 \div 4\tfrac{2}{7} = 5 \div \frac{30}{7} = \frac{7}{\underset{6}{\cancel{30}}} \times \cancel{5} = \frac{7}{6} = 1\tfrac{1}{6}. \text{ } \textit{Ans.}$$

16. Divide $4\frac{2}{3}$ by $\frac{7}{8}$.

$$4\tfrac{2}{3} \div \tfrac{7}{8} = \frac{14}{3} \div \frac{7}{8} = \frac{8}{\cancel{7}} \times \frac{\overset{2}{\cancel{14}}}{3} = \frac{16}{3} = 5\tfrac{1}{3}. \text{ } \textit{Ans.}$$

17. Divide $8\frac{3}{5}$ by $6\frac{1}{7}$.

$$8\tfrac{3}{5} \div 6\tfrac{1}{7} = \frac{43}{5} \div \frac{43}{7} = \frac{7}{\cancel{43}} \times \frac{\cancel{43}}{5} = \frac{7}{5} = 1\tfrac{2}{5}. \text{ } \textit{Ans.}$$

18. Divide $8\frac{4}{5}$ by $1\frac{1}{10}$.

$$8\tfrac{4}{5} \div 1\tfrac{1}{10} = \frac{44}{5} \div \frac{11}{10} = \frac{\overset{2}{\cancel{10}}}{\cancel{11}} \times \frac{\overset{4}{\cancel{44}}}{\cancel{5}} = 8. \text{ } \textit{Ans.}$$

19. Divide 100 by $6\frac{2}{3}$.

$$100 \div 6\tfrac{2}{3} = 100 \div \frac{20}{3} = \frac{3}{\cancel{20}} \times \overset{5}{\cancel{100}} = 15. \text{ } \textit{Ans.}$$

20. Divide $\frac{14}{15}$ by $\frac{12}{25}$.

$$\frac{14}{15} \div \frac{12}{25} = \frac{\overset{5}{\cancel{25}}}{\underset{6}{\cancel{12}}} \times \frac{\overset{7}{\cancel{14}}}{\underset{3}{\cancel{15}}} = \frac{35}{18} = 1\tfrac{17}{18}. \text{ } \textit{Ans.}$$

21. Divide $3\frac{1}{8}$ by 5.

$$3\tfrac{1}{8} \div 5 = \frac{25}{8} \div 5 = \frac{1}{\cancel{5}} \times \frac{\overset{5}{\cancel{25}}}{8} = \frac{5}{8}. \text{ } \textit{Ans.}$$

22. Divide 100 by $33\frac{1}{3}$.

$$100 \div 33\tfrac{1}{3} = 100 \div \frac{100}{3} = \frac{3}{\cancel{100}} \times \cancel{100} = 3. \text{ } \textit{Ans.}$$

23. Divide 100 by $37\frac{1}{2}$.

$$100 \div 37\tfrac{1}{2} = 100 \div \frac{75}{2} = \frac{2}{\underset{3}{\cancel{75}}} \times \overset{4}{\cancel{100}} = \frac{8}{3} = 2\tfrac{2}{3}. \textit{ Ans.}$$

24. Divide $7\frac{1}{7}$ by $6\frac{1}{4}$.

$$7\tfrac{1}{7} \div 6\tfrac{1}{4} = \frac{50}{7} \div \frac{25}{4} = \frac{4}{\cancel{25}} \times \frac{\overset{2}{\cancel{50}}}{7} = \frac{8}{7} = 1\tfrac{1}{7}. \textit{ Ans.}$$

25. Divide $\frac{1}{9}$ by $\frac{1}{11}$.

$$\frac{1}{9} \div \frac{1}{11} = 11 \times \frac{1}{9} = \frac{11}{9} = 1\tfrac{2}{9}. \textit{ Ans.}$$

26. Divide $6\frac{2}{5}$ by 32.

$$6\tfrac{2}{5} \div 32 = \frac{32}{5} \div 32 = \frac{1}{\cancel{32}} \times \frac{\cancel{32}}{5} = \frac{1}{5}. \textit{ Ans.}$$

27. Divide $3\frac{1}{7}$ by $3\frac{3}{7}$.

$$3\tfrac{1}{7} \div 3\tfrac{3}{7} = \frac{22}{7} \div \frac{24}{7} = \frac{\cancel{7}}{\underset{12}{\cancel{24}}} \times \frac{\overset{11}{\cancel{22}}}{\cancel{7}} = \frac{11}{12}. \textit{ Ans.}$$

28. Divide $1\frac{7}{15}$ by $\frac{11}{25}$.

$$1\tfrac{7}{15} \div \frac{11}{25} = \frac{22}{15} \div \frac{11}{25} = \frac{\overset{5}{\cancel{25}}}{\cancel{11}} \times \frac{\overset{2}{\cancel{22}}}{\underset{3}{\cancel{15}}} = \frac{10}{3} = 3\tfrac{1}{3}. \textit{ Ans.}$$

29. Divide $11\frac{1}{5}$ by $\frac{8}{9}$.

$$11\tfrac{1}{5} \div \frac{8}{9} = \frac{56}{5} \div \frac{8}{9} = \frac{9}{\cancel{8}} \times \frac{\overset{7}{\cancel{56}}}{5} = \frac{63}{5} = 12\tfrac{3}{5}. \textit{ Ans.}$$

30. Divide 100 by $83\frac{1}{3}$.

$$100 \div 83\tfrac{1}{3} = 100 \div \frac{250}{3} = \frac{3}{\underset{5}{\cancel{250}}} \times \overset{2}{\cancel{100}} = \frac{6}{5} = 1\tfrac{1}{5}. \textit{ Ans.}$$

31. Divide 50 by $16\frac{2}{3}$.

$$50 \div 16\tfrac{2}{3} = 50 \div \frac{50}{3} = \frac{3}{\cancel{50}} \times \cancel{50} = 3. \textit{ Ans.}$$

32. Divide $\frac{15}{22}$ by $1\frac{1}{2}$.

$$\frac{15}{22} \div 1\tfrac{1}{2} = \frac{15}{22} \div \frac{3}{2} = \frac{\cancel{2}}{\cancel{3}} \times \frac{\overset{5}{\cancel{15}}}{\underset{11}{\cancel{22}}} = \frac{5}{11}. \textit{ Ans.}$$

33. Divide $1\frac{13}{38}$ by $1\frac{2}{19}$.

$$1\tfrac{13}{38} \div 1\tfrac{2}{19} = \frac{51}{38} \div \frac{21}{19} = \frac{\cancel{19}}{\underset{7}{\cancel{21}}} \times \frac{\overset{17}{\cancel{51}}}{\underset{2}{\cancel{38}}} = \frac{17}{14} = 1\tfrac{3}{14}. \quad \textit{Ans.}$$

34. Divide $20\frac{1}{4}$ by 5.

$$20\tfrac{1}{4} \div 5 = \frac{81}{4} \div 5 = \frac{1}{5} \times \frac{81}{4} = \frac{81}{20} = 4\tfrac{1}{20}. \quad \textit{Ans.}$$

35. Divide $16\frac{2}{3}$ by $\frac{4}{9}$.

$$16\tfrac{2}{3} \div \frac{4}{9} = \frac{50}{3} \div \frac{4}{9} = \frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{4}}} \times \frac{\overset{25}{\cancel{50}}}{\cancel{3}} = \frac{75}{2} = 37\tfrac{1}{2}. \quad \textit{Ans.}$$

36. Divide $22\frac{2}{9}$ by $16\frac{2}{3}$.

$$22\tfrac{2}{9} \div 16\tfrac{2}{3} = \frac{200}{9} \div \frac{50}{3} = \frac{\cancel{3}}{\cancel{50}} \times \frac{\overset{4}{\cancel{200}}}{\underset{3}{\cancel{9}}} = \frac{4}{3} = 1\tfrac{1}{3}. \quad \textit{Ans.}$$

37. Divide $20\frac{5}{9}$ by $1\frac{10}{27}$.

$$20\tfrac{5}{9} \div 1\tfrac{10}{27} = \frac{185}{9} \div \frac{37}{27} = \frac{\overset{3}{\cancel{27}}}{\cancel{37}} \times \frac{\overset{5}{\cancel{185}}}{\cancel{9}} = 15. \quad \textit{Ans.}$$

38. Divide $16\frac{2}{3}$ by $11\frac{1}{9}$.

$$16\tfrac{2}{3} \div 11\tfrac{1}{9} = \frac{50}{3} \div \frac{100}{9} = \frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{100}}} \times \frac{\cancel{50}}{\cancel{3}} = \frac{3}{2} = 1\tfrac{1}{2}. \quad \textit{Ans.}$$

39. Divide $33\frac{1}{3}$ by $28\frac{4}{7}$.

$$33\tfrac{1}{3} \div 28\tfrac{4}{7} = \frac{100}{3} \div \frac{200}{7} = \frac{7}{\underset{2}{\cancel{200}}} \times \frac{\cancel{100}}{3} = \frac{7}{6} = 1\tfrac{1}{6}. \quad \textit{Ans.}$$

40. Divide $47\frac{7}{9}$ by $17\frac{1}{5}$.

$$47\tfrac{7}{9} \div 17\tfrac{1}{5} = \frac{430}{9} \div \frac{86}{5} = \frac{5}{\cancel{86}} \times \frac{\overset{5}{\cancel{430}}}{9} = \frac{25}{9} = 2\tfrac{7}{9}. \quad \textit{Ans.}$$

41. Divide $18\frac{4}{7}$ by $1\frac{5}{21}$.

$$18\tfrac{4}{7} \div 1\tfrac{5}{21} = \frac{130}{7} \div \frac{26}{21} = \frac{\overset{3}{\cancel{21}}}{\cancel{26}} \times \frac{\overset{5}{\cancel{130}}}{\cancel{7}} = 15. \quad \textit{Ans.}$$

42. Divide $37\frac{4}{5}$ by $1\frac{4}{17}$.

$$37\tfrac{4}{5} \div 1\tfrac{4}{17} = \frac{189}{5} \div \frac{21}{17} = \frac{17}{\cancel{21}} \times \frac{\overset{9}{\cancel{189}}}{5} = \frac{153}{5} = 30\tfrac{3}{5}. \quad \textit{Ans.}$$

43. Divide $3\frac{4}{5}$ of $2\frac{1}{2}$ by $1\frac{1}{2}$ of $2\frac{1}{9}$.

$$3\tfrac{4}{5} \text{ of } 2\tfrac{1}{2} \div 1\tfrac{1}{2} \text{ of } 2\tfrac{1}{9} = \frac{19}{5} \text{ of } \frac{5}{2} \div \frac{3}{2} \text{ of } \frac{19}{9} = \frac{\cancel{19}}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{\overset{3}{\cancel{9}}}{\cancel{19}} = 3. \quad \textit{Ans.}$$

44. Divide $2\frac{2}{7}$ by $3\frac{1}{3}$ of $1\frac{1}{15}$.

$$2\tfrac{2}{7} \div 3\tfrac{1}{3} \text{ of } 1\tfrac{1}{15} = \frac{16}{7} \div \frac{10}{3} \text{ of } \frac{16}{15} = \frac{\cancel{16}}{7} \times \frac{3}{\underset{2}{\cancel{10}}} \times \frac{\overset{3}{\cancel{15}}}{\cancel{16}} = \frac{9}{14}. \quad \textit{Ans.}$$

45. Divide $2\frac{9}{11}$ of $5\frac{1}{2}$ by $7\frac{3}{4}$.

$$2\tfrac{9}{11} \text{ of } 5\tfrac{1}{2} \div 7\tfrac{3}{4} = \frac{31}{11} \text{ of } \frac{11}{2} \div \frac{31}{4} = \frac{\cancel{31}}{\cancel{11}} \times \frac{\cancel{11}}{\cancel{2}} \times \frac{\overset{2}{\cancel{4}}}{\cancel{31}} = 2. \quad \textit{Ans.}$$

46. Divide $5\frac{3}{5}$ of $8\frac{1}{3}$ of $1\frac{4}{7}$ by $2\frac{1}{10}$ of $5\frac{5}{9}$.

$$5\tfrac{3}{5} \text{ of } 8\tfrac{1}{3} \text{ of } 1\tfrac{4}{7} \div 2\tfrac{1}{10} \text{ of } 5\tfrac{5}{9} = \frac{28}{5} \text{ of } \frac{25}{3} \text{ of } \frac{11}{7} \div \frac{21}{10} \text{ of } \frac{50}{9}$$

$$= \frac{\overset{4}{\cancel{28}}}{\cancel{5}} \times \frac{\cancel{25}}{\cancel{3}} \times \frac{11}{\cancel{7}} \times \frac{\overset{\cancel{2}}{\cancel{10}}}{\underset{7}{\cancel{21}}} \times \frac{\overset{\cancel{3}}{\cancel{9}}}{\underset{\cancel{2}}{\cancel{50}}} = \frac{44}{7} = 6\tfrac{2}{7}. \quad \textit{Ans.}$$

Exercise 54. Page 121.

Find the quotient of :

1. $31\frac{7}{8} \div 5 = 6\frac{3}{8}$.
2. $16\frac{7}{9} \div 6 = 2\frac{43}{54}$.
3. $14\frac{2}{7} \div 2 = 7\frac{1}{7}$.
4. $33\frac{1}{3} \div 7 = 4\frac{16}{21}$.
5. $42\frac{6}{7} \div 6 = 7\frac{1}{7}$.
6. $49\frac{4}{5} \div 7 = 7\frac{4}{35}$.
7. $52\frac{4}{7} \div 8 = 6\frac{4}{7}$.
8. $44\frac{4}{11} \div 12 = 3\frac{23}{33}$.
9. $48\frac{24}{25} \div 12 = 4\frac{2}{5}$.
10. $24\frac{4}{9} \div 11 = 2\frac{2}{9}$.
11. $19\frac{3}{5} \div 7 = 2\frac{4}{5}$.
12. $29\frac{1}{6} \div 8 = 3\frac{31}{48}$.

Exercise 55. Page 122.

Find the value of :

1. $2\frac{1}{5} \text{ of } 2\frac{1}{2} \div \frac{3}{14} \text{ of } 3\frac{2}{3} = \frac{\cancel{11}}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{2}} \times \frac{\overset{7}{\cancel{14}}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{11}} = 7.$

2. $\frac{5}{9} \text{ of } 6\frac{2}{3} \text{ of } \frac{6}{25} \div 5\frac{1}{2} = \frac{\cancel{5}}{9} \times \frac{\overset{4}{\cancel{20}}}{\cancel{3}} \times \frac{\overset{2}{\cancel{6}}}{\underset{\cancel{5}}{\cancel{25}}} \times \frac{2}{11} = \frac{16}{99}.$

3. $\frac{3}{10} \div \frac{2}{5} \text{ of } 2\frac{1}{4} \text{ of } 1\frac{5}{7} = \frac{\cancel{3}}{\underset{2}{\cancel{10}}} \times \frac{\cancel{5}}{2} \times \frac{\cancel{4}}{9} \times \frac{7}{\underset{\cancel{4}}{\cancel{12}}} = \frac{7}{36}.$

4. $\frac{3}{10} \div \left(\frac{2}{5} \times 2\frac{1}{4} \times 1\frac{5}{7}\right) = \frac{\cancel{3}}{\underset{2}{\cancel{10}}} \times \frac{\cancel{5}}{2} \times \frac{\cancel{4}}{9} \times \frac{7}{\underset{\cancel{4}}{\cancel{12}}} = \frac{7}{36}.$

5. $\frac{7}{9} \text{ of } \frac{15}{16} \div 1\frac{1}{27} \text{ of } 1\frac{11}{35} = \frac{\cancel{7}}{\cancel{9}} \times \frac{15}{16} \times \frac{\overset{3}{\cancel{27}}}{\underset{4}{\cancel{28}}} \times \frac{35}{46} = \frac{1565}{2944}.$

6. $\frac{3}{4} \text{ of } \frac{5}{6} \div \left(\frac{5}{8} \times \frac{4}{11}\right) = \frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{5}}{\underset{\cancel{2}}{\cancel{6}}} \times \frac{\overset{\cancel{2}}{\cancel{8}}}{\cancel{5}} \times \frac{11}{4} = \frac{11}{4} = 2\frac{3}{4}.$

7. $\frac{3}{7} \text{ of } \frac{14}{27} \div \frac{11}{13} \text{ of } \frac{26}{27} = \frac{\cancel{3}}{\cancel{7}} \times \frac{\overset{\cancel{2}}{\cancel{14}}}{\underset{\cancel{9}}{\cancel{27}}} \times \frac{\cancel{13}}{11} \times \frac{\overset{3}{\cancel{27}}}{\underset{\cancel{2}}{\cancel{26}}} = \frac{3}{11}.$

8. $\frac{3}{8} \text{ of } \frac{32}{33} \div \frac{18}{19} \text{ of } \frac{76}{81} = \frac{\cancel{3}}{\cancel{8}} \times \frac{\overset{\cancel{4}}{\cancel{32}}}{\underset{11}{\cancel{33}}} \times \frac{\cancel{19}}{\underset{2}{\cancel{18}}} \times \frac{\overset{9}{\cancel{81}}}{\underset{\cancel{4}}{\cancel{76}}} = \frac{9}{22}.$

9. $\frac{2}{7} \text{ of } 1\frac{7}{16} \div \frac{27}{43} \text{ of } \frac{53}{81} = \frac{\cancel{2}}{7} \times \frac{23}{\underset{8}{\cancel{16}}} \times \frac{43}{\cancel{27}} \times \frac{\overset{3}{\cancel{81}}}{53} = \frac{2967}{2968}.$

10. $\frac{4}{7} \text{ of } \frac{23}{30} \div \frac{8}{35} \text{ of } 4 = \frac{\cancel{4}}{\cancel{7}} \times \frac{23}{\underset{6}{\cancel{30}}} \times \frac{\overset{\cancel{5}}{\cancel{35}}}{8} \times \frac{1}{\cancel{4}} = \frac{23}{48}.$

11. $\frac{9}{10} \text{ of } \frac{110}{111} \div \frac{3}{4} \text{ of } 1\frac{1}{11} = \frac{\overset{\cancel{3}}{\cancel{9}}}{\cancel{10}} \times \frac{\overset{11}{\cancel{110}}}{111} \times \frac{\cancel{4}}{\cancel{3}} \times \frac{11}{\underset{\cancel{3}}{\cancel{12}}} = \frac{121}{111} = 1\frac{10}{111}.$

12. $\frac{2}{5} \text{ of } \frac{26}{27} \text{ of } \frac{5}{13} \div \left(\frac{1}{2} \times \frac{3}{4} \text{ of } \frac{4}{9}\right) = \frac{2}{\cancel{5}} \times \frac{\overset{2}{\cancel{26}}}{\underset{3}{\cancel{27}}} \times \frac{\cancel{5}}{\cancel{13}} \times 2 \times \frac{\cancel{4}}{3} \times \frac{\cancel{9}}{\cancel{4}} = \frac{8}{9}.$

13. $\frac{2}{5} \text{ of } \frac{5}{6} \text{ of } \frac{16}{25} \div \frac{4}{3} \text{ of } \frac{1}{15} \text{ of } 1\frac{7}{11}$

$$= \frac{2}{\cancel{5}} \times \frac{\cancel{5}}{\underset{\cancel{2}}{\cancel{6}}} \times \frac{\overset{\overset{2}{\cancel{4}}}{\cancel{16}}}{\underset{5}{\cancel{25}}} \times \frac{\cancel{3}}{\cancel{4}} \times \overset{\cancel{3}}{\cancel{15}} \times \frac{11}{\underset{\underset{3}{\cancel{6}}}{\cancel{18}}} = \frac{22}{15} = 1\frac{7}{15}.$$

14. $\left(\frac{7}{9} \div \frac{11}{18}\right) \div \left(5\frac{6}{17} \div 4\frac{33}{34}\right) = \frac{\cancel{7}}{\cancel{9}} \times \frac{\overset{2}{\cancel{18}}}{11} \times \frac{\cancel{17}}{\underset{13}{\cancel{91}}} \times \frac{\overset{13}{\cancel{169}}}{\underset{2}{\cancel{34}}} = \frac{13}{11} = 1\frac{2}{11}.$

15. $\left(14\frac{2}{3} \div 4\frac{8}{9}\right) \div \left(3\frac{11}{12} \div 9\frac{2}{5}\right) = \frac{\cancel{44}}{\cancel{3}} \times \frac{\overset{3}{\cancel{9}}}{\cancel{44}} \times \frac{12}{\cancel{47}} \times \frac{\cancel{47}}{5} = \frac{36}{5} = 7\frac{1}{5}.$

16. $\frac{3}{5}$ of $\frac{10}{33}$ of $8\frac{1}{4} \div 3\frac{1}{11}$ of $\frac{1}{17}$ of $5\frac{1}{2}$

$$= \frac{3}{\cancel{5}} \times \frac{\overset{2}{\cancel{10}}}{\cancel{33}} \times \frac{\cancel{33}}{\underset{2}{\cancel{4}}} \times \frac{\cancel{11}}{\underset{2}{\cancel{34}}} \times \cancel{17} \times \frac{2}{\cancel{11}} = \frac{3}{2} = 1\frac{1}{2}.$$

Exercise 56. Page 122.

1. If $\frac{5}{6}$ of a ton of hay costs \$15, what is the cost of one ton?

$$\$15 \div \frac{5}{6} = \frac{6}{\cancel{5}} \times \$\overset{3}{\cancel{15}} = \$18. \textit{ Ans.}$$

2. 15 is $\frac{5}{6}$ of what number?

$$15 \div \frac{5}{6} = \frac{6}{\cancel{5}} \times \overset{3}{\cancel{15}} = 18. \textit{ Ans.}$$

3. If $\frac{6}{7}$ of a roll of carpeting is worth \$75, what is the whole roll worth?

$$\$75 \div \frac{6}{7} = \frac{7}{\underset{2}{\cancel{6}}} \times \$\overset{25}{\cancel{75}} = \frac{\$175}{2} = \$87.50. \textit{ Ans.}$$

4. A man sold $6\frac{2}{3}$ yards of cloth, which was $\frac{4}{15}$ of the whole piece. How many yards were there in the piece?

$$6\frac{2}{3} \div \frac{4}{15} = \frac{\overset{5}{\cancel{15}}}{\cancel{4}} \times \frac{\overset{5}{\cancel{20}}}{\cancel{3}} = 25. \textit{ Ans.}$$

5. A farmer sold $\frac{3}{7}$ of his hay for \$195.60. What was the value of his entire crop of hay?

$$\$195.60 \div \frac{3}{7} = \frac{7}{\cancel{3}} \times \$\overset{65.20}{\cancel{195.60}} = \$456.40. \textit{ Ans.}$$

6. $21\frac{2}{3}$ is $\frac{13}{17}$ of what number?

$$21\tfrac{2}{3} \div \tfrac{13}{17} = \frac{17}{\cancel{13}} \times \frac{\overset{5}{\cancel{65}}}{3} = \frac{85}{3} = 28\tfrac{1}{3}. \textit{ Ans.}$$

7. $6\frac{2}{9}$ is $\frac{14}{27}$ of what number?

$$6\tfrac{2}{9} \div \tfrac{14}{27} = \frac{\overset{3}{\cancel{27}}}{\cancel{14}} \times \frac{\overset{4}{\cancel{56}}}{\cancel{9}} = 12. \textit{ Ans.}$$

8. $2\frac{13}{22}$ is $\frac{19}{11}$ of what number?

$$2\tfrac{13}{22} \div \tfrac{19}{11} = \frac{\cancel{11}}{\cancel{19}} \times \frac{\overset{3}{\cancel{57}}}{\underset{2}{\cancel{22}}} = \frac{3}{2} = 1\tfrac{1}{2}. \textit{ Ans.}$$

9. If $\frac{3}{7}$ of an acre of land is worth \$32, what is the value of an acre?

$$\$32 \div \frac{3}{7} = \frac{7}{3} \times \$32 = \$\frac{224}{3} = \$74\tfrac{2}{3}. \textit{ Ans.}$$

10. If $\frac{4}{5}$ of a bushel of wheat is worth 48 cents, what is the value of $2\frac{7}{12}$ bushels of wheat?

$$48 \div \frac{4}{5} = \frac{5}{\cancel{4}} \times \overset{12}{\cancel{48}} = 60. \qquad 2\tfrac{7}{12} \times 60 = \frac{31}{\cancel{12}} \times \overset{5}{\cancel{60}} = 155.$$

155 cents = \$1.55. *Ans.*

11. If $\frac{5}{7}$ of a ton of hay is worth \$15, what is the value of $7\frac{1}{3}$ tons of hay?

$$\$15 \div \frac{5}{7} = \frac{7}{\cancel{5}} \times \$\overset{3}{\cancel{15}} = \$21. \qquad 7\tfrac{1}{3} \times \$21 = \$154. \textit{ Ans.}$$

12. If $\frac{5}{6}$ of a cord of wood is worth \$4, find the value of 7 cords of wood.

$$\$4 \div \frac{5}{6} = \frac{6}{5} \times \$4 = \$\frac{24}{5} = \$4\tfrac{4}{5}. \qquad 7 \times \$4\tfrac{4}{5} = \$33\tfrac{3}{5} = \$33.60. \textit{ Ans.}$$

13. If $\frac{4}{11}$ of a barrel of apples is worth 44 cents, what is the value of 12 barrels of apples?

$$44 \div \frac{4}{11} = \frac{11}{\cancel{4}} \times \overset{11}{\cancel{44}} = 121. \qquad 12 \times 121 = 1452.$$

1452 cents = \$14.52. *Ans.*

14. \$125 is $\frac{1}{4}$ more than (that is, $\frac{5}{4}$ of) what sum of money?

$$\$125 \div \frac{5}{4} = \frac{4}{\cancel{5}} \times \$\overset{25}{\cancel{125}} = \$100. \quad \textit{Ans.}$$

15. \$132 is $\frac{1}{4}$ less than what sum of money?

$$\$132 \div \frac{3}{4} = \frac{4}{\cancel{3}} \times \$\overset{44}{\cancel{132}} = \$176. \quad \textit{Ans.}$$

16. 495 is $\frac{1}{8}$ more than what number?

$$495 \div \frac{9}{8} = \frac{8}{\cancel{9}} \times \overset{55}{\cancel{495}} = 440. \quad \textit{Ans.}$$

17. 217 is $\frac{1}{8}$ less than what number?

$$217 \div \frac{7}{8} = \frac{8}{\cancel{7}} \times \overset{31}{\cancel{217}} = 248. \quad \textit{Ans.}$$

18. 495 is $\frac{2}{13}$ less than what number?

$$495 \div \frac{11}{13} = \frac{13}{\cancel{11}} \times \overset{45}{\cancel{495}} = 585. \quad \textit{Ans.}$$

19. 495 is $\frac{2}{13}$ more than what number?

$$495 \div \frac{15}{13} = \frac{13}{\cancel{15}} \times \overset{33}{\cancel{495}} = 429. \quad \textit{Ans.}$$

20. If $\frac{4}{5}$ of a yard of silk is worth \$1, find the value of 4 yards of silk.

$$\$1 \div \frac{4}{5} = \frac{5}{4} \times \$1 = \$\frac{5}{4}. \qquad \cancel{4} \times \$\frac{5}{\cancel{4}} = \$5. \quad \textit{Ans.}$$

21. If $\frac{2}{3}$ of a yard of linen is worth 60 cents, what is the value of $2\frac{1}{3}$ yards of linen?

$$60 \div \frac{2}{3} = \frac{3}{\cancel{2}} \times \overset{30}{\cancel{60}} = 90. \qquad 2\tfrac{1}{3} \times 90 = 210. \qquad 210 \text{ cents} = \$2.10. \quad \textit{Ans.}$$

22. If a man who owned $\frac{3}{4}$ of a schooner sold $\frac{1}{3}$ of his share for \$1200, what was the value of the schooner?

$$\frac{\cancel{3}}{4} \times \frac{1}{\cancel{3}} = \frac{1}{4}. \qquad \$1200 \div \frac{1}{4} = 4 \times \$1200 = \$4800. \quad \textit{Ans.}$$

[redacted]

23. One fourth of one third of three sevenths of a number is 60. What is the number?

$$\frac{1}{4} \times \frac{1}{\cancel{3}} \times \frac{\cancel{3}}{7} = \frac{1}{28}. \qquad 60 \div \frac{1}{28} = 28 \times 60 = 1680. \textit{ Ans.}$$

24. Three fourths of two ninths of six sevenths of a number is $12\frac{5}{7}$. What is the number?

$$\frac{\cancel{3}}{\underset{2}{\cancel{4}}} \times \frac{\cancel{2}}{\underset{3}{\cancel{9}}} \times \frac{\overset{3}{\cancel{6}}}{7} = \frac{1}{7}. \qquad 12\tfrac{5}{7} \div \frac{1}{7} = 7 \times 12\tfrac{5}{7} = 89. \textit{ Ans.}$$

25. If $\frac{5}{16}$ of the goods in a store were sold for \$1000, what was the value of the whole stock of goods?

$$\$1000 \div \frac{5}{16} = \frac{16}{\cancel{5}} \times \$\overset{200}{\cancel{1000}} = \$3200. \textit{ Ans.}$$

26. If $\frac{5}{32}$ of a farm is worth \$1200, what is the value of the whole farm?

$$\$1200 \div \frac{5}{32} = \frac{32}{\cancel{5}} \times \$\overset{240}{\cancel{1200}} = \$7680. \textit{ Ans.}$$

Exercise 57. Page 125.

1. Change $\frac{1}{2}$, $\frac{2}{5}$, $\frac{5}{6}$ to similar fractions.

The L. C. D. = 30.

$$\therefore \frac{1}{2}, \frac{2}{5}, \frac{5}{6} = \frac{15 \quad 12 \quad 25}{30}. \textit{ Ans.}$$

2. Change $\frac{2}{3}$, $\frac{5}{9}$, $\frac{7}{8}$, $\frac{9}{10}$ to similar fractions.

The L. C. D. = 360.

$$\therefore \frac{2}{3}, \frac{5}{9}, \frac{7}{8}, \frac{9}{10} = \frac{240 \quad 200 \quad 315 \quad 324}{360}. \textit{ Ans.}$$

3. Change $\frac{5}{6}$, $\frac{1}{8}$, $\frac{5}{21}$, $\frac{19}{35}$ to similar fractions.

The L. C. D. = 840.

$$\therefore \frac{5}{6}, \frac{1}{8}, \frac{5}{21}, \frac{19}{35} = \frac{700 \quad 105 \quad 200 \quad 456}{840}. \textit{ Ans.}$$

4. Change $\frac{2}{15}$, $\frac{7}{20}$, $\frac{3}{25}$, $\frac{8}{45}$ to similar fractions.

The L. C. D. = 900.

$$\therefore \frac{2}{15}, \frac{7}{20}, \frac{3}{25}, \frac{8}{45} = \frac{120 \quad 315 \quad 108 \quad 160}{900}. \textit{ Ans.}$$

[redacted]

5. Change $\frac{12}{25}$, $\frac{17}{40}$, $\frac{13}{60}$, $\frac{19}{75}$ to similar fractions.

The L. C. D. = 600.

$$\therefore \frac{12}{25}, \frac{17}{40}, \frac{13}{60}, \frac{19}{75} = \frac{288 \quad 255 \quad 130 \quad 152}{600}. \quad \textit{Ans.}$$

6. Change $\frac{3}{8}$, $\frac{7}{30}$, $\frac{4}{35}$, $\frac{3}{28}$, $\frac{19}{24}$ to similar fractions.

The L. C. D. = 840.

$$\therefore \frac{3}{8}, \frac{7}{30}, \frac{4}{35}, \frac{3}{28}, \frac{19}{24} = \frac{315 \quad 196 \quad 96 \quad 90 \quad 665}{840}. \quad \textit{Ans.}$$

7. Change $\frac{11}{16}$, $\frac{7}{18}$, $\frac{13}{20}$, $\frac{23}{30}$, $\frac{17}{54}$ to similar fractions.

The L. C. D. = 2160.

$$\therefore \frac{11}{16}, \frac{7}{18}, \frac{13}{20}, \frac{23}{30}, \frac{17}{54} = \frac{1485 \quad 840 \quad 1404 \quad 1656 \quad 680}{2160}. \quad \textit{Ans.}$$

8. Change $\frac{4}{5}$, $\frac{8}{9}$, $\frac{11}{12}$, $\frac{13}{15}$ to similar fractions.

The L. C. D. = 180.

$$\therefore \frac{4}{5}, \frac{8}{9}, \frac{11}{12}, \frac{13}{15} = \frac{144 \quad 160 \quad 165 \quad 156}{180}. \quad \textit{Ans.}$$

9. Change $\frac{5}{6}$, $\frac{5}{18}$, $\frac{13}{24}$, $\frac{19}{30}$ to similar fractions.

The L. C. D. = 360.

$$\therefore \frac{5}{6}, \frac{5}{18}, \frac{13}{24}, \frac{19}{30} = \frac{300 \quad 100 \quad 195 \quad 228}{360}. \quad \textit{Ans.}$$

10. Change $\frac{7}{8}$, $\frac{17}{24}$, $\frac{19}{32}$, $\frac{11}{48}$ to similar fractions.

The L. C. D. = 96.

$$\therefore \frac{7}{8}, \frac{17}{24}, \frac{19}{32}, \frac{11}{48} = \frac{84 \quad 68 \quad 57 \quad 22}{96}. \quad \textit{Ans.}$$

11. Change $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{12}$, $\frac{15}{16}$ to similar fractions.

The L. C. D. = 48.

$$\therefore \frac{2}{3}, \frac{5}{6}, \frac{7}{12}, \frac{15}{16} = \frac{32 \quad 40 \quad 28 \quad 45}{48}. \quad \textit{Ans.}$$

12. Change $\frac{2}{7}$, $\frac{3}{14}$, $\frac{5}{18}$, $\frac{7}{9}$, $\frac{2}{21}$ to similar fractions.

The L. C. D. = 126.

$$\therefore \frac{2}{7}, \frac{3}{14}, \frac{5}{18}, \frac{7}{9}, \frac{2}{21} = \frac{36 \quad 27 \quad 35 \quad 98 \quad 12}{126}. \quad \textit{Ans.}$$

[redacted]

13. Change $\frac{3}{8}$, $\frac{3}{4}$, $\frac{3}{16}$, $\frac{3}{64}$, $\frac{3}{256}$ to similar fractions.

The L. C. D. = 256.

$$\therefore \frac{3}{8}, \frac{3}{4}, \frac{3}{16}, \frac{3}{64}, \frac{3}{256} = \frac{96 \quad 192 \quad 48 \quad 12 \quad 3}{256}. \quad \textit{Ans.}$$

14. Change $\frac{3}{5}$, $\frac{7}{15}$, $\frac{2}{9}$, $\frac{11}{24}$, $\frac{7}{8}$, $\frac{17}{45}$ to similar fractions.

The L. C. D. = 360.

$$\therefore \frac{3}{5}, \frac{7}{15}, \frac{2}{9}, \frac{11}{24}, \frac{7}{8}, \frac{17}{45} = \frac{216 \quad 168 \quad 80 \quad 165 \quad 315 \quad 136}{360}. \quad \textit{Ans.}$$

15. Change $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{7}$, $\frac{7}{12}$, $\frac{13}{18}$, $\frac{4}{27}$ to similar fractions.

The L. C. D. = 756.

$$\therefore \frac{2}{3}, \frac{3}{4}, \frac{5}{7}, \frac{7}{12}, \frac{13}{18}, \frac{4}{27} = \frac{504 \quad 567 \quad 540 \quad 441 \quad 546 \quad 112}{756}. \quad \textit{Ans.}$$

16. Change $\frac{11}{12}$, $\frac{9}{10}$, $\frac{14}{15}$, $\frac{5}{6}$, $\frac{17}{20}$, $\frac{29}{30}$ to similar fractions.

The L. C. D. = 60.

$$\therefore \frac{11}{12}, \frac{9}{10}, \frac{14}{15}, \frac{5}{6}, \frac{17}{20}, \frac{29}{30} = \frac{55 \quad 54 \quad 56 \quad 50 \quad 51 \quad 58}{60}. \quad \textit{Ans.}$$

17. Change $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{6}{11}$, $\frac{7}{44}$, $\frac{9}{22}$ to similar fractions.

The L. C. D. = 88.

$$\therefore \frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{6}{11}, \frac{7}{44}, \frac{9}{22} = \frac{44 \quad 66 \quad 55 \quad 48 \quad 14 \quad 36}{88}. \quad \textit{Ans.}$$

18. Change $\frac{9}{14}$, $\frac{7}{10}$, $\frac{13}{28}$, $\frac{17}{70}$, $\frac{3}{4}$, $\frac{31}{56}$ to similar fractions.

The L. C. D. = 280.

$$\therefore \frac{9}{14}, \frac{7}{10}, \frac{13}{28}, \frac{17}{70}, \frac{3}{4}, \frac{31}{56} = \frac{180 \quad 196 \quad 130 \quad 68 \quad 210 \quad 155}{280}. \quad \textit{Ans.}$$

19. Which is the greater, $\frac{13}{20}$ or $\frac{17}{25}$? $\frac{5}{6}$ or $\frac{7}{9}$? $\frac{3}{5}$ or $\frac{7}{12}$?

The L. C. D. = 100.

$$\frac{13}{20} = \frac{65}{100},$$

$$\frac{17}{25} = \frac{68}{100}.$$

$\therefore$ $\frac{17}{25}$ is the greater.

The L. C. D. = 18.

$$\frac{5}{6} = \frac{15}{18},$$

$$\frac{7}{9} = \frac{14}{18}.$$

$\therefore$ $\frac{5}{6}$ is the greater.

The L. C. D. = 60.

$$\frac{3}{5} = \frac{36}{60},$$

$$\frac{7}{12} = \frac{35}{60}.$$

$\therefore$ $\frac{3}{5}$ is the greater.

20. Arrange the fractions $\frac{7}{12}$, $\frac{11}{18}$, $\frac{13}{24}$ in order of magnitude.

The L. C. D. = 72.

$$\frac{7}{12}, \frac{11}{18}, \frac{13}{24} = \frac{42 \quad 44 \quad 39}{72}. \qquad \frac{13}{24}, \frac{7}{12}, \frac{11}{18}. \text{ Ans.}$$

21. Arrange the fractions $\frac{5}{12}$, $\frac{8}{15}$, $\frac{4}{11}$, $\frac{7}{18}$ in order of magnitude.

The L. C. D. = 1980.

$$\frac{5}{12}, \frac{8}{15}, \frac{4}{11}, \frac{7}{18} = \frac{825 \quad 1056 \quad 720 \quad 770}{1980}.$$

$$\frac{4}{11}, \frac{7}{18}, \frac{5}{12}, \frac{8}{15}. \text{ Ans.}$$

22. Arrange the fractions $\frac{3}{7}$, $\frac{4}{9}$, $\frac{9}{19}$, $\frac{10}{23}$ in order of magnitude.

The L. C. D. = 27,531.

$$\frac{3}{7}, \frac{4}{9}, \frac{9}{19}, \frac{10}{23} = \frac{11799 \quad 12236 \quad 13041 \quad 11970}{27531}. \qquad \frac{3}{7}, \frac{10}{23}, \frac{4}{9}, \frac{9}{19}. \text{ Ans.}$$

Exercise 58. Page 127.

Find the sum of:

1. $\frac{1}{2} + \frac{3}{2}$.

$$\frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2.$$

2. $\frac{1}{3} + \frac{2}{3} + \frac{1}{3}$.

$$\frac{1}{3} + \frac{2}{3} + \frac{1}{3} = \frac{4}{3} = 1\frac{1}{3}.$$

3. $\frac{1}{4} + \frac{1}{4} + \frac{3}{4}$.

$$\frac{1}{4} + \frac{1}{4} + \frac{3}{4} = \frac{5}{4} = 1\frac{1}{4}.$$

4. $1\frac{1}{2} + 2\frac{1}{2}$.

$$1\tfrac{1}{2} + 2\tfrac{1}{2} = 3\tfrac{1+1}{2} = 4.$$

5. $1\frac{1}{3} + 2\frac{2}{3}$.

$$1\tfrac{1}{3} + 2\tfrac{2}{3} = 3\tfrac{1+2}{3} = 4.$$

6. $3\frac{1}{4} + \frac{3}{4}$.

$$3\tfrac{1}{4} + \tfrac{3}{4} = 3\tfrac{1+3}{4} = 4.$$

7. $2\frac{3}{5} + 3\frac{4}{5}$.

$$2\tfrac{3}{5} + 3\tfrac{4}{5} = 5\tfrac{3+4}{5} = 6\tfrac{2}{5}.$$

8. $1\frac{7}{8} + \frac{3}{8}$.

$$1\tfrac{7}{8} + \tfrac{3}{8} = 1\tfrac{7+3}{8} = 2\tfrac{1}{4}.$$

9. $\frac{9}{17} + \frac{3}{17} + \frac{14}{17} + \frac{11}{17}$.

$$\frac{9}{17} + \frac{3}{17} + \frac{14}{17} + \frac{11}{17} = \frac{37}{17} = 2\tfrac{3}{17}.$$

10. $8\frac{9}{17} + 6\frac{3}{17} + 5\frac{14}{17} + \frac{11}{17}$.

$$8\tfrac{9}{17} + 6\tfrac{3}{17} + 5\tfrac{14}{17} + \tfrac{11}{17} = 19\tfrac{37}{17} = 21\tfrac{3}{17}.$$

[redacted]

11. $\frac{4}{5} + \frac{5}{6}$.

The L. C. D. = 30.

$$\frac{4}{5} + \frac{5}{6} = \frac{24 + 25}{30} = \frac{49}{30} = 1\tfrac{19}{30}.$$

12. $\frac{3}{4} + \frac{7}{8}$.

The L. C. D. = 8.

$$\frac{3}{4} + \frac{7}{8} = \frac{6 + 7}{8} = \frac{13}{8} = 1\tfrac{5}{8}.$$

13. $\frac{1}{2} + \frac{1}{6}$.

The L. C. D. = 6.

$$\frac{1}{2} + \frac{1}{6} = \frac{3 + 1}{6} = \frac{4}{6} = \frac{2}{3}.$$

14. $\frac{4}{15} + \frac{11}{20}$.

The L. C. D. = 60.

$$\frac{4}{15} + \frac{11}{20} = \frac{16 + 33}{60} = \frac{49}{60}.$$

15. $\frac{5}{16} + \frac{11}{24}$.

The L. C. D. = 48.

$$\frac{5}{16} + \frac{11}{24} = \frac{15 + 22}{48} = \frac{37}{48}.$$

16. $12\frac{5}{8} + 7\frac{3}{16}$.

The L. C. D. = 16.

$$12\tfrac{5}{8} + 7\tfrac{3}{16} = 19\tfrac{10+3}{16} = 19\tfrac{13}{16}.$$

17. $85\frac{7}{12} + 27\frac{11}{18}$.

The L. C. D. = 36.

$$85\tfrac{7}{12} + 27\tfrac{11}{18} = 112\tfrac{21+22}{36} = 112\tfrac{43}{36} = 113\tfrac{7}{36}.$$

18. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$.

The L. C. D. = 60.

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{30 + 20 + 15 + 12}{60} = \frac{77}{60} = 1\tfrac{17}{60}.$$

19. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5}$.

The L. C. D. = 60.

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} = \frac{30 + 40 + 45 + 48}{60} = \frac{163}{60} = 2\tfrac{43}{60}.$$

20. $\frac{5}{6} + \frac{11}{12} + \frac{8}{15} + \frac{7}{20} + \frac{13}{30}$.

The L. C. D. = 60.

$$\frac{5}{6} + \frac{11}{12} + \frac{8}{15} + \frac{7}{20} + \frac{13}{30} = \frac{50 + 55 + 32 + 21 + 26}{60} = \frac{184}{60} = 3\tfrac{1}{15}.$$

21. $5\frac{17}{20} + 11\frac{19}{30} + 24\frac{21}{40} + \frac{9}{50} + 17\frac{8}{15} + 14 + 11\frac{5}{12}$.

The L. C. D. = 600.

$$5\tfrac{17}{20} + 11\tfrac{19}{30} + 24\tfrac{21}{40} + \tfrac{9}{50} + 17\tfrac{8}{15} + 14 + 11\tfrac{5}{12}$$

$$= 82\tfrac{510+380+315+108+320+250}{600} = 82\tfrac{1883}{600} = 85\tfrac{83}{600}.$$

22. $9\frac{4}{7} + 15\frac{11}{28} + 163\frac{17}{63} + 1\frac{11}{42} + 10\frac{1}{4}$.

The L. C. D. = 252.

$$9\tfrac{4}{7} + 15\tfrac{11}{28} + 163\tfrac{17}{63} + 1\tfrac{11}{42} + 10\tfrac{1}{4} = 198\tfrac{144+99+68+66+63}{252}$$
$$= 198\tfrac{440}{252} = 199\tfrac{47}{63}.$$

23. $3\frac{3}{5} + 4\frac{2}{3} + 1\frac{5}{6} + 2$.

The L. C. D. = 30.

$$3\tfrac{3}{5} + 4\tfrac{2}{3} + 1\tfrac{5}{6} + 2 = 10\tfrac{18+20+25}{30} = 10\tfrac{63}{30} = 12\tfrac{3}{30} = 12\tfrac{1}{10}.$$

24. $1\frac{3}{20} + 2\frac{2}{25} + 5\frac{7}{30} + \frac{4}{15}$.

The L. C. D. = 300.

$$1\tfrac{3}{20} + 2\tfrac{2}{25} + 5\tfrac{7}{30} + \tfrac{4}{15} = 8\tfrac{45+24+70+80}{300} = 8\tfrac{219}{300} = 8\tfrac{73}{100}.$$

25. $\frac{2}{7} + 1\frac{4}{9} + 2 + 3\frac{3}{8} + 4\frac{5}{12}$.

The L. C. D. = 504.

$$\tfrac{2}{7} + 1\tfrac{4}{9} + 2 + 3\tfrac{3}{8} + 4\tfrac{5}{12} = 10\tfrac{144+224+189+210}{504} = 10\tfrac{767}{504} = 11\tfrac{263}{504}.$$

26. $4\frac{4}{9} + 3\frac{3}{8} + 2\frac{2}{7} + 1\frac{1}{6} + \frac{9}{14}$.

The L. C. D. = 504.

$$4\tfrac{4}{9} + 3\tfrac{3}{8} + 2\tfrac{2}{7} + 1\tfrac{1}{6} + \tfrac{9}{14} = 10\tfrac{224+189+144+84+324}{504} = 10\tfrac{965}{504} = 11\tfrac{461}{504}.$$

27. $\frac{11}{35} + \frac{7}{40} + 10 + \frac{23}{60}$.

The L. C. D. = 840.

$$\tfrac{11}{35} + \tfrac{7}{40} + 10 + \tfrac{23}{60} = 10\tfrac{264+147+322}{840} = 10\tfrac{733}{840}.$$

28. $\frac{27}{50} + \frac{29}{60} + \frac{31}{80} + \frac{33}{100} + \frac{37}{240}$.

The L. C. D. = 1200.

$$\frac{27}{50} + \frac{29}{60} + \frac{31}{80} + \frac{33}{100} + \frac{37}{240} = \frac{648 + 580 + 465 + 396 + 185}{1200} = \frac{2274}{1200} = 1\tfrac{179}{200}.$$

29. $2 + \frac{2}{3} + 1\frac{3}{4} + 4\frac{8}{9} + 5\frac{13}{24}$.

The L. C. D. = 72.

$$2 + \tfrac{2}{3} + 1\tfrac{3}{4} + 4\tfrac{8}{9} + 5\tfrac{13}{24} = 12\tfrac{48+54+64+39}{72} = 12\tfrac{205}{72} = 14\tfrac{61}{72}.$$

[redacted]

30. $3\frac{5}{8}+6+\frac{4}{11}+2\frac{3}{10}+5\frac{5}{16}+\frac{9}{20}$.

The L. C. D. = 880.

$$3\frac{5}{8}+6+\frac{4}{11}+2\frac{3}{10}+5\frac{5}{16}+\frac{9}{20}=16\frac{550+320+264+275+396}{880}=16\frac{1805}{880}$$
$$=18\frac{45}{880}=18\frac{9}{176}.$$

31. $\frac{8}{15}+\frac{7}{18}+3\frac{17}{20}+1\frac{19}{24}+2\frac{19}{120}$.

The L. C. D. = 360.

$$\frac{8}{15}+\frac{7}{18}+3\frac{17}{20}+1\frac{19}{24}+2\frac{19}{120}=6\frac{192+140+306+285+57}{360}=6\frac{980}{360}$$
$$=8\frac{260}{360}=8\frac{13}{18}.$$

32. $\frac{5}{14}+\frac{6}{11}+9\frac{1}{2}$.

The L. C. D. = 154.

$$\frac{5}{14}+\frac{6}{11}+9\frac{1}{2}=9\frac{55+84+77}{154}=9\frac{216}{154}=10\frac{62}{154}=10\frac{31}{77}.$$

33. $20\frac{5}{12}+11\frac{7}{20}+5\frac{1}{8}+305$.

The L. C. D. = 120.

$$20\frac{5}{12}+11\frac{7}{20}+5\frac{1}{8}+305=341\frac{50+42+15}{120}=341\frac{107}{120}.$$

34. $\frac{11}{38}+\frac{14}{57}+\frac{17}{76}$.

The L. C. D. = 228.

$$\frac{11}{38}+\frac{14}{57}+\frac{17}{76}=\frac{66+56+51}{228}=\frac{173}{228}.$$

35. $\frac{5}{17}+\frac{11}{34}+\frac{14}{51}+\frac{19}{68}$.

The L. C. D. = 204.

$$\frac{5}{17}+\frac{11}{34}+\frac{14}{51}+\frac{19}{68}=\frac{60+66+56+57}{204}=\frac{239}{204}=1\frac{35}{204}.$$

36. $317\frac{2}{5}+17\frac{8}{51}+4\frac{9}{10}+\frac{7}{15}+6\frac{2}{3}+\frac{5}{17}$.

The L. C. D. = 510.

$$317\frac{2}{5}+17\frac{8}{51}+4\frac{9}{10}+\frac{7}{15}+6\frac{2}{3}+\frac{5}{17}=344\frac{204+80+459+238+340+150}{510}$$
$$=344\frac{1471}{510}=346\frac{451}{510}.$$

37. $4\frac{7}{15}+8\frac{5}{21}+4\frac{7}{11}+5\frac{2}{7}+5\frac{4}{5}+\frac{2}{3}$.

The L. C. D. = 1155.

$$4\frac{7}{15}+8\frac{5}{21}+4\frac{7}{11}+5\frac{2}{7}+5\frac{4}{5}+\frac{2}{3}=26\frac{539+275+735+330+924+770}{1155}$$
$$=26\frac{3573}{1155}=29\frac{108}{1155}=29\frac{36}{385}.$$

[redacted]

38. $3\frac{2}{3} + 5\frac{3}{40} + 8\frac{7}{240} + \frac{39}{60} + 1\frac{29}{2880}$.

The L. C. D. = 2880.

$$3\frac{2}{3} + 5\frac{3}{40} + 8\frac{7}{240} + \frac{39}{60} + 1\frac{29}{2880} = 17\frac{1920+216+84+1872+29}{2880}$$
$$= 17\frac{4121}{2880} = 18\frac{1241}{2880}.$$

39. $4\frac{5}{13} + 7\frac{5}{39} + 5\frac{47}{78} + 275\frac{37}{156} + 2\frac{57}{91}$.

The L. C. D. = 1092.

$$4\frac{5}{13} + 7\frac{5}{39} + 5\frac{47}{78} + 275\frac{37}{156} + 2\frac{57}{91} = 293\frac{420+140+658+259+684}{1092}$$
$$= 293\frac{2161}{1092} = 294\frac{1069}{1092}.$$

40. $\frac{17}{33} + 7\frac{5}{12} + 6\frac{2}{11} + 400\frac{3}{28} + 51\frac{25}{56}$.

The L. C. D. = 1848.

$$\frac{17}{33} + 7\frac{5}{12} + 6\frac{2}{11} + 400\frac{3}{28} + 51\frac{25}{56} = 464\frac{952+770+336+198+825}{1848}$$
$$= 464\frac{3081}{1848} = 465\frac{1233}{1848}.$$

41. $3\frac{2}{3} + 1\frac{1}{4} + 2\frac{1}{9} + 3\frac{5}{6} + 107\frac{5}{18} + 2\frac{7}{36}$.

The L. C. D. = 36.

$$3\frac{2}{3} + 1\frac{1}{4} + 2\frac{1}{9} + 3\frac{5}{6} + 107\frac{5}{18} + 2\frac{7}{36} = 118\frac{24+9+4+30+10+7}{36} = 118\frac{84}{36}$$
$$= 120\frac{12}{36} = 120\frac{1}{3}.$$

42. $5\frac{1}{14} + 5\frac{3}{5} + 2\frac{1}{7} + 7\frac{8}{21} + 12\frac{4}{15}$.

The L. C. D. = 210.

$$5\frac{1}{14} + 5\frac{3}{5} + 2\frac{1}{7} + 7\frac{8}{21} + 12\frac{4}{15} = 31\frac{15+126+30+80+56}{210} = 31\frac{307}{210}$$
$$= 32\frac{97}{210}.$$

43. $4\frac{3}{4} + 2\frac{1}{8} + 3\frac{3}{8} + 7\frac{1}{6} + 8\frac{15}{16}$.

The L. C. D. = 48.

$$4\frac{3}{4} + 2\frac{1}{8} + 3\frac{3}{8} + 7\frac{1}{6} + 8\frac{15}{16} = 24\frac{36+6+18+8+45}{48} = 24\frac{113}{48} = 26\frac{17}{48}.$$

44. $6\frac{1}{2} + 7\frac{2}{3} + 8\frac{3}{4} + 9\frac{5}{6} + 8\frac{11}{18}$.

The L. C. D. = 36.

$$6\frac{1}{2} + 7\frac{2}{3} + 8\frac{3}{4} + 9\frac{5}{6} + 8\frac{11}{18} = 38\frac{18+24+27+30+22}{36} = 38\frac{121}{36} = 41\frac{13}{36}.$$

45. $7\frac{5}{6} + 8\frac{3}{4} + 5\frac{15}{16} + 7\frac{11}{12} + 9\frac{1}{2}$.

The L. C. D. = 48.

$$7\frac{5}{6}+8\frac{3}{4}+5\frac{15}{16}+7\frac{11}{12}+9\frac{1}{2}=36\frac{40+36+45+44+24}{48}=36\frac{189}{48}=39\frac{45}{48}=39\frac{15}{16}.$$

[redacted]

46. $5\frac{1}{2} + 6\frac{2}{3} + 7\frac{11}{24} + 9\frac{17}{48} + 3\frac{11}{16} + 2\frac{1}{6}$.

The L. C. D. $= 48$.

$$5\frac{1}{2} + 6\frac{2}{3} + 7\frac{11}{24} + 9\frac{17}{48} + 3\frac{11}{16} + 2\frac{1}{6} = 32\frac{24+32+22+17+33+8}{48} = 32\frac{136}{48}$$
$$= 34\frac{40}{48} = 34\frac{5}{6}.$$

47. $9\frac{3}{4} + 10\frac{6}{7} + 11\frac{2}{3} + 5\frac{17}{42} + 7\frac{8}{21} + 18\frac{5}{6}$.

The L. C. D. $= 84$.

$$9\frac{3}{4} + 10\frac{6}{7} + 11\frac{2}{3} + 5\frac{17}{42} + 7\frac{8}{21} + 18\frac{5}{6} = 60\frac{63+72+56+34+32+70}{84}$$
$$= 60\frac{327}{84} = 63\frac{75}{84} = 63\frac{25}{28}.$$

48. $\frac{13}{21} + \frac{3}{49} + \frac{16}{35} + \frac{11}{63} + \frac{4}{7} + \frac{4}{15} + \frac{17}{45}$.

The L. C. D. $= 2205$.

$$\frac{13}{21} + \frac{3}{49} + \frac{16}{35} + \frac{11}{63} + \frac{4}{7} + \frac{4}{15} + \frac{17}{45}$$
$$= \frac{1365 + 135 + 1008 + 385 + 1260 + 588 + 833}{2205}$$
$$= \frac{5574}{2205} = 2\frac{1164}{2205} = 2\frac{388}{735}.$$

Exercise 59. Page 129.

Find the value of :

1. $52\frac{1}{3} - 46 = 6\frac{1}{3}$.

2. $\frac{6}{9} - \frac{3}{9} = \frac{6-3}{9} = \frac{3}{9} = \frac{1}{3}$.

3. $\frac{3}{4} - \frac{2}{3} = \frac{9-8}{12} = \frac{1}{12}$.

4. $\frac{8}{15} - \frac{5}{12} = \frac{32-25}{60} = \frac{7}{60}$.

5. $\frac{11}{18} - \frac{3}{14} = \frac{77-27}{126} = \frac{50}{126} = \frac{25}{63}$.

6. $4 - \frac{1}{2} = 3\frac{1}{2}$.

7. $7 - \frac{2}{3} = 6\frac{1}{3}$.

8. $3 - \frac{5}{6} = 2\frac{1}{6}$.

9. $8 - \frac{6}{7} = 7\frac{1}{7}$.

10. $5 - \frac{4}{5} = 4\frac{1}{5}$.

11. $5 - \frac{7}{9} = 4\frac{2}{9}$.

12. $6\frac{1}{3} - 5\frac{1}{6} = 1\frac{2-1}{6} = 1\frac{1}{6}$.

13. $4\frac{2}{5} - 3\frac{3}{7} = 1\frac{14-15}{35}$
$= \frac{49-15}{35} = \frac{34}{35}$.

14. $7\frac{1}{3} - 2\frac{3}{10} = 5\frac{10-9}{30} = 5\frac{1}{30}$.

15. $7\frac{2}{5} - 4\frac{8}{9} = 3\frac{18-40}{45}$
$= 2\frac{63-40}{45} = 2\frac{23}{45}$.

16. $6\frac{2}{3} - 2\frac{3}{4} = 4\frac{8-9}{12}$
$= 3\frac{20-9}{12} = 3\frac{11}{12}$.

17. $9\frac{4}{5} - 4\frac{5}{6} = 5\frac{24-25}{30}$
$= 4\frac{54-25}{30} = 4\frac{29}{30}$.

18. $4\frac{2}{3} - \frac{1}{2} = 4\frac{4-3}{6} = 4\frac{1}{6}$.

19. $6\frac{3}{4} - 4\frac{2}{3} = 2\frac{9-8}{12} = 2\frac{1}{12}$.

20. $7\frac{1}{2} - 2\frac{3}{4} = 5\frac{2-3}{4} = 4\frac{6-3}{4} = 4\frac{3}{4}$.

21. $8\frac{1}{5} - 4\frac{4}{7} = 4\frac{7-20}{35}$
$= 3\frac{42-20}{35} = 3\frac{22}{35}$.

[redacted]

22. $85\frac{7}{22}-27\frac{11}{18}=58\frac{63-121}{198}=57\frac{261-121}{198}=57\frac{140}{198}=57\frac{70}{99}$.

23. $8\frac{7}{10}-2\frac{11}{16}=6\frac{56-55}{80}=6\frac{1}{80}$.

24. $10-3\frac{5}{8}=6\frac{3}{8}$.

25. $120\frac{21}{32}-110\frac{13}{24}=10\frac{63-52}{96}=10\frac{11}{96}$.

26. $5\frac{17}{28}-\frac{27}{35}=5\frac{85-108}{140}=4\frac{225-108}{140}=4\frac{117}{140}$.

27. $13\frac{3}{40}-2\frac{15}{44}=11\frac{33-150}{440}=10\frac{473-150}{440}=10\frac{323}{440}$.

28. $2\frac{151}{240}-1\frac{163}{192}=1\frac{604-815}{960}=\frac{1564-815}{960}=\frac{749}{960}$.

29. $4-1\frac{2817}{4000}=2\frac{4000-2817}{4000}=2\frac{1183}{4000}$.

30. $1473-279\frac{11}{12}=1193\frac{1}{12}$.

31. $1473\frac{5}{13}-279\frac{11}{12}=1194\frac{60-143}{156}=1193\frac{216-143}{156}=1193\frac{73}{156}$.

32. $1473\frac{7}{18}-279\frac{11}{12}=1194\frac{14-33}{36}=1193\frac{50-33}{36}=1193\frac{17}{36}$.

33. $278\frac{15}{16}-30\frac{5}{12}=248\frac{45-20}{48}=248\frac{25}{48}$.

34. $125\frac{5}{22}-10\frac{17}{33}=115\frac{15-34}{66}=114\frac{81-34}{66}=114\frac{47}{66}$.

35. $118\frac{5}{11}-17\frac{3}{14}=101\frac{70-33}{154}=101\frac{37}{154}$.

36. $94\frac{5}{11}-91\frac{13}{14}=3\frac{70-143}{154}=2\frac{224-143}{154}=2\frac{81}{154}$.

37. $7\frac{5}{21}-2\frac{11}{14}=5\frac{10-33}{42}=4\frac{52-33}{42}=4\frac{19}{42}$.

38. $\frac{235}{357}-\frac{13}{51}=\frac{235-91}{357}=\frac{144}{357}=\frac{48}{119}$.

39. $\frac{17}{63}-\frac{29}{108}=\frac{204-203}{756}=\frac{1}{756}$.

40. $\frac{9}{38}-\frac{43}{209}=\frac{99-86}{418}=\frac{13}{418}$.

41. $\frac{146}{273}-\frac{268}{637}=\frac{1022-804}{1911}=\frac{218}{1911}$.

42. $\frac{359}{360}-\frac{199}{200}=\frac{1795-1791}{1800}=\frac{4}{1800}=\frac{1}{450}$.

[redacted]

Exercise 60. Page 130.

1. Simplify $3\frac{2}{5} - 2\frac{5}{8} + 4\frac{3}{10} + 1\frac{7}{9} - 5\frac{8}{15}$.

$$3\tfrac{2}{5} + 4\tfrac{3}{10} + 1\tfrac{7}{9} = 8\tfrac{36+27+70}{90} = 8\tfrac{133}{90} = 9\tfrac{43}{90}.$$

$$2\tfrac{5}{8} + 5\tfrac{8}{15} = 7\tfrac{75+64}{120} = 7\tfrac{139}{120} = 8\tfrac{19}{120}.$$

$$9\tfrac{43}{90} - 8\tfrac{19}{120} = 1\tfrac{172-57}{360} = 1\tfrac{115}{360} = 1\tfrac{23}{72}. \textit{ Ans.}$$

2. Simplify $1\frac{5}{11} - \frac{11}{12} + 7\frac{3}{8} - 2\frac{1}{3} - 1\frac{11}{16}$.

$$1\tfrac{5}{11} + 7\tfrac{3}{8} = 8\tfrac{40+33}{88} = 8\tfrac{73}{88}.$$

$$\tfrac{11}{12} + 2\tfrac{1}{3} + 1\tfrac{11}{16} = 3\tfrac{44+16+33}{48} = 3\tfrac{93}{48} = 4\tfrac{45}{48} = 4\tfrac{15}{16}.$$

$$8\tfrac{73}{88} - 4\tfrac{15}{16} = 4\tfrac{146-165}{176} = 3\tfrac{322-165}{176} = 3\tfrac{157}{176}. \textit{ Ans.}$$

3. Simplify $12 - 3\frac{2}{7} - 1\frac{3}{10} - 4\frac{5}{28} + 2\frac{13}{20} - 4\frac{3}{5}$.

$$12 + 2\tfrac{13}{20} = 14\tfrac{13}{20}.$$

$$3\tfrac{2}{7} + 1\tfrac{3}{10} + 4\tfrac{5}{28} + 4\tfrac{3}{5} = 12\tfrac{40+42+25+84}{140} = 12\tfrac{191}{140} = 13\tfrac{51}{140}.$$

$$14\tfrac{13}{20} - 13\tfrac{51}{140} = 1\tfrac{91-51}{140} = 1\tfrac{40}{140} = 1\tfrac{2}{7}. \textit{ Ans.}$$

4. Simplify $43\frac{7}{15} - 1\frac{1}{3} - 1\frac{31}{48} - 1\frac{23}{24} - 2\frac{13}{48} - 2\frac{7}{12} - 2\frac{43}{48} - 3\frac{5}{12}$.

$$1\tfrac{1}{3} + 1\tfrac{31}{48} + 1\tfrac{23}{24} + 2\tfrac{13}{48} + 2\tfrac{7}{12} + 2\tfrac{43}{48} + 3\tfrac{5}{12} = 12\tfrac{16+31+46+13+28+43+20}{48}$$
$$= 12\tfrac{197}{48} = 16\tfrac{5}{48}.$$

$$43\tfrac{7}{15} - 16\tfrac{5}{48} = 27\tfrac{112-25}{240} = 27\tfrac{87}{240} = 27\tfrac{29}{80}. \textit{ Ans.}$$

5. Simplify $\frac{1}{2} + \frac{4}{13} + 7\frac{9}{40} + 8\frac{14}{39} + 7\frac{1}{4} + 8\frac{3}{10} + 4\frac{1}{12} - 36\frac{1}{40}$.

$$\tfrac{1}{2} + \tfrac{4}{13} + 7\tfrac{9}{40} + 8\tfrac{14}{39} + 7\tfrac{1}{4} + 8\tfrac{3}{10} + 4\tfrac{1}{12}$$
$$= 34\tfrac{780+480+351+560+390+468+130}{1560} = 34\tfrac{3159}{1560} = 36\tfrac{39}{1560} = 36\tfrac{1}{40}.$$

$$36\tfrac{1}{40} - 36\tfrac{1}{40} = 0. \textit{ Ans.}$$

6. Simplify $(8\frac{5}{18} + 1\frac{10}{27} + 17\frac{11}{36} + 40) - (30\frac{13}{40} + 11\frac{11}{20})$.

$$8\tfrac{5}{18} + 1\tfrac{10}{27} + 17\tfrac{11}{36} + 40 = 66\tfrac{30+40+33}{108} = 66\tfrac{103}{108}.$$

$$30\tfrac{13}{40} + 11\tfrac{11}{20} = 41\tfrac{13+22}{40} = 41\tfrac{35}{40} = 41\tfrac{7}{8}.$$

$$66\tfrac{103}{108} - 41\tfrac{7}{8} = 25\tfrac{206-189}{216} = 25\tfrac{17}{216}. \textit{ Ans.}$$

[redacted]

7. Simplify $(172\frac{19}{78} + 93\frac{14}{117}) + (172\frac{19}{78} - 93\frac{14}{117})$.

$$(172\tfrac{19}{78} + 93\tfrac{14}{117}) + (172\tfrac{19}{78} - 93\tfrac{14}{117})$$
$$= 172\tfrac{19}{78} + 93\tfrac{14}{117} + 172\tfrac{19}{78} - 93\tfrac{14}{117}$$
$$= 172\tfrac{19}{78} + 172\tfrac{19}{78} = 344\tfrac{19+19}{78} = 344\tfrac{19}{39}.\ \textit{Ans.}$$

8. Simplify $(172\frac{19}{78} + 93\frac{14}{117}) - (172\frac{19}{78} - 93\frac{14}{117})$.

$$(172\tfrac{19}{78} + 93\tfrac{14}{117}) - (172\tfrac{19}{78} - 93\tfrac{14}{117})$$
$$= 172\tfrac{19}{78} + 93\tfrac{14}{117} - 172\tfrac{19}{78} + 93\tfrac{14}{117}$$
$$= 93\tfrac{14}{117} + 93\tfrac{14}{117} = 186\tfrac{28}{117}.\ \textit{Ans.}$$

9. Simplify $(\frac{3}{13} - \frac{2}{39}) + (\frac{5}{78} + \frac{7}{156})$.

$$\tfrac{3}{13} - \tfrac{2}{39} = \tfrac{9-2}{39} = \tfrac{7}{39}.$$
$$\tfrac{5}{78} + \tfrac{7}{156} = \tfrac{10+7}{156} = \tfrac{17}{156}.$$
$$\tfrac{7}{39} + \tfrac{17}{156} = \tfrac{28+17}{156} = \tfrac{45}{156} = \tfrac{15}{52}.\ \textit{Ans.}$$

10. Simplify $\frac{4}{9} - \frac{3}{11} - 2\frac{3}{4} + 3\frac{2}{3} + 7\frac{7}{12} - 1\frac{3}{5} - \frac{3}{22}$.

$$\tfrac{4}{9} + 3\tfrac{2}{3} + 7\tfrac{7}{12} = 10\tfrac{16+24+21}{36} = 10\tfrac{61}{36} = 11\tfrac{25}{36}.$$
$$\tfrac{3}{11} + 2\tfrac{3}{4} + 1\tfrac{3}{5} + \tfrac{3}{22} = 3\tfrac{60+165+132+30}{220} = 3\tfrac{387}{220} = 4\tfrac{167}{220}.$$
$$11\tfrac{25}{36} - 4\tfrac{167}{220} = 7\tfrac{1375-1503}{1980} = 6\tfrac{3355-1503}{1980} = 6\tfrac{1852}{1980} = 6\tfrac{463}{495}.\ \textit{Ans.}$$

11. Simplify $\frac{3}{10} - \frac{7}{100} - \frac{9}{1000} - \frac{5}{10000}$.

$$\tfrac{7}{100} + \tfrac{9}{1000} + \tfrac{5}{10000} = \tfrac{700+90+5}{10000} = \tfrac{795}{10000} = \tfrac{159}{2000}.$$
$$\tfrac{3}{10} - \tfrac{159}{2000} = \tfrac{600-159}{2000} = \tfrac{441}{2000}.\ \textit{Ans.}$$

12. Simplify $9\frac{5}{8} - 7 - \frac{3}{4} - \frac{5}{6}$.

$$7 + \tfrac{3}{4} + \tfrac{5}{6} = 7\tfrac{9+10}{12} = 7\tfrac{19}{12} = 8\tfrac{7}{12}.$$
$$9\tfrac{5}{8} - 8\tfrac{7}{12} = 1\tfrac{15-14}{24} = 1\tfrac{1}{24}.\ \textit{Ans.}$$

13. Simplify $5\frac{2}{3} + 8\frac{3}{4} - 1\frac{3}{5} - 4\frac{7}{9}$.

$$5\tfrac{2}{3} + 8\tfrac{3}{4} = 13\tfrac{8+9}{12} = 13\tfrac{17}{12} = 14\tfrac{5}{12}.$$
$$1\tfrac{3}{5} + 4\tfrac{7}{9} = 5\tfrac{27+35}{45} = 5\tfrac{62}{45} = 6\tfrac{17}{45}.$$
$$14\tfrac{5}{12} - 6\tfrac{17}{45} = 8\tfrac{75-68}{180} = 8\tfrac{7}{180}.\ \textit{Ans.}$$

[redacted]

14. Simplify $6\frac{3}{4} - 5\frac{2}{3} + 4\frac{2}{5} - 4\frac{5}{12}$.

$$6\tfrac{3}{4} + 4\tfrac{2}{5} = 10\tfrac{15+8}{20} = 10\tfrac{23}{20} = 11\tfrac{3}{20}.$$

$$5\tfrac{2}{3} + 4\tfrac{5}{12} = 9\tfrac{8+5}{12} = 9\tfrac{13}{12} = 10\tfrac{1}{12}.$$

$$11\tfrac{3}{20} - 10\tfrac{1}{12} = 1\tfrac{9-5}{60} = 1\tfrac{4}{60} = 1\tfrac{1}{15}.\ \textit{Ans.}$$

15. Simplify $14\frac{7}{18} + 9\frac{3}{5} - 6\frac{3}{4} - 12\frac{4}{5} - 3\frac{3}{5}$.

$$14\tfrac{7}{18} + 9\tfrac{3}{5} = 23\tfrac{35+54}{90} = 23\tfrac{89}{90}.$$

$$6\tfrac{3}{4} + 12\tfrac{4}{5} + 3\tfrac{3}{5} = 21\tfrac{15+16+12}{20} = 21\tfrac{43}{20} = 23\tfrac{3}{20}.$$

$$23\tfrac{89}{90} - 23\tfrac{3}{20} = \tfrac{178-27}{180} = \tfrac{151}{180}.\ \textit{Ans.}$$

16. Simplify $20\frac{2}{3} - 2\frac{5}{8} - 9\frac{5}{9} + 10\frac{3}{10} - 14\frac{7}{12}$.

$$20\tfrac{2}{3} + 10\tfrac{3}{10} = 30\tfrac{20+9}{30} = 30\tfrac{29}{30}.$$

$$2\tfrac{5}{8} + 9\tfrac{5}{9} + 14\tfrac{7}{12} = 25\tfrac{45+40+42}{72} = 25\tfrac{127}{72} = 26\tfrac{55}{72}.$$

$$30\tfrac{29}{30} - 26\tfrac{55}{72} = 4\tfrac{348-275}{360} = 4\tfrac{73}{360}.\ \textit{Ans.}$$

17. Simplify $95\frac{2}{3} - 9\frac{7}{20} - 8\frac{3}{7} - 14\frac{2}{5} + 74\frac{2}{9}$.

$$95\tfrac{2}{3} + 74\tfrac{2}{9} = 169\tfrac{6+2}{9} = 169\tfrac{8}{9}.$$

$$9\tfrac{7}{20} + 8\tfrac{3}{7} + 14\tfrac{2}{5} = 31\tfrac{49+60+56}{140} = 31\tfrac{165}{140} = 32\tfrac{25}{140} = 32\tfrac{5}{28}.$$

$$169\tfrac{8}{9} - 32\tfrac{5}{28} = 137\tfrac{224-45}{252} = 137\tfrac{179}{252}.\ \textit{Ans.}$$

18. Simplify $12\frac{3}{4} + 23\frac{3}{8} - (4\frac{3}{10} + 12\frac{2}{5} + 7\frac{14}{15})$.

$$12\tfrac{3}{4} + 23\tfrac{3}{8} = 35\tfrac{6+3}{8} = 35\tfrac{9}{8} = 36\tfrac{1}{8}.$$

$$4\tfrac{3}{10} + 12\tfrac{2}{5} + 7\tfrac{14}{15} = 23\tfrac{9+12+28}{30} = 23\tfrac{49}{30} = 24\tfrac{19}{30}.$$

$$36\tfrac{1}{8} - 24\tfrac{19}{30} = 12\tfrac{15-76}{120} = 11\tfrac{135-76}{120} = 11\tfrac{59}{120}.\ \textit{Ans.}$$

19. Simplify $16\frac{2}{15} + 18\frac{5}{24} - (5\frac{2}{5} + 9\frac{9}{40} + 14\frac{5}{24})$.

$$16\tfrac{2}{15} + 18\tfrac{5}{24} = 34\tfrac{16+25}{120} = 34\tfrac{41}{120}.$$

$$5\tfrac{2}{5} + 9\tfrac{9}{40} + 14\tfrac{5}{24} = 28\tfrac{48+27+25}{120} = 28\tfrac{100}{120}.$$

$$34\tfrac{41}{120} - 28\tfrac{100}{120} = 6\tfrac{41-100}{120} = 5\tfrac{161-100}{120} = 5\tfrac{61}{120}.\ \textit{Ans.}$$

20. Simplify $97\frac{8}{9} - (20 + 9\frac{3}{4} + 18\frac{2}{25} + 24\frac{49}{50})$.

$$20 + 9\tfrac{3}{4} + 18\tfrac{2}{25} + 24\tfrac{49}{50} = 71\tfrac{75+8+98}{100} = 71\tfrac{181}{100} = 72\tfrac{81}{100}.$$

$$97\tfrac{8}{9} - 72\tfrac{81}{100} = 25\tfrac{800-729}{900} = 25\tfrac{71}{900}.\ \textit{Ans.}$$

21. Simplify $2\frac{12}{16}+3\frac{15}{35}-(1\frac{53}{70}+1\frac{31}{32}+\frac{49}{56})$.

$$2\tfrac{12}{16}+3\tfrac{15}{35}=2\tfrac{3}{4}+3\tfrac{3}{7}=5\tfrac{21+12}{28}=5\tfrac{33}{28}=6\tfrac{5}{28}.$$

$$1\tfrac{53}{70}+1\tfrac{31}{32}+\tfrac{49}{56}=2\tfrac{848+1085+980}{1120}=2\tfrac{2913}{1120}=4\tfrac{673}{1120}.$$

$$6\tfrac{5}{28}-4\tfrac{673}{1120}=2\tfrac{200-673}{1120}=1\tfrac{1320-673}{1120}=1\tfrac{647}{1120}.\ \textit{Ans.}$$

22. Simplify $\frac{143}{100}+\frac{2471}{1000}-\frac{82648}{100000}$.

$$\tfrac{143}{100}+\tfrac{2471}{1000}=\tfrac{1430+2471}{1000}=\tfrac{3901}{1000}=3\tfrac{901}{1000}.$$

$$3\tfrac{901}{1000}-\tfrac{82648}{100000}=3\tfrac{90100-82648}{100000}=3\tfrac{7452}{100000}.\ \textit{Ans.}$$

Exercise 61. Page 132.

1. Simplify $\dfrac{2\frac{3}{11}}{3\frac{3}{4}}$.

$$\frac{2\frac{3}{11}}{3\frac{3}{4}}=\frac{\overset{5}{\cancel{25}}}{11}\times\frac{4}{\underset{3}{\cancel{15}}}=\frac{20}{33}.$$

2. Simplify $\dfrac{3}{7\frac{1}{8}}$.

$$\frac{3}{7\frac{1}{8}}=\cancel{3}\times\frac{8}{\underset{19}{\cancel{57}}}=\frac{8}{19}.$$

3. Simplify $\dfrac{17\frac{1}{7}}{13\frac{1}{3}}$.

$$\frac{17\frac{1}{7}}{13\frac{1}{3}}=\frac{\overset{3}{\cancel{120}}}{7}\times\frac{3}{\cancel{40}}=\frac{9}{7}=1\tfrac{2}{7}.$$

4. Simplify $\dfrac{\frac{5}{9}}{8\frac{1}{3}}$.

$$\frac{\frac{5}{9}}{8\frac{1}{3}}=\frac{\cancel{5}}{\underset{3}{\cancel{9}}}\times\frac{\cancel{3}}{\underset{5}{\cancel{25}}}=\frac{1}{15}.$$

5. Simplify $\dfrac{5\frac{1}{9}}{8\frac{4}{11}}$.

$$\frac{5\frac{1}{9}}{8\frac{4}{11}}=\frac{\cancel{46}}{9}\times\frac{11}{\underset{2}{\cancel{92}}}=\frac{11}{18}.$$

6. Simplify $\dfrac{1\frac{4}{5}\text{ of }3\frac{1}{7}}{4\frac{1}{8}\text{ of }\frac{9}{10}}$.

$$\frac{1\frac{4}{5}\text{ of }3\frac{1}{7}}{4\frac{1}{8}\text{ of }\frac{9}{10}}=\frac{\cancel{9}}{\cancel{5}}\times\frac{\overset{2}{\cancel{22}}}{7}\times\frac{8}{\underset{3}{\cancel{33}}}\times\frac{\overset{2}{\cancel{10}}}{\cancel{9}}$$
$$=\frac{32}{21}=1\tfrac{11}{21}.$$

7. Simplify $\dfrac{2\frac{1}{2}-1\frac{5}{9}}{1\frac{5}{6}-1\frac{5}{8}}$.

$$\frac{2\frac{1}{2}-1\frac{5}{9}}{1\frac{5}{6}-1\frac{5}{8}}=\frac{180-112}{132-117}=\frac{68}{15}=4\tfrac{8}{15}.$$

8. Simplify $\dfrac{10\frac{2}{5}-1\frac{5}{7}}{7\frac{1}{8}-3\frac{3}{40}}$.

$$\frac{10\frac{2}{5}-1\frac{5}{7}}{7\frac{1}{8}-3\frac{3}{40}}=\frac{2912-480}{1995-861}=\frac{2432}{1134}=2\tfrac{164}{1134}=2\tfrac{82}{567}.$$

[redacted]

9. Simplify $\dfrac{\frac{3}{7} \text{ of } 2\frac{1}{17}}{1\frac{2}{3} \div 2\frac{3}{7}}$.

$$\frac{\frac{3}{7} \text{ of } 2\frac{1}{17}}{1\frac{2}{3} \div 2\frac{3}{7}} = \frac{3}{\cancel{7}} \times \frac{\overset{\cancel{7}}{\cancel{35}}}{\cancel{17}} \times \frac{3}{\cancel{5}} \times \frac{\cancel{17}}{7} = \frac{9}{7} = 1\tfrac{2}{7}.$$

10. Simplify $\dfrac{6\frac{3}{4} - 1\frac{5}{14}}{2\frac{1}{6} + 1\frac{3}{7}}$.

$$\frac{6\frac{3}{4} - 1\frac{5}{14}}{2\frac{1}{6} + 1\frac{3}{7}} = \frac{567 - 114}{182 + 120} = \frac{453}{302} = 1\tfrac{151}{302} = 1\tfrac{1}{2}.$$

11. Simplify $\dfrac{5\frac{4}{5} + 2\frac{3}{7}}{4\frac{2}{3} - 3\frac{11}{13}}$.

$$5\tfrac{4}{5} + 2\tfrac{3}{7} = 7\tfrac{28+15}{35} = 7\tfrac{43}{35} = 8\tfrac{8}{35}.$$

$$4\tfrac{2}{3} - 3\tfrac{11}{13} = 1\tfrac{26-33}{39} = \frac{65 - 33}{39} = \frac{32}{39}.$$

$$\frac{5\frac{4}{5} + 2\frac{3}{7}}{4\frac{2}{3} - 3\frac{11}{13}} = \frac{8\frac{8}{35}}{\frac{32}{39}} = \frac{\overset{9}{\cancel{288}}}{35} \times \frac{39}{\cancel{32}} = \frac{351}{35} = 10\tfrac{1}{35}.$$

12. Simplify $\dfrac{8\frac{3}{4}}{14} - \dfrac{\frac{2}{3}}{1\frac{1}{7}}$.

$$\frac{8\frac{3}{4}}{14} = \frac{\overset{5}{\cancel{35}}}{4} \times \frac{1}{\underset{2}{\cancel{14}}} = \frac{5}{8}.$$

$$\frac{\frac{2}{3}}{1\frac{1}{7}} = \frac{2}{3} \times \frac{7}{\underset{4}{\cancel{8}}} = \frac{7}{12}.$$

$$\frac{8\frac{3}{4}}{14} - \frac{\frac{2}{3}}{1\frac{1}{7}} = \frac{5}{8} - \frac{7}{12} = \frac{15 - 14}{24} = \frac{1}{24}.$$

13. Simplify $\dfrac{3\frac{3}{7}}{11\frac{1}{4}}$ of $\dfrac{3\frac{3}{8}}{2\frac{2}{5}}$.

$$\frac{3\frac{3}{7}}{11\frac{1}{4}} \text{ of } \frac{3\frac{3}{8}}{2\frac{2}{5}} = \frac{\overset{\cancel{2}}{\cancel{24}}}{7} \times \frac{\cancel{4}}{\underset{\cancel{5}}{\cancel{45}}} \times \frac{\overset{3}{\cancel{27}}}{\underset{\cancel{2}}{\cancel{8}}} \times \frac{\cancel{5}}{\cancel{12}} = \frac{3}{7}.$$

14. Simplify $\dfrac{5\frac{8}{9} - 4\frac{11}{12}}{5\frac{3}{8} - 2\frac{17}{18}}$.

$$\frac{5\frac{8}{9} - 4\frac{11}{12}}{5\frac{3}{8} - 2\frac{17}{18}} = \frac{424 - 354}{387 - 212} = \frac{70}{175} = \frac{2}{5}.$$

15. Simplify $\dfrac{2\frac{3}{4}+2\frac{7}{8}}{4\frac{3}{4}-3\frac{1}{7}}$.

$$\frac{2\frac{3}{4}+2\frac{7}{8}}{4\frac{3}{4}-3\frac{1}{7}}=\frac{154+161}{266-176}=\frac{315}{90}=3\tfrac{45}{90}=3\tfrac{1}{2}.$$

16. Simplify $\dfrac{2\frac{3}{5}\times\frac{9}{11}}{3\frac{5}{7}\div 4\frac{1}{8}}$.

$$\frac{2\frac{3}{5}\times\frac{9}{11}}{3\frac{5}{7}\div 4\frac{1}{8}}=\frac{\cancel{13}}{5}\times\frac{9}{\cancel{11}}\times\frac{7}{\underset{2}{\cancel{26}}}\times\frac{\overset{3}{\cancel{33}}}{8}=\frac{189}{80}=2\tfrac{29}{80}.$$

17. Simplify $\dfrac{\frac{17}{20}+\frac{11}{15}+\frac{7}{10}+\frac{4}{5}}{\frac{17}{20}-\frac{11}{15}+\frac{7}{10}-\frac{4}{5}}$.

$$\frac{\frac{17}{20}+\frac{11}{15}+\frac{7}{10}+\frac{4}{5}}{\frac{17}{20}-\frac{11}{15}+\frac{7}{10}-\frac{4}{5}}=\frac{51+44+42+48}{51-44+42-48}=185.$$

18. Simplify $\dfrac{4\frac{1}{7}-2\frac{1}{4}}{6\frac{1}{2}-2\frac{1}{7}}$.

$$\frac{4\frac{1}{7}-2\frac{1}{4}}{6\frac{1}{2}-2\frac{1}{7}}=\frac{116-63}{182-60}=\frac{53}{122}.$$

19. Simplify $\dfrac{2\frac{27}{40}-4\frac{4}{7}+3\frac{1}{8}}{5\frac{6}{7}-4\frac{7}{8}+\frac{2}{5}}$.

$$\frac{2\frac{27}{40}-4\frac{4}{7}+3\frac{1}{8}}{5\frac{6}{7}-4\frac{7}{8}+\frac{2}{5}}=\frac{749-1280+875}{1640-1365+112}=\frac{344}{387}=\frac{8}{9}.$$

20. Simplify $\dfrac{1\frac{1}{4}\times 1\frac{2}{7}+\frac{1}{3}\text{ of }2\frac{1}{4}-\frac{13}{28}\times 2}{\frac{13}{28}\text{ of }2+\frac{1}{3}\text{ of }2\frac{1}{4}-1\frac{1}{4}\text{ of }1\frac{2}{7}}$.

$$\frac{1\frac{1}{4}\times 1\frac{2}{7}+\frac{1}{3}\text{ of }2\frac{1}{4}-\frac{13}{28}\times 2}{\frac{13}{28}\text{ of }2+\frac{1}{3}\text{ of }2\frac{1}{4}-1\frac{1}{4}\text{ of }1\frac{2}{7}}=\frac{\frac{45}{28}+\frac{3}{4}-\frac{13}{14}}{\frac{13}{14}+\frac{3}{4}-\frac{45}{28}}=\frac{45+21-26}{26+21-45}=\frac{40}{2}=20.$$

21. Simplify $2\frac{1}{4}\times\dfrac{10\frac{3}{4}-4\frac{11}{12}}{6\frac{3}{16}+7\frac{2}{3}}\times\dfrac{3\frac{5}{11}}{1\frac{2}{5}\times 9\frac{1}{11}}$.

$$10\tfrac{3}{4}-4\tfrac{11}{12}=6\tfrac{9-11}{12}=5\tfrac{21-11}{12}=5\tfrac{10}{12}=5\tfrac{5}{6}.$$

$$6\tfrac{3}{16}+7\tfrac{2}{3}=13\tfrac{9+32}{48}=13\tfrac{41}{48}.$$

$$2\tfrac{1}{4}\times\frac{10\frac{3}{4}-4\frac{11}{12}}{6\frac{3}{16}+7\frac{2}{3}}\times\frac{3\frac{5}{11}}{1\frac{2}{5}\times 9\frac{1}{11}}=2\tfrac{1}{4}\times\frac{5\frac{5}{6}}{13\frac{41}{48}}\times\frac{3\frac{5}{11}}{1\frac{2}{5}\times 9\frac{1}{11}}$$

$$=\frac{9}{\cancel{4}}\times\frac{\cancel{35}}{\cancel{6}}\times\frac{\overset{2}{\overset{\cancel{8}}{\cancel{48}}}}{\underset{\cancel{35}}{\cancel{665}}}\times\frac{\overset{2}{\cancel{38}}}{\cancel{11}}\times\frac{\cancel{5}}{7}\times\frac{\cancel{11}}{\underset{5}{\underset{\cancel{10}}{\underset{\cancel{20}}{\cancel{100}}}}}=\frac{9}{35}.$$

[redacted]

22. Simplify $\dfrac{8\frac{7}{8} - 7\frac{6}{7} + 5\frac{5}{6} - 4\frac{4}{5}}{9\frac{9}{10} - 8\frac{13}{15} + 7\frac{7}{8} - 6\frac{6}{7}}$.

$$\frac{8\frac{7}{8} - 7\frac{6}{7} + 5\frac{5}{6} - 4\frac{4}{5}}{9\frac{9}{10} - 8\frac{13}{15} + 7\frac{7}{8} - 6\frac{6}{7}} = \frac{7455 - 6600 + 4900 - 4032}{8316 - 7448 + 6615 - 5760} = \frac{1723}{1723} = 1.$$

23. Simplify $\dfrac{\frac{1}{7}}{8} \times \dfrac{\frac{8}{7}}{9\frac{1}{2}} \times \dfrac{7\frac{1}{9}}{\frac{8}{9}} \times \dfrac{4\frac{3}{4}}{7\frac{3}{14}} \times \dfrac{3}{27} \times 1\frac{1}{8}$.

$$\frac{\frac{1}{7}}{8} \times \frac{\frac{8}{7}}{9\frac{1}{2}} \times \frac{7\frac{1}{9}}{\frac{8}{9}} \times \frac{4\frac{3}{4}}{7\frac{3}{14}} \times \frac{3}{27} \times 1\frac{1}{8}$$

$$= \frac{1}{7} \times \frac{1}{\cancel{8}} \times \frac{\cancel{8}}{\cancel{7}} \times \frac{\cancel{2}}{\cancel{19}} \times \frac{\overset{\cancel{8}}{\cancel{64}}}{\cancel{9}} \times \frac{\cancel{9}}{\cancel{8}} \times \frac{\cancel{19}}{\underset{\cancel{2}}{\cancel{4}}} \times \frac{\overset{\cancel{2}}{\cancel{14}}}{101} \times \frac{\cancel{3}}{\underset{\cancel{3}}{\cancel{27}}} \times \frac{\cancel{9}}{\cancel{8}} = \frac{1}{707}.$$

24. Simplify $\dfrac{27}{37\frac{4}{5}} \times \dfrac{87\frac{2}{9}}{98\frac{1}{8}} \times \dfrac{\frac{7}{8}}{2\frac{1}{2}} \times \dfrac{89\frac{5}{11}}{128}$.

$$\frac{27}{37\frac{4}{5}} \times \frac{87\frac{2}{9}}{98\frac{1}{8}} \times \frac{\frac{7}{8}}{2\frac{1}{2}} \times \frac{89\frac{5}{11}}{128} = \cancel{27} \times \frac{\cancel{5}}{\underset{\cancel{7}}{\cancel{189}}} \times \frac{\cancel{785}}{\underset{3}{\cancel{9}}} \times \frac{\cancel{8}}{\cancel{785}} \times \frac{\cancel{7}}{\cancel{8}} \times \frac{2}{\cancel{5}} \times \frac{\overset{\overset{41}{\cancel{123}}}{\cancel{984}}}{11} \times \frac{1}{\underset{\underset{8}{\cancel{16}}}{\cancel{128}}} = \frac{41}{264}.$$

25. Simplify $\dfrac{4\frac{1}{17}}{6\frac{1}{19}} \times \dfrac{170}{399} \div \dfrac{12\frac{2}{3}}{7\frac{7}{9}}$.

$$\frac{4\frac{1}{17}}{6\frac{1}{19}} \times \frac{170}{399} \div \frac{12\frac{2}{3}}{7\frac{7}{9}} = \frac{\overset{\cancel{3}}{\cancel{69}}}{\cancel{17}} \times \frac{\cancel{19}}{\underset{\cancel{5}}{\cancel{115}}} \times \frac{\overset{\overset{\cancel{2}}{\cancel{10}}}{\cancel{170}}}{\underset{57}{\cancel{399}}} \times \frac{\overset{10}{\cancel{70}}}{\underset{\cancel{3}}{\cancel{9}}} \times \frac{\cancel{3}}{\underset{\cancel{2}}{\cancel{38}}} = \frac{10}{57}.$$

26. Simplify $\left(1 - \dfrac{426}{697} + \dfrac{2\frac{1}{2}}{8\frac{1}{2}}\right) \div \dfrac{3\frac{1}{2}}{5\frac{1}{8}}$.

$$1 - \frac{426}{697} + \frac{2\frac{1}{2}}{8\frac{1}{2}} = \frac{271}{697} + \frac{5}{17} = \frac{271 + 205}{697} = \frac{476}{697} = \frac{28}{41}.$$

$$\left(1 - \frac{426}{697} + \frac{2\frac{1}{2}}{8\frac{1}{2}}\right) \div \frac{3\frac{1}{2}}{5\frac{1}{8}} = \frac{28}{41} \div \frac{3\frac{1}{2}}{5\frac{1}{8}} = \frac{\overset{\cancel{7}}{\cancel{28}}}{\cancel{41}} \times \frac{\cancel{2}}{\cancel{7}} \times \frac{\cancel{41}}{\underset{\cancel{4}}{\cancel{8}}} = 1.$$

[redacted]

27. Simplify $\dfrac{\frac{1}{6} \text{ of } 1\frac{13}{16} + 1\frac{1}{6} \text{ of } 6\frac{1}{4} - 1\frac{1}{3} \text{ of } 5\frac{4}{9}}{\frac{1}{6} \text{ of } 2\frac{5}{6} \text{ of } 5\frac{2}{3}}$.

$$\frac{1}{6} \text{ of } 1\tfrac{13}{16} = \frac{1}{6} \times \frac{29}{16} = \frac{29}{96}.$$

$$1\tfrac{1}{6} \text{ of } 6\tfrac{1}{4} = \frac{7}{6} \times \frac{25}{4} = \frac{175}{24}.$$

$$1\tfrac{1}{3} \text{ of } 5\tfrac{4}{9} = \frac{4}{3} \times \frac{49}{9} = \frac{196}{27}.$$

$$\frac{1}{6} \text{ of } 2\tfrac{5}{6} \text{ of } 5\tfrac{2}{3} = \frac{1}{6} \times \frac{17}{6} \times \frac{17}{3} = \frac{289}{108}.$$

$$\frac{\frac{1}{6} \text{ of } 1\frac{13}{16} + 1\frac{1}{6} \text{ of } 6\frac{1}{4} - 1\frac{1}{3} \text{ of } 5\frac{4}{9}}{\frac{1}{6} \text{ of } 2\frac{5}{6} \text{ of } 5\frac{2}{3}} = \frac{\frac{29}{96} + \frac{175}{24} - \frac{196}{27}}{\frac{289}{108}}$$

$$= \frac{261 + 6300 - 6272}{2312} = \frac{289}{2312} = \frac{1}{8}.$$

28. Simplify $\dfrac{\frac{37}{68} \times \frac{51}{111} \times \frac{152}{187} \times \frac{153}{171}}{5\frac{1}{2}}$.

$$\frac{\frac{37}{68} \times \frac{51}{111} \times \frac{152}{187} \times \frac{153}{171}}{5\frac{1}{2}} = \frac{\cancel{37}}{\underset{4}{\cancel{68}}} \times \frac{\overset{\cancel{17}}{\cancel{51}}}{\underset{\cancel{37}}{\cancel{111}}} \times \frac{\overset{\overset{2}{\cancel{8}}}{\cancel{152}}}{\underset{11}{\cancel{187}}} \times \frac{\overset{\cancel{17}}{\cancel{153}}}{\underset{\cancel{19}}{\cancel{171}}} \times \frac{2}{11} = \frac{4}{121}.$$

29. Simplify $\dfrac{\frac{3}{11} \times 9\frac{3}{13} \times 3\frac{1}{7} \times 9\frac{1}{10}}{\frac{4}{17} \times 3\frac{9}{19} \times 12\frac{1}{7} \times 2\frac{10}{33} \times \frac{7}{20}}$.

$$\frac{\frac{3}{11} \times 9\frac{3}{13} \times 3\frac{1}{7} \times 9\frac{1}{10}}{\frac{4}{17} \times 3\frac{9}{19} \times 12\frac{1}{7} \times 2\frac{10}{33} \times \frac{7}{20}}$$

$$= \frac{3}{\cancel{11}} \times \frac{\overset{\overset{\overset{3}{\cancel{6}}}{\cancel{30}}}{\cancel{120}}}{\cancel{13}} \times \frac{\overset{2}{\cancel{22}}}{\cancel{7}} \times \frac{\overset{7}{\cancel{91}}}{\cancel{10}} \times \frac{\cancel{17}}{\cancel{4}} \times \frac{\cancel{19}}{\underset{2}{\cancel{66}}} \times \frac{\cancel{7}}{\underset{\cancel{5}}{\cancel{85}}} \times \frac{\cancel{33}}{\underset{\underset{2}{\cancel{4}}}{\cancel{76}}} \times \frac{\overset{2}{\cancel{20}}}{\cancel{7}} = 9.$$

30. Simplify $\dfrac{2\frac{3}{4} \times 7\frac{7}{11}}{\frac{1}{2} \times \frac{3}{4} \times 18\frac{2}{3}}$.

$$\frac{2\frac{3}{4} \times 7\frac{7}{11}}{\frac{1}{2} \times \frac{3}{4} \times 18\frac{2}{3}} = \frac{\cancel{11}}{\cancel{4}} \times \frac{\overset{3}{\cancel{84}}}{\cancel{11}} \times \cancel{2} \times \frac{\cancel{4}}{\cancel{3}} \times \frac{\cancel{3}}{\underset{\cancel{28}}{\cancel{56}}} = 3.$$

[redacted]

Exercise 62. Page 133.

1. What fraction of 8 is 3?

$\frac{3}{8}$. *Ans.*

2. What fraction of 3 is 8?

$\frac{8}{3}$. *Ans.*

3. What fraction of 9 is 7?

$\frac{7}{9}$. *Ans.*

4. What fraction of 7 is 9?

$\frac{9}{7}$. *Ans.*

5. What fraction of 8 is 12?

$\frac{12}{8}=\frac{3}{2}$. *Ans.*

6. What fraction of 12 is 8?

$\frac{8}{12}=\frac{2}{3}$. *Ans.*

7. What fraction of $2\frac{1}{5}$ is $\frac{3}{5}$?

$\frac{\frac{3}{5}}{2\frac{1}{5}}=\frac{3}{11}$. *Ans.*

8. What fraction of $\frac{3}{5}$ is $2\frac{1}{5}$?

$\frac{2\frac{1}{5}}{\frac{3}{5}}=\frac{11}{3}$. *Ans.*

9. What fraction of $2\frac{3}{4}$ is $1\frac{1}{4}$?

$\frac{1\frac{1}{4}}{2\frac{3}{4}}=\frac{5}{11}$. *Ans.*

10. What fraction of $1\frac{1}{4}$ is $2\frac{3}{4}$?

$\frac{2\frac{3}{4}}{1\frac{1}{4}}=\frac{11}{5}$. *Ans.*

11. What fraction of $2\frac{1}{3}$ is $7\frac{1}{8}$?

$\frac{7\frac{1}{8}}{2\frac{1}{3}}=\frac{171}{56}$. *Ans.*

12. What fraction of $7\frac{1}{3}$ is $2\frac{1}{8}$?

$\frac{2\frac{1}{8}}{7\frac{1}{3}}=\frac{51}{176}$. *Ans.*

13. What fraction of $3\frac{1}{3}$ is $8\frac{1}{7}$?

$\frac{8\frac{1}{7}}{3\frac{1}{3}}=\frac{171}{70}$. *Ans.*

14. What fraction of \$2 is \$$1\frac{1}{2}$?

$\frac{\$1\frac{1}{2}}{\$2}=\frac{3}{4}$. *Ans.*

15. What fraction of \$$2\frac{1}{2}$ is \$5?

$\frac{\$5}{\$2\frac{1}{2}}=\frac{2}{1}$. *Ans.*

16. What fraction of \$$\frac{3}{4}$ is \$$\frac{1}{4}$?

$\frac{\$\frac{1}{4}}{\$\frac{3}{4}}=\frac{1}{3}$. *Ans.*

17. What fraction of \$$\frac{5}{4}$ is \$$\frac{3}{8}$?

$\frac{\$\frac{3}{8}}{\$\frac{5}{4}}=\frac{3}{10}$. *Ans.*

18. What fraction of \$$2\frac{3}{4}$ is \$$\frac{1}{3}$?

$\frac{\$\frac{1}{3}}{\$2\frac{3}{4}}=\frac{4}{33}$. *Ans.*

19. What fraction of \$$\frac{1}{2}$ is \$$\frac{1}{10}$?

$\frac{\$\frac{1}{10}}{\$\frac{1}{2}}=\frac{1}{5}$. *Ans.*

20. What fraction of \$1 is \$$\frac{7}{8}$?

$\frac{\$\frac{7}{8}}{\$1}=\frac{7}{8}$. *Ans.*

[redacted]

21. What fraction of \$10 is \$$\frac{2}{3}$?

$$\frac{\$\frac{2}{3}}{\$10}=\frac{1}{15}.\quad \textit{Ans.}$$

22. What fraction of \$100 is \$6?

$$\frac{\$6}{\$100}=\frac{3}{50}.\quad \textit{Ans.}$$

23. What fraction of \$100 is \$$4\frac{1}{2}$?

$$\frac{\$4\frac{1}{2}}{\$100}=\frac{9}{200}.\quad \textit{Ans.}$$

24. What fraction of \$4 is \$25?

$$\frac{\$25}{\$4}=\frac{25}{4}.\quad \textit{Ans.}$$

25. What fraction of $100\frac{5}{9}$ is $8\frac{4}{9}$?

$$\frac{8\frac{4}{9}}{100\frac{5}{9}}=\frac{76}{905}.\quad \textit{Ans.}$$

26. What fraction of 21 is $\frac{15}{19}$ of $3\frac{4}{5}$?

$$\frac{\frac{15}{19}\times 3\frac{4}{5}}{21}=\frac{\overset{3}{\cancel{15}}}{\cancel{19}}\times\frac{\cancel{19}}{\cancel{5}}\times\frac{1}{\underset{7}{\cancel{21}}}=\frac{1}{7}.\quad \textit{Ans.}$$

27. What fraction of $18\frac{117}{256}$ is $\frac{5}{8}$ of $33\frac{3}{4}$?

$$\frac{\frac{5}{8}\times 33\frac{3}{4}}{18\frac{117}{256}}=\frac{\cancel{5}}{\cancel{8}}\times\frac{\cancel{135}}{\cancel{4}}\times\frac{\overset{\overset{8}{\cancel{32}}}{\cancel{256}}}{\underset{\underset{7}{\cancel{945}}}{\cancel{4725}}}=\frac{8}{7}.\quad \textit{Ans.}$$

28. What fraction of $3\frac{1}{3}$ is $\frac{2}{3}\times 1\frac{1}{3}$?

$$\frac{\frac{2}{3}\times 1\frac{1}{3}}{3\frac{1}{3}}=\frac{2}{3}\times\frac{4}{\cancel{3}}\times\frac{\cancel{3}}{\underset{5}{\cancel{10}}}=\frac{4}{15}.\quad \textit{Ans.}$$

29. What fraction of $3\frac{1}{11}\times 5\frac{1}{27}$ is 1720?

$$\frac{1720}{3\frac{1}{11}\times 5\frac{1}{27}}=\overset{215}{\cancel{1720}}\times\frac{11}{34}\times\frac{27}{\underset{17}{\cancel{136}}}=\frac{63855}{578}.\quad \textit{Ans.}$$

30. What fraction of $3\frac{1}{2}\times\frac{8}{9}$ of $\frac{4}{7}$ is $1\frac{3}{5}$?

$$\frac{1\frac{3}{5}}{3\frac{1}{2}\times\frac{8}{9}\times\frac{4}{7}}=\frac{\cancel{8}}{5}\times\frac{2}{\cancel{7}}\times\frac{9}{\cancel{8}}\times\frac{\cancel{7}}{\underset{2}{\cancel{4}}}=\frac{9}{10}.\quad \textit{Ans.}$$

31. What part of $\frac{28}{59}\times\frac{59}{63}$ is $\frac{1}{6}\times 4\times\frac{2}{3}$?

$$\frac{\frac{1}{6}\times 4\times\frac{2}{3}}{\frac{28}{59}\times\frac{59}{63}}=\frac{1}{\underset{3}{\cancel{6}}}\times\cancel{4}\times\frac{2}{\cancel{3}}\times\frac{\cancel{59}}{\underset{7}{\cancel{28}}}\times\frac{\overset{\overset{3}{\cancel{9}}}{\cancel{63}}}{\cancel{59}}=\frac{1}{1}.\quad \textit{Ans.}$$

[redacted]

32. What part of $13\frac{5}{8} \times \frac{2}{3} \times \frac{9}{65}$ is $\frac{2}{3}$ of $1\frac{44}{65}$ of $1\frac{1}{8}$?

$$\frac{\frac{2}{3} \times 1\frac{44}{65} \times 1\frac{1}{8}}{13\frac{5}{8} \times \frac{2}{3} \times \frac{9}{65}} = \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{109}}{\cancel{65}} \times \frac{\cancel{9}}{\cancel{8}} \times \frac{\cancel{8}}{\cancel{109}} \times \frac{\cancel{3}}{\cancel{2}} \times \frac{\cancel{65}}{\cancel{9}} = \frac{1}{1}. \quad \textit{Ans.}$$

33. What part of $\frac{17}{20} + \frac{11}{15} + \frac{7}{10} + \frac{4}{5}$ is $\frac{17}{20} - \frac{11}{15} + \frac{7}{10} - \frac{4}{5}$?

$$\frac{\frac{17}{20} - \frac{11}{15} + \frac{7}{10} - \frac{4}{5}}{\frac{17}{20} + \frac{11}{15} + \frac{7}{10} + \frac{4}{5}} = \frac{51 - 44 + 42 - 48}{51 + 44 + 42 + 48} = \frac{1}{185}. \quad \textit{Ans.}$$

34. What part of $4\frac{1}{7} - 2\frac{1}{4}$ is $6\frac{1}{2} - 2\frac{1}{7}$?

$$\frac{6\frac{1}{2} - 2\frac{1}{7}}{4\frac{1}{7} - 2\frac{1}{4}} = \frac{182 - 60}{116 - 63} = \frac{122}{53}. \quad \textit{Ans.}$$

35. What part of $17\frac{2}{3} - 12\frac{6}{7}$ is $5 - \frac{1}{13} - \frac{4}{39} - \frac{1}{25}$?

$$\frac{5 - \frac{1}{13} - \frac{4}{39} - \frac{1}{25}}{17\frac{2}{3} - 12\frac{6}{7}} = \frac{34125 - 525 - 700 - 273}{120575 - 87750} = \frac{32627}{32825}. \quad \textit{Ans.}$$

36. What part of $24 - 17\frac{4}{13}$ is $7 + \frac{2}{15} - \frac{5}{81} - \frac{11}{45}$?

$$\frac{7 + \frac{2}{15} - \frac{5}{81} - \frac{11}{45}}{24 - 17\frac{4}{13}} = \frac{36855 + 702 - 325 - 1287}{126360 - 91125} = \frac{35945}{35235} = \frac{7189}{7047}. \quad \textit{Ans.}$$

37. What part of $\frac{3}{7}$ of $2\frac{1}{17}$ is $1\frac{2}{3} \div 2\frac{3}{7}$?

$$\frac{1\frac{2}{3} \div 2\frac{3}{7}}{\frac{3}{7} \times 2\frac{1}{17}} = \frac{\cancel{7}}{\cancel{17}} \times \frac{\cancel{5}}{3} \times \frac{7}{3} \times \frac{\cancel{17}}{\underset{\cancel{7}}{\cancel{35}}} = \frac{7}{9}. \quad \textit{Ans.}$$

38. What part of

$$\left(\frac{7}{4 - \frac{5}{6}} - \frac{5}{6 - \frac{3}{8}}\right) \div \left(\frac{4}{7 - \frac{4}{7}} + \frac{2}{4 - \frac{2}{5}}\right) \text{ is}$$

$$\left(14 - \frac{1}{\frac{1}{2} - \frac{6}{31}}\right) \div \left(\frac{1}{\frac{1}{2} - \frac{27}{59}} - 13\right)?$$

$$\frac{\left(14 - \frac{1}{\frac{1}{2} - \frac{6}{31}}\right) \div \left(\frac{1}{\frac{1}{2} - \frac{27}{59}} - 13\right)}{\left(\frac{7}{4 - \frac{5}{6}} - \frac{5}{6 - \frac{3}{8}}\right) \div \left(\frac{4}{7 - \frac{4}{7}} + \frac{2}{4 - \frac{2}{5}}\right)}$$

$$= \frac{(14 - \frac{62}{19}) \div (\frac{118}{5} - 13)}{(\frac{42}{19} - \frac{8}{9}) \div (\frac{28}{45} + \frac{5}{9})} = \frac{\frac{204}{19} \times \frac{5}{53}}{\frac{226}{171} \times \frac{45}{53}} = \frac{102}{113}. \quad \textit{Ans.}$$

[redacted]

Exercise 63. Page 134.

Reduce to a common fraction or to a mixed number:

1. $0.125 = \frac{125}{1000} = \frac{1}{8}$. *Ans.*
2. $0.625 = \frac{625}{1000} = \frac{5}{8}$. *Ans.*
3. $0.675 = \frac{675}{1000} = \frac{27}{40}$. *Ans.*
4. $10.864 = 10\frac{864}{1000} = 10\frac{108}{125}$. *Ans.*
5. $50.84 = 50\frac{84}{100} = 50\frac{21}{25}$. *Ans.*
6. $3.00025 = 3\frac{25}{100000} = 3\frac{1}{4000}$. *Ans.*
7. $8.1075 = 8\frac{1075}{10000} = 8\frac{43}{400}$. *Ans.*
8. $35.01024 = 35\frac{1024}{100000} = 35\frac{128}{12500} = 35\frac{32}{3125}$. *Ans.*
9. $7.015625 = 7\frac{15625}{1000000} = 8\frac{1}{64}$. *Ans.*
10. $20.100256 = 20\frac{100256}{1000000} = 20\frac{12532}{125000} = 20\frac{3133}{31250}$. *Ans.*
11. $10.012575 = 10\frac{12575}{1000000} = 10\frac{503}{40000}$. *Ans.*
12. $104.235 = 104\frac{235}{1000} = 104\frac{47}{200}$. *Ans.*
13. $50.0004 = 50\frac{4}{10000} = 50\frac{1}{2500}$. *Ans.*
14. $100.001 = 100\frac{1}{1000}$. *Ans.*
15. $8.00725 = 8\frac{725}{100000} = 8\frac{29}{4000}$. *Ans.*
16. $20.018375 = 20\frac{18375}{1000000} = 20\frac{147}{8000}$. *Ans.*
17. $125.6048 = 125\frac{6048}{10000} = 125\frac{756}{1250} = 125\frac{378}{625}$. *Ans.*
18. $0.128 = \frac{128}{1000} = \frac{16}{125}$. *Ans.*
19. $0.73125 = \frac{73125}{100000} = \frac{2925}{4000} = \frac{117}{160}$. *Ans.*
20. $1.1875 = 1\frac{1875}{10000} = 1\frac{75}{400} = 1\frac{3}{16}$. *Ans.*
21. $0.603125 = \frac{603125}{1000000} = \frac{24125}{40000} = \frac{965}{1600} = \frac{193}{320}$. *Ans.*
22. $6.03125 = 6\frac{3125}{100000} = 6\frac{125}{4000} = 6\frac{1}{32}$. *Ans.*
23. $60.3125 = 60\frac{3125}{10000} = 60\frac{125}{400} = 60\frac{5}{16}$. *Ans.*
24. $7.0315 = 7\frac{315}{10000} = 7\frac{63}{2000}$. *Ans.*
25. $12.0625 = 12\frac{625}{10000} = 12\frac{25}{400} = 12\frac{1}{16}$. *Ans.*
26. $4.7168 = 4\frac{7168}{10000} = 4\frac{896}{1250} = 4\frac{448}{625}$. *Ans.*
27. $0.0425 = \frac{425}{10000} = \frac{17}{400}$. *Ans.*
28. $6.46875 = 6\frac{46875}{100000} = 6\frac{1875}{4000} = 6\frac{75}{160} = 6\frac{15}{32}$. *Ans.*
29. $0.00256 = \frac{256}{100000} = \frac{32}{12500} = \frac{8}{3125}$. *Ans.*
30. $0.000375 = \frac{375}{1000000} = \frac{15}{40000} = \frac{3}{8000}$. *Ans.*

[redacted]

Exercise 64. Page 135.

1. Reduce $\frac{7}{8}$ to a decimal.

$8)\underline{7.0}$
0.875 *Ans.*

2. Reduce $\frac{15}{16}$ to a decimal.

0.9375 *Ans.*
$16)\overline{15.0}$
144
60
48
120
112
80
80

3. Reduce $\frac{9}{32}$ to a decimal.

0.28125 *Ans.*
$32)\overline{9.0}$
64
260
256
40
32
80
64
160
160

4. Reduce $\frac{9}{25}$ to a decimal.

$$\frac{9}{25}=\frac{36}{100}=0.36. \quad \textit{Ans.}$$

5. Reduce $\frac{5}{64}$ to a decimal.

0.078125 *Ans.*
$64)\overline{5.00}$
448
520
512
80
64
160
128
320
320

6. Reduce $4\frac{11}{800}$ to a decimal.

$8\cancel{0}\cancel{0})\underline{0.11}$
0.01375
4.01375. *Ans.*

7. Reduce $5\frac{5}{32000}$ to a decimal.

$\frac{5}{32000}=\frac{1}{6400}$.

0.00015625
$64\cancel{0}\cancel{0})\overline{0.0100}$
64
360
320
400
384
160
128
320
320

5.00015625. *Ans.*

[redacted]

8. Reduce $9\frac{123}{25600}$ to a decimal.

```
          0.0048046875
256̸0̸0̸)1.230
           1024
           ----
            2060
            2048
            ----
             1200
             1024
             ----
              1760
              1536
              ----
               2240
               2048
               ----
                1920
                1792
                ----
                 1280
                 1280
                 ----
```

9.0048046875. *Ans.*

9. Reduce $11\frac{19}{4000}$ to a decimal.

```
4̸0̸0̸0̸)0.019
      0.00475
```

11.00475. *Ans.*

10. Reduce $\frac{9}{125}$ to a decimal.

```
      0.072  Ans.
125)9.00
     875
     ---
      250
      250
      ---
```

11. Reduce $\frac{17}{4000}$ to a decimal.

```
4̸0̸0̸0̸)0.017
      0.00425  Ans.
```

12. Reduce $\frac{119}{128}$ to a decimal.

```
       0.9296875  Ans.
128)119.0
    1152
    ----
      380
      256
      ---
      1240
      1152
      ----
        880
        768
        ---
        1120
        1024
        ----
          960
          896
          ---
           640
           640
           ---
```

13. Reduce $\frac{13}{625}$ to a decimal.

```
      0.0208  Ans.
625)13.00
    1250
    ----
      5000
      5000
      ----
```

14. Reduce $\frac{11}{256}$ to a decimal.

```
      0.04296875  Ans.
256)11.00
    10.24
    -----
      760
      512
      ---
      2480
      2304
      ----
       1760
       1536
       ----
        2240
        2048
        ----
         1920
         1792
         ----
          1280
          1280
          ----
```

[redacted]

15. Reduce $\frac{3}{160}$ to a decimal.

$$\begin{array}{r l}
 & 0.01875 \quad \textit{Ans.} \\
16\cancel{0}) & 0.30 \\
 & 16 \\
 & 140 \\
 & 128 \\
 & 120 \\
 & 112 \\
 & 80 \\
 & 80
\end{array}$$

16. Reduce $\frac{124}{16}$ to a decimal.

$\frac{124}{16} = 7\frac{12}{16} = 7\frac{3}{4} = 7.75.$ *Ans.*

17. Reduce $\frac{2}{3}$ of $1\frac{4}{5}$ to a decimal.

$$\frac{2}{3} \text{ of } 1\frac{4}{5} = \frac{2}{\cancel{3}} \text{ of } \frac{\overset{3}{\cancel{9}}}{5} = \frac{6}{5} = \frac{12}{10} = 1.2. \quad \textit{Ans.}$$

18. Reduce $\frac{3}{4}$ of $\frac{5}{8}$ of $\frac{7}{10}$ to a decimal.

$$\frac{3}{4} \text{ of } \frac{\cancel{5}}{8} \text{ of } \frac{7}{\underset{2}{\cancel{10}}} = \frac{21}{64}. \quad \textit{Ans.}$$

$$\begin{array}{r l}
 & 0.328125 \quad \textit{Ans.} \\
64) & 21.0 \\
 & 192 \\
 & 180 \\
 & 128 \\
 & 520 \\
 & 512 \\
 & 80 \\
 & 64 \\
 & 160 \\
 & 128 \\
 & 320 \\
 & 320
\end{array}$$

19. Reduce $3\frac{3}{5}$ of $4\frac{1}{9}$ to a decimal.

$$3\frac{3}{5} \text{ of } 4\frac{1}{9} = \frac{\overset{2}{\cancel{18}}}{5} \times \frac{37}{\cancel{9}} = \frac{74}{5} = \frac{148}{10}$$
$$= 14.8. \quad \textit{Ans.}$$

20. Reduce $\frac{29}{32}$ of $\frac{49}{64}$ to a decimal.

$$\frac{29}{32} \text{ of } \frac{49}{64} = \frac{1421}{2048}.$$

$$\begin{array}{r l}
 & 0.69384765625 \quad \textit{Ans.} \\
2048) & 1421.0 \\
 & 12288 \\
 & 19220 \\
 & 18432 \\
 & 7880 \\
 & 6144 \\
 & 17360 \\
 & 16384 \\
 & 9760 \\
 & 8192 \\
 & 15680 \\
 & 14336 \\
 & 13440 \\
 & 12288 \\
 & 11520 \\
 & 10240 \\
 & 12800 \\
 & 12288 \\
 & 5120 \\
 & 4096 \\
 & 10240 \\
 & 10240
\end{array}$$

[redacted]

Exercise 65. Page 135.

Simplify by common fractions, then by reducing the common fractions to decimals, and show that the results in each example agree:

1. $7\frac{2}{5} + 4\frac{5}{8} + 9\frac{13}{20} + 11\frac{29}{32}$.

$$7\tfrac{2}{5} + 4\tfrac{5}{8} + 9\tfrac{13}{20} + 11\tfrac{29}{32} = 31\tfrac{413}{160} = 33\tfrac{93}{160} = 33.58125.$$

$$7\tfrac{2}{5} + 4\tfrac{5}{8} + 9\tfrac{13}{20} + 11\tfrac{29}{32} = 7.4 + 4.625 + 9.65 + 11.90625 = 33.58125.$$

2. $84\frac{13}{20} + 19\frac{11}{21} + \frac{41}{50}$.

$$84\tfrac{13}{20} + 19\tfrac{11}{21} + \tfrac{41}{50} = 103\tfrac{1365+1100+1722}{2100} = 103\tfrac{4187}{2100} = 104\tfrac{2087}{2100}$$
$$= 104.993809\tfrac{11}{21}.$$

$$84\tfrac{13}{20} + 19\tfrac{11}{21} + \tfrac{41}{50} = 84.65 + 19.523809\tfrac{11}{21} + 0.82 = 104.993809\tfrac{11}{21}.$$

3. $4\frac{27}{64} + 13\frac{17}{20} + 42\frac{37}{50} + 2\frac{13}{16} + 1\frac{1}{2}$.

$$4\tfrac{27}{64} + 13\tfrac{17}{20} + 42\tfrac{37}{50} + 2\tfrac{13}{16} + 1\tfrac{1}{2} = 62\tfrac{675+1360+1184+1300+800}{1600}$$
$$= 62\tfrac{5319}{1600} = 65\tfrac{519}{1600} = 65.324375.$$

$$4\tfrac{27}{64}+13\tfrac{17}{20}+42\tfrac{37}{50}+2\tfrac{13}{16}+1\tfrac{1}{2} = 4.421875+13.85+42.74+2.8125+1.5$$
$$= 65.324375.$$

4. $5\frac{7}{8} + 13\frac{4}{5} + 19\frac{7}{16} + 7\frac{3}{20}$.

$$5\tfrac{7}{8} + 13\tfrac{4}{5} + 19\tfrac{7}{16} + 7\tfrac{3}{20} = 44\tfrac{70+64+35+12}{80} = 44\tfrac{181}{80} = 46\tfrac{21}{80} = 46.2625.$$

$$5\tfrac{7}{8} + 13\tfrac{4}{5} + 19\tfrac{7}{16} + 7\tfrac{3}{20} = 5.875 + 13.8 + 19.4375 + 7.15 = 46.2625.$$

5. $5\frac{5}{10} + \frac{2}{3}$ of $1\frac{4}{5} + \frac{7}{8}$ of $2\frac{2}{7} + \frac{3}{4}$ of $\frac{5}{8}$.

$$5\tfrac{5}{10} + \tfrac{2}{3} \times 1\tfrac{4}{5} + \tfrac{7}{8} \times 2\tfrac{2}{7} + \tfrac{3}{4} \times \tfrac{5}{8} = 5\tfrac{1}{2} + 1\tfrac{1}{5} + 2 + \tfrac{15}{32} = 8\tfrac{80+32+75}{160}$$
$$= 8\tfrac{187}{160} = 9\tfrac{27}{160} = 9.16875.$$

$$5\tfrac{5}{10} + \tfrac{2}{3} \times 1\tfrac{4}{5} + \tfrac{7}{8} \times 2\tfrac{2}{7} + \tfrac{3}{4} \times \tfrac{5}{8}$$
$$= 5.5 + 0.666\tfrac{2}{3} \times 1.8 + 0.875 \times 2.285714\tfrac{2}{7} + 0.75 \times 0.625$$
$$= 5.5 + 1.2 + 2 + 0.46875 = 9.16875.$$

6. $1\frac{5}{12}$ of $2\frac{5}{8}$.

$$1\tfrac{5}{12} \times 2\tfrac{5}{8} = \frac{17}{\cancel{12}_{4}} \times \frac{\cancel{21}^{7}}{8} = \frac{119}{32} = 3\tfrac{23}{32} = 3.71875.$$

$$1\tfrac{5}{12} \times 2\tfrac{5}{8} = 1.4166\tfrac{2}{3} \times 2.625 = 3.71875.$$

7. $3\frac{5}{16} + 2\frac{19}{20}$.

$$3\tfrac{5}{16} + 2\tfrac{19}{20} = 5\tfrac{25+76}{80} = 5\tfrac{101}{80} = 6\tfrac{21}{80} = 6.2625.$$

$$3\tfrac{5}{16} + 2\tfrac{19}{20} = 3.3125 + 2.95 = 6.2625.$$

[redacted]

8. $7\frac{2}{5} - 4\frac{5}{8}$.

$$7\tfrac{2}{5} - 4\tfrac{5}{8} = 3\tfrac{16-25}{40} = 2\tfrac{31}{40} = 2.775.$$
$$7\tfrac{2}{5} - 4\tfrac{5}{8} = 7.4 - 4.625 = 2.775.$$

9. $82\frac{1}{5} - 37\frac{11}{16}$.

$$82\tfrac{1}{5} - 37\tfrac{11}{16} = 45\tfrac{16-55}{80} = 44\tfrac{41}{80} = 44.5125.$$
$$82\tfrac{1}{5} - 37\tfrac{11}{16} = 82.2 - 37.6875 = 44.5125.$$

10. $100 - 17\frac{113}{625}$.

$$100 - 17\tfrac{113}{625} = 82\tfrac{512}{625} = 82.8192.$$
$$100 - 17\tfrac{113}{625} = 100 - 17.1808 = 82.8192.$$

11. $5\frac{1}{2} - 1\frac{1}{2}$ of $1\frac{13}{24}$.

$$5\tfrac{1}{2} - 1\tfrac{1}{2} \times 1\tfrac{13}{24} = 5\tfrac{1}{2} - 2\tfrac{5}{16} = 3\tfrac{3}{16} = 3.1875.$$
$$5\tfrac{1}{2} - 1\tfrac{1}{2} \times 1\tfrac{13}{24} = 5.5 - 1.5 \times 1.5416\tfrac{2}{3} = 5.5 - 2.3125 = 3.1875.$$

12. $\frac{14}{25} - \frac{11}{64}$.

$$\tfrac{14}{25} - \tfrac{11}{64} = \tfrac{896-275}{1600} = \tfrac{621}{1600} = 0.388125.$$
$$\tfrac{14}{25} - \tfrac{11}{64} = 0.56 - 0.171875 = 0.388125.$$

13. $8\frac{1}{5} - 1\frac{1}{2}$ of $\frac{3}{16}$.

$$8\tfrac{1}{5} - 1\tfrac{1}{2} \times \tfrac{3}{16} = 8\tfrac{1}{5} - \tfrac{9}{32} = 8\tfrac{32-45}{160} = 7\tfrac{147}{160} = 7.91875.$$
$$8\tfrac{1}{5} - 1\tfrac{1}{2} \times \tfrac{3}{16} = 8.2 - 1.5 \times 0.1875 = 8.2 - 0.28125 = 7.91875.$$

14. $\frac{19}{64} \times 1000$.

$$\tfrac{19}{64} \times 1000 = \tfrac{2375}{8} = 296\tfrac{7}{8} = 296.875.$$
$$\tfrac{19}{64} \times 1000 = 0.296875 \times 1000 = 296.875.$$

Exercise 66. Page 137.

1. Reduce $\frac{5}{9}$ to a decimal.

$$9\overline{)5.}$$
$$0.5 \qquad 0.\dot{5}.\ \textit{Ans.}$$

2. Reduce $\frac{5}{11}$ to a decimal.

$$11\overline{)5.}$$
$$0.45 \qquad 0.\dot{4}\dot{5}.\ \textit{Ans.}$$

3. Reduce $3\frac{5}{12}$ to a decimal.

$$12\overline{)5.}$$
$$0.416 \qquad 3.41\dot{6}.\ \textit{Ans.}$$

4. Reduce $\frac{11}{60}$ to a decimal.

$$6\cancel{0}\overline{)1.1}$$
$$0.183 \qquad 0.18\dot{3}.\ \textit{Ans.}$$

5. Reduce $3\frac{17}{48}$ to a decimal.

$$
\begin{array}{r}
0.35416 \\
48\overline{)17.0} \\
144 \\
\hline
260 \\
240 \\
\hline
200 \\
192 \\
\hline
80 \\
48 \\
\hline
320 \\
288 \\
\hline
32
\end{array}
\qquad 3.3541\dot{6}.\ \textit{Ans.}
$$

[redacted]

6. Reduce $2\frac{5}{37}$ to a decimal.

```
    0.135
37)5.0
   37
   130
   111
    190
    185
      5
```

$2.\dot{1}3\dot{5}$. *Ans.*

7. Reduce $\frac{3}{3700}$ to a decimal.

```
      0.00081
37ØØ)0.0300
        296
         40
         37
          3
```

$0.00\dot{0}8\dot{1}$. *Ans.*

8. Reduce $11\frac{11}{84}$ to a decimal.

```
   0.13095238
84)11.0
   84
   260
   252
    800
    756
     440
     420
      200
      168
       320
       252
        680
        672
          8
```

$11.13\dot{0}9523\dot{8}$. *Ans.*

9. Reduce $9\frac{11}{108}$ to a decimal.

```
     0.10185
108)11.0
    108
      200
      108
       920
       864
        560
        540
         20
```

$9.10\dot{1}8\dot{5}$. *Ans.*

10. Reduce $11\frac{4}{35}$ to a decimal.

```
   0.1142857
35)4.0
   35
    50
    35
    150
    140
     100
      70
     300
     280
      200
      175
       250
       245
         5
```

$11.1\dot{1}4285\dot{7}$. *Ans.*

[redacted]

11. Reduce $\frac{15}{56}$ to a decimal.

```
       0.267857142
56)15.0
    112
     380
     336
      440
      392
       480
       448
        320
        280
         400
         392
           80
           56
          240
          224
           160
           112
            48
```

0.267$\dot{8}$5714$\dot{2}$. *Ans.*

12. Reduce $\frac{8}{21}$ to a decimal.

```
     0.380952
21)8.0
    63
    170
    168
     200
     189
      110
      105
        50
        42
         8
```

0.$\dot{3}$8095$\dot{2}$. *Ans.*

13. Reduce $\frac{13}{33}$ to a decimal.

```
      0.39
33)13.0
     99
     310
     297
      13
```

0.$\dot{3}\dot{9}$. *Ans.*

14. Reduce $\frac{37}{70}$ to a decimal.

```
7𝟎)3.7
   0.5285714
```

0.5$\dot{2}$8571$\dot{4}$. *Ans.*

15. Reduce $2\frac{58}{255}$ to a decimal.

```
      0.22745098039215686
255)58.0
    510
     700
     510
     1900
     1785
      1150
      1020
       1300
       1275
        2500
        2295
         2050
         2040
          1000
           765
          2350
          2295
            550
            510
             400
             255
            1450
            1275
             1750
             1530
              2200
              2040
               1600
               1530
                 70
```

2.2$\dot{2}$74509803921568$\dot{6}$. *Ans.*

[redacted]

16. Reduce $5\frac{6}{26}$ to a decimal.

$$5\tfrac{6}{26} = 5\tfrac{3}{13}.$$

```
    0.230769
13)3.0
   26
    40
    39
     100
      91
       90
       78
       120
       117
         3
```

$5.\dot{2}3076\dot{9}$. *Ans.*

17. If $\dfrac{117}{5^7 \times 2^8}$ is expressed as a decimal, how many decimal places will the quotient contain?

As 7 is the highest power of 2 or 5 in the denominator, and as there are no other factors than 2 or 5, there will be seven decimal places in the quotient.

18. If $\dfrac{119}{2^5 \times 13}$ is expressed as a decimal, how many decimal places will *precede* the repetend?

As 5 is the highest power of 2 or 5 in the denominator, and as there is another factor than 2 or 5, five decimal places will precede the repetend.

19. If $\dfrac{57}{5^2 \times 7}$ is reduced to a decimal, how many decimal places will precede the repetend?

As 2 is the highest power of 2 or 5 in the denominator, and as there is another factor than 2 or 5, two decimal places will precede the repetend.

[redacted]

Exercise 67. Page 138.

Reduce to a common fraction or to a mixed number:

1. $0.2\dot{4}\dot{5} = \frac{245-2}{990} = \frac{243}{990} = \frac{27}{110}$.
2. $0.4\dot{2}\dot{5} = \frac{425-4}{990} = \frac{421}{990}$.
3. $53.00\dot{2}4\dot{3} = 53\frac{243}{99900} = 53\frac{9}{3700}$.
4. $7.2\dot{0}1\dot{1} = 7\frac{2011-2}{9990} = 7\frac{2009}{9990}$.
5. $2.53\dot{0}\dot{6} = 2\frac{5306-53}{9900} = 2\frac{5253}{9900} = 2\frac{1751}{3300}$.
6. $0.004\dot{2}\dot{6} = \frac{426-4}{99000} = \frac{422}{99000} = \frac{211}{49500}$.
7. $31.2\dot{0}\dot{3} = 31\frac{203-2}{990} = 31\frac{201}{990} = 31\frac{67}{330}$.
8. $0.\dot{3}5\dot{1} = \frac{351}{999} = \frac{13}{37}$.
9. $1.41\dot{6} = 1\frac{416-41}{900} = 1\frac{375}{900} = 1\frac{5}{12}$.
10. $0.5\dot{5}7\dot{5} = \frac{5575-5}{9990} = \frac{5570}{9990} = \frac{557}{999}$.
11. $2.0\dot{8}\dot{1} = 2\frac{81}{990} = 2\frac{9}{110}$.
12. $5.12\dot{2}9\dot{7} = 5\frac{12297-12}{99900} = 5\frac{12285}{99900} = 5\frac{91}{740}$.
13. $0.35\dot{9}\dot{0} = \frac{3590-35}{9900} = \frac{3555}{9900} = \frac{79}{220}$.
14. $4.3\dot{1}6\dot{2} = 4\frac{3162-3}{9990} = 4\frac{3159}{9990} = 4\frac{117}{370}$.
15. $0.7\dot{2}8\dot{3} = \frac{7283-7}{9990} = \frac{7276}{9990} = \frac{3638}{4995}$.
16. $5.1\dot{4}2885\dot{7} = 5\frac{142857-1}{999990} = 5\frac{142856}{999990} = 5\frac{71428}{499995}$.
17. $0.2\dot{3}6\dot{8} = \frac{2368-2}{9990} = \frac{2366}{9990} = \frac{1183}{4995}$.
18. $1.1\dot{3}\dot{6} = 1\frac{136-1}{990} = 1\frac{135}{990} = 1\frac{27}{198} = 1\frac{3}{22}$.
19. $1.\dot{5}3\dot{1} = 1\frac{531}{999} = 1\frac{59}{111}$.
20. $3.28\dot{9}6\dot{3} = 3\frac{28963-28}{99900} = 3\frac{28935}{99900} = 3\frac{643}{2220}$.
21. $5.878\dot{3} = 5\frac{8783-878}{9000} = 5\frac{7905}{9000} = 5\frac{527}{600}$.
22. $1.6\dot{9}40\dot{8} = 1\frac{69408-6}{99990} = 1\frac{69402}{99990} = 1\frac{11567}{16665}$.
23. $0.48\dot{3}2\dot{4} = \frac{48324-48}{99900} = \frac{48276}{99900} = \frac{447}{925}$.
24. $0.00\dot{1}221\dot{3} = \frac{12213}{9999900} = \frac{1357}{1111100}$.

[redacted]

Exercise 68. Page 140.

1. Find the G. C. M. and L. C. M. of $\frac{7}{9}$, $\frac{14}{27}$, $\frac{18}{45}$.

$$\tfrac{18}{45} = \tfrac{2}{5}.$$

The G. C. M. of 7, 14, 2 $= 1$.
The L. C. M. of 9, 27, 5 $= 135$.
$\therefore$ the G. C. M. required $= \frac{1}{135}$.
The L. C. M. of 7, 14, 2 $= 14$.
The G. C. M. of 9, 27, 5 $= 1$.
$\therefore$ the L. C. M. required $= 14$.

2. Find the G. C. M. and L. C. M. of $2\frac{2}{9}$, $2\frac{2}{5}$, $\frac{4}{40}$.

$$2\tfrac{2}{9} = \tfrac{20}{9},\ 2\tfrac{2}{5} = \tfrac{12}{5},\ \tfrac{4}{40} = \tfrac{1}{10}.$$

The G. C. M. of 20, 12, 1 $= 1$.
The L. C. M. of 9, 5, 10 $= 90$.
$\therefore$ the G. C. M. required $= \frac{1}{90}$.
The L. C. M. of 20, 12, 1 $= 60$.
The G. C. M. of 9, 5, 10 $= 1$.
$\therefore$ the L. C. M. required $= 60$.

3. Find the G. C. M. and L. C. M. of $33\frac{3}{7}$, $50\frac{5}{8}$.

$$33\tfrac{3}{7} = \tfrac{234}{7},\ 50\tfrac{5}{8} = \tfrac{405}{8}.$$

The G. C. M. of 234, 405 $= 9$.
The L. C. M. of 7, 8 $= 56$.
$\therefore$ the G. C. M. required $= \frac{9}{56}$.
The L. C. M. of 234, 405 $= 10,530$.
The G. C. M. of 7, 8 $= 1$.
$\therefore$ the L. C. M. required $= 10,530$

4. Find the G. C. M. and L. C. M. of $\frac{7}{24}$, $\frac{35}{36}$, $\frac{49}{60}$.

The G. C. M. of 7, 35, 49 $= 7$.
The L. C. M. of 24, 36, 60 $= 360$.
$\therefore$ the G. C. M. required $= \frac{7}{360}$.
The L. C. M. of 7, 35, 49 $= 245$.
The G. C. M. of 24, 36, 60 $= 12$.
$\therefore$ the L. C. M. required $= \frac{245}{12} = 20\frac{5}{12}$.

[redacted]

5. Find the G. C. M. and L. C. M. of $5\frac{1}{2}$, $7\frac{1}{3}$, $8\frac{1}{4}$, $4\frac{8}{9}$, $9\frac{1}{6}$, $6\frac{5}{12}$.

$5\frac{1}{2}, 7\frac{1}{3}, 8\frac{1}{4}, 4\frac{8}{9}, 9\frac{1}{6}, 6\frac{5}{12} = \frac{11}{2}, \frac{22}{3}, \frac{33}{4}, \frac{44}{9}, \frac{55}{6}, \frac{77}{12}$.

The G. C. M. of 11, 22, 33, 44, 55, 77 = 11.
The L. C. M. of 2, 3, 4, 9, 6, 12 = 36.
∴ the G. C. M. required = $\frac{11}{36}$.
The L. C. M. of 11, 22, 33, 44, 55, 77 = 4620.
The G. C. M. of 2, 3, 4, 9, 6, 12 = 1.
∴ the L. C. M. required = 4620.

6. Find the G. C. M. and L. C. M. of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{10}$, $\frac{1}{12}$.

The G. C. M. of 1, 1, 1, 1, 1, 1, 1 = 1.
The L. C. M. of 2, 3, 4, 5, 6, 10, 12 = 60.
∴ the G. C. M. required = $\frac{1}{60}$.
The L. C. M. of 1, 1, 1, 1, 1, 1, 1 = 1.
The G. C. M. of 2, 3, 4, 5, 6, 10, 12 = 1.
∴ the L. C. M. required = 1.

7. Find the G. C. M. and L. C. M. of $50\frac{1}{2}$, $67\frac{1}{3}$, $44\frac{8}{9}$, $84\frac{1}{6}$, 707.

$50\frac{1}{2}, 67\frac{1}{3}, 44\frac{8}{9}, 84\frac{1}{6}, 707 = \frac{101}{2}, \frac{202}{3}, \frac{404}{9}, \frac{505}{6}, \frac{707}{1}$.

The G. C. M. of 101, 202, 404, 505, 707 = 101.
The L. C. M. of 2, 3, 9, 6, 1 = 18.
∴ the G. C. M. required = $\frac{101}{18} = 5\frac{11}{18}$.
The L. C. M. of 101, 202, 404, 505, 707 = 14,140.
The G. C. M. of 2, 3, 9, 6, 1 = 1.
∴ the L. C. M. required = 14,140.

8. Find the G. C. M. and L. C. M. of $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, $\frac{8}{9}$, $\frac{9}{10}$.

The G. C. M. of 4, 5, 6, 7, 8, 9 = 1.
The L. C. M. of 5, 6, 7, 8, 9, 10 = 2520.
∴ the G. C. M. required = $\frac{1}{2520}$.
The L. C. M. of 4, 5, 6, 7, 8, 9 = 2520.
The G. C. M. of 5, 6, 7, 8, 9, 10 = 1.
∴ the L. C. M. required = 2520.

9. Find the G. C. M. and L. C. M. of $1\frac{1}{14}$, $1\frac{19}{21}$, $4\frac{2}{7}$, $\frac{25}{42}$.

$1\frac{1}{14}, 1\frac{19}{21}, 4\frac{2}{7}, \frac{25}{42} = \frac{15}{14}, \frac{40}{21}, \frac{30}{7}, \frac{25}{42}$.

The G. C. M. of 15, 40, 30, 25 = 5.
The L. C. M. of 14, 21, 7, 42 = 42.
∴ the G. C. M. required = $\frac{5}{42}$.
The L. C. M. of 15, 40, 30, 25 = 600.
The G. C. M. of 14, 21, 7, 42 = 7.
∴ the L. C. M. required = $\frac{600}{7} = 85\frac{5}{7}$.

[redacted]

10. Find the G. C. M. and L. C. M. of $18\frac{2}{5}$, $57\frac{1}{2}$.

$$18\tfrac{2}{5} = \tfrac{92}{5},\ 57\tfrac{1}{2} = \tfrac{115}{2}.$$

The G. C. M. of 92, 115 $= 23$.
The L. C. M. of 5, 2 $= 10$.
$\therefore$ the G. C. M. required $= \frac{23}{10} = 2\frac{3}{10}$.
The L. C. M. of 92, 115 $= 460$.
The G. C. M. of 5, 2 $= 1$.
$\therefore$ the L. C. M. required $= 460$.

11. Find the G. C. M. and L. C. M. of $134\frac{3}{4}$, $128\frac{1}{3}$, $115\frac{1}{2}$.

$$134\tfrac{3}{4},\ 128\tfrac{1}{3},\ 115\tfrac{1}{2} = \tfrac{539}{4},\ \tfrac{385}{3},\ \tfrac{231}{2}.$$

The G. C. M. of 539, 385, 231 $= 77$.
The L. C. M. of 4, 3, 2 $= 12$.
$\therefore$ the G. C. M. required $= \frac{77}{12} = 6\frac{5}{12}$.
The L. C. M. of 539, 385, 231 $= 8085$.
The G. C. M. of 4, 3, 2 $= 1$.
$\therefore$ the L. C. M. required $= 8085$.

12. Find the G. C. M. and L. C. M. of $2\frac{22}{25}$, $1\frac{37}{75}$, $\frac{63}{100}$.

$$2\tfrac{22}{25},\ 1\tfrac{37}{75},\ \tfrac{63}{100} = \tfrac{72}{25},\ \tfrac{112}{75},\ \tfrac{63}{100}.$$

The G. C. M. of 72, 112, 63 $= 1$.
The L. C. M. of 25, 75, 100 $= 300$.
$\therefore$ the G. C. M. required $= \frac{1}{300}$.
The L. C. M. of 72, 112, 63 $= 1008$.
The G. C. M. of 25, 75, 100 $= 25$.
$\therefore$ the L. C. M. required $= \frac{1008}{25} = 40\frac{8}{25}$.

13. A, B, and C start together to walk in the same direction round a circular island. It takes A $2\frac{1}{3}$ days, B $2\frac{5}{6}$ days, C $2\frac{7}{8}$ days to walk round the island. They walk until they all meet at the point of starting. In how many days will they be together at the point of starting?

$$2\tfrac{1}{3},\ 2\tfrac{5}{6},\ 2\tfrac{7}{8} = \tfrac{7}{3},\ \tfrac{17}{6},\ \tfrac{23}{8}.$$

The L. C. M. of 7, 17, 23 $= 2737$.
The G. C. M. of 3, 6, 8 $= 1$.
$\therefore$ the L. C. M. required $= 2737$.

2737 days. *Ans.*

[redacted]

14. If the step of a man is $2\frac{1}{3}$ feet, and that of a horse is $2\frac{3}{4}$ feet, find the smallest number of feet which is an exact number of steps for a man and for a horse.

$$2\tfrac{1}{3},\ 2\tfrac{3}{4},=\tfrac{7}{3},\ \tfrac{11}{4}.$$

The L. C. M. of 7, 11 $= 77$.
The G. C. M. of 3, 4 $= 1$.
$\therefore$ the L. C. M. required $= 77$.

77 feet. *Ans.*

15. Find the largest number that is contained without remainder in $2\frac{5}{9}$, $6\frac{7}{18}$, $11\frac{1}{2}$, and $19\frac{1}{6}$.

$$2\tfrac{5}{9},\ 6\tfrac{7}{18},\ 11\tfrac{1}{2},\ 19\tfrac{1}{6}=\tfrac{23}{9},\ \tfrac{115}{18},\ \tfrac{23}{2},\ \tfrac{115}{6}.$$

The G. C. M. of 23, 115, 23, 115 $= 23$.
The L. C. M. of 9, 18, 2, 6 $= 18$.
$\therefore$ the G. C. M. required $=\frac{23}{18}=1\frac{5}{18}$. *Ans.*

Exercise 69. Page 141.

1. Simplify $\frac{2709}{6966}$, $\frac{43785}{56835}$, $\frac{2436}{567216}$, $\frac{4087}{5063}$.

$$\frac{2709}{6966}=\frac{301}{774}=\frac{7}{18}. \textit{ Ans.} \qquad \frac{43785}{56835}=\frac{8757}{11367}=\frac{973}{1263}. \textit{ Ans.}$$

$$\frac{2436}{567216}=\frac{203}{47268}. \textit{ Ans.} \qquad \frac{4087}{5063}=\frac{67}{83}. \textit{ Ans.}$$

2. Which is greater, and by how much, $\frac{7}{9}$ or $\frac{19}{24}$?

$$\frac{7}{9},\ \frac{19}{24}=\frac{56}{72},\ \frac{57}{72}. \qquad \therefore\ \frac{19}{24} \text{ is greater by } \frac{1}{72}.$$

3. Find the sum of $3\frac{2}{5}$, $2\frac{4}{11}$, $5\frac{1}{2}$, $7\frac{7}{10}$, $1\frac{3}{22}$.

$$3\tfrac{2}{5}+2\tfrac{4}{11}+5\tfrac{1}{2}+7\tfrac{7}{10}+1\tfrac{3}{22}=18\tfrac{44+40+55+77+15}{110}$$
$$=18\tfrac{231}{110}=20\tfrac{11}{110}=20\tfrac{1}{10}. \textit{ Ans.}$$

4. Simplify $5\frac{1}{2}-3\frac{3}{7}+2\frac{9}{10}-1\frac{3}{5}$.

$$5\tfrac{1}{2}+2\tfrac{9}{10}=7\tfrac{5+9}{10}=7\tfrac{14}{10}=8\tfrac{4}{10}=8\tfrac{2}{5}.$$
$$3\tfrac{3}{7}+1\tfrac{3}{5}=4\tfrac{15+21}{35}=4\tfrac{36}{35}=5\tfrac{1}{35}.$$
$$8\tfrac{2}{5}-5\tfrac{1}{35}=3\tfrac{14-1}{35}=3\tfrac{13}{35}. \textit{ Ans.}$$

5. Simplify $1\frac{4}{5}+3\frac{5}{6}-2\frac{7}{12}+4\frac{3}{20}-3\frac{7}{15}$.

$$1\tfrac{4}{5}+3\tfrac{5}{6}+4\tfrac{3}{20}=8\tfrac{48+50+9}{60}=8\tfrac{107}{60}=9\tfrac{47}{60}.$$
$$2\tfrac{7}{12}+3\tfrac{7}{15}=5\tfrac{35+28}{60}=5\tfrac{63}{60}=6\tfrac{3}{60}.$$
$$9\tfrac{47}{60}-6\tfrac{3}{60}=3\tfrac{44}{60}=3\tfrac{11}{15}. \textit{ Ans.}$$

[redacted]

6. Simplify $\dfrac{3\frac{1}{2}+3\frac{5}{6}}{4\frac{1}{3}-2\frac{7}{12}}$.

$$\frac{3\frac{1}{2}+3\frac{5}{6}}{4\frac{1}{3}-2\frac{7}{12}}=\frac{42+46}{52-31}=\frac{88}{21}=4\tfrac{4}{21}.\ \textit{Ans.}$$

7. Simplify $7\div 2\frac{3}{4}$; $\dfrac{7}{1\frac{3}{8}}$; $\dfrac{95\frac{1}{2}}{8\frac{7}{11}}$; $15\div\frac{2}{3}$; $\dfrac{16}{5\frac{1}{3}}$; $7\frac{4}{11}\div 9$; $43\frac{1}{4}\div 37\frac{1}{3}$; $\dfrac{6\frac{7}{11}}{18\frac{1}{4}}$; $5\frac{4}{5}\div 4\frac{5}{6}$; $\dfrac{\frac{3}{8}\text{ of }4\frac{1}{2}}{\frac{7}{4}\times\frac{3}{2}}$; $106\div 8\frac{5}{6}$; $\dfrac{17}{4\frac{7}{17}}$.

$7\div 2\frac{3}{4}=\frac{4}{11}\times 7=\frac{28}{11}=2\frac{6}{11}$.

$\frac{7}{1\frac{3}{8}}=\frac{8}{11}\times 7=\frac{56}{11}=5\frac{1}{11}$.

$\frac{95\frac{1}{2}}{8\frac{7}{11}}=\frac{11}{95}\times\frac{191}{2}=\frac{2101}{190}=11\frac{11}{190}$.

$15\div\frac{2}{3}=\frac{3}{2}\times 15=\frac{45}{2}=22\frac{1}{2}$.

$\frac{16}{5\frac{1}{3}}=\frac{3}{\cancel{16}}\times\cancel{16}=3$.

$7\frac{4}{11}\div 9=\frac{1}{\cancel{9}}\times\frac{\overset{9}{\cancel{81}}}{11}=\frac{9}{11}$.

$43\frac{1}{4}\div 37\frac{1}{3}=\frac{3}{112}\times\frac{173}{4}=\frac{519}{448}=1\frac{71}{448}$.

$\frac{6\frac{7}{11}}{18\frac{1}{4}}=\frac{4}{\cancel{73}}\times\frac{\cancel{73}}{11}=\frac{4}{11}$.

$5\frac{4}{5}\div 4\frac{5}{6}=\frac{6}{\cancel{29}}\times\frac{\cancel{29}}{5}=\frac{6}{5}=1\frac{1}{5}$.

$\frac{\frac{3}{8}\text{ of }4\frac{1}{2}}{\frac{7}{4}\times\frac{3}{2}}=\frac{\cancel{3}}{\underset{2}{\cancel{8}}}\times\frac{9}{\cancel{2}}\times\frac{\cancel{4}}{7}\times\frac{\cancel{2}}{\cancel{3}}=\frac{9}{14}$.

$106\div 8\frac{5}{6}=\frac{6}{\cancel{53}}\times\overset{2}{\cancel{106}}=12$.

$\frac{17}{4\frac{7}{17}}=\frac{17}{75}\times 17=\frac{289}{75}=3\frac{64}{75}$.

8. Simplify $7\frac{19}{32}\times 8$; $43\frac{13}{53}\times 6\frac{5}{8}$; $6\frac{5}{6}\div 8\frac{1}{5}$; $5\frac{1}{17}\times 51$; $\frac{11}{12}$ of $\frac{11}{13}$; $\frac{30}{49}$ of $\frac{7}{15}$ of $\frac{7}{8}$ of $\frac{2}{3}$ of $\frac{2}{5}$; $\frac{17}{19}$ of $\frac{228}{561}$; $\frac{1}{2}\times\frac{3}{4}\times\frac{7}{11}\times\frac{8}{9}\times\frac{3}{7}$.

$$7\tfrac{19}{32}\times 8=\frac{243}{\underset{4}{\cancel{32}}}\times\cancel{8}=\frac{243}{4}=60\tfrac{3}{4}.$$

$$43\tfrac{13}{53}\times 6\tfrac{5}{8}=\frac{\overset{573}{\cancel{2292}}}{\cancel{53}}\times\frac{\cancel{53}}{\underset{2}{\cancel{8}}}=\frac{573}{2}=286\tfrac{1}{2}.$$

$6\frac{5}{6}\div 8\frac{1}{5}=\frac{5}{\cancel{41}}\times\frac{\cancel{41}}{6}=\frac{5}{6}$.

$5\frac{1}{17}\times 51=\frac{86}{\cancel{17}}\times\overset{3}{\cancel{51}}=258$.

$\frac{11}{12}$ of $\frac{11}{13}=\frac{121}{156}$.

$\frac{\cancel{17}}{\cancel{19}}$ of $\frac{\overset{\overset{4}{\cancel{12}}}{\cancel{228}}}{\underset{\underset{11}{\cancel{33}}}{\cancel{561}}}=\frac{4}{11}$.

$\frac{\overset{2}{\cancel{30}}}{\cancel{49}}$ of $\frac{\cancel{7}}{\cancel{15}}$ of $\frac{\cancel{7}}{\underset{\underset{2}{\cancel{4}}}{\cancel{8}}}$ of $\frac{\cancel{2}}{3}$ of $\frac{\cancel{2}}{5}=\frac{1}{15}$.

$\frac{1}{\cancel{2}}\times\frac{\cancel{3}}{\cancel{4}}\times\frac{\cancel{7}}{11}\times\frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{9}}}\times\frac{\cancel{3}}{\cancel{7}}=\frac{1}{11}$.

[redacted]

9. By what must $\frac{1}{6}$ be multiplied to obtain $\frac{1}{2}$? $\frac{1}{6}$ to obtain $\frac{2}{3}$? $\frac{1}{2}$ to obtain $\frac{5}{6}$? $\frac{2}{3}$ to obtain $\frac{5}{6}$? $\frac{3}{5}$ to obtain $\frac{7}{8}$?

$$\frac{1}{2}\div\frac{1}{6}=\overset{3}{\cancel{6}}\times\frac{1}{\cancel{2}}=3.\ \textit{Ans.}\qquad \frac{5}{6}\div\frac{1}{2}=\cancel{2}\times\frac{5}{\underset{3}{\cancel{6}}}=\frac{5}{3}=1\tfrac{2}{3}.\ \textit{Ans.}$$

$$\frac{2}{3}\div\frac{1}{6}=\overset{2}{\cancel{6}}\times\frac{2}{\cancel{3}}=4.\ \textit{Ans.}\qquad \frac{5}{6}\div\frac{2}{3}=\frac{\cancel{3}}{2}\times\frac{5}{\underset{2}{\cancel{6}}}=\frac{5}{4}=1\tfrac{1}{4}.\ \textit{Ans.}$$

$$\frac{7}{8}\div\frac{3}{5}=\frac{5}{3}\times\frac{7}{8}=\frac{35}{24}=1\tfrac{11}{24}.\ \textit{Ans.}$$

10. By what must $\frac{1}{6}$ be divided to obtain $\frac{1}{2}$? $\frac{2}{3}$ to obtain $\frac{1}{6}$? $\frac{7}{8}$ to obtain $\frac{4}{5}$? $\frac{3}{5}$ to obtain $\frac{7}{8}$? 8 to obtain $7\frac{19}{32}$?

$$\frac{1}{6}\div\frac{1}{2}=\cancel{2}\times\frac{1}{\underset{3}{\cancel{6}}}=\frac{1}{3}.\ \textit{Ans.}\qquad \frac{7}{8}\div\frac{4}{5}=\frac{5}{4}\times\frac{7}{8}=\frac{35}{32}=1\tfrac{3}{32}.\ \textit{Ans.}$$

$$\frac{2}{3}\div\frac{1}{6}=\overset{2}{\cancel{6}}\times\frac{2}{\cancel{3}}=4.\ \textit{Ans.}\qquad \frac{3}{5}\div\frac{7}{8}=\frac{8}{7}\times\frac{3}{5}=\frac{24}{35}.\ \textit{Ans.}$$

$$8\div7\tfrac{19}{32}=\frac{32}{243}\times8=\frac{256}{243}=1\tfrac{13}{243}.\ \textit{Ans.}$$

11. What number exceeds $5\frac{2}{9}$ by $4\frac{7}{8}$?

$$5\tfrac{2}{9}+4\tfrac{7}{8}=9\tfrac{16+63}{72}=9\tfrac{79}{72}=10\tfrac{7}{72}.\ \textit{Ans.}$$

12. From what must $6\frac{3}{5}$ be subtracted to leave $\frac{1}{2}$ of $3\frac{1}{9}$?

$$\frac{1}{2}\text{ of }3\tfrac{1}{9}=\frac{1}{\cancel{2}}\times\frac{\overset{14}{\cancel{28}}}{9}=\frac{14}{9}=1\tfrac{5}{9}.$$

$$6\tfrac{3}{5}+1\tfrac{5}{9}=7\tfrac{27+25}{45}=7\tfrac{52}{45}=8\tfrac{7}{45}.\ \textit{Ans.}$$

13. What fraction falls short of $\frac{7}{12}$ by $\frac{3}{20}$?

$$\frac{7}{12}-\frac{3}{20}=\frac{35-9}{60}=\frac{26}{60}=\frac{13}{30}.\ \textit{Ans.}$$

14. What fraction must be added to $\frac{5}{76}$ to make $\frac{11}{57}$?

$$\frac{11}{57}-\frac{5}{76}=\frac{44-15}{228}=\frac{29}{228}.\ \textit{Ans.}$$

[redacted]

15. Reduce to decimals: $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{3}$; $\frac{3}{4}$; $\frac{1}{8}$; $\frac{3}{8}$; $\frac{5}{8}$; $\frac{7}{8}$; $\frac{1}{16}$; $\frac{3}{16}$; $\frac{5}{16}$; $\frac{7}{16}$; $\frac{9}{16}$; $\frac{11}{16}$; $\frac{13}{16}$; $\frac{15}{16}$; $\frac{1}{6}$; $\frac{5}{6}$; $\frac{3}{7}$; $\frac{5}{9}$; $\frac{3}{11}$; $\frac{7}{40}$.

2)1.	4)1.	3)1.0	4)3.	8)1.
0.5	0.25	0.3	0.75	0.125
$\therefore \frac{1}{2}=0.5.$	$\therefore \frac{1}{4}=0.25.$	$\therefore \frac{1}{3}=0.\dot{3}.$	$\therefore \frac{3}{4}=0.75.$	$\therefore \frac{1}{8}=0.125.$

8)3.	8)5.	8)7.	16)1.	0.0625 × 3
0.375	0.625	0.875	0.0625	0.1875
$\therefore \frac{3}{8}=0.375.$	$\therefore \frac{5}{8}=0.625.$	$\therefore \frac{7}{8}=0.875.$	$\therefore \frac{1}{16}=0.0625.$	$\therefore \frac{3}{16}=0.1875.$

0.0625 × 5	0.0625 × 7	0.0625 × 9	0.0625 × 11
0.3125	0.4375.	0.5625	0.6875
$\therefore \frac{5}{16}=0.3125.$	$\therefore \frac{7}{16}=0.4375.$	$\therefore \frac{9}{16}=0.5625.$	$\therefore \frac{11}{16}=0.6875.$

0.0625 × 13	0.0625 × 15	6)1.	6)5.
1875	3125	0.16	0.83
625	625	$\therefore \frac{1}{6}=0.1\dot{6}.$	$\therefore \frac{5}{6}=0.8\dot{3}.$
0.8125	0.9375		
$\therefore \frac{13}{16}=0.8125.$	$\therefore \frac{15}{16}=0.9375.$		

7)3.	9)5.	11)3.	40)0.7
0.428571	0.5	0.27	0.175
$\therefore \frac{3}{7}=0.\dot{4}2857\dot{1}.$	$\therefore \frac{5}{9}=0.\dot{5}.$	$\therefore \frac{3}{11}=0.\dot{2}\dot{7}.$	$\therefore \frac{7}{40}=0.175.$

16. Reduce to common fractions: 0.16; 0.016; 0.125; 0.13; 0.725; 0.625; 0.00625; 0.8125; 0.03125; 0.08; $0.5\dot{4}$; $0.01\dot{6}$; $0.5\dot{4}3\dot{7}$; $0.0\dot{2}\dot{7}$; $0.27\dot{7}$; $0.68\dot{4}9\dot{4}$; $1.3\dot{4}\dot{5}$.

$0.16 = \frac{16}{100} = \frac{4}{25}.$

$0.016 = \frac{16}{1000} = \frac{2}{125}.$

$0.125 = \frac{125}{1000} = \frac{1}{8}.$

$0.13 = \frac{13}{100}.$

$0.725 = \frac{725}{1000} = \frac{29}{40}.$

$0.625 = \frac{625}{1000} = \frac{25}{40} = \frac{5}{8}.$

$0.00625 = \frac{625}{100000} = \frac{25}{4000} = \frac{1}{160}.$

$0.8125 = \frac{8125}{10000} = \frac{325}{400} = \frac{13}{16}.$

[redacted]

$$\begin{aligned}
0.03125 &= \tfrac{3125}{100000} = \tfrac{125}{4000} = \tfrac{5}{160} = \tfrac{1}{32}.\\
0.08 &= \tfrac{8}{100} = \tfrac{2}{25}.\\
0.\dot{5}\dot{4} &= \tfrac{54}{99} = \tfrac{6}{11}.\\
0.01\dot{6} &= \tfrac{16-1}{900} = \tfrac{15}{900} = \tfrac{1}{60}.\\
0.5\dot{4}3\dot{7} &= \tfrac{5437-5}{9990} = \tfrac{5432}{9990} = \tfrac{2716}{4995}.\\
0.0\dot{2}\dot{7} &= \tfrac{27}{990} = \tfrac{3}{110}.\\
0.27\dot{7} &= \tfrac{277-27}{900} = \tfrac{250}{900} = \tfrac{5}{18}.\\
0.68\dot{4}9\dot{4} &= \tfrac{68494-68}{99900} = \tfrac{68426}{99900} = \tfrac{34213}{49950}.\\
1.3\dot{4}\dot{5} &= 1\tfrac{345-3}{990} = 1\tfrac{342}{990} = 1\tfrac{171}{495} = 1\tfrac{19}{55}.
\end{aligned}$$

17. Simplify $\dfrac{2.8 \text{ of } 2.\dot{2}\dot{7}}{1.1\dot{3}\dot{6}}$.

$$2.\dot{2}\dot{7} = 2\tfrac{27}{99} = 2\tfrac{3}{11}; \quad 1.1\dot{3}\dot{6} = 1\tfrac{136-1}{990} = 1\tfrac{135}{990} = 1\tfrac{15}{110} = 1\tfrac{3}{22}.$$

$$\therefore \quad \frac{2.8 \text{ of } 2.\dot{2}\dot{7}}{1.1\dot{3}\dot{6}} = \frac{2\frac{4}{5} \times 2\frac{3}{11}}{1\frac{3}{22}} = \frac{14}{5} \times \frac{\cancel{25}}{\cancel{11}} \times \frac{\overset{2}{\cancel{22}}}{\cancel{25}} = \frac{28}{5} = 5\tfrac{3}{5} = 5.6. \quad \textit{Ans.}$$

18. Multiply $6.9\dot{5}\dot{4}$ by $5.\dot{3}0\dot{3}$, and express the result as a whole number and common fraction.

$$6.9\dot{5}\dot{4} = 6\tfrac{954-9}{990} = 6\tfrac{945}{990} = 6\tfrac{105}{110} = 6\tfrac{21}{22};$$
$$5.\dot{3}0\dot{3} = 5\tfrac{303-3}{990} = 5\tfrac{300}{990} = 5\tfrac{10}{33};$$

$$\therefore \quad 6.9\dot{5}\dot{4} \times 5.\dot{3}0\dot{3} = 6\tfrac{21}{22} \times 5\tfrac{10}{33} = \frac{\overset{51}{\cancel{153}}}{22} \times \frac{175}{\underset{11}{\cancel{33}}} = \frac{8925}{242} = 36\tfrac{213}{242}. \quad \textit{Ans.}$$

19. Simplify $1\frac{1}{2}$ of $2\frac{4}{5} + 6\frac{7}{8} \div 2\frac{3}{4}$ and reduce the result to a decimal.

$$1\tfrac{1}{2} \text{ of } 2\tfrac{4}{5} + 6\tfrac{7}{8} \div 2\tfrac{3}{4} = \frac{3}{\cancel{2}} \times \frac{\overset{7}{\cancel{14}}}{5} + \frac{\cancel{4}}{\cancel{11}} \times \frac{\overset{5}{\cancel{55}}}{\underset{2}{\cancel{8}}} = \frac{21}{5} + \frac{5}{2}$$
$$= 4\tfrac{1}{5} + 2\tfrac{1}{2} = 6\tfrac{2+5}{10} = 6\tfrac{7}{10} = 6.7. \quad \textit{Ans.}$$

20. From what number can $4\frac{17}{36}$ be taken 9 times and leave no remainder?

$$9 \times 4\tfrac{17}{36} = \cancel{9} \times \frac{161}{\underset{4}{\cancel{36}}} = \frac{161}{4} = 40\tfrac{1}{4}. \quad \textit{Ans.}$$

21. Of what fraction is $17\frac{1}{3}$ the 7th part?

$$17\tfrac{1}{3} \div \frac{1}{7} = \frac{52}{3} \times 7 = \frac{364}{3}. \quad \textit{Ans.}$$

22. Add $\frac{4}{5}$, 0.35, $\frac{5}{8}$, $\frac{3}{4}$, 0.112, 45.28.

$$\tfrac{4}{5} + 0.35 + \tfrac{5}{8} + \tfrac{3}{4} + 0.112 + 45.28$$
$$= 0.8 + 0.35 + 0.625 + 0.75 + 0.112 + 45.28 = 47.917. \quad \textit{Ans.}$$

[redacted]

23. Reduce to decimals $\frac{13}{15}$; $\frac{3}{11}$; $\frac{4}{35}$; $\frac{17}{60}$; $\frac{14}{19}$; $\frac{5}{13}$.

$$\begin{array}{r} 0.86 \\ 15\overline{)13.0} \\ \underline{120} \\ 100 \\ \underline{90} \\ 10 \end{array}$$

$\therefore \frac{13}{15} = 0.8\dot{6}$.

$$\begin{array}{r} 11\underline{)3.} \\ 0.27 \end{array}$$

$\therefore \frac{3}{11} = 0.\dot{2}\dot{7}$.

$$\begin{array}{r} 0.736842105263157894 \\ 19\overline{)14.0} \\ \underline{133} \\ 70 \\ \underline{57} \\ 130 \\ \underline{114} \\ 160 \\ \underline{152} \\ 80 \\ \underline{76} \\ 40 \\ \underline{38} \\ 20 \\ \underline{19} \\ 100 \\ \underline{95} \\ 50 \\ \underline{38} \\ 120 \\ \underline{114} \\ 60 \\ \underline{57} \\ 30 \\ \underline{19} \\ 110 \\ \underline{95} \\ 150 \\ \underline{133} \\ 170 \\ \underline{152} \\ 180 \\ \underline{171} \\ 90 \\ \underline{76} \\ 14 \end{array}$$

$$\begin{array}{r} 0.1142857 \\ 35\overline{)4.0} \\ \underline{35} \\ 50 \\ \underline{35} \\ 150 \\ \underline{140} \\ 100 \\ \underline{70} \\ 300 \\ \underline{280} \\ 200 \\ \underline{175} \\ 250 \\ \underline{245} \\ 5 \end{array}$$

$\therefore \frac{4}{35} = 0.1\dot{1}42857\dot{}$.

$$\begin{array}{r} 6\cancel{0}\underline{)1.7} \\ 0.283 \end{array}$$

$\therefore \frac{17}{60} = 0.28\dot{3}$.

$$\begin{array}{r} 0.384615 \\ 13\overline{)5.0} \\ \underline{39} \\ 110 \\ \underline{104} \\ 60 \\ \underline{52} \\ 80 \\ \underline{78} \\ 20 \\ \underline{13} \\ 70 \\ \underline{65} \\ 5 \end{array}$$

$\therefore \frac{5}{13} = 0.\dot{3}8461\dot{5}$.

$\therefore \frac{14}{19} = 0.\dot{7}3684210526315789\dot{4}$.

[redacted]

24. What part of $\frac{15}{73}$ is $\frac{3}{1241}$?

$$\frac{3}{1241} \div \frac{15}{73} = \frac{\cancel{73}}{\underset{5}{\cancel{15}}} \times \frac{\cancel{3}}{\underset{17}{\cancel{1241}}} = \frac{1}{85}. \quad \textit{Ans.}$$

25. Divide 0.0015 by 0.012, and express the result as a common fraction in lowest terms.

$$012)\underline{1.5}$$
$$0.125 \qquad\qquad 0.125 = \tfrac{1}{8}. \quad \textit{Ans.}$$

26. Reduce to decimals: $\frac{3}{32}$; $\frac{3}{32000}$; $\frac{17}{74}$; $\frac{1}{7}$.

```
   0.09375            0.2297              7)1.
32)3.00            74)17.0                 0.142857
   288                148
    120                220
     96                148
    240                 720
    224                 666
     160                 540
     160                 518
                          22
```

$\therefore \frac{3}{32} = 0.09375.$

$\frac{3}{32000} = 0.00009375.$

$\therefore \frac{17}{74} = 0.2\dot{2}9\dot{7}.$

$\therefore \frac{1}{7} = 0.\dot{1}4285\dot{7}.$

27. The product of two factors is $\frac{5}{8}$, and one factor is $1\frac{1}{4}$; find the other factor.

$$\frac{5}{8} \div 1\tfrac{1}{4} = \frac{\cancel{4}}{\cancel{5}} \times \frac{\cancel{5}}{\underset{2}{\cancel{8}}} = \frac{1}{2}. \quad \textit{Ans.}$$

28. The dividend is $\frac{11}{12}$, the quotient $6\frac{1}{2}$; find the divisor.

$$\frac{11}{12} \div 6\tfrac{1}{2} = \frac{\cancel{2}}{13} \times \frac{11}{\underset{6}{\cancel{12}}} = \frac{11}{78}. \quad \textit{Ans.}$$

29. The dividend is $12\frac{17}{72}$, quotient 3, remainder $1\frac{5}{12}$; find the divisor.

$$(12\tfrac{17}{72} - 1\tfrac{5}{12}) \div 3 = 10\tfrac{59}{72} \div 3 = \tfrac{1}{3} \times \tfrac{779}{72} = \tfrac{779}{216} = 3\tfrac{131}{216}. \quad \textit{Ans.}$$

30. Find the G. C. M. and the L. C. M. of 833, 1127, 1421, 343.

7	833	1127	1421	343
7	119	161	203	49
	17	23	29	7

The G. C. M. $= 7 \times 7 = 49$. *Ans.*

The L. C. M. $= 7^3 \times 17 \times 23 \times 29 = 3{,}889{,}277$. *Ans.*

[redacted]

31. Arrange in order of magnitude $\frac{9}{11}$, $\frac{32}{45}$, $\frac{19}{27}$, $\frac{17}{20}$, $\frac{29}{33}$.

$\frac{9}{11}=\frac{4860}{5940}$. $\quad \frac{32}{45}=\frac{4224}{5940}$. $\quad \frac{19}{27}=\frac{4180}{5940}$. $\quad \frac{17}{20}=\frac{5049}{5940}$. $\quad \frac{29}{33}=\frac{5220}{5940}$.

$\frac{17}{20}$, $\frac{9}{11}$, $\frac{32}{45}$, $\frac{19}{27}$, $\frac{29}{33}$. *Ans.*

32. Find the L. C. M. of $\frac{15}{17}$, $\frac{26}{51}$, $\frac{65}{102}$.

The L. C. M. of 15, 26, 65 $=390$.
The G. C. M. of 17, 51, 102 $=17$.
$\therefore$ the L. C. M. required $=\frac{390}{17}=22\frac{16}{17}$.

33. Find the G. C. M. of $\frac{65}{68}$, $\frac{39}{2}$, $\frac{91}{64}$, and $6\frac{1}{2}$.

The G. C. M. of 65, 39, 91, 13 $=13$.
The L. C. M. of 68, 2, 64, 2 $=1088$.
$\therefore$ the G. C. M. required $=\frac{13}{1088}$.

34. Reduce to common fractions: $7.2\dot{0}1\dot{1}$; $6.9\dot{5}\dot{4}$.

$7.2\dot{0}1\dot{1}=7\frac{2011-2}{9990}=7\frac{2009}{9990}$. $\qquad 6.9\dot{5}\dot{4}=6\frac{954-9}{990}=6\frac{945}{990}=6\frac{105}{110}=6\frac{21}{22}$.

35. Simplify $\dfrac{3\frac{7}{9}\times 1\frac{1}{17}+4\frac{1}{12}-3\frac{9}{16}}{5\frac{1}{9}-7\frac{7}{8}\div 28\frac{7}{20}+\frac{1}{3}}$.

$$3\tfrac{7}{9}\times 1\tfrac{1}{17}=\frac{\overset{2}{\cancel{34}}}{\cancel{9}}\times\frac{\overset{2}{\cancel{18}}}{\cancel{17}}=4; \qquad 7\tfrac{7}{8}\div 28\tfrac{7}{20}=\frac{\cancel{63}}{\underset{2}{\cancel{8}}}\times\frac{\overset{5}{\cancel{20}}}{\underset{9}{\cancel{567}}}=\frac{5}{18}.$$

$$\frac{3\frac{7}{9}\times 1\frac{1}{17}+4\frac{1}{12}-3\frac{9}{16}}{5\frac{1}{9}-7\frac{7}{8}\div 28\frac{7}{20}+\frac{1}{3}}=\frac{4+4\frac{1}{12}-3\frac{9}{16}}{5\frac{1}{9}-\frac{5}{18}+\frac{1}{3}}=\frac{4\frac{25}{48}}{5\frac{1}{6}}=\frac{7}{8}. \text{ Ans.}$$

36. Simplify $\dfrac{6\frac{3}{4}+5\frac{1}{2}\times 3\frac{1}{7}-7\frac{1}{4}}{3\frac{1}{5}+2\frac{1}{2}-4\frac{1}{10}}$.

$$5\tfrac{1}{2}\times 3\tfrac{1}{7}=\frac{11}{\cancel{2}}\times\frac{\overset{11}{\cancel{22}}}{7}=\frac{121}{7}=17\tfrac{2}{7}.$$

$$\frac{6\frac{3}{4}+5\frac{1}{2}\times 3\frac{1}{7}-7\frac{1}{4}}{3\frac{1}{5}+2\frac{1}{2}-4\frac{1}{10}}=\frac{6\frac{3}{4}+17\frac{2}{7}-7\frac{1}{4}}{3\frac{1}{5}+2\frac{1}{2}-4\frac{1}{10}}=\frac{945+2420-1015}{448+350-574}$$
$$=\frac{2350}{224}=10\tfrac{110}{224}=10\tfrac{55}{112}. \text{ Ans.}$$

37. Simplify $\dfrac{2\frac{4}{5}-1\frac{1}{2}+9\frac{1}{11}}{4\frac{1}{5}-2\frac{1}{4}+13\frac{7}{11}}$.

$$\frac{2\frac{4}{5}-1\frac{1}{2}+9\frac{1}{11}}{4\frac{1}{5}-2\frac{1}{4}+13\frac{7}{11}}=\frac{616-330+2000}{924-495+3000}=\frac{2286}{3429}=\frac{2}{3}. \text{ Ans.}$$

38. Simplify $\dfrac{(3.71-1.908)\times 7.03}{2.\dot{2}-\frac{74}{333}}$.

$$\frac{(3.71-1.908)\times 7.03}{2.\dot{2}-\frac{74}{333}}=\frac{1.802\times 7.03}{2\frac{2}{9}-\frac{2}{9}}=\frac{12.66806}{2}=6.33403. \text{ Ans.}$$

[redacted]

39. Simplify $\dfrac{5\frac{5}{8} \div \frac{2}{3}}{1\frac{1}{5} \text{ of } \frac{5}{9} \div 10\frac{1}{3}} \times \frac{2}{5} \text{ of } \dfrac{1\frac{1}{2} \text{ of } 4\frac{1}{9}}{13\frac{7}{8} \text{ of } 5\frac{1}{3}}.$

$$\frac{5\frac{5}{8} \div \frac{2}{3}}{1\frac{1}{5} \text{ of } \frac{5}{9} \div 10\frac{1}{3}} \times \frac{2}{5} \text{ of } \frac{1\frac{1}{2} \text{ of } 4\frac{1}{9}}{13\frac{7}{8} \text{ of } 5\frac{1}{3}}$$

$$= \frac{\overset{9}{\cancel{45}}}{\cancel{8}} \times \frac{\cancel{3}}{2} \times \frac{\cancel{5}}{\underset{2}{\cancel{6}}} \times \frac{9}{\cancel{5}} \times \frac{31}{\cancel{3}} \times \frac{\cancel{2}}{\cancel{5}} \times \frac{\cancel{3}}{\cancel{2}} \times \frac{\cancel{37}}{\cancel{9}} \times \frac{\cancel{8}}{\underset{\cancel{3}}{\cancel{111}}} \times \frac{\cancel{3}}{16} = \frac{279}{64} = 4\tfrac{23}{64}.$$

40. Simplify $1\frac{1}{2}$ of $2\frac{4}{5} + 6\frac{7}{8} \div 2\frac{3}{4} + \left(5\frac{1}{2} + \dfrac{0.24 + 0.5\dot{3}}{2.2 - 0.6\dot{4}}\right).$

$$1\tfrac{1}{2} \text{ of } 2\tfrac{4}{5} = \frac{3}{\cancel{2}} \times \frac{\overset{7}{\cancel{14}}}{5} = \frac{21}{5} = 4\tfrac{1}{5}; \qquad 6\tfrac{7}{8} \div 2\tfrac{3}{4} = \frac{\overset{5}{\cancel{55}}}{\underset{2}{\cancel{8}}} \times \frac{\cancel{4}}{\cancel{11}} = \frac{5}{2} = 2\tfrac{1}{2};$$

$$\frac{0.24 + 0.5\dot{3}}{2.2 - 0.6\dot{4}} = \frac{\frac{24}{100} + \frac{48}{90}}{2\frac{1}{5} - \frac{58}{90}} = \frac{\frac{6}{25} + \frac{8}{15}}{2\frac{1}{5} - \frac{29}{45}} = \frac{54 + 120}{495 - 145} = \frac{174}{350} = \frac{87}{175}.$$

$$1\tfrac{1}{2} \text{ of } 2\tfrac{4}{5} + 6\tfrac{7}{8} \div 2\tfrac{3}{4} + \left(5\tfrac{1}{2} + \frac{0.24 + 0.5\dot{3}}{2.2 - 0.6\dot{4}}\right) = 4\tfrac{1}{5} + 2\tfrac{1}{2} + 5\tfrac{1}{2} + \tfrac{87}{175}$$

$$= 11\tfrac{70+175+175+174}{350} = 11\tfrac{594}{350} = 12\tfrac{244}{350} = 12\tfrac{122}{175}. \textit{ Ans.}$$

41. Simplify 0.9 of $\frac{5}{8}$ of $\frac{4}{7}$ of $15\frac{3}{4}$.

$$0.9 \text{ of } \tfrac{5}{8} \text{ of } \tfrac{4}{7} \text{ of } 15\tfrac{3}{4} = \frac{9}{\underset{2}{\cancel{10}}} \times \frac{\cancel{5}}{\underset{2}{\cancel{8}}} \times \frac{\cancel{4}}{\cancel{7}} \times \frac{\overset{9}{\cancel{63}}}{4} = \frac{81}{16} = 5\tfrac{1}{16}. \textit{ Ans.}$$

42. What part of $\frac{2}{3}$ is $\frac{1}{2}$?

$$\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}. \quad \textit{Ans.}$$

43. What part of 0.390625 is 0.05?

$$\frac{0.05}{0.390625} = \frac{\cancel{5}}{\cancel{100}} \times \frac{\overset{16}{\overset{\cancel{10000}}{\cancel{1000000}}}}{\underset{125}{\underset{\cancel{78125}}{\cancel{390625}}}} = \frac{16}{125}. \quad \textit{Ans.}$$

44. What fraction of $0.20\dot{4}\dot{5}$ is $0.\dot{0}\dot{9}$?

$$\frac{0.\dot{0}\dot{9}}{0.20\dot{4}\dot{5}} = \frac{\frac{9}{99}}{\frac{2025}{9900}} = \frac{\cancel{9}}{\underset{\cancel{11}}{\cancel{99}}} \times \frac{\overset{4}{\overset{\cancel{900}}{\cancel{9900}}}}{\underset{9}{\cancel{2025}}} = \frac{4}{9}. \quad \textit{Ans.}$$

[redacted]

45. Reduce to decimals: $\frac{49}{67}$; $\frac{14}{37}$; $\frac{62}{73}$.

```
      0.731343283582089552238805970149253
67)49.0
   469
    210
    201
      90
      67
      230
      201
       290
       268
        220
        201
         190
         134
          560
          536
           240
           201
            390
            335
             550        540
             536        536
              140        400
              134        335
               600        650
               536        603
                640        470
                603        469
                 370        100
                 335         67
                  350        330
                  335        268
                   150        620
                   134        603
                    160        170
                    134        134
                     260        360
                     201        335
                      590        250
                      536        201
                       540        49
```

$\therefore \frac{49}{67} = 0.\dot{7}3134328358208955223880597014925\dot{3}$.

```
      0.84931506
73)62.0
   584
    360
    292
     680
     657
      230
      219
       110
        73
       370
       365
        500
        438
         62
```

$\therefore \frac{62}{73} = 0.\dot{8}493150\dot{6}$.

```
     0.378
37)14.0
   111
    290
    259
     310
     296
      14
```

$\therefore \frac{14}{37} = 0.\dot{3}7\dot{8}$.

[redacted]

46. The G. C. M. of three numbers is 15, and their L. C. M. is 450. What are the numbers?

The G. C. M. $= 15 = 3 \times 5$.

The L. C. M. $= 450 = 2 \times 3^2 \times 5^2 = (3 \times 5) \times 2 \times 3 \times 5$.

$\therefore$ the numbers are $15 \times 2 = 30$, $15 \times 3 = 45$, $15 \times 5 = 75$.

47. A merchant, after selling $5\frac{1}{6}$ yards and $3\frac{1}{2}$ yards from a remnant of calico, found that he had $7\frac{3}{5}$ yards left. What was the entire length of the remnant?

$$5\tfrac{1}{6} + 3\tfrac{1}{2} + 7\tfrac{3}{5} = 15\tfrac{5+15+18}{30} = 15\tfrac{38}{30} = 16\tfrac{8}{30} = 16\tfrac{4}{15}.$$

$16\frac{4}{15}$ yards. *Ans.*

48. If $3\frac{3}{4}$ yards of cloth are required for a coat, how many coats can be made from $56\frac{1}{4}$ yards of cloth?

$$56\tfrac{1}{4} \div 3\tfrac{3}{4} = \frac{\overset{15}{\cancel{225}}}{\cancel{4}} \times \frac{\cancel{4}}{\cancel{15}} = 15. \quad \textit{Ans.}$$

49. A grocer bought a hogshead of sugar weighing 744 pounds at $4\frac{7}{8}$ cents per pound, and sold it at $5\frac{1}{4}$ cents per pound. How much did he gain?

$$5\tfrac{1}{4} - 4\tfrac{7}{8} = \frac{10-7}{8} = \frac{3}{8}. \qquad \overset{93}{\cancel{744}} \times \frac{3}{\cancel{8}} \text{ cents} = 279 \text{ cents} = \$2.79. \quad \textit{Ans.}$$

50. A man, after selling $\frac{2}{9}$ of his field, sold $\frac{2}{7}$ of the remainder and then had $13\frac{1}{3}$ acres left. How many acres did he own at first?

$$1 - \frac{2}{9} = \frac{7}{9}; \quad \frac{2}{7} \text{ of } \frac{7}{9} = \frac{2}{9}; \quad \frac{2}{9} + \frac{2}{9} = \frac{4}{9}; \quad 1 - \frac{4}{9} = \frac{5}{9}.$$

$$13\tfrac{1}{3} \text{ acres} \div \frac{5}{9} = \frac{\overset{3}{\cancel{9}}}{\cancel{5}} \times \frac{\overset{8}{\cancel{40}}}{\cancel{3}} \text{ acres} = 24 \text{ acres}. \quad \textit{Ans.}$$

51. A railroad train passed over $\frac{7}{12}$ of its route in $3\frac{1}{2}$ hours. In how many hours would it pass over the entire route? In how many hours over $\frac{2}{5}$ of the route? $\frac{7}{8}$? $\frac{9}{14}$?

$$3\tfrac{1}{2} \div \frac{7}{12} = \frac{\cancel{7}}{\cancel{2}} \times \frac{\overset{6}{\cancel{12}}}{\cancel{7}} = 6. \quad \textit{Ans.} \qquad \frac{7}{\underset{4}{\cancel{8}}} \text{ of } \overset{3}{\cancel{6}} = \frac{21}{4} = 5\tfrac{1}{4}. \quad \textit{Ans.}$$

$$\frac{2}{5} \text{ of } 6 = \frac{12}{5} = 2\tfrac{2}{5}. \quad \textit{Ans.} \qquad \frac{9}{\underset{7}{\cancel{14}}} \text{ of } \overset{3}{\cancel{6}} = \frac{27}{7} = 3\tfrac{6}{7}. \quad \textit{Ans.}$$

52. A boy, being asked to find the value of [illegible] − [illegible] − [illegible] − [illegible], gave as his answer 30. How great was his error?

$$[illegible] = [illegible] = [illegible]$$

$$30 - [illegible] = [illegible] \quad \textit{Ans.}$$

53. The meter is equal to [illegible] feet, very nearly. Express in centimeters the value of 4[illegible] feet.

$$[illegible] = 1\tfrac{1}{4}. \qquad 1\tfrac{1}{4}^{m} = 125^{cm}. \quad \textit{Ans.}$$

54. For a piano cover a lady bought 2⅝ yards of plush at $3½ per yard, the same amount of lining flannel at $⅞ per yard, 1¼ yards of [illegible] at $1½ per yard, and 1⅛ yards of fringe at $1¼ per yard. If the making cost $5, what was the cost of the piano cover?

$$2\tfrac{5}{8} \times \$3\tfrac{1}{2} = \frac{21}{8} \times \$\frac{7}{2} = \$\frac{147}{16} = \$9\tfrac{3}{16} = \$9.19.$$

$$2\tfrac{5}{8} \times \$\frac{7}{8} = \frac{21}{8} \times \$\frac{7}{8} = \$\frac{147}{64} = \$2\tfrac{19}{64} = \$2.30.$$

$$1\tfrac{1}{4} \times \$1\tfrac{1}{2} = \frac{5}{4} \times \$\frac{3}{2} = \$\frac{15}{8} = \$1\tfrac{7}{8} = \$1.88.$$

$$1\tfrac{1}{8} \times \$1\tfrac{1}{4} = \frac{9}{8} \times \$\frac{5}{4} = \$\frac{45}{32} = \$1\tfrac{13}{32} = \$1.41.$$

$$\$9.19 + \$2.30 + \$1.88 + \$1.41 + \$5.00 = \$19.78. \quad \textit{Ans.}$$

55. A mason built [illegible] yards of wall on Monday, 4[illegible] yards on Tuesday, [illegible] yards on Wednesday, and 7[illegible] yards on Thursday. If he is paid $0.80 per yard, how much has he earned in the four days together?

$$[illegible] = 23\tfrac{23}{120}.$$

$$\$0.80 = \$\frac{4}{5}.$$

$$23\tfrac{23}{120} \times \$\frac{4}{5} = \frac{2783}{120} \times \$\frac{4}{5} = \$\frac{[illegible]}{[illegible]} = \$18.55. \quad \textit{Ans.}$$

[redacted]

56. A coal dealer sold 100 tons of coal. If he shipped by six cars $14\frac{1}{2}$, $14\frac{1}{10}$, $14\frac{3}{14}$, $14\frac{8}{35}$, $14\frac{7}{16}$, $14\frac{1}{70}$ tons respectively, how many tons must he load on the seventh car to complete his shipment?

$$14\tfrac{1}{2} + 14\tfrac{1}{10} + 14\tfrac{3}{14} + 14\tfrac{8}{35} + 14\tfrac{7}{16} + 14\tfrac{1}{70}$$
$$= 84\tfrac{280+56+120+128+245+8}{560}$$
$$= 84\tfrac{837}{560} = 85\tfrac{277}{560}. \qquad 100 - 85\tfrac{277}{560} = 14\tfrac{283}{560}. \text{ Ans.}$$

57. The moon's diameter is $\frac{3}{11}$ that of the earth, and the sun's diameter is 110 times that of the earth. What fraction of the sun's diameter is the moon's diameter?

$$\frac{3}{11} \div 110 = \frac{3}{11} \times \frac{1}{110} = \frac{3}{1210}. \text{ Ans.}$$

58. If a silver rupee in Calcutta is worth $\$\frac{12}{25}$, what is the value in dollars and cents of a fan costing $4\frac{7}{8}$ rupees?

$$4\tfrac{7}{8} \times \$\frac{12}{25} = \frac{39}{\overset{}{\cancel{8}}_{2}} \times \$\frac{\overset{3}{\cancel{12}}}{25} = \$\frac{117}{50} = \$2.34. \text{ Ans.}$$

59. If a man can do $\frac{2}{11}$ of a piece of work in 25 days, what fraction of the work can he do in $62\frac{6}{7}$ days?

$$25 \text{ days} \div \tfrac{2}{11} = \tfrac{11}{2} \times 25 \text{ days} = \tfrac{275}{2} \text{ days.}$$

$$62\tfrac{6}{7} \div \frac{275}{2} = \frac{\overset{8}{\cancel{440}}}{7} \times \frac{2}{\underset{5}{\cancel{275}}} = \frac{16}{35}. \text{ Ans.}$$

60. I paid a tailor $\$3\frac{1}{2}$ a yard for $5\frac{1}{4}$ yards of broadcloth. On measuring it, I found that there were only $4\frac{7}{8}$ yards. How much money ought the tailor to return?

$$5\tfrac{1}{4} - 4\tfrac{7}{8} = 1\tfrac{2-7}{8} = \frac{10-7}{8} = \frac{3}{8}.$$

$$\frac{3}{8} \text{ of } \$3\tfrac{1}{2} = \frac{3}{8} \times \$\frac{7}{2} = \$\frac{21}{16} = \$1.31. \text{ Ans.}$$

61. From a tank full of water $\frac{2}{3}$ of the water was drawn off. Then 35 gallons were added, and the tank was just half full. What is the capacity of the tank?

$$\frac{2}{3} - \frac{1}{2} = \frac{1}{6}.$$

$$35 \text{ gallons} \div \frac{1}{6} = 6 \times 35 \text{ gallons} = 210 \text{ gallons. Ans.}$$

62. What number exceeds the sum of its fourth, fifth, sixth, and seventh parts by 101?

$$\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}=\frac{105+84+70+60}{420}=\frac{319}{420}.$$

$$1-\frac{319}{420}=\frac{101}{420}.$$

$$101 \div \frac{101}{420}=\cancel{101} \times \frac{420}{\cancel{101}}=420. \quad \textit{Ans.}$$

63. A trader bought wheat at 75 cents a bushel, and sold it at 71 cents a bushel. How many cents did he lose on every dollar he paid?

He lost 75 cents − 71 cents = 4 cents on every 75 cents.

Therefore, he lost $\frac{4}{75}$.

$$\frac{4}{\underset{3}{\cancel{75}}} \text{ of } \overset{4}{\cancel{100}} \text{ cents} = \frac{16}{3} \text{ cents} = 5\tfrac{1}{3} \text{ cents. } \textit{Ans.}$$

64. How many bushels of potatoes at $\$\frac{2}{5}$ per bushel will pay for 16 bushels of wheat at $\$\frac{13}{20}$ per bushel?

$$\overset{4}{\cancel{16}} \times \$\frac{13}{\underset{5}{\cancel{20}}} = \$\frac{52}{5}.$$

$$\$\frac{52}{5} \div \$\frac{2}{5} = \frac{\overset{26}{\cancel{52}}}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{2}} = 26. \quad \textit{Ans.}$$

65. From a piece of calico containing $35\frac{7}{8}$ yards, there have been sold at different times $12\frac{3}{4}$ yards, $2\frac{1}{2}$ yards, $2\frac{3}{16}$ yards, and $8\frac{5}{8}$ yards. How many yards remain?

$$12\tfrac{3}{4}+2\tfrac{1}{2}+2\tfrac{3}{16}+8\tfrac{5}{8}=24\tfrac{12+8+3+10}{16}=24\tfrac{33}{16}=26\tfrac{1}{16}.$$

$$35\tfrac{7}{8}-26\tfrac{1}{16}=9\tfrac{14-1}{16}=9\tfrac{13}{16}. \quad \textit{Ans.}$$

66. If gun metal is composed of $90\frac{1}{2}$ parts of copper to $9\frac{1}{2}$ parts of tin by weight, how many ounces of tin are there in one pound (16 ounces) of gun metal? how many ounces of copper in one pound?

$$\frac{9\frac{1}{2}}{90\frac{1}{2}+9\frac{1}{2}}=\frac{9\frac{1}{2}}{100}=\frac{19}{200}.$$

$$\frac{19}{\underset{25}{\cancel{200}}} \times \overset{2}{\cancel{16}} \text{ ounces} = \frac{38}{25} \text{ ounces} = 1\tfrac{13}{25} \text{ ounces, tin. } \textit{Ans.}$$

16 ounces − $1\frac{13}{25}$ ounces = $14\frac{12}{25}$ ounces, copper. *Ans.*

[redacted]

67. One man mows $\frac{1}{3}$ of a field, a second $\frac{2}{7}$ of it, and a third $\frac{5}{21}$ of it. What fraction of the field remains to be mowed?

$$\frac{1}{3}+\frac{2}{7}+\frac{5}{21}=\frac{7+6+5}{21}=\frac{18}{21}=\frac{6}{7}.$$

$$1-\frac{6}{7}=\frac{1}{7}.\quad \textit{Ans.}$$

68. Bell metal by weight consists of 4 parts of copper to 1 part of tin. What is the cost of a bell weighing 12,400 pounds, if the copper costs 19 cents per pound, the tin $22\frac{1}{2}$ cents per pound, and the cost of making is \$500?

$$\frac{4}{1+4}=\frac{4}{5},\text{ copper.}\qquad \frac{1}{1+4}=\frac{1}{5},\text{ tin.}$$

$$\frac{4}{\cancel{5}}\times\overset{2480}{\cancel{12400}}\text{ pounds}=9920\text{ pounds, copper.}$$

$$\frac{1}{\cancel{5}}\times\overset{2480}{\cancel{12400}}\text{ pounds}=2480\text{ pounds, tin.}$$

$$\begin{array}{r}9920\\ \underline{0.19}\\ 89280\\ \underline{9920}\\ 1884.80\end{array}\qquad\begin{array}{r}2480\\ \underline{0.22\frac{1}{2}}\\ 1240\\ 4960\\ \underline{4960}\\ 558.00\end{array}\qquad\begin{array}{r}\$1884.80\\ 558.00\\ \underline{500.00}\\ \$2942.80\end{array}\ \textit{Ans.}$$

69. If an ore loses $\frac{17}{40}$ of its weight in roasting, and $\frac{8}{19}$ of the remainder in smelting, how many tons of ore must be mined to obtain 466 tons of pure metal?

$$1-\frac{17}{40}=\frac{23}{40}.\qquad \frac{\cancel{8}}{19}\times\frac{23}{\underset{5}{\cancel{40}}}=\frac{23}{95}.\qquad \frac{23}{40}-\frac{23}{95}=\frac{437-184}{760}=\frac{253}{760}.$$

$$466\div\frac{253}{760}=466\times\frac{760}{253}=\frac{354160}{253}=1399\tfrac{213}{253}.\quad \textit{Ans.}$$

70. The amount of starch in potatoes is $\frac{11}{50}$ of their weight, but the amount that can usually be extracted is only $\frac{2}{15}$. How many pounds of starch can be obtained from 100 pounds of potatoes, and how many pounds of starch will be left in the potatoes?

$$\frac{2}{\underset{3}{\cancel{15}}}\times\overset{20}{\cancel{100}}=\frac{40}{3}=13\tfrac{1}{3}.\quad \textit{Ans.}$$

$$\frac{11}{\cancel{50}}\times\overset{2}{\cancel{100}}=22.\qquad 22-13\tfrac{1}{3}=8\tfrac{2}{3}.\quad \textit{Ans.}$$

[redacted]

1000 ÷ 5/6 = 1200. 1200 × $0.22½ = $270.
$270 + $2.25 = $272.25, … 1000 × $0.30 = $300 …
$300 − $272.25 = $27.75, Ans.

71. How many pairs of trousers, each pair requiring $2\frac{3}{8}$ yards, can be made from $33\frac{1}{4}$ yards of cloth?

$$33\tfrac{1}{4} \div 2\tfrac{3}{8} = \frac{\overset{7}{\cancel{133}}}{\cancel{4}} \times \frac{\overset{2}{\cancel{8}}}{\cancel{19}} = 14. \quad \textit{Ans.}$$

72. If $3\frac{1}{2}$ yards of cloth are required for a shirt, how many shirts can be made from 12 pieces of cloth, each piece measuring $47\frac{1}{4}$ yards?

$$\frac{12 \times 47\frac{1}{4}}{3\frac{1}{2}} = \overset{3}{\cancel{12}} \times \frac{\overset{27}{\cancel{189}}}{\cancel{4}} \times \frac{2}{\cancel{7}} = 162. \quad \textit{Ans.}$$

73. Green coffee when roasted loses $\frac{1}{6}$ of its weight. If a dealer buys green coffee at $22\frac{1}{2}$ cents a pound, and sells it roasted at 30 cents a pound, what will be his gain in selling 1000 pounds of roasted coffee, the cost of roasting the whole quantity being $2.25?

$$1000 \times \$0.22\tfrac{1}{2} = \$225. \qquad \$225 + \$2.25 = \$227.25, \text{ cost.}$$

$$\frac{5}{\cancel{6}} \times 1000 \times \$\overset{0.05}{\cancel{0.30}} = \$250, \text{ selling price.}$$

$$\$250 - \$227.25 = \$22.75. \quad \textit{Ans.}$$

74. If an iron bar, when heated 1 degree, expands $\frac{1}{143460}$ of its length, what is the length at 212 degrees of a bar whose length at 32 degrees is $10\frac{5}{6}$ feet?

$$212 - 32 = 180. \qquad 180 \times \frac{1}{143460} \times 10\tfrac{5}{6} = \cancel{180} \times \frac{1}{\underset{797}{\cancel{143460}}} \times \frac{65}{6} = \frac{65}{4782}.$$

$$10\tfrac{5}{6} + \tfrac{65}{4782} = 10\tfrac{3985+65}{4782} = 10\tfrac{4050}{4782} = 10\tfrac{675}{797}. \qquad 10\tfrac{675}{797} \text{ feet. } \textit{Ans.}$$

75. If a horse eats $\frac{7}{16}$ of a ton of hay in 30 days, how long will $4\frac{9}{10}$ tons of hay last 5 horses?

$$1 \text{ horse in 1 day eats } \frac{1}{30} \times \frac{7}{16} \text{ tons} = \frac{7}{480} \text{ tons.}$$

$$5 \text{ horses in 1 day eat } \cancel{5} \times \frac{7}{\underset{96}{\cancel{480}}} \text{ tons} = \frac{7}{96} \text{ tons.}$$

$$4\tfrac{9}{10} \div \frac{7}{96} = \frac{\overset{7}{\cancel{49}}}{\underset{5}{\cancel{10}}} \times \frac{\overset{48}{\cancel{96}}}{\cancel{7}} = \frac{336}{5} = 67\tfrac{1}{5}. \qquad 67\tfrac{1}{5} \text{ days. } \textit{Ans.}$$

[redacted]

76. If 4 is added to both terms of the fraction $\frac{11}{16}$, by how much is the value of the fraction increased?

$$\frac{11+4}{16+4}=\frac{15}{20}=\frac{3}{4}.\qquad \frac{3}{4}-\frac{11}{16}=\frac{12-11}{16}=\frac{1}{16}.\ \textit{Ans.}$$

77. If 4 is subtracted from both terms of the fraction $\frac{11}{16}$, by how much is the value of the fraction decreased?

$$\frac{11-4}{16-4}=\frac{7}{12}.\qquad \frac{11}{16}-\frac{7}{12}=\frac{33-28}{48}=\frac{5}{48}.\ \textit{Ans.}$$

78. Find the least number of apples that arranged in groups of 8, 9, 10, or 12 will have just 6 over in each case.

The L. C. M. of 8, 9, 10, and 12 is 360.

$$\begin{array}{r|llll} 2 & 8 & 9 & 10 & 12 \\ \hline 2 & 4 & 9 & 5 & 6 \\ \hline & 2 & 9 & 5 & \cancel{3} \end{array}$$

$360 + 6 = 366.$ *Ans.*

79. The diameter of a bicycle wheel is $2\frac{1}{3}$ feet, and the circumference is $3\frac{1}{7}$ times the diameter. How many times does the wheel turn in going 1 mile (5280 feet)?

$$\frac{5280}{3\frac{1}{7}\times 2\frac{1}{3}}=\overset{240}{\cancel{5280}}\times\frac{\cancel{7}}{\cancel{22}}\times\frac{3}{\cancel{7}}=720.\ \textit{Ans.}$$

80. What is the least number of yards of carpet in a roll that can be cut into lengths of exactly $13\frac{1}{3}$ yards, 8 yards, or $11\frac{3}{7}$ yards?

$$13\tfrac{1}{3},\ 8,\ 11\tfrac{3}{7}=\frac{40}{3},\ \frac{8}{1},\ \frac{80}{7}.$$

The L. C. M. of 40, 8, 80 = 80.
The G. C. M. of 3, 1, 7 = 1.
∴ the L. C. M. of the fractions = 80.

80 yards. *Ans.*

81. What is the length of the longest chain that will exactly measure the sides of a field whose lengths are respectively $135\frac{1}{9}$ yards, $118\frac{2}{9}$ yards, 152 yards, and $202\frac{2}{3}$ yards?

$$135\tfrac{1}{9},\ 118\tfrac{2}{9},\ 152,\ 202\tfrac{2}{3}=\frac{1216}{9},\ \frac{1064}{9},\ \frac{152}{1},\ \frac{608}{3}.$$

The G. C. M. of 1216, 1064, 152, 608 = 152.
The L. C. M. of 9, 9, 1, 3 = 9.
∴ the G. C. M. of the fractions $= \frac{152}{9}$.

$\frac{152}{9}$ yards $= 16\frac{8}{9}$ yards. *Ans.*

82. Find the least multiplier of $\frac{7}{9}$, $\frac{14}{27}$, and $\frac{28}{45}$ that will make each product an integral number.

The least multiplier that will make $\frac{7}{9}$ an integer is $\frac{9}{7}$.

The least multiplier that will make $\frac{14}{27}$ an integer is $\frac{27}{14}$.

The least multiplier that will make $\frac{28}{45}$ an integer is $\frac{45}{28}$.

The L. C. M. of 9, 27, 45 $= 135$.

The G. C. M. of 7, 14, 28 $= 7$.

$\therefore$ the L. C. M. of the multipliers $= \frac{135}{7}$.

$\frac{135}{7} = 19\frac{2}{7}$. *Ans.*

83. Find the least integral number that is exactly divisible by $5\frac{1}{4}$, $3\frac{1}{2}$, and 7.

$$5\tfrac{1}{4},\ 3\tfrac{1}{2},\ 7 = \frac{21}{4},\ \frac{7}{2},\ \frac{7}{1}.$$

The L. C. M. of 21, 7, 7 $= 21$.

The G. C. M. of 4, 2, 1 $= 1$.

$\therefore$ the L. C. M. of the fractions $= 21$.

Since 21 is integral, 21 is the number required.

84. Four bells commence tolling together, and toll at intervals of 1, $1\frac{1}{8}$, $1\frac{1}{12}$, and $1\frac{3}{10}$ seconds, respectively. In how many seconds will all four toll again at the same instant?

$$1,\ 1\tfrac{1}{8},\ 1\tfrac{1}{12},\ 1\tfrac{3}{10} = \frac{1}{1},\ \frac{9}{8},\ \frac{13}{12},\ \frac{13}{10}.$$

The L. C. M. of 1, 9, 13, 13 $= 117$.

The G. C. M. of 1, 8, 12, 10 $= 1$.

$\therefore$ the L. C. M. of the fractions $= 117$.

117 seconds. *Ans.*

85. What number multiplied by $\frac{7}{11}$ of $\frac{9}{14}$ of $29\frac{1}{3}$ will give $102\frac{2}{3}$ for the product?

$$\frac{102\frac{2}{3}}{\frac{7}{11}\text{ of }\frac{9}{14}\text{ of }29\frac{1}{3}} = \frac{\overset{77}{\cancel{308}}}{\cancel{3}} \times \frac{\cancel{11}}{\cancel{7}} \times \frac{\overset{2}{\cancel{14}}}{9} \times \frac{\cancel{3}}{\underset{\underset{\cancel{4}}{\cancel{8}}}{\cancel{88}}} = \frac{77}{9} = 8\tfrac{5}{9}.\ \textit{Ans.}$$

[redacted]

86. How many miles an hour must a man walk to go 28 miles in $7\frac{7}{15}$ hours?

$$28 \div 7\tfrac{7}{15} = \cancel{28} \times \frac{15}{\underset{4}{\cancel{112}}} = \frac{15}{4} = 3\tfrac{3}{4}. \ \textit{Ans.}$$

87. If the rent of $5\frac{5}{12}$ acres of land is \$$21\frac{2}{3}$, what will be the rent of $19\frac{9}{16}$ acres at the same rate?

$$19\tfrac{9}{16} \times \$\frac{21\frac{2}{3}}{5\frac{5}{12}} = \frac{313}{\underset{4}{\cancel{16}}} \times \$\frac{\cancel{65}}{\cancel{3}} \times \frac{\overset{4}{\cancel{12}}}{\cancel{65}} = \$\frac{313}{4} = \$78\tfrac{1}{4}. \ \textit{Ans.}$$

88. If the English acre is $\frac{31\frac{1}{4}}{49}$ of an Irish acre, how many English acres are there in $218\frac{3}{4}$ Irish acres?

$$218\tfrac{3}{4} \div \frac{31\frac{1}{4}}{49} = \frac{\overset{7}{\cancel{875}}}{\cancel{4}} \times \frac{\cancel{4}}{\cancel{125}} \times 49 = 343. \ \textit{Ans.}$$

89. Resolve the denominator of $\frac{25}{48}$ into its prime factors; from the result state the number of figures the equivalent decimal will have, and the number that will precede the repetend.

$$48 = 2^4 \times 3.$$

Since the highest power of 2 or 5 in the denominator is the fourth, the repetend will be preceded by 4 figures. Since the only factor of the denominator besides 2 and 5 is 3, the repetend will consist of 1 figure. Therefore, the decimal will contain 5 figures.

90. Find the greatest common measure of 9083, 9207, 8897.

```
9083            3³ | 9207            7)8897
                11 |  341               1271
                   ----
                       31
  293                                  41
31)9083                             31)1271
   62                                  124
   ---                                 ---
   288                                  31
   279                                  31
   ---                                  --
    93
    93
    --
```

∴ the G. C. M. = 31. *Ans.*

[redacted]

Exercise 70. Page 149.

1. Reduce 3 pk. 5 qt. 1 pt. to pints.

pk.	qt.	pt.
3	5	1
8		
29		
2		
59		

59 pt. *Ans.*

2. Reduce 4234 pt. (dry measure) to higher units.

```
2 | 4234 pt.
8 | 2117 qt.
4 |  264 pk. . . . 5 qt.
      66 bu.
```

66 bu. 5 qt. *Ans.*

3. Reduce 24 gal. 2 qt. 1 pt. 2 gi. to gills.

gal.	qt.	pt.	gi.
24	2	1	2
4			
98			
2			
197			
4			
790			

790 gi. *Ans.*

4. Reduce 3047 gills to higher units.

```
4 | 3047 gi.
2 |  761 pt. . . . 3 gi.
4 |  380 qt. . . . 1 pt.
      95 gal.
```

95 gal. 1 pt. 3 gi. *Ans.*

5. Reduce 1715½ bu. to pints.

```
  1715½
     64
  -----
     32
   6860
  10290
 ------
 109792
```

109,792 pt. *Ans.*

6. Reduce 508 dry quarts to higher units.

```
8 | 508 qt.
4 |  63 pk. . . . 4 qt.
     15 bu. . . . 3 pk.
```

15 bu. 3 pk. 4 qt. *Ans.*

7. Reduce 1016 liquid pints to higher units.

```
2 | 1016 pt.
4 |  508 qt.
     127 gal.
```

127 gal. *Ans.*

8. Reduce 44 gal. 3 qt. 1 pt. to pints.

gal.	qt.	pt.
44	3	1
4		
179		
2		
359		

359 pt. *Ans.*

[redacted]

9. Reduce 44 bu. 3 pk. 7 qt. 1 pt. to pints.

bu.	pk.	qt.	pt.
44	3	7	1
4			
179			
8			
1439			
2			
2879			

2879 pt. *Ans.*

10. Reduce 272 liquid quarts to dry quarts.

$$272 \times \frac{57\frac{3}{4}}{67\frac{1}{5}} = \cancelto{17}{272} \times \frac{\cancelto{11}{231}}{4} \times \frac{5}{\cancelto{16}{336}}$$

$$= \frac{935}{4} = 233\tfrac{3}{4}.$$

$233\frac{3}{4}$ qt. *Ans.*

11. Reduce 429 dry quarts to liquid quarts.

$$429 \times \frac{67\frac{1}{5}}{57\frac{3}{4}} = \cancelto{39}{429} \times \frac{\cancelto{16}{336}}{5} \times \frac{4}{\cancelto{11}{231}} = \frac{2496}{5} = 499\tfrac{1}{5}.$$

$499\frac{1}{5}$ qt. *Ans.*

Exercise 71. Page 150.

1. Add 5 bu. 3 pk. 6 qt. 1 pt.; 6 bu. 2 pk. 7 qt.; 7 bu. 1 pk. 1 qt. 1 pt.; 1 pk. 7 qt.; 2 bu. 3 pk. 1 pt.

bu.	pk.	qt.	pt.
5	3	6	1
6	2	7	0
7	1	1	1
0	1	7	0
2	3	0	1
23	0	6	1

23 bu. 6 qt. 1 pt. *Ans.*

2. Add 50 gal. 3 qt. 1 pt. 3 gi.; 12 gal. 1 qt. 1 pt. 1 gi.; 5 gal. 2 qt. 1 pt. 2 gi.; 75 gal. 3 qt. 1 pt. 3 gi.; 80 gal. 3 qt. 1 gi.; 17 gal. 1 qt. 1 pt. 3 gi.

gal.	qt.	pt.	gi.
50	3	1	3
12	1	1	1
5	2	1	2
75	3	1	3
80	3	0	1
17	1	1	3
243	1	0	1

243 gal. 1 qt. 1 gi. *Ans.*

3. Add 4 gal. 3 qt. 1 pt.; 3 gal. 2 qt. $1\frac{1}{2}$ pt.; 12 gal. 3 qt.; 14 gal. $1\frac{1}{2}$ pt.; 5 gal. 2 qt. 1 pt.

gal.	qt.	pt.
4	3	1
3	2	$1\frac{1}{2}$
12	3	0
14	0	$1\frac{1}{2}$
5	2	1
41	0	1

41 gal. 1 pt. *Ans.*

4. Subtract 5 bu. 1 pk. 6 qt. 1 pt. from 5 bu. 3 pk. 3 qt.

bu.	pk.	qt.	pt.
5	3	3	0
5	1	6	1
	1	4	1

1 pk. 4 qt. 1 pt. *Ans.*

[redacted]

5 Subtract 2 gal. ... qt. ... pt. from ... gal. ... qt.

gal.	qt.	pt.
...	2	0
2	...	1
...	...	1

... gal. 2 qt. 1 pt. *Ans.*

7 Find the difference between 2 bu. ... and ... qt. 1 pt.

bu.	pk.	qt.	pt.
2	0	0	0
...	...	...	...
1	...	2	...

1 bu. ... pk. 2 qt. 1 pt. *Ans.*

6 Add ... bu. ... pk. 7 ... 2 ... 2 ... pk. 0 ... 1 ... 4 ... 2 ... 4 ... 10 ... 5 ... pk. 2 ...

bu.	pk.	qt.	pt.
...	...	...	...
2	2		
10	...		...
13	2	...	...
16	1	...	
5	...	2	...
50	2	...	...

50 bu. 2 pk. 7 qt. 1 pt. *Ans.*

Exercise 72 Page 151

1 Multiply 19 gal. 3 qt. 1 pt. by 70.

gal.	qt.	pt.
19	3	1
		70
13...	...	0

1384 gal. 1 qt. *Ans.*

2 M... 2 pk. 6 qt. ...

... *Ans.*

... *Ans.*

4 Multiply 26 gal. 2 qt. 1 pt. 3 gi. by 12.

gal.	qt.	pt.	gi.
26	2	1	...
			12
320	2	...	...

320 gal. 2 qt. 1 pt. *Ans.*

5 Multiply 12 bu. 3 pk. 7 qt. 1 pt. by 2...

bu.	pk.	qt.	pt.
12	3	7	...
			...
324	2	3	...

324 bu. 2 pk. 3 qt. 1 pt. *Ans.*

6 Divide 34 gal. 3 qt. 1 pt. ... by 7

gal.	qt.	pt.	gi.
...	...	0	...
...	...	...	

... gal. ... qt. 1 pt. ... gi. *Ans.*

[redacted]

7. Divide 147 gal. 2 qt. 1 pt. 2 gi. by 17.

	gal.	qt.	pt.	gi.
17	147	2	1	2
	8	2	1	2

8 gal. 2 qt. 1 pt. 2 gi. *Ans.*

8. Divide 54 bu. 3 pk. 2 qt. 1 pt. by 11.

	bu.	pk.	qt.	pt.
11	54	3	2	1
	4	3	7	1

4 bu. 3 pk. 7 qt. 1 pt. *Ans.*

9. Divide 34 bu. 3 pk. 5 qt. 1 pt. by 15.

	bu.	pk.	qt.	pt.
15	34	3	5	1
	2	1	2	1

2 bu. 1 pk. 2 qt. 1 pt. *Ans.*

Exercise 73. Page 152.

1. Reduce 27,587 gr. to higher troy units.

24	27587 gr.		
20	1149 dwt.	. . .	11 gr.
12	57 oz.	. . .	9 dwt.
	4 lb.	. . .	9 oz.

4 lb. 9 oz. 9 dwt. 11 gr. *Ans.*

2. Reduce 34,652 pounds avoirdupois to long tons, etc.

112	34652 lb.		
20	309 l. cwt.	. . .	44 lb.
	15 l. t.	. . . .	9 l. cwt.

15 l. t. 9 l. cwt. 44 lb. *Ans.*

3. Reduce 136,851 ounces avoirdupois to higher units.

16	136851 oz.		
100	8553 lb.	. . . . .	3 oz.
20	85 cwt.	. . .	53 lb.
	4 t.	. . . .	5 cwt.

4 t. 5 cwt. 53 lb. 3 oz. *Ans.*

4. Reduce 864,205 gr. to higher troy units.

24	864205 gr.		
20	36008 dwt.	. . .	13 gr.
12	1800 oz.	. . . . .	8 dwt.
	150 lb.		

150 lb. 8 dwt. 13 gr. *Ans.*

5. Reduce 864,205 gr. to higher avoirdupois units.

$$864{,}205 \text{ gr.} = \frac{864205}{7000} \text{ lb.}$$
$$= 123\tfrac{641}{1400} \text{ lb.}$$
$$= 123 \text{ lb. } 7\tfrac{57}{175} \text{ oz. } \textit{Ans.}$$

6. Reduce 5 lb. 7 oz. 6 dwt. 12 gr. to grains.

lb.	oz.	dwt.	gr.
5	7	6	12
12			
67			
20			
1346			
24			
32316			

32,316 gr. Ans.

[redacted]

$613\frac{1}{35}$ lb. Ans.

7. Reduce 745 lb. avoirdupois to troy measures.

745 lb. avoir. = 745 × 7000 gr.
= 5,215,000 gr.

24	5215000 gr.		
20	217291 dwt.	. . .	16 gr.
12	10864 oz.	. . . . .	11 dwt.
	905 lb.	. . . .	4 oz.

905 lb. 4 oz. 11 dwt. 16 gr. *Ans.*

8. Reduce 745 lb. troy to avoirdupois measures.

$745 \text{ lb. troy} = \overset{149}{\cancel{745}} \times \dfrac{\overset{144}{\cancel{5760}}}{\underset{\underset{37}{\cancel{175}}}{\cancel{7000}}} \text{ lb. avoir.}$

5

= ~~$579\frac{11}{37}$ lb.~~

~~= 579 lb. $14\frac{10}{37}$ oz. Ans.~~

9. Reduce 1,440,445 oz. avoirdupois to higher units.

16	1440445 oz.		
100	90027 lb.	. . . . .	13 oz.
20	900 cwt.	. . . .	27 lb.
	45 t.		

45 t. 27 lb. 13 oz. *Ans.*

10. Reduce 5,640,773 oz. avoirdupois to higher units.

16	5640773 oz.		
100	352548 lb.	. . . . .	5 oz.
20	3525 cwt.	. . . .	48 lb.
	176 t.	. . . . . .	5 cwt.

176 t. 5 cwt. 48 lb. 5 oz. *Ans.*

11. Add 48 t. 13 cwt. 75 lb. 6 oz.; 25 t. 12 cwt. 27 lb. 8 oz.; 51 t. 10 cwt. 44 lb.; 80 t. 5 cwt. 6 oz.; 19 cwt. 27 lb.; 25 lb. 8 oz.; 5 t. 5 cwt. 5 lb.

t.	cwt.	lb.	oz.
48	13	75	6
25	12	27	8
51	10	44	0
80	5	0	6
	19	27	0
		25	8
5	5	5	0
212	6	4	12

212 t. 6 cwt. 4 lb. 12 oz. *Ans.*

12. Add 13 lb. 4 oz. 8 dwt. 6 gr.; 25 lb. 8 oz. 13 dwt. 20 gr.; 8 lb. 11 oz. 14 gr.; 20 lb. 16 dwt. 8 gr.; 15 lb. 9 oz. 12 dwt.; 4 oz. 3 dwt.

lb.	oz.	dwt.	gr.
13	4	8	6
25	8	13	20
8	11	0	14
20	0	16	8
15	9	12	0
	4	3	0
84	2	14	0

84 lb. 2 oz. 14 dwt. *Ans.*

13. Subtract 23 lb. 8 oz. 19 dwt. 10 gr. from 58 lb. 6 oz. 17 dwt. 21 gr.

lb.	oz.	dwt.	gr.
58	6	17	21
23	8	19	10
34	9	18	11

34 lb. 9 oz. 18 dwt. 11 gr. *Ans.*

14. Subtract 17 t. 7 cwt. 17 lb. 6 oz. from 25 t. 13 cwt. 15 lb. 12 oz.

t.	cwt.	lb.	oz.
25	13	15	12
17	7	17	6
8	5	98	6

8 t. 5 cwt. 98 lb. 6 oz. *Ans.*

[redacted]

15. Multiply 3 lb. 4 oz. 8 dwt. 10 gr. by 10.

lb.	oz.	dwt.	gr.
3	4	8	10
			10
33	8	4	4

33 lb. 8 oz. 4 dwt. 4 gr. *Ans.*

16. Multiply 5 t. 10 cwt. 67 lb. 4 oz. by 15.

t.	cwt.	lb.	oz.
5	10	67	4
			15
83	0	8	12

83 t. 8 lb. 12 oz. *Ans.*

17. Divide 17 t. 19 cwt. 79 lb. 8 oz. by 8.

	t.	cwt.	lb.	oz.
8	17	19	79	8
	2	4	97	7

2 t. 4 cwt. 97 lb. 7 oz. *Ans.*

18. Divide 60 lb. 6 oz. 10 dwt. 20 gr. by 7.

	lb.	oz.	dwt.	gr.
7	60	6	10	20
	8	7	15	20

8 lb. 7 oz. 15 dwt. 20 gr. *Ans.*

19. How many bags each holding 2 bu. 1 pk. 3 qt. are required to hold 234 bu. 1 pk. 4 qt. of corn?

bu.	pk.	qt.	bu.	pk.	qt.
2	1	3	234	1	4
4			4		
9			937		
8			8		
75			7500		

$7500 \div 75 = 100.$ *Ans.*

20. What is the value at $4\frac{1}{2}$ cents a pound of a calf weighing 184 lb. 6 oz.?

184 lb. 6 oz. $= 184\frac{6}{16}$ lb. $= 184\frac{3}{8}$ lb.

$184\frac{3}{8} \times \$0.045 = 0.045 \times \$184\frac{3}{8}$.

$$\begin{array}{r} \$184\frac{3}{8} \\ 0.045 \\ \hline 16\frac{7}{8} \\ 920 \\ 736 \\ \hline \$8.296\frac{7}{8} \end{array}$$

\$8.30. *Ans.*

21. How many tablespoons each weighing 2 oz. 17 dwt. 12 gr. can be made from 155 oz. 5 dwt. of silver?

oz.	dwt.	gr.	oz.	dwt.
2	17	12	155	5
20			20	
57			3105	
24			24	
1380			74520	

$$\begin{array}{r} 54 \\ 138\cancel{0})\overline{7452\cancel{0}} \\ 690 \\ \hline 552 \\ 552 \\ \hline \end{array}$$

54. *Ans.*

[redacted]

Exercise 74. Page 155.

1. Reduce 3 yd. 2 ft. to inches.

yd.	ft.
3	2
3	
11	
12	
132	132 in. *Ans.*

2. Reduce 4 mi. 124 rd. 3 yd. 2 ft. to feet.

mi.	rd.	yd.	ft.
4	124	3	2
320			
1404			
5½			
7725			
3			
23177		23,177 ft. *Ans.*	

3. Reduce 27 rd. 4 yd. 9 in. to inches.

rd.	yd.	ft.	in.
27	4	0	9
5½			
152½			
3			
457½			
12			
5499		5499 in. *Ans.*	

4. Reduce 290 leagues to feet.

leagues.	knots.	ft.
290	0	0
3		
870		
6086		
5294820	5,294,820 ft. *Ans.*	

5. Reduce 82,976,432 in. to higher units.

12	82976432	in.	
3	6914702	ft.	8 in.
5½	2304900	yd. . . .	2 ft.
	2		
11	4609800	half yd.	[=4 yd.
320	419072	rd. . . .	8 half yd.
	1309	mi. . . . 192 rd.	

1309 mi. 192 rd. 4 yd. 2 ft. 8 in. *Ans.*

6. Reduce 7 mi. 3 yd. 1 ft. 6 in. to inches.

mi.	rd.	yd.	ft.	in.
7	0	3	1	6
320				
2240				
5½				
12323				
3				
36970				
12				
443646	443,646 in. *Ans.*			

7. Reduce 22 mi. 222 rd. 4 ft. 8 in. to inches.

mi.	rd.	ft.	in.
22	222	4	8
320			
7262			
16½			
119827			
12			
1437932	1,437,932 in. *Ans.*		

[redacted]

8. Reduce 712 mi. to feet.

mi.
712
5280
3759360
3,759,360 ft. *Ans.*

9. Reduce 540,451 ft. to higher units.

3 | 540451 ft.
$5\frac{1}{2}$ | 180150 yd. . . . . 1 ft.
2
11 | 360300 half yd. [= 3 yd.
320 | 32754 rd. . . . . 6 half yd.
102 mi. . . . 114 rd.

102 mi. 114 rd. 3 yd. 1 ft. *Ans.*

10. Reduce 271,256 in. to higher units.

12 | 271256 in.
3 | 22604 ft. . . . . 8 in.
$5\frac{1}{2}$ | 7534 yd. . . . . 2 ft.
2
11 | 15068 half yd. [= $4\frac{1}{2}$ yd.
320 | 1369 rd. . . . . 9 half yd.
4 mi. . . . 89 rd.

mi.	rd.	yd.	ft.	in.
4	89	$4\cancel{\frac{1}{2}}$	2	8
			1	6
4	89	5	1	2

4 mi. 89 rd. 5 yd. 1 ft. 2 in. *Ans.*

11. Reduce 723,964 ft. to higher units.

3 | 723964 ft.
$5\frac{1}{2}$ | 241321 yd. . . . . 1 ft.
2
11 | 482642 half yd. [= 3 yd.
320 | 43876 rd. . . . . 6 half yd.
137 mi. . . . 36 rd.

137 mi. 36 rd. 3 yd. 1 ft. *Ans.*

12. Reduce 233,205 in. to higher units.

12 | 233205 in.
3 | 19433 ft. . . . 9 in.
$5\frac{1}{2}$ | 6477 yd. . . . 2 ft.
2
11 | 12954 half yd. [= $3\frac{1}{2}$ yd.
320 | 1177 rd. . . . 7 half yd.
3 mi. . . . 217 rd.

mi.	rd.	yd.	ft.	in.
3	217	$3\cancel{\frac{1}{2}}$	2	9
			1	6
3	217	4	1	3

3 mi. 217 rd. 4 yd. 1 ft. 3 in. *Ans.*

13. How many feet high is a horse 16 hands high?

1 hand = 4 in. = $\frac{1}{3}$ ft.

$16 \times \frac{1}{3}$ ft. = $\frac{16}{3}$ ft. = $5\frac{1}{3}$ ft. *Ans.*

14. Add 6 mi. 120 rd. 3 yd. 2 ft. 2 in.; 18 mi. 15 rd. 1 yd. 1 ft. 6 in.; 3 mi. 215 rd. 2 yd. 2 ft. 3 in.; 7 mi. 95 rd. 1 yd. 1 ft. 8 in.

mi.	rd.	yd.	ft.	in.
6	120	3	2	2
18	15	1	1	6
3	215	2	2	3
7	95	1	1	8
35	126	$3\cancel{\frac{1}{2}}$	1	7
			1	6
35	126	4	0	1

35 mi. 126 rd. 4 yd. 1 in. Ans.

15. Subtract 3 mi. 217 rd. 4 yd. 1 ft. 3 in. from 4 mi. 100 rd. 3 yd. 2 in.

mi.	rd.	yd.	ft.	in.
4	100	3	0	2
3	217	4	1	3
	202	$3\cancel{4}$	1	11
			1	6
	202	4	0	5

202 rd. 4 yd. 5 in. *Ans.*

16. Multiply 5 mi. 126 rd. 9 ft. 6 in. by 7125.

mi.	rd.	ft.	in.
5	126	9	6
			7125
38443	92	4	6

38,443 mi. 92 rd. 4 ft. 6 in. *Ans.*

17. Divide 54 mi. 124 rd. 1 yd. 2 ft. 6 in. by 33.

	mi.	rd.	yd.	ft.	in.
33	54	124	1	2	6
	1	207	2	0	8

1 mi. 207 rd. 2 yd. 8 in. *Ans.*

18. If a man builds 1 rd. 1 yd. 1 ft. 6 in. of stone wall in one day, how much will he build in 26 days?

rd.	yd.	ft.	in.
1	1	1	6
			26
33	$\cancel{1}$	0	0
		1	6
33	0	1	6

33 rd. 1 ft. 6 in. *Ans.*

19. A man builds 25 rd. 2 yd. 1 ft. 6 in. of wall in 20 days. How much does he build per day?

	rd.	yd.	ft.	in.
20	25	2	1	6
	1	1	1	6

1 rd. 1 yd. 1 ft. 6 in. *Ans.*

Exercise 75. Page 156.

1. Reduce 92,638 sq. yd. to square inches.

$$\begin{array}{r} \text{sq. yd.} \\ 92638 \\ 9 \\ \hline 833742 \\ 144 \\ \hline 120058848 \end{array}$$

120,058,848 sq. in. *Ans.*

[redacted]

2. Reduce 1,223,527 sq. in. to higher units.

144 | 1223527 sq. in.
9 | 8496 sq. ft. . . . 103 sq. in.
$30\frac{1}{4}$ | 944 sq. yd.
4
121 | 3776 quarter sq. yd.
31 sq. rd. . . . 25 quarter sq. yd. = $6\frac{1}{4}$ sq. yd.

sq. rd.	sq. yd.	sq. ft.	sq. in.
31	$6\cancel{\frac{1}{4}}$	0	103
		2	36
31	6	2	139

31 sq. rd. 6 sq. yd. 2 sq. ft. 139 sq. in. *Ans*

3. Reduce 721 sq. mi. to square rods.

sq. mi.
721
640
461440
160
73830400 — 73,830,400 sq. rd. *Ans.*

4. Reduce 34,729 sq. yd. to higher units.

$30\frac{1}{4}$ | 34729 sq. yd.
4
121 | 138916 quarter sq. yd.
160 | 1148 sq. rd. . . . 8 quarter sq. yd. = 2 sq. yd.
7 A. . . . . 28 sq. rd.

7 A. 28 sq. rd. 2 sq. yd. *Ans.*

5. Reduce to square inches 3 A. 107 sq. rd. 27 sq. yd. 7 sq. ft. 23 sq. in.

A.	sq. rd.	sq. yd.	sq. ft.	sq. in.
3	107	27	7	23

160
587
$30\frac{1}{4}$
$17783\frac{3}{4}$
9
$160060\frac{3}{4}$
144
23048771 — 23,048,771 sq. in. Ans.

[redacted]

6. Reduce 99,894,712 sq. in. to higher units.

144 | 99894712 sq. in.
9 | 693713 sq. ft. . . . 40 sq. in.
$30\frac{1}{4}$ | 77079 sq yd. . . . 2 sq. ft.
4
121 | 308316 quarter sq. yd.
160 | 2548 sq. rd. . . . 8 quarter sq. yd. = 2 sq. yd.
15 A. . . . . 148 sq. rd.

15 A. 148 sq. rd. 2 sq. yd. 2 sq. ft. 40 sq. in. *Ans.*

7. Reduce 15,376 sq. yd. to higher units.

$30\frac{1}{4}$ | 15376
4
121 | 61504 quarter sq. yd.
160 | 508 sq. rd. . . . 36 quarter sq. yd. = 9 sq. yd.
3 A. . . . . . 28 sq. rd.

3 A. 28 sq. rd. 9 sq. yd. *Ans.*

8. Reduce 562,934 sq. in. to higher units.

144 | 562934 sq. in.
9 | 3909 sq. ft. . . . . 38 sq. in.
$30\frac{1}{4}$ | 434 sq. yd. . . . 3 sq. ft.
4
121 | 1736 quarter sq. yd.
14 sq. rd. . . . 42 quarter sq. yd. = $10\frac{1}{2}$ sq. yd.

sq. rd.	sq. yd.	sq. ft.	sq. in.
14	$10\cancel{\frac{1}{2}}$	3	38
		4	72
14	10	7	110

14 sq. rd. 10 sq. yd. 7 sq. ft. 110 sq. in. *Ans.*

9. Add 74 A. 21 sq. rd. 5 sq. yd. 4 sq. ft. 100 sq. in.; 123 A. 23 sq. rd. 13 sq. yd. 5 sq. ft. 83 sq. in.; 112 A. 106 sq. rd. 17 sq. yd. 8 sq. ft. 7 sq. in.; 541 A. 50 sq. rd. 23 sq. yd. 24 sq. in.

sq. mi.	A.	sq. rd.	sq. yd.	sq. ft.	sq. in.
	74	21	5	4	100
	123	23	13	5	83
	112	106	17	8	7
	541	50	23	0	24
1	211	41	$29\cancel{\frac{1}{4}}$	0	70
				6	108
1	211	41	29	7	34

1 sq. mi. 211 A. 41 sq. rd. 29 sq. yd. 7 sq. ft. 34 sq. in. *Ans.*

[redacted]

10. From 20 A. take 13 A. 150 sq. rd. 98 sq. ft. 10 sq. in.

A.	sq. rd.	sq. ft.	sq. in.
20	0	0	0
13	150	98	10
6	9	$173\cancel{\frac{1}{4}}$	134
			36
6	9	174	26

6 A. 9 sq. rd. 174 sq. ft. 26 sq. in. *Ans.*

11. Multiply 27 A. 76 sq. rd. 22 sq. yd. 5 sq. ft. by 90.

sq. mi.	A.	sq. rd.	sq. yd.	sq. ft.	sq. in.
	27	76	22	5	
				90	
3	553	27	$3\cancel{\frac{1}{4}}$	0	
				2	36
3	553	27	3	2	36

3 sq. mi. 553 A. 27 sq. rd. 3 sq. yd. 2 sq. ft. 36 sq. in. *Ans.*

12. Divide 74,128 sq. mi. 517 A. 80 sq. rd. by 10,000.

	sq. mi.	A.	sq. rd.
10000	74128	517	80
	7	264	39

7 sq. mi. 264 A. 39 sq. rd. *Ans.*

Exercise 76. Page 157.

1. Reduce 10 ch. to inches.

1 ch. = 100 × 7.92 in. = 792 in.
10 ch. = 10 × 792 in. = 7920 in. *Ans.*

2. Reduce 3168 in. to chains.

	4
792	3168
	3168

4 ch. *Ans.*

3. How many acres are there in a township?

1 tp. = 36 sq. mi. = 36 × 640 A.
= 23,040 A. *Ans.*

$$\begin{array}{r} 640 \\ 36 \\ \hline 3840 \\ 1920 \\ \hline 23040 \end{array}$$

4. Reduce 6400 sq. ch. to acres; to square miles.

10	6400 sq. ch.
640	640 A.
	1 sq. mi.

640 A.; 1 sq. mi. Ans.

5. Reduce 82,426 sq. ch. to higher units.

10	82426 sq. ch.	
640	8242 A.	6 sq. ch.
	12 sq. mi. . . . 562 A.	

12 sq. mi. 562 A. 6 sq. ch. *Ans.*

6. Add 4 sq. mi. 412 A. 6 sq. ch. 8 sq. rd.; 7 sq. mi. 88 A. 2 sq. ch. 11 sq. rd.; 3 sq. mi. 367 A. 7 sq. ch. 2 sq. rd.; 11 sq. mi. 344 A. 9 sq. ch. 15 sq. rd.

sq. mi.	A.	sq. ch.	sq. rd.
4	412	6	8
7	88	2	11
3	367	7	2
11	344	9	15
26	573	6	4

26 sq. mi. 573 A. 6 sq. ch. 4 sq. rd. *Ans.*

7. Subtract 1 mi. 75 ch. 85 l. from 4 mi. 44 ch. 38 l.

mi.	ch.	l.
4	44	38
1	75	85
2	48	53

2 mi. 48 ch. 53 l. *Ans.*

8. What is the area of a field if it can be divided into 12 lots each containing 2 sq. ch. 7 sq. rd.?

A.	sq. ch.	sq. rd.
	2	7
		12
2	9	4

2 A. 9 sq. ch. 4 sq. rd. *Ans.*

9. Multiply 3 sq. mi. 172 A. 5 sq. ch. 7 sq. rd. by 11.

sq. mi.	A.	sq. ch.	sq. rd.
3	172	5	7
			11
35	617	9	13

35 sq. mi. 617 A. 9 sq. ch. 13 sq. rd. *Ans.*

10. Divide 6 sq. mi. 422 A. 2 sq. ch. 13 sq. rd. by 5.

	sq. mi.	A.	sq. ch.	sq. rd.
5	6	422	2	13
	1	212	4	9

1 sq. mi. 212 A. 4 sq. ch. 9 sq. rd. *Ans.*

11. A field is divided into 47 gardens each containing 1 sq. ch. 9 sq. rd. What is the area of the field?

A.	sq. ch.	sq. rd.
	1	9
		47
7	3	7

7 A. 3 sq. ch. 7 sq. rd. *Ans.*

12. A field containing 5 A. 4 sq. ch. 11 sq. rd. is divided into 25 equal lots. What is the area of each lot?

	A.	sq. ch.	sq. rd.
25	5	4	11
		2	3

2 sq. ch. 3 sq. rd. *Ans.*

[redacted]

13. Find the rent of 8 sq. ch. 10 sq. rd. at $2 an acre.

sq. ch.	sq. rd.
8	10
16	
138	

8 sq. ch. 10 sq. rd. = $\frac{138}{160}$ A.

$$\frac{\cancel{138}^{\,69}}{\cancel{160}_{\,\cancel{80}\ 40}} \times \$\cancel{2} = \$\frac{69}{40} = \$1.73.\ \textit{Ans.}$$

14. If a field contains 3 A. 6 sq. ch. 12 sq. rd., what is it worth at 14 cents a square foot?

A.	sq. ch.	sq. rd.
3	6	12
10		
36		160083
16		0.14
588		640332
272¼		160083
160083		22411.62

$22,411.62. *Ans.*

Exercise 77. Page 158.

1. Reduce 25 cu. yd. 5 cu. ft. 143 cu. in. to cubic inches.

cu. yd.	cu. ft.	cu. in.
25	5	143
27		
680		
1728		
1175183		

1,175,183 cu. in. *Ans.*

2. Reduce 921,730 cu. in. to higher units.

1728	921730 cu. in.	
27	533 cu. ft. . . .	706 cu. in.
	19 cu. yd. . . .	20 cu. ft.

19 cu. yd. 20 cu. ft. 706 cu. in. *Ans.*

3. Wood cut in lengths of 4 ft. is piled 3½ ft. high. How long must the pile be to contain 2 cords?

$$\frac{2\times 128}{4\times 3\frac{1}{2}} = 2\times \cancel{128}^{\,32}\times\frac{1}{\cancel{4}}\times\frac{2}{7}$$

$$= \frac{128}{7} = 18\tfrac{2}{7}.$$

18$\frac{2}{7}$ ft. *Ans.*

4. How many cords in a pile of 4-ft. wood 43 ft. long and 6 ft. high?

$$\frac{\cancel{4}\times 43\times \cancel{6}^{\,3}}{\cancel{128}_{\,\cancel{32}\ 16}} = \frac{129}{16} = 8\tfrac{1}{16}.$$

8$\frac{1}{16}$ cd. *Ans.*

5. Add 130 cu. yd. 5 cu. ft. 820 cu. in.; 56 cu. yd. 20 cu. ft. 304 cu. in.; 37 cu. yd. 4 cu. ft. 86 cu. in.; 8 cu. yd. 10 cu. ft. 129 cu. in.; 12 cu. yd. 19 cu. ft. 175 cu. in.

cu. yd.	cu. ft.	cu. in.
130	5	820
56	20	304
37	4	86
8	10	129
12	19	175
245	4	1514

245 cu. yd. 4 cu. ft. 1514 cu. in. *Ans.*

[redacted]

6. Subtract 32 cu. yd. 13 cu. ft. 1600 cu. in. from 39 cu. yd. 17 cu. ft. 1400 cu. in.

cu. yd.	cu. ft.	cu. in.
39	17	1400
32	13	1600
7	3	1528

7 cu. yd. 3 cu. ft. 1528 cu. in. *Ans.*

7. Multiply 12 cd. 4 cd. ft. by 14.

cd.	cd. ft.
12	4
	14
175	0

175 cd. *Ans.*

8. Divide 5 cu. yd. 10 cu. ft. 371 cu. in. by 6.

	cu. yd.	cu. ft.	cu. in.
6	5	10	371
		24	$349\frac{5}{6}$

24 cu. ft. $349\frac{5}{6}$ cu. in. *Ans.*

Exercise 78. Page 160.

1. Reduce £583 6 s. 8 d. to pence.

£	s.	d.
583	6	8
20		
11666		
12		
140000		

140,000 d. *Ans.*

2. Reduce £79 18 s. $11\frac{1}{4}$ d. to farthings.

£	s.	d.
79	18	$11\frac{1}{4}$
20		
1598		
12		
$19187\frac{1}{4}$		
4		
76750		

76,750 farthings. *Ans.*

3. Reduce 28,572 d. to higher units.

12 | 28572 d.
20 | 2381 s.
£119 . . . 1 s.

£119 1 s. *Ans.*

4. Reduce 27,281 crowns to guineas.

27281
5
136405

21 | 136405 s.
6495 guineas . . . 10 s.

6495 guineas 10 shillings. *Ans.*

5. Reduce 1,716,114 guineas to pounds.

1716114
21
1716114
3432228
36038394

20 | 36038394 s.
£1801919 . . . 14 s.

£1,801,919 14 s. *Ans.*

6. Reduce 706,126 d. to higher units.

12 | 706126 d.
20 | 58843 s. . . . 10 d.
£2942 . . . 3 s.

£2942 3 s. 10 d. *Ans.*

[redacted]

7. Add £35 2s. 6¾d.; £18 5s. 4d.; £27 3s. 10d.; £12 5d.; £6 7s. 8d.; £14 19s. 11d.; £29 16s. 2d.

£	s.	d.
35	2	6¾
18	5	4
27	3	10
12	0	5
6	7	8
14	19	11
29	16	2
143	15	10¾

£143 15s. 10¾d. *Ans.*

8. Subtract £92 15s. 1¼d. from £120 13s. 4d.

£	s.	d.
120	13	4
92	15	1¼
27	18	2¾

£27 18s. 2¾d. *Ans.*

9. Multiply £31 2s. 6½d. by 8.

£	s.	d.
31	2	6½
		8
249	0	4

£249 4d. *Ans.*

10. Divide £394 2s. 10½d. by £5 2s. 4½d.

£	s.	d.		£	s.	d.
394	2	10½		5	2	4½
20				20		
7882				102		
12				12		
94594½				1228½		

```
         77
12285)945945
      85995
       85995
       85995
```

77. *Ans.*

11. Divide £108 15s. 4d. by 13.

	£	s.	d.
13	108	15	4
	8	7	4

£8 7s. 4d. *Ans.*

12. Find the value in United States money of the money in a box containing 35 sovereigns, 27 half-sovereigns, 13 crowns, 41 half-crowns, and 85 shillings.

35 sovereigns = 35 × 20s. = 700s.
27 half-sovereigns = 27 × 10s. = 270s.
13 crowns = 13 × 5s. = 65s.
41 half-crowns = 41 × 2.5s. = 102.5s.
85 shillings = 85s.
1222.5s.

20 | 1222.5s.
£61.125
= £61⅛.

$4.8665
61⅛
6083⅛
48665
291990
$297.4648⅛ $297.46. Ans.

[redacted]

Exercise 79. Page 163.

1. Reduce 6 hr. 17 min. 25 sec. to seconds.

hr.	min.	sec.
6	17	25
60		
377		
60		
22645		

22,645 sec. *Ans.*

2. Reduce 1 yr. 13 dy. 8 hr. 4 min. to minutes.

yr.	dy.	hr.	min.
1	13	8	4
365			
378			
24			
9080			
60			
544804			

544,804 min. *Ans.*

3. Reduce 48,567 min. to higher units.

60) 48567 min.
24) 809 hr. . . . 27 min.
33 dy. . . . 17 hr.

33 dy. 17 hr. 27 min. *Ans.*

4. Reduce 7,423,922 sec. to higher units.

60) 7423922 sec.
60) 123732 min. . . . 2 sec.
24) 2062 hr. . . . 12 min.
85 dy. . . . 22 hr.

85 dy. 22 hr. 12 min. 2 sec. *Ans.*

5. How many minutes are there from midnight of March 7 to midnight of June 20?

		dy.
Mar.	24 dy.	105
Apr.	30	24
May	31	2520
June	20	60
	105 dy.	151200

151,200 min. *Ans.*

6. Find the number of seconds from eight o'clock Monday morning till six o'clock the next Saturday evening.

		hr.
Mon.	16 hr.	130
Tues.	24	60
Wed.	24	7800
Thu.	24	60
Fri.	24	468000
Sat.	18	
	130 hr.	

468,000 sec. *Ans.*

7. Which of the years 1600, 1656, 1700, 1734, 1800, 1818, 1880, 1900, 1924, 2000 are leap years?

1600; 1656; 1880; 1924; 2000. *Ans.*

8. Add 8 dy. 14 hr. 21 min. 37 sec.; 44 dy. 17 hr. 13 min. 32 sec.; 208 dy. 9 hr. 47 min. 43 sec.; 161 dy. 12 hr. 53 min. 54 sec.; 88 dy. 22 hr. 17 min. 50 sec.

yr.	dy.	hr.	min.	sec.
	8	14	21	37
	44	17	13	32
	208	9	47	43
	161	12	53	54
	88	22	17	50
1	147	4	34	36

1 yr. 147 dy. 4 hr. 34 min. 36 sec. *Ans.*

[redacted]

9. Subtract 2 yr. 213 dy. 17 hr. 48 min. 48 sec. from 3 yr. 147 dy. 14 hr. 14 min. 32 sec.

yr.	dy.	hr.	min.	sec.
3	147	14	14	32
2	213	17	48	48
	298	20	25	44

298 dy. 20 hr. 25 min. 44 sec. *Ans.*

10. Multiply 34 dy. 10 hr. 13 min. 12 sec. by 108.

yr.	dy.	hr.	min.	sec.
	34	10	13	12
				108
10	67	23	45	36

10 yr. 67 dy. 23 hr. 45 min. 36 sec. *Ans.*

11. Divide 16 yr. 357 dy. 17 hr. 20 min. 48 sec. by 18.

	yr.	dy.	hr.	min.	sec.
18	16	357	17	20	48
		344	7	37	$49\frac{1}{3}$

344 dy. 7 hr. 37 min. $49\frac{1}{3}$ sec. *Ans.*

12. Divide 22 wk. 2 dy. by 11 hr. 31 min. 12 sec.

wk.	dy.	hr.	min.	sec.
22	2	11	31	12
7		60		
156		691		
24		60		
3744		41472		
60				
224640				
60				
13478400				

$$\begin{array}{r} 325 \\ 41472\overline{)13478400} \\ 124416 \\ \hline 103680 \\ 82944 \\ \hline 207360 \\ 207360 \\ \hline \end{array}$$

325. *Ans.*

Exercise 80. Page 164.

1. Napoleon was born Aug. 15, 1769, and died at the age of 51 yr. 8 mo. 20 dy. What was the date of his death?

yr.	mo.	dy.
1769	8	15
51	8	20
1821	5	5

May 5, 1821. *Ans.*

2. Daniel Webster was born Jan. 18, 1782, and died Oct. 24, 1852. How old was he when he died?

yr.	mo.	dy.
1852	10	24
1782	1	18
70	9	6

70 yr. 9 mo. 6 dy. *Ans.*

[redacted]

3. A note dated July 14, 1897 has 63 days to run. When is the note due?

The number of days in July = 17
The number of days in Aug. = 31
The number of days in Sept. = 15
63

Sept. 15, 1897. *Ans.*

4. A note dated Feb. 11, 1896 has 93 days to run. When is the note due?

The number of days in Feb. = 18
The number of days in Mar. = 31
The number of days in Apr. = 30
The number of days in May = 14
93

May 14, 1896. *Ans.*

5. A note dated Feb. 11, 1897 has 63 days to run. When is the note due?

The number of days in Feb. = 17
The number of days in Mar. = 31
The number of days in Apr. = 15
63

Apr. 15, 1897. *Ans.*

6. In the morning of July 5 a man went into the country for his vacation, and returned in the evening of Sept. 26. Express in weeks and days the length of his vacation.

July 5 and Sept. 26 are both included in the vacation.

The number of days in July = 27
The number of days in Aug. = 31
The number of days in Sept. = 26
84

84 dy. = 12 wk. *Ans.*

7. Find the difference in time between Oct. 12, 1492, and July 4, 1776.

yr.	mo.	dy.
1776	7	4
1492	10	12
283	8	22

283 yr. 8 mo. 22 dy. *Ans.*

8. Jan. 1, 1859, fell on Saturday. What day of the week was Jan. 1, 1860? Jan. 1, 1861?

The year from Jan. 1, 1859, to Jan. 1, 1860, contained 365 days; that is, 52 wk. 1 dy.

Therefore, Jan. 1, 1860, fell *one day later*, or on Sunday.

The year from Jan. 1, 1860, to Jan. 1, 1861, contained 366 days; that is, 52 wk. 2 dy.

Therefore, Jan. 1, 1861, fell *two days later*, or on Tuesday.

Sunday; Tuesday. *Ans.*

[redacted]

Exercise 81. Page 166.

1. Reduce 2° 30′ 25″ to seconds.

°	′	″
2	30	25
60		
150		
60		
9025		

9025″. *Ans.*

2. Reduce 15° 3′ 22″ to seconds.

°	′	″
15	3	22
60		
903		
60		
54202		

54,202″. *Ans.*

3. Reduce 56,760″ to higher units.

60 | 56760″
60 | 946′
15° . . . 46′

15° 46′. *Ans.*

4. Reduce 212,221″ to higher units.

60 | 212221″
60 | 3537′ . . . 1″
58° . . . 57′

58° 57′ 1″. *Ans.*

5. Add 60° 50′ 50″; 20° 41′ 52″; 30° 25′ 20″; 20° 32′ 43″.

°	′	″
60	50	50
20	41	52
30	25	20
20	32	43
132	30	45

132° 30′ 45″. *Ans.*

6. Subtract 58° 33′ 36″ from 90° 11′ 21″.

°	′	″
90	11	21
58	33	36
31	37	45

31° 37′ 45″. *Ans.*

7. Multiply 12° 14′ 32″ by 48.

°	′	″
12	14	32
		48
587	37	36

587° 37′ 36″. *Ans.*

8. Divide 321° 49′ 24″ by 22.

	°	′	″
22	321	49	24
	14	37	42

14° 37′ 42″. *Ans.*

9. Divide 38° 37′ 42″ by 5° 31′ 6″.

°	′	″	°	′	″
38	37	42	5	31	6
60			60		
2317			331		
60			60		
139062			19866		

7 *Ans.*
19866)139062
139062

[redacted]

Exercise 82. Page 168.

1. Find the value of $\frac{4}{5}$ of a mile.

$\frac{4}{5}$ mi. $= \frac{4}{5}$ of 320 rd. $=$ 256 rd. *Ans.*

2. Find the value of $\frac{3}{16}$ of an acre.

$\frac{3}{16}$ A. $= \frac{3}{16}$ of 160 sq. rd.
$=$ 30 sq. rd. *Ans.*

3. Find the value of $\frac{5}{8}$ of a hundredweight.

$\frac{5}{8}$ cwt. $= \frac{5}{8}$ of 100 lb. $= 62\frac{1}{2}$ lb.
$\frac{1}{2}$ lb. $= \frac{1}{2}$ of 16 oz. $=$ 8 oz.
62 lb. 8 oz. *Ans.*

4. Find the value of $\frac{2}{3}$ of a pound sterling.

£$\frac{2}{3} = \frac{2}{3}$ of 20 s. $= 13\frac{1}{3}$ s.
$\frac{1}{3}$ s. $= \frac{1}{3}$ of 12 d. $=$ 4 d.
13 s. 4 d. *Ans.*

5. Find the value of $\frac{9}{11}$ of a mile.

$\frac{9}{11}$ mi. $= \frac{9}{11}$ of 320 rd. $= 261\frac{9}{11}$ rd.
$\frac{9}{11}$ rd. $= \frac{9}{11}$ of $5\frac{1}{2}$ yd. $= 4\frac{1}{2}$ yd.
$\frac{1}{2}$ yd. $= \frac{1}{2}$ of 3 ft. $= 1\frac{1}{2}$ ft.
$\frac{1}{2}$ ft. $= \frac{1}{2}$ of 12 in. $=$ 6 in.
261 rd. 4 yd. 1 ft. 6 in. *Ans.*

6. Find the value of $\frac{7}{11}$ of an acre.

$\frac{7}{11}$ A. $= \frac{7}{11}$ of 160 sq. rd. $= 101\frac{9}{11}$ sq. rd.
$\frac{9}{11}$ sq. rd. $= \frac{9}{11}$ of $30\frac{1}{4}$ sq. yd. $= 24\frac{3}{4}$ sq. yd.
$\frac{3}{4}$ sq. yd. $= \frac{3}{4}$ of 9 sq. ft. $= 6\frac{3}{4}$ sq. ft.
$\frac{3}{4}$ sq. ft. $= \frac{3}{4}$ of 144 sq. in. $=$ 108 sq. in.
101 sq. rd. 24 sq. yd. 6 sq. ft. 108 sq. in. *Ans.*

7. Find the value of $\frac{4}{9}$ of a degree.

$\frac{4}{9}^\circ = \frac{4}{9}$ of $60' = 26\frac{2}{3}'$.
$\frac{2}{3}' = \frac{2}{3}$ of $60'' = 40''$.
$26'\ 40''$. *Ans.*

8. Find the value of $\frac{1}{3}$ of a year.

$\frac{1}{3}$ yr. $= \frac{1}{3}$ of 365 dy. $= 121\frac{2}{3}$ dy.
$\frac{2}{3}$ dy. $= \frac{2}{3}$ of 24 hr. $=$ 16 hr.
121 dy. 16 hr. *Ans.*

9. Find the value of 0.15625 of a bushel.

$$\begin{array}{r} 0.15625 \\ 4 \\ \hline 0.62500 \\ 8 \\ \hline 5.000 \end{array}$$

5 qt. *Ans.*

10. Find the value of 0.625 of a gallon.

$$\begin{array}{r} 0.625 \\ 4 \\ \hline 2.500 \\ 2 \\ \hline 1.0 \end{array}$$

2 qt. 1 pt. *Ans.*

[redacted]

11. Find the value of 0.875 of a leap year.

0.875
366
5250
5250
2625
320.250
24
100
60
6.00

320 dy. 6 hr. *Ans.*

12. Find the value of 0.325 of a pound troy.

0.325
12
650
325
3.900
20
18.0

3 oz. 18 dwt. *Ans.*

13. Find the value of $6\frac{2}{5}$ of 3 A. $101\frac{1}{3}$ sq. rd.

A.	sq. rd.	sq. yd.	sq. ft.	sq. in.
3	$101\frac{1}{3}$			
	2			
7	$42\frac{2}{3}$			
		20	1	72
5 | 7	42	20	1	72
1	72	16	1	$28\frac{4}{5}$

A.	sq. rd.	sq. yd.	sq. ft.	sq. in.
3	$101\frac{1}{3}$			
	6			
21	128			
1	72	16	1	$28\frac{4}{5}$
23	40	16	1	$28\frac{4}{5}$

23 A. 40 sq. rd. 16 sq. yd. 1 sq. ft. $28\frac{4}{5}$ sq. in. *Ans.*

14. Find the value of $1\frac{3}{7}$ of 7 hr. 21 min. 27 sec.

	hr.	min.	sec.
	7	21	27
			3
7	22	4	21
	3	9	$11\frac{4}{7}$
	7	21	27
	10	30	$38\frac{4}{7}$

10 hr. 30 min. $38\frac{4}{7}$ sec. *Ans.*

15. Find the value of 10.0175 of 1 dy. 13 hr.

1 dy. 13 hr. = 37 hr.

10.0175
37
701225
300525
370.6475
60
38.8500
60
51.00

370 hr. 38 min. 51 sec. =
15 dy. 10 hr. 38 min. 51 sec. *Ans.*

16. Find the value of $17\frac{7}{12}$ of 10 yd. 2 ft. $3\frac{1}{5}$ in.

	yd.	ft.	in.
	10	2	$3\frac{1}{5}$
			7
12	75	0	$10\frac{2}{5}$
	6	0	$9\frac{13}{15}$

yd.	ft.	in.
10	2	$3\frac{1}{5}$
		17
182	2	$6\frac{2}{5}$
6	0	$9\frac{13}{15}$
189	0	$4\frac{4}{15}$

189 yd. $4\frac{4}{15}$ in. =
34 rd. 2 yd. $4\frac{4}{15}$ in. *Ans.*

[redacted]

17. Find the value of 0.01284 of 14 mi.

$$\begin{array}{r} 0.01284 \\ 14 \\ \hline 5136 \\ 1284 \\ \hline 0.17976 \\ 320 \\ \hline 359520 \\ 53928 \\ \hline 57.52320 \\ 5\frac{1}{2} \\ \hline 2616 \\ 26160 \\ \hline 2.8776 \\ 3 \\ \hline 2.6328 \\ 12 \\ \hline 7.5936 \end{array}$$

57 rd. 2 yd. 2 ft. 7.5936 in. *Ans.*

18. Find the value of 0.42776 of 12 t. 10 cwt.

12 t. 10 cwt. = 12.5 t.

$$\begin{array}{r} 0.42776 \\ 12.5 \\ \hline 213880 \\ 85552 \\ 42776 \\ \hline 5.347000 \\ 20 \\ \hline 6.940 \\ 100 \\ \hline 94.00 \end{array}$$

5 t. 6 cwt. 94 lb. *Ans.*

19. Find the value of $\frac{2}{5}$ of 1 lb. + $3\frac{2}{9}$ oz. + $5\frac{2}{3}$ dwt.

$\frac{2}{5}$ of 1 lb. = $\frac{2}{5}$ lb. = $\frac{2}{5}$ of 12 oz. = $4\frac{4}{5}$ oz.

$4\frac{4}{5}$ oz. + $3\frac{2}{9}$ oz. = $8\frac{1}{45}$ oz.

$\frac{1}{45}$ oz. = $\frac{1}{45}$ of 20 dwt. = $\frac{4}{9}$ dwt.

$\frac{4}{9}$ dwt. + $5\frac{2}{3}$ dwt. = $6\frac{1}{9}$ dwt.

$\frac{1}{9}$ dwt. = $\frac{1}{9}$ of 24 gr. = $2\frac{2}{3}$ gr.

8 oz. 6 dwt. $2\frac{2}{3}$ gr. *Ans.*

20. Find the value of 0.35 of 4 lb. 5 oz. 6 dwt. 16 gr.

$0.35 = \frac{35}{100} = \frac{7}{20}$.

	lb.	oz.	dwt.	gr.
	4	5	6	16
				7
20	31	1	6	16
	1	6	13	8

1 lb. 6 oz. 13 dwt. 8 gr. *Ans.*

21. Find the value of 3.726 mi. − 33.57 rd.

$$\begin{array}{r} 3.726 \\ 320 \\ \hline 14520 \\ 2178 \\ \hline 232.320 \end{array}$$

$$\begin{array}{cr} \text{mi.} & \text{rd.} \\ 3 & 232.32 \\ & 33.57 \\ \hline 3 & 198.75 \\ & 5.5 \\ \hline & 375 \\ & 375 \\ \hline & 4.125 \\ & 3 \\ \hline & 0.375 \\ & 12 \\ \hline & 4.500 \end{array}$$

3 mi. 198 rd. 4 yd. 4.5 in. *Ans.*

[redacted]

22. Find the value of $\frac{3}{73}$ of a year + $\frac{9}{56}$ of a week + $\frac{7}{12}$ of an hour.

$\frac{3}{73}$ yr. = $\frac{3}{73}$ of 365 dy. = 15 dy.
$\frac{9}{56}$ wk. = $\frac{9}{56}$ of 7 dy. = $1\frac{1}{8}$ dy.
$\frac{1}{8}$ dy. = $\frac{1}{8}$ of 24 hr. = 3 hr.
$\frac{7}{12}$ hr. = $\frac{7}{12}$ of 60 min. = 35 min.

dy.	hr.	min.
15		
1	3	
		35
16	3	35

16 dy. 3 hr. 35 min. *Ans.*

23. Find the value of 5.268 of 2 dy. + 2.829 of 16 hr. + 0.9528 of 25 min.

5.268	2.829	0.9528
2	16	25
10.536	16974	47640
24	2829	19056
2144	45.264	23.8200
1072	12.864	7.68
12.864	58.128	31.50
	60	60
	7.680	30.0

10 dy. 58 hr. 31 min. 30 sec.
= 12 dy. 10 hr. 31 min. 30 sec. *Ans.*

24. Find the value of $\frac{3}{16}$ of a mile + $\frac{2}{3}$ of 40 rd. + $\frac{3}{8}$ of a yard.

$\frac{3}{16}$ mi. = $\frac{3}{16}$ of 320 rd. = 60 rd.
$\frac{2}{3}$ of 40 rd. = $26\frac{2}{3}$ rd.
60 rd. + $26\frac{2}{3}$ rd. = $86\frac{2}{3}$ rd.
$\frac{2}{3}$ rd. = $\frac{2}{3}$ of $5\frac{1}{2}$ yd. = $3\frac{2}{3}$ yd.
$3\frac{2}{3}$ yd. + $\frac{3}{8}$ yd. = $4\frac{1}{24}$ yd.
$\frac{1}{24}$ yd. = $\frac{1}{24}$ of 3 ft. = $\frac{1}{8}$ ft.
$\frac{1}{8}$ ft. = $\frac{1}{8}$ of 12 in. = $1\frac{1}{2}$ in.
86 rd. 4 yd. $1\frac{1}{2}$ in. *Ans.*

25. Find the value of $\frac{3}{4}$ of 2 cwt. 84 lb. + $\frac{3}{7}$ of 5 cwt. 98 lb. + $\frac{2}{5}$ of $7\frac{1}{2}$ lb.

$\frac{3}{4}$ of 2 cwt. 84 lb. = $\frac{3}{4}$ of 284 lb.
= 213 lb.
$\frac{3}{7}$ of 5 cwt. 98 lb. = $\frac{3}{7}$ of 598 lb.
= $256\frac{2}{7}$ lb.
$\frac{2}{5}$ of $7\frac{1}{2}$ lb. = 3 lb.
213 lb. + $256\frac{2}{7}$ lb. + 3 lb. = $472\frac{2}{7}$ lb.
$\frac{2}{7}$ lb. = $\frac{2}{7}$ of 16 oz. = $4\frac{4}{7}$ oz.
472 lb. $4\frac{4}{7}$ oz.
= 4 cwt. 72 lb. $4\frac{4}{7}$ oz. *Ans.*

26. Find the value of $\frac{3}{8}$ of 21 ft. 7 in. + 0.855 of 16 ft. 2 in. + 0.365 of 1 ft.

21 ft. 7 in. = 259 in.; 16 ft. 2 in.
= 194 in.; 1 ft. = 12 in.

259	194	0.365
0.375	0.855	12
1295	970	730
1813	970	365
777	1552	4.380
97.125	165.870	

97.125 in.
165.87
4.38
12 | 267.375 in.
3 | 22 ft. . . . 3.375 in.
7 yd. . . . 1 ft.
7 yd. 1 ft. $3\frac{3}{8}$ in. *Ans.*

27. Find the value of 0.9 of 4 A. 17 sq. rd. − $\frac{11}{12}$ of 3 A. 15 sq. rd.

	A.	sq. rd.		A.	sq. rd.
	4	17		3	15
		9			11
10	36	153	12	34	5
	3	111.3		2	133.75

[redacted]

A.	sq. rd.	
3	111.3	30.25
2	133.75	0.55
	137.55	15125
		15125
		16.6375
		9
		5.7375
		144
		29500
		29500
		7375
		106.2000

137 sq. rd. 16 sq. yd. 5 sq. ft. 106.2 sq. in. *Ans.*

28. Find the value of 0.652 of 2 cu. yd. 7 cu. ft. – 0.888 of 1 cu. yd. 2 cu. ft.

2 cu. yd. 7 cu. ft. = 61 cu. ft.
1 cu. yd. 2 cu. ft. = 29 cu. ft.

0.652	0.888
61	29
652	7992
3912	1776
39.772	25.752

39.772
25.752
14.02
1728
34.56

14 cu. ft. 34.56 cu. in. *Ans.*

29. Find the value of 0.456 of 12 bu. 3 pk. – 0.654 of 5 bu. 2 pk.

12 bu. 3 pk. = 51 pk.;
5 bu. 2 pk. = 22 pk.

0.456	0.654	23.256
51	22	14.388
456	1308	8.868
2280	1308	8
23.256	14.388	6.944
		2
		1.888

8 pk. 6 qt. 1.888 pt.
= 2 bu. 6 qt. 1.888 pt. *Ans.*

Exercise 83. Page 169.

1. Express a pound avoirdupois as the fraction of a pound troy.

1 lb. av. = 7000 gr. 1 lb. troy = 5760 gr.

$$\frac{7000}{5760} = \frac{175}{144}. \quad \textit{Ans.}$$

2. Express an ounce avoirdupois as the fraction of an ounce troy.

1 oz. av. $= \frac{1}{16}$ of 7000 gr. $= 437\frac{1}{2}$ gr.
1 oz. troy $= \frac{1}{12}$ of 5760 gr. = 480 gr.

$$\frac{437\frac{1}{2}}{480} = \frac{875}{960} = \frac{175}{192}. \quad \textit{Ans.}$$

[redacted]

3. Express 363 sq. yd. as the fraction of an acre.

$$\frac{363}{160 \times 30\frac{1}{4}} = \overset{3}{\cancel{363}} \times \frac{1}{\underset{40}{\cancel{160}}} \times \frac{\cancel{4}}{\cancel{121}} = \frac{3}{40}. \quad \textit{Ans.}$$

4. Express $\frac{3}{5}$ of £2 1 s. 3 d. + $\frac{5}{11}$ of £1 4 s. 9 d. as the fraction of £2 14 s.

$$\begin{array}{r|ccc}
 & £ & s. & d. \\
 & 2 & 1 & 3 \\
 & & & 3 \\
\hline
5 & 6. & 3 & 9 \\
\hline
 & 1 & 4 & 9
\end{array}
\qquad
\begin{array}{r|ccc}
 & £ & s. & d. \\
 & 1 & 4 & 9 \\
 & & & 5 \\
\hline
11 & 6 & 3 & 9 \\
\hline
 & & 11 & 3
\end{array}$$

$$\begin{array}{ccc}
£ & s. & d. \\
1 & 4 & 9 \\
 & 11 & 3 \\
\hline
1 & 16 & 0 \\
20 & & \\
\hline
36 & &
\end{array}
\qquad
\begin{array}{cc}
£ & s. \\
2 & 14 \\
20 & \\
\hline
54 &
\end{array}$$

$$\frac{36}{54} = \frac{2}{3}. \quad \textit{Ans.}$$

5. Express 2 mi. 138 rd. 1 yd. as the fraction of 3 mi. 265 rd. 3 yd. 1 ft. 6 in.

3 mi. 265 rd. 3 yd. 1 ft. 6 in. = 3 mi. 265 rd. $3\frac{1}{2}$ yd.

$$\begin{array}{ccc}
\text{mi.} & \text{rd.} & \text{yd.} \\
2 & 138 & 1 \\
320 & & \\
\hline
778 & & \\
5\frac{1}{2} & & \\
\hline
4280 & &
\end{array}
\qquad
\begin{array}{ccc}
\text{mi.} & \text{rd.} & \text{yd.} \\
3 & 265 & 3\frac{1}{2} \\
320 & & \\
\hline
1225 & & \\
5\frac{1}{2} & & \\
\hline
6741 & &
\end{array}
\qquad
\frac{4280}{6741} = \frac{40}{63}. \quad \textit{Ans.}$$

6. Express $\frac{2}{7}$ of 560 lb. as the fraction of 5 long tons.

$$\frac{\frac{2}{7} \text{ of } 560}{5 \times 2240} = \frac{2}{7} \times \cancel{560} \times \frac{1}{5} \times \frac{1}{\underset{\underset{2}{\cancel{4}}}{\cancel{2240}}} = \frac{1}{70}. \quad \textit{Ans.}$$

7. Express $\frac{2}{3}$ of 200 rd. as the fraction of 4 mi.

$$\frac{\frac{2}{3} \text{ of } 200}{4 \times 320} = \frac{2}{3} \times \overset{5}{\cancel{200}} \times \frac{1}{\underset{2}{\cancel{4}}} \times \frac{1}{\underset{8}{\cancel{320}}} = \frac{5}{48}. \quad \textit{Ans.}$$

[redacted]

8. Express $\frac{10}{27}$ of 2 dy. 2 hr. 24 min. as the fraction of 2 wk. 1 dy.

dy.	hr.	min.	wk.	dy.
2	2	24	2	1
24			7	
50			15	
60			24	
3024			360	
			60	
			21600	

$$\frac{\frac{10}{27}\text{ of }3024}{21600}=\frac{\cancel{10}}{\cancel{27}}\times\overset{\overset{7}{\cancel{112}}}{\cancel{3024}}\times\frac{1}{\underset{\underset{135}{\cancel{1350}}}{\cancel{21600}}}=\frac{7}{135}.\ \textit{Ans.}$$

9. Express $\frac{4}{5}$ of the difference between 3 yd. 2 ft. 11 in. and 10 yd. 7 in. as the fraction of 8 yd.

yd.	ft.	in.
10	0	7
3	2	11
6	0	8

6 yd. 8 in. $= 6\frac{8}{36}$ yd. $= 6\frac{2}{9}$ yd.

$$\frac{\frac{4}{5}\text{ of }6\frac{2}{9}}{8}=\frac{4}{5}\times\frac{\overset{7}{\cancel{56}}}{9}\times\frac{1}{\cancel{8}}=\frac{28}{45}.\ \textit{Ans.}$$

10. Express $\frac{10}{21}$ of the difference between $\frac{5}{8}$ of 7 hr. and $\frac{7}{25}$ of 15 min. as the fraction of 12 hr. 18 min.

$\frac{5}{8}$ of 7 hr. $=\frac{35}{8}$ hr. $=4\frac{3}{8}$ hr. $\frac{7}{25}$ of 15 min. $=\frac{7}{25}$ of $\frac{1}{4}$ hr. $=\frac{7}{100}$ hr.

$4\frac{3}{8}$ hr. $-\frac{7}{100}$ hr. $=4\frac{61}{200}$ hr. 12 hr. 18 min. $=12\frac{18}{60}$ hr. $=12\frac{3}{10}$ hr.

$$\frac{\frac{10}{21}\text{ of }4\frac{61}{200}}{12\frac{3}{10}}=\frac{\cancel{10}}{\cancel{21}}\times\frac{\overset{41}{\cancel{861}}}{\underset{\underset{2}{\cancel{20}}}{\cancel{200}}}\times\frac{\cancel{10}}{\underset{3}{\cancel{123}}}=\frac{1}{6}.\ \textit{Ans.}$$

11. Express $\frac{2}{5}$ pt. as the fraction of a gallon.

1 gal. = 8 pt.

$$\frac{\frac{2}{5}}{8}=\frac{\cancel{2}}{5}\times\frac{1}{\underset{4}{\cancel{8}}}=\frac{1}{20}.\ \textit{Ans.}$$

12. Express 16 s. $3\frac{3}{4}$ d. as the decimal of a pound.

12 | 3.75 d.
20 | 16.3125 s.
£0.815625. *Ans.*

[redacted]

13. Express 233 rd. 9 ft. 10.8 in. as the decimal of a mile.

$$\begin{array}{r|l}
12 & 10.8 \text{ in.} \\
16\frac{1}{2} & 9.9 \text{ ft.} \\
320 & \underline{233.6 \text{ rd.}} \\
 & 0.73 \text{ mi. } \textit{Ans.}
\end{array}$$

14. Express 71 sq. rd. 54 sq. ft. 64.8 sq. in. as the decimal of an acre.

$$\begin{array}{r|l}
144 & 64.8 \text{ sq. in.} \\
272\frac{1}{4} & 54.45 \text{ sq. ft.} \\
160 & \underline{71.20} \text{ sq. rd.} \\
 & 0.445 \text{ A. } \textit{Ans.}
\end{array}$$

15. Express 15 hr. 14 min. 6 sec. as the decimal of 2 days.

$$\begin{array}{r|l}
60 & 6. \quad \text{sec.} \\
60 & 14.1 \quad \text{min.} \\
24 & \underline{15.235} \quad \text{hr.} \\
 & 0.6348 \text{ dy.}
\end{array}$$

$$\frac{0.6348}{2} = 0.3174. \ \textit{Ans.}$$

16. Express 38 sq. rd. 21 sq. yd. 5 sq. ft. 108 sq. in. as the decimal of an acre.

$$\begin{array}{r|l}
144 & 108. \quad \text{sq. in.} \\
9 & 5.75 \text{ sq. ft.} \\
30\frac{1}{4} & 21.639 \text{ sq. yd.} \\
160 & \underline{38.715} \text{ sq. rd.} \\
 & 0.242 \text{ A. } \textit{Ans.}
\end{array}$$

17. Express 3 mi. 242 rd. 2 yd. 2 ft. 3 in. as the decimal of 7 mi. 160 rd.

$$\begin{array}{r|l}
12 & 3. \quad \text{in.} \\
3 & 2.25 \quad \text{ft.} \\
5\frac{1}{2} & 2.75 \quad \text{yd.} \\
320 & \underline{242.5} \quad \text{rd.} \\
 & 3.7578 \text{ mi.}
\end{array}
\qquad
\begin{array}{r|l}
320 & \underline{160 \text{ rd.}} \\
 & 7.5 \text{ mi.}
\end{array}$$

$$\frac{3.7578}{7.5} = 0.501. \ \textit{Ans.}$$

18. Express 5 hr. 13 min. 30 sec. as the decimal of a week.

$$\begin{array}{r|l}
60 & 30. \quad \text{sec.} \\
60 & 13.5 \quad \text{min.} \\
24 & 5.225 \quad \text{hr.} \\
7 & \underline{0.2177} \text{ dy.} \\
 & 0.0311 \text{ wk. } \textit{Ans.}
\end{array}$$

19. Express 27° 14′ 45″ as the decimal of 90°.

$$\begin{array}{r|l}
60 & 45.'' \\
60 & \underline{14.75'} \\
 & 27.246^\circ
\end{array}$$

$$\frac{27.246}{90} = 0.303. \ \textit{Ans.}$$

20. Express 54 dy. 2 hr. 40 min. as the decimal of $365\frac{1}{4}$ days.

$$2 \text{ hr. } 40 \text{ min.} = 2\tfrac{2}{3} \text{ hr.} = \frac{2\frac{2}{3}}{24} \text{ dy.}$$
$$= \frac{\cancel{8}}{\cancel{72}} \text{ dy.} = \tfrac{1}{9} \text{ dy.}$$

$$\frac{54\frac{1}{9}}{365\frac{1}{4}} = \frac{\cancel{487}}{9} \times \frac{4}{\underset{3}{\cancel{1461}}} = \frac{4}{27} = 0.\dot{1}4\dot{8}. \ \textit{Ans.}$$

$$\begin{array}{r}
0.148 \\
27\overline{)4.0} \\
\underline{27} \\
130 \\
\underline{108} \\
220 \\
\underline{216} \\
4
\end{array}$$

144)35.0
288

620
576

440
432

800
720

80

Express 44,920.9025 hr. as mal of a year.

805

$$\frac{\cancel{9025}}{24} = \frac{8984.1805}{1752} = 5.128. \quad \textit{Ans.}$$

Express 14.52 sq. yd. as the of a square chain.

sq. ch. = 16 sq. rd.
= 484 sq. yd.

$$\frac{14.52}{484} = 0.03. \quad \textit{Ans.}$$

988
24

23727

269
24

6471

$\frac{6471}{23727} = \frac{3}{11}$.

26. What part of 2 mi. is $\frac{2}{3}$ 6 rd. 3 yd. 2 in.?

$2 \text{ in.} = \frac{2}{36} \text{ yd.} = \frac{1}{18} \text{ yd.}$

$3\frac{1}{18} \text{ yd.} = \frac{3\frac{1}{18}}{5\frac{1}{2}} \text{ rd.} = \frac{5}{9} \text{ rd.}$

$$\frac{\frac{2}{3} \text{ of } 6\frac{5}{9}}{2 \times 320} = \frac{\cancel{2}}{3} \times \frac{59}{9} \times \frac{1}{\cancel{2}} \times \frac{1}{320} =$$

27. What part of a bushe 1 pk. 2 qt. 1 pt.?

1 bu. = 64 pt.

pk.	qt.	pt.
1	2	1
8		
10		
2		

[redacted]

29. What part of 5 tons is 3 t. 240 lb.?

3 t. 240 lb. = 6240 lb.

$$\frac{\overset{78}{\cancel{6240}}}{5 \times \underset{25}{\cancel{2000}}} = \frac{78}{125}. \quad \textit{Ans.}$$

30. What part of an acre is 38 sq. rd. 194 sq. ft. 108 sq. in.?

38 sq. rd. 194 sq. ft. 108 sq. in.
= 38 sq. rd. 194¾ sq. ft.

sq. rd.	sq. ft.
38	194¾
272¼	
10540¼	

$$\frac{10540\frac{1}{4}}{43560} = \frac{42161}{174240}. \quad \textit{Ans.}$$

31. Express 2 lb. 9 oz. 21 dwt. as the decimal of 4 lb. 7 oz. 19 dwt.

lb.	oz.	dwt.	lb.	oz.	dwt.
2	9	21	4	7	19
12			12		
33			55		
20			20		
681			1119		

```
      0.60858  Ans.
1119)681.0
     6714
      9600
      8952
       6480
       5595
        8850
```

32. Express 17 wk. 6 dy. 22 hr. 39 min. as the decimal of 35 wk. 3 dy. 15 hr. 25 min.

wk.	dy.	hr.	min.	wk.	dy.	hr.	min.
17	6	22	39	35	3	15	25
7				7			
125				248			
24				24			
3022				5967			
60				60			
181359				358045			

```
                 0.50652
358045)181359.0
        1790225
         2336500
         2148270
          1882300
          1790225
            920750
            716090
            204660
```

0.50653. *Ans.*

33. What part of 61 ft. 3 in. is 8 ft. 7 in.?

ft.	in.	ft.	in.
61	3	8	7
12		12	
735		103	

$\frac{103}{735}$. *Ans.*

Exercise 84. Page 172.

Find the difference in longitude between two cities, if the difference in time is:

1. 1 hr. 15 min.

1 hr.	15 min.	
	15	
18°	45′	

2. 2 hr. 11 min.

2 hr.	11 min.	
	15	
32°	45′	

3. 5 hr. 10 min. 10 sec.

5 hr.	10 min.	10 sec.
		15
77°	32′	30″

4. 3 hr. 25 min. 35 sec.

3 hr.	25 min.	35 sec.
		15
51°	23′	45″

5. 6 hr. 12 min. 30 sec.

6 hr.	12 min.	30 sec.
		15
93°	7′	30″

6. 4 hr. 8 min. 12 sec.

4 hr.	8 min.	12 sec.
		15
62°	3′	

7. 18 hr. 10 min.

18 hr.	10 min.	
	15	
272°	30′	

8. 15 hr. 15 min. 15 sec.

15 hr.	15 min.	15 sec.
		15
228°	48′	45″

Find the difference in time between two cities, if the difference in longitude is:

9. 9° 20′.

15	9°	20′	
		37 min.	20 sec.

10. 70° 30′.

15	70°	30′
	4 hr.	42 min.

11. 56° 36′ 12″.

15	56°	36′	12″
	3 hr.	46 min.	24.8 sec.

12. 108° 32′ 36″.

15	108°	32′	36″
	7 hr.	14 min.	10.4 sec.

13. 120° 14′ 30″.

15	120°	14′	30″
	8 hr.	0 min.	58 sec.

14. 100° 45′ 54″.

15	100°	45′	54″
	6 hr.	43 min.	3.6 sec.

15. 2° 2′ 2″.

15	2°	2′	2″
		8 min.	$8\frac{2}{15}$ sec.

16. 75° 10′.

15	75°	10′	
	5 hr.	0 min.	40 sec.

[redacted]

17. Find the difference in time between New York, longitude 74° 0′ 3″ west, and San Francisco, longitude 122° 26′ 15″ west.

	122°	26′	15″ W.
	74°	0′	3″ W.
15	48°	26′	12″
	3 hr.	13 min.	44.8 sec.

18. The difference in time between Berlin and New York is 5 hr. 49 min. 35 sec. What is the difference in longitude?

5 hr.	49 min.	35 sec.
		15
87°	23′	45″

Exercise 85. Page 174.

The longitude of some public building in:

(1) Berlin is 13° 23′ 43″ E.
(2) Rome, 12° 27′ 14″ E.
(3) Constantinople, 28° 59′ E.
(4) Pekin, 116° 23′ 45″ E.
(5) San Francisco, 122° 26′ 15″ W.
(6) St. Louis, 90° 15′ 15″ W.
(7) Jerusalem, 35° 32′ E.
(8) Bombay, 72° 54′ E.
(9) Calcutta, 88° 19′ 2″ E.
(10) Chicago, 87° 35′ W.
(11) New York, 74° 0′ 3″ W.
(12) Montreal, 73° 25′ W.

What is the clock-time at each of the above cities:

1. When is it noon at Greenwich?

(1)

15	13°	23′	43″
		53 min.	$34\frac{13}{15}$ sec.

53 min. $34\frac{13}{15}$ sec. past 12 P.M. *Ans.*

(2)

15	12°	27′	14″
		49 min.	$48\frac{14}{15}$ sec.

49 min. $48\frac{14}{15}$ sec. past 12 P.M. *Ans.*

(3)

15	28°	59′	
	1 hr.	55 min.	56 sec.

55 min. 56 sec. past 1 P.M. *Ans.*

(4)

15	116°	23′	45″
	7 hr.	45 min.	35 sec.

45 min. 35 sec. past 7 P.M. *Ans.*

(5)

15	122°	26′	15″
	8 hr.	9 min.	45 sec.

hr.	min.	sec.
12	0	0
8	9	45
3	50	15

50 min. 15 sec. past 3 A.M. *Ans.*

(6)

15	90°	15′	15″
	6 hr.	1 min.	1 sec.

hr.	min.	sec.
12	0	0
6	1	1
5	58	59

58 min. 59 sec. past 5 A.M. *Ans.*

[redacted]

(7)

15	35°	32′	
	2 hr.	22 min.	8 sec.

22 min. 8 sec. past 2 P.M. *Ans.*

(8)

15	72°	54′	
	4 hr.	51 min.	36 sec.

51 min. 36 sec. past 4 P.M. *Ans.*

(9)

15	88°	19′	2″
	5 hr.	53 min.	16$\frac{2}{15}$ sec.

53 min. 16$\frac{2}{15}$ sec. past 5 P.M. *Ans.*

(10)

15	87°	35′	
	5 hr.	50 min.	20 sec.

hr.	min.	sec.
12	0	0
5	50	20
6	9	40

9 min. 40 sec. past 6 A.M. *Ans.*

(11)

15	74°	0′	3″
	4 hr.	56 min.	$\frac{1}{5}$ sec.

hr.	min.	sec.
12	0	0
4	56	$\frac{1}{5}$
7	3	59$\frac{4}{5}$

3 min. 59$\frac{4}{5}$ sec. past 7 A.M. *Ans.*

(12)

15	73°	25′	
	4 hr.	53 min.	40 sec.

hr.	min.	sec.
12	0	0
4	53	40
7	6	20

6 min. 20 sec. past 7 A.M. *Ans.*

2. When it is half-past 4 P.M. at Chicago?

(1)

	87°	35′	0″ W.
	13°	23′	43″ E.
15	100°	58′	43″
	6 hr.	43 min.	54$\frac{13}{15}$ sec.

hr.	min.	sec.
4	30	
6	43	54$\frac{13}{15}$
11	13	54$\frac{13}{15}$

13 min. 54$\frac{13}{15}$ sec. past 11 P.M. *Ans.*

(2)

	87°	35′	0″ W.
	12°	27′	14″ E.
15	100°	2′	14″
	6 hr.	40 min.	8$\frac{14}{15}$ sec.

hr.	min.	sec.
4	30	
6	40	8$\frac{14}{15}$
11	10	8$\frac{14}{15}$

10 min. 8$\frac{14}{15}$ sec. past 11 P.M. *Ans.*

(3)

	87°	35′ W.	
	28°	59′ E.	
15	116°	34′	
	7 hr.	46 min.	16 sec.

hr.	min.	sec.
4	30	
7	46	16
12	16	16

16 min. 16 sec. past 12 A.M. of the following day. *Ans.*

[redacted]

(4)

	87°	35′	0″ W.
	116°	23′	45″ E.
15	203°	58′	45″
	13 hr.	35 min.	55 sec.

hr.	min.	sec.
4	30	
13	35	55
18	5	55
12	0	0
6	5	55

5 min. 55 sec. past 6 A.M. of the following day. *Ans.*

(5)

	122°	26′	15″ W.
	87°	35′	W.
15	34°	51′	15″
	2 hr.	19 min.	25 sec.

hr.	min.	sec.
4	30	
2	19	25
2	10	35

10 min. 35 sec. past 2 P.M. *Ans.*

(6)

	90°	15′	15″ W.
	87°	35′	W.
15	2°	40′	15″
		10 min.	41 sec.

hr.	min.	sec.
4	30	
	10	41
4	19	19

19 min. 19 sec. past 4 P.M. *Ans.*

(7)

	87°	35′ W.	
	35°	32′ E.	
15	123°	7′	
	8 hr.	12 min.	28 sec.

hr.	min.	sec.
4	30	
8	12	28
12	42	28

42 min. 28 sec. past 12 A.M. of the following day. *Ans.*

(8)

	87°	35′ W.	
	72°	54′ E.	
15	160°	29′	
	10 hr.	41 min.	56 sec.

hr.	min.	sec.
4	30	
10	41	56
15	11	56
12	0	0
3	11	56

11 min. 56 sec. past 3 A.M. of the following day. *Ans.*

(9)

	87°	35′	0″ W.
	88°	19′	2″ E.
15	175°	54′	2″
	11 hr.	43 min.	$36\frac{2}{15}$ sec.

hr.	min.	sec.
4	30	
11	43	$36\frac{2}{15}$
16	13	$36\frac{2}{15}$
12	0	0
4	13	$36\frac{2}{15}$

13 min. $36\frac{2}{15}$ sec. past 4 A.M. of the following day. *Ans.*

[redacted]

(10)

30 min. past 4 P.M. *Ans.*

(11)

	87°	35′	0″ W.
	74°	0′	3″ W.
15	13°	34′	57″
		54 min.	$19\frac{4}{5}$ sec.

hr.	min.	sec.
4	30	
	54	$19\frac{4}{5}$
5	24	$19\frac{4}{5}$

24 min. $19\frac{4}{5}$ sec. past 5 P.M. *Ans.*

(12)

	87°	35′ W.	
	73°	25′ W.	
15	14°	10′	
		56 min.	40 sec.

hr.	min.	sec.
4	30	
	56	40
5	26	40

26 min. 40 sec. past 5 P.M. *Ans.*

3. When it is eight o'clock A.M. at Constantinople?

(1)

	28°	59′	0″ E.
	13°	23′	43″ E.
15	15°	35′	17″
	1 hr.	2 min.	$21\frac{2}{15}$ sec.

hr.	min.	sec.
8	0	0
1	2	$21\frac{2}{15}$
6	57	$38\frac{13}{15}$

57 min. $38\frac{13}{15}$ sec. past 6 A.M. *Ans.*

(2)

	28°	59′	0″ E.
	12°	27′	14″ E.
15	16°	31′	46″
	1 hr.	6 min.	$7\frac{1}{15}$ sec.

hr.	min.	sec.
8	0	0
1	6	$7\frac{1}{15}$
6	53	$52\frac{14}{15}$

53 min. $52\frac{14}{15}$ sec. past 6 A.M. *Ans.*

(3)

8 A.M. *Ans.*

(4)

	116°	23′	45″ E.
	28°	59′	E.
15	87°	24′	45″
	5 hr.	49 min.	39 sec.

hr.	min.	sec.
8	0	0
5	49	39
13	49	39
12	0	0
1	49	39

49 min. 39 sec. past 1 P.M. *Ans.*

(5)

	28°	59′	0″ E.
	122°	26′	15″ W.
15	151°	25′	15″
	10 hr.	5 min.	41 sec.

hr.	min.	sec.
8	0	0
10	5	41
9	54	19

54 min. 19 sec. past 9 P.M. of the previous day. *Ans.*

(6)

	28°	59′	0″ E.
	90°	15′	15″ W.
15	119°	14′	15″
	7 hr.	56 min.	57 sec.

hr.	min.	sec.
8	0	0
7	56	57
	3	3

3 min. 3 sec. past 12 A.M. *Ans.*

[redacted]

(7)

	35°	32′ E.	
	28°	59′ E.	
15	6°	33′	
		26 min.	12 sec.

hr.	min.	sec.
8	0	0
	26	12
8	26	12

26 min. 12 sec. past 8 A.M. *Ans.*

(8)

	72°	54′ E.	
	28°	59′ E.	
15	43°	55′	
	2 hr.	55 min.	40 sec.

hr.	min.	sec.
8	0	0
2	55	40
10	55	40

55 min. 40 sec. past 10 A.M. *Ans.*

(9)

	88°	19′	2″ E.
	28°	59′	E.
15	59°	20′	2″
	3 hr.	57 min.	$20\frac{2}{15}$ sec.

hr.	min.	sec.
8	0	0
3	57	$20\frac{2}{15}$
11	57	$20\frac{2}{15}$

57 min. $20\frac{2}{15}$ sec. past 11 A.M. *Ans.*

(10)

	28°	59′ E.	
	87°	35′ W.	
15	116°	34′	
	7 hr.	46 min.	16 sec.

hr.	min.	sec.
8	0	0
7	46	16
	13	44

13 min. 44 sec. past 12 A.M. *Ans.*

(11)

	28°	59′	0″ E.
	74°	0′	3″ W.
15	102°	59′	3″
	6 hr.	51 min.	$56\frac{1}{5}$ sec.

hr.	min.	sec.
8	0	0
6	51	$56\frac{1}{5}$
1	8	$3\frac{4}{5}$

8 min. $3\frac{4}{5}$ sec. past 1 A.M. *Ans.*

(12)

	28°	59′ E.	
	73°	25′ W.	
15	102°	24′	
	6 hr.	49 min.	36 sec.

hr.	min.	sec.
8	0	0
6	49	36
1	10	24

10 min. 24 sec. past 1 A.M. *Ans.*

When it is noon at Greenwich the time at:

(1) Boston, Mass., is 7 hr. 15 min. 46 sec. A.M.
(2) Columbia, S.C., 6 hr. 35 min. 32 sec. A.M.
(3) Salt Lake, 4 hr. 30 min. A.M.
(4) Albany, N.Y., 7 hr. 5 min. 1 sec. A.M.
(5) Harrisburg, Penn., 6 hr. 52 min. 40 sec. A.M.

[redacted]

(6) New Orleans, La., 6 hr. A.M.
(7) Columbus, O., 6 hr. 27 min. 48 sec. A.M.
(8) Washington, D.C., 6 hr. 51 min. 44 sec. A.M.
(9) Springfield, Ill., 6 hr. 1 min. 48 sec. A.M.

4. What is the longitude of each of the above cities?

(1)

hr.	min.	sec.
12	0	0
7	15	46
4	44	14
		15
71°	3′	30″

71° 3′ 30″ W. *Ans.*

(2)

hr.	min.	sec.
12	0	0
6	35	32
5	24	28
		15
81°	7′	

81° 7′ W. *Ans.*

(3)

hr.	min.
12	0
4	30
7	30
	15
112°	30′

112° 30′ W. *Ans.*

(4)

hr.	min.	sec.
12	0	0
7	5	1
4	54	59
		15
73°	44′	45″

73° 44′ 45″ W. *Ans.*

(5)

hr.	min.	sec.
12	0	0
6	52	40
5	7	20
		15
76°	50′	

76° 50′ W. *Ans.*

(6) 90° W. *Ans.*

(7)

hr.	min.	sec.
12	0	0
6	27	48
5	32	12
		15
83°	3′	

83° 3′ W. *Ans.*

(8)

hr.	min.	sec.
12	0	0
6	51	44
5	8	16
		15
77°	4′	

77° 4′ W. *Ans.*

(9)

hr.	min.	sec.
12	0	0
6	1	48
5	58	12
		15
89°	33′	

89° 33′ W. *Ans.*

[redacted]

Exercise 86. Page 175.

1. Reduce 7 gal. 3 qt. 1 pt. to gallons and the decimal of a gallon.

$$\begin{array}{r|l} 2 & 1.\text{ pt.} \\ 4 & 3.5\text{ qt.} \\ \hline & 7.875\text{ gal. } \textit{Ans.} \end{array}$$

2. Reduce £4.375 to pounds, shillings, and pence.

$$\begin{array}{r} £4.375 \\ 20 \\ \hline 7.5\text{ s.} \\ 12 \\ \hline 6\text{ d.} \end{array}$$

£4 7 s. 6 d. *Ans.*

3. Reduce 7.6875 gal. to gallons, quarts, and pints.

$$\begin{array}{r} 7.6875 \\ 4 \\ \hline 2.75 \\ 2 \\ \hline 1.5 \end{array}$$

7 gal. 2 qt. 1.5 pt. *Ans.*

4. If \$4.85 is equal to a pound, reduce to pounds, shillings, and pence \$5.875; \$7.38; \$17.85; \$21.75.

$$\begin{array}{r} 1.2\frac{11}{97} \\ 485\overline{)587.5} \\ 485 \\ \hline 1025 \\ 970 \\ \hline 55 \end{array}$$

$0.2\frac{11}{97} = \frac{41}{194}$.

£$\frac{41}{194}$ = $\frac{41}{194}$ of 20 s. = $4\frac{22}{97}$ s.

$\frac{22}{97}$ s. = $\frac{22}{97}$ of 12 d. = $2\frac{70}{97}$ d.

£1 4 s. $2\frac{70}{97}$ d. *Ans.*

$$\begin{array}{r} 1\frac{253}{485} \\ 485\overline{)738} \\ 485 \\ \hline 253 \end{array}$$

£$\frac{253}{485}$ = $\frac{253}{485}$ of 20 s. = $10\frac{42}{97}$ s.

$\frac{42}{97}$ s. = $\frac{42}{97}$ of 12 d. = $5\frac{19}{97}$ d.

£1 10 s. $5\frac{19}{97}$ d. *Ans.*

$$\begin{array}{r} 3\frac{66}{97} \\ 485\overline{)1785} \\ 1455 \\ \hline 330 \end{array}$$

£$\frac{66}{97}$ = $\frac{66}{97}$ of 20 s. = $13\frac{59}{97}$ s.

$\frac{59}{97}$ s. = $\frac{59}{97}$ of 12 d. = $7\frac{29}{97}$ d.

£3 13 s. $7\frac{29}{97}$ d. *Ans.*

$$\begin{array}{r} 4\frac{47}{97} \\ 485\overline{)2175} \\ 1940 \\ \hline 235 \end{array}$$

£$\frac{47}{97}$ = $\frac{47}{97}$ of 20 s. = $9\frac{67}{97}$ s.

$\frac{67}{97}$ s. = $\frac{67}{97}$ of 12 d. = $8\frac{28}{97}$ d.

£4 9 s. $8\frac{28}{97}$ d. *Ans.*

[redacted]

5. How many square yards in 3.7156 A.?

```
    3.7156
       160
  --------
   594.496
      30¼
  --------
 17983.504
```

6. If 2 qt. of linseed oil are mixed with ½ pt. spirits of turpentine, what fraction of the mixture is turpentine? How much turpentine in one pint of the mixture?

2 qt. + ½ pt. = 4½ pt.

$$\frac{\frac{1}{2}}{4\frac{1}{2}} = \frac{\cancel{2}}{9} \times \frac{1}{\cancel{2}} = \frac{1}{9}. \quad \textit{Ans.}$$

$\frac{1}{9}$ of 1 pt. = $\frac{1}{9}$ pt. *Ans.*

7. Reduce 5.1732 mi. to yards, feet, and inches.

```
   5.1732
     1760
 --------
 9104.832
        3
 --------
    2.496
       12
 --------
    5.952
```

9104 yd. 2 ft. 5.952 in. *Ans.*

8. If a man walks 88 mi. in 26 hr., how many feet does he walk in a second?

$$\frac{\overset{\overset{11}{\cancel{22}}}{\cancel{88}} \times \cancel{5280}}{\underset{13}{\cancel{26}} \times \cancel{60} \times \underset{15}{\cancel{60}}} \text{ ft.} = \frac{968}{195} \text{ ft.}$$

$= 4\frac{188}{195}$ ft. *Ans.*

9. Of a mixture of sand and lime 0.27 of the weight is lime. How many ounces of lime in a pound of the mixture? How many troy grains of lime in an avoirdupois pound of the mixture?

```
  16 oz.        7000 gr.
  0.27          0.27
  ----          ----
   112          1890 gr. Ans.
   32
  ----
  4.32 oz. Ans.
```

10. A gill of water is put into a quart measure, and the measure then filled with milk. What part of the mixture is water?

1 gi. = $\frac{1}{8}$ qt.

$\therefore$ $\frac{1}{8}$ is water.

11. Reduce 555 ft. to the decimal of a mile.

```
        0.1051136
 5280)555.5
       528
       ----
        2700
        2640
        ----
          600
          528
          ----
           720
           528
           ----
           1920
           1584
           ----
            3360
            3168
            ----
             192
```

$0.10511\dot{3}\dot{6}$ mi. *Ans.*

[redacted]

12. Reduce 1 mi. 13 rd. 2 yd. 2 ft. 6 in. to inches.

mi.	rd.	yd.	ft.	in.
1	13	2	2	6

$$\begin{array}{r} 1 \\ \underline{320} \\ 333 \\ \underline{5\frac{1}{2}} \\ 1833\frac{1}{2} \\ \underline{3} \\ 5502\frac{1}{2} \\ \underline{12} \\ 66036 \end{array}$$

66,036 in. *Ans.*

13. How many cubic inches in $2\frac{1}{2}$ cu. ft.?

$$\begin{array}{r} 1728 \text{ cu. in.} \\ \underline{2\frac{1}{2}} \\ 864 \\ \underline{3456} \\ 4320 \text{ cu. in.} \end{array}$$

4320 cu. in. *Ans.*

14. How many pounds avoirdupois does a cubic yard of water weigh if a cubic foot weighs 1000 oz.?

$$\begin{array}{r} 1000 \text{ oz.} \\ \underline{27} \\ 16\overline{)27000} \text{ oz.} \\ 1687\frac{1}{2} \text{ lb.} \end{array}$$

$1687\frac{1}{2}$ lb. *Ans.*

15. Express the weight of a cubic yard of water as the decimal of a ton.

$$1687\tfrac{1}{2} \text{ lb.} = \frac{1687.5}{2000} \text{ t.}$$
$$= 0.84375 \text{ t. } \textit{Ans.}$$

16. What is the weight of 7 bu. $3\frac{1}{2}$ pk. of potatoes?

$$3\tfrac{1}{2} \text{ pk.} = \frac{3\frac{1}{2}}{4} \text{ bu.} = \tfrac{7}{8} \text{ bu.}$$

$$\begin{array}{r} 60 \text{ lb.} \\ \underline{7\frac{7}{8}} \\ 52\frac{1}{2} \\ \underline{420} \\ 472\frac{1}{2} \text{ lb.} \end{array}$$

$472\frac{1}{2}$ lb. *Ans.*

17. A farmer sowed 5 bu. 1 pk. 1 qt. of seed, and harvested from it 103 bu. 3 pk. 5 qt. How much did he raise from a bushel of seed?

bu.	pk.	qt.
5	1	1
4		
21		
8		
169		

bu.	pk.	qt.
103	3	5
4		
415		
8		
3325		

$\frac{3325}{169}$ of 1 bu. $= \frac{3325}{169}$ bu. $= 19\frac{114}{169}$ bu.

$\frac{114}{169}$ bu. $= \frac{114}{169}$ of 4 pk. $= 2\frac{118}{169}$ pk.

$\frac{118}{169}$ pk. $= \frac{118}{169}$ of 8 qt. $= 5\frac{99}{169}$ qt. $= 5.6$ qt.

19 bu. 2 pk. 5.6 qt. *Ans.*

[redacted]

18. How many bushels in 5 tons of oats?

$$\frac{5 \times \overset{125}{\cancel{2000}}}{\underset{2}{\cancel{32}}} = \frac{625}{2} = 312\tfrac{1}{2}. \ \textit{Ans.}$$

19. How many bottles, each holding 1 pt. 3 gi., can be filled from a barrel of cider?

1 pt. 3 gi. $= 1\frac{3}{4}$ pt. $= \frac{7}{32}$ gal.

$$\frac{31\frac{1}{2}}{\frac{7}{32}} = \frac{\overset{16}{\cancel{32}}}{\cancel{7}} \times \frac{\overset{9}{\cancel{63}}}{\cancel{2}} = 144. \ \textit{Ans.}$$

20. If a steamer makes 13 mi. 6 rd. an hour, how far will she go between 6 A.M. and 6 P.M.? How many hours will she require to make 113 miles?

mi.	rd.
13	6
	12
156	72

156 mi. 72 rd. *Ans.*

13 mi. 6 rd. $= 13\frac{3}{160}$ mi.

$$\frac{113}{13\frac{3}{160}} = \frac{160}{2083} \times 113$$

$$= \frac{18080}{2083} = 8\tfrac{1416}{2083}. \ \textit{Ans.}$$

21. If a train runs at the average rate of 111 rd. a minute, how many hours will it require to run from Boston to Buffalo, 498 miles?

$$\frac{\overset{166}{\cancel{498}} \times \overset{16}{\cancel{320}}}{\underset{3}{\cancel{60}} \times \underset{37}{\cancel{111}}} = \frac{2656}{111} = 23\tfrac{103}{111}.$$

$\frac{103}{111}$ hr. $= \frac{103}{111}$ of 60 min.
$= 56$ min. nearly.

23 hr. 56 min. nearly. *Ans.*

22. What is the cost of 12 A. 146 sq. rd. of land at $16.25 an acre?

146 sq. rd. $= \frac{73}{80}$ A.

$$12\tfrac{73}{80} \times \$16\tfrac{1}{4} = \frac{1033}{\underset{16}{\cancel{80}}} \times \$\frac{\overset{13}{\cancel{65}}}{4}$$

$$= \$\frac{13429}{64} = \$209.83. \ \textit{Ans.}$$

23. What is the cost of 8 t. 3 cwt. 27 lb. of coal at $5.75 a ton?

$$\begin{array}{r|l} 100 & 27.\text{ lb.} \\ 20 & \underline{3.27\text{ cwt.}} \\ & 8.1635\text{ t.} \end{array}$$

$$\begin{array}{r} 8.1635 \\ \underline{5.75} \\ 408175 \\ 571445 \\ \underline{408175} \\ 46.940125 \end{array}$$

$46.94. *Ans.*

24. What is the cost of 7 t. 1560 lb. of hay at $15.50 a ton?

1560 lb. $= \frac{1560}{2000}$ t. $= \frac{78}{100}$ t.

$$\begin{array}{r} \$15.50 \\ \underline{7.78} \\ 12400 \\ 10850 \\ \underline{10850} \\ \$120.59 \end{array}$$

$120.59 *Ans.*

25. What is the cost of a car load of wheat weighing 20,000 lb., at $1.05 a bushel?

$\$1.05 = \$1\frac{1}{20} = \$\frac{21}{20}$.

$$\frac{\overset{\overset{50}{\cancel{1000}}}{\cancel{20000}}}{\underset{\underset{\cancel{20}}{}}{\cancel{60}}} \times \$\frac{\overset{7}{\cancel{21}}}{\cancel{20}} = \$350. \ \textit{Ans.}$$

[redacted]

26. Reduce 5 rd. 4 yd. 2½ ft. to the decimal of a mile.

$$\begin{array}{r|l} 3 & 2.5 \text{ ft.} \\ 5\frac{1}{2} & 4.8333 \text{ yd.} \\ 320 & 5.8787 \text{ rd.} \\ \hline & 0.0184 \text{ mi. } \textit{Ans.} \end{array}$$

27. Reduce 9 sq. ch. 11.25 sq. rd. to the decimal of an acre.

$$\begin{array}{r|l} 16 & 11.25 \text{ sq. rd.} \\ 10 & 9.703125 \text{ sq. ch.} \\ \hline & 0.9703125 \text{ A. } \textit{Ans.} \end{array}$$

28. Reduce 0.09375 bu. to quarts.

$$\begin{array}{r} 0.09375 \\ 32 \\ \hline 18750 \\ 28125 \\ \hline 3. \end{array} \qquad 3 \text{ qt. } \textit{Ans.}$$

29. Reduce 7560 chains to miles.

$$\begin{array}{r|l} 80 & 7560 \text{ ch.} \\ \hline & 94.5 \text{ mi. } \textit{Ans.} \end{array}$$

30. How many gross are 2000 pens?

$$\frac{\overset{125}{\cancel{\overset{500}{\cancel{2000}}}}}{\underset{3}{\cancel{12}} \times \underset{3}{\cancel{12}}} = \frac{125}{9} = 13\tfrac{8}{9}. \ \textit{Ans.}$$

31. Find the cost of 27.248 A., at $93.75 an acre.

$$\begin{array}{r} 27.248 \\ 93.75 \\ \hline 136240 \\ 190736 \\ 81744 \\ 245232 \\ \hline 2554.5 \end{array}$$

$2554.50. *Ans.*

32. Which is the greater, 2.8 of 3 ft. 11 in. or 3.11 of 2 ft. 8 in., and by how much?

3 ft. 11 in. = 47 in.;

2 ft. 8 in. = 32 in.

$$\begin{array}{r} 47 \text{ in.} \\ 2.8\phantom{ \text{ in.}} \\ \hline 376\phantom{ \text{ in.}} \\ 94\phantom{0 \text{ in.}} \\ \hline 131.6 \text{ in.} \end{array} \qquad \begin{array}{r} 32 \text{ in.} \\ 3.11\phantom{ \text{ in.}} \\ \hline 32\phantom{ \text{ in.}} \\ 32\phantom{0 \text{ in.}} \\ 96\phantom{00 \text{ in.}} \\ \hline 99.52 \text{ in.} \end{array}$$

131.6 in. − 99.52 in. = 32.08 in.
= 2 ft. 8.08 in.

Therefore, 2.8 of 3 ft. 11 in. is the greater by 2 ft. 8.08 in. *Ans.*

33. Reduce 171 lb. 6 oz. troy to the decimal of a ton avoirdupois.

171 lb. 6 oz. = 171½ lb.

$$\frac{171\frac{1}{2} \times 5760}{7000} = \frac{\overset{49}{\cancel{343}}}{\cancel{2}} \times \frac{\overset{2880}{\cancel{5760}}}{\underset{1000}{\cancel{7000}}} = 141.12.$$

$$\begin{array}{r|l} 100 & 141.12 \text{ lb.} \\ 20 & 1.4112 \text{ cwt.} \\ \hline & 0.07056 \text{ t. } \textit{Ans.} \end{array}$$

34. Express 14.52 sq. yd. as the decimal of a square chain.

$$14.52 \text{ sq. yd.} = \frac{14.52}{30\frac{1}{4}} \text{ sq. rd.}$$

$$= \frac{14.52}{16 \times 30\frac{1}{4}} \text{ sq. ch.}$$

$$= \frac{14.52}{484} \text{ sq. ch.}$$

$$= 0.03 \text{ sq. ch. } \textit{Ans.}$$

[redacted]

35. If a sovereign is equal to 25.22 francs, or to $4.85, what decimal of a dollar is a franc?

$$\begin{array}{r} 0.192 \text{ } \textit{Ans.} \\ 2522)\overline{485.} \\ 2522 \\ \hline 23280 \\ 22698 \\ \hline 5820 \\ 5044 \\ \hline 776 \end{array}$$

36. If 0.327 of some work is done in 3 hr. 38 min., how long will the whole work require?

3 hr. 38 min. = 218 min.

$$218 \text{ min.} \div 0.327 = \frac{1000}{\underset{3}{\cancel{327}}} \text{ of } \overset{2}{\cancel{218}} \text{ min.}$$

$$= \frac{2000}{3} \text{ min.} = 666\tfrac{2}{3} \text{ min.}$$

= 11 hr. 6 min. 40 sec. *Ans.*

37. A can run a mile in 7.68 min.; B can run at the rate of 7.68 mi. an hour. Which is the faster runner?

$$\begin{array}{r} 7.8 \\ 768)\overline{6000.} \\ 5376 \\ \hline 6240 \\ 6144 \\ \hline 96 \end{array}$$

Therefore, A is the faster runner.

38. How many miles an hour does a person walk who takes 2 steps a second and 1900 steps in a mile?

$$\frac{\overset{6}{\cancel{60}} \times \overset{6}{\cancel{60}} \times 2}{\underset{\underset{19}{\cancel{190}}}{\cancel{1900}}} = \frac{72}{19} = 3\tfrac{15}{19}. \text{ } \textit{Ans.}$$

39. If an ounce troy of gold is worth $20, what is the value of a pound avoirdupois?

$$\begin{array}{r} \$20 \\ 12 \\ \hline \$240 \end{array} \text{ per lb. troy.}$$

$$\frac{\overset{175}{\cancel{7000}}}{\underset{\underset{3}{\cancel{144}}}{\cancel{5760}}} \times \$\overset{5}{\cancel{240}} = \$\frac{875}{3} = \$291.67.$$

Ans.

40. Two stars cross the meridian at 6 hr. 4 min. 42.3 sec. and 7 hr. 2 min. 57.21 sec., respectively. What is the interval between the observations?

hr.	min.	sec.
7	2	57.21
6	4	42.3
	58	14.91

58 min. 14.91 sec. *Ans.*

41. How long will it take to fill $\frac{13}{25}$ of a cistern, when the whole requires 6 hr. 10 min.?

6 hr. 10 min. = $6\frac{1}{6}$ hr.

$\frac{13}{25}$ of $6\frac{1}{6} = \frac{13}{25} \times \frac{37}{6} = \frac{481}{150} = 3\frac{31}{150}$.

$\frac{31}{150}$ hr. = $\frac{31}{150}$ of 60 min. = $12\frac{2}{5}$ min.

$\frac{2}{5}$ min. = $\frac{2}{5}$ of 60 sec. = 24 sec.

3 hr. 12 min. 24 sec. *Ans.*

[redacted]

42. The circumference of a circle is 6 yd. 1 ft. 5.1 in. What is the length of 55°?

yd.	ft.	in.
6	1	5.1
3		
19		
12		
233.1		

$$\frac{55}{360} \text{ of } 233.1 = \frac{\overset{11}{\cancel{55}}}{\underset{\underset{8}{\cancel{72}}}{\cancel{360}}} \times \frac{\overset{259}{\cancel{2331}}}{10}$$

$$= \frac{2849}{80} = 35\tfrac{49}{80}.$$

$35\frac{49}{80}$ in. = 2 ft. $11\frac{49}{80}$ in. *Ans.*

43. Multiply 2 t. 16 cwt. $63\frac{2}{5}$ lb. by $1\frac{4}{9}$.

	t.	cwt.	lb.
	2	16	$63\frac{2}{5}$
			4
9	11	6	$53\frac{3}{5}$
	1	5	$17\frac{1}{15}$
	2	16	$63\frac{2}{5}$
	4	1	$80\frac{7}{15}$

4 t. 1 cwt. $80\frac{7}{15}$ lb. *Ans.*

44. Into how many shares has £ 120 been divided when each share is £ 3 8 s. $6\frac{6}{7}$ d.

$$6\tfrac{6}{7}\text{ d.} = \frac{6\frac{6}{7}}{12}\text{s.} = \tfrac{4}{7}\text{ s.}$$

$$8\tfrac{4}{7}\text{ s.} = £\frac{8\frac{4}{7}}{20} = £\,\tfrac{3}{7}.$$

$$\frac{120}{3\frac{3}{7}} = \frac{7}{\cancel{24}} \times \overset{5}{\cancel{120}} = 35. \quad \textit{Ans.}$$

45. If $\frac{14}{15}$ of one line is equal to $\frac{8}{9}$ of another line, which is the greater? What fraction of the greater is the less?

$$\frac{14}{15}, \frac{8}{9} = \frac{42, 40}{45}.$$

∴ the second line is the greater. *Ans.*

$$\frac{\frac{8}{9}}{\frac{14}{15}} = \frac{\overset{4}{\cancel{8}}}{\underset{3}{\cancel{9}}} \times \frac{\overset{5}{\cancel{15}}}{\underset{7}{\cancel{14}}} = \frac{20}{21}. \quad \textit{Ans.}$$

46. Multiply 5 mi. 206 rd. 2 ft. 2 in. by 786.

mi.	rd.	ft.	in.
5	206	2	2
			786
4436	99	$3\cancel{\frac{1}{2}}$	0
			6
4436	99	3	6

4436 mi. 99 rd. 3 ft. 6 in. *Ans.*

47. The returns of a gold mine are 241 t. of ore yielding 2 oz. 1 dwt. 15 gr. of fine gold a ton, and 193 t. yielding 1 oz. 12 dwt. 9 gr. a ton. Find the value of the whole yield, at $19.45 an ounce.

lb.	oz.	dwt.	gr.
	2	1	15
			241
41	9	11	15

lb.	oz.	dwt.	gr.
	1	12	9
			193
26	0	8	9
41	9	11	15
67	10		

67 lb. 10 oz. = 814 oz.

```
 $19.45
    814
   7780
  1945
 15560
$15832.30  Ans.
```

[redacted]

48. Divide 93 long tons 56 lb. by 23 lb. 5 oz.

t.	lb.	lb.	oz.
93	56	23	5
2240		16	
208376		373	
16			
3334016			

$$\begin{array}{r} 8938\tfrac{142}{373} \quad \textit{Ans.} \\ 373\overline{)3334016} \\ 2984 \\ \hline 3500 \\ 3357 \\ \hline 1431 \\ 1119 \\ \hline 3126 \\ 2984 \\ \hline 142 \end{array}$$

49. Telegraph poles on railroads are generally erected at intervals of 88 yd. Show that if a passenger counts the number of poles which the train passes in three minutes, that number will express the number of miles an hour the train is going.

Since 1 mi. = 1760 yd., 88 yd. = $\frac{1}{20}$ of a mile, and there are 20 poles to the mile. Since 1 hr. = 60 min., 3 min. = $\frac{1}{20}$ of an hour. Hence, the number of poles passed by in 3 min. expresses the rate of the train in miles per hour.

50. If Greenwich time is 5 hr. 8 min. 16 sec. later than Washington time, and Chicago is 87° 35′ W., what is the difference between Washington time and Chicago time?

$$\begin{array}{r|lll} 15 & 87^\circ & 35' & \\ \hline & 5 \text{ hr.} & 50 \text{ min.} & 20 \text{ sec.} \end{array}$$

hr.	min.	sec.	
5	50	20	
5	8	16	
	42	4	42 min. 4 sec. *Ans.*

[redacted]

51. What fraction of 21 cu. yd. 11 cu. ft. 1215 cu. in. is 3 cu. yd. 1 cu. ft. 1161 cu. in.?

cu. yd.	cu. ft.	cu. in.	cu. yd.	cu. ft.	cu. in.
21	11	1215	3	1	1161
27			27		
578			82		
1728			1728		
999999			142857		

$\frac{142857}{999999} = \frac{1}{7}$. *Ans.*

52. How many minutes in the first three months of 1895? How many in the first three months of 1896?

Jan. 31 dy.	Jan. 31 dy.
Feb. 28	Feb. 29
Mar. 31	Mar. 31
90 dy.	91 dy.

24	24
60	60
1440	1440
90	91
129600	1440
	12960
	131040

129,600 min.; 131,040 min. *Ans.*

53. A knot is $\frac{1}{60}$ of a degree, and a mile is 0.01477 of a degree. Find in miles the value of a knot to five decimals.

$$1 \text{ knot} = \tfrac{1}{60}^\circ = 0.01\dot{6}^\circ.$$

$$1 \text{ mi.} = 0.01477^\circ.$$

```
               1.12841
   01477)01666.66666
          1477
           1896
           1477
            4196
            2954
            12426
            11816
              6106
              5908
               1986
               1477
                509
```

1.12841 mi. *Ans.*

[redacted]

54. The captain of a steamer, sailing from Liverpool, found on taking an observation that the sun crossed his meridian at 42 min. 5 sec. past one o'clock P.M. by Greenwich time. Find his longitude.

Time on the steamer was 1 hr. 42 min. 5 sec. later than Greenwich time.

$$\begin{array}{lll} 1 \text{ hr.} & 42 \text{ min.} & 5 \text{ sec.} \\ & & 15 \\ \hline 25^\circ & 31' & 15'' \end{array}$$

$25^\circ\ 31'\ 15''$ W. *Ans.*

55. If a walk 6 ft. wide is made round a park 600 ft. square within the enclosure, how many square yards will the walk contain?

$$600 \text{ ft.} = 200 \text{ yd.}; \ 6 \text{ ft.} = 2 \text{ yd.}$$

$$2(200 + 196) \times 2 = 1584.$$

1584 sq. yd. *Ans.*

56. How many pickets 3 in. wide, placed 3 in. apart, will be required to fence a rectangular lot 231 ft. long and 99 ft. wide? What will they cost at \$3.25 per hundred?

Each picket occupies with its space 3 in. + 3 in. = 6 in. = $\frac{1}{2}$ ft.

Perimeter $= 2 \times (231 + 99)$ ft. $= 660$ ft.

$660 + \frac{1}{2} = 2 \times 660 = 1320$. *Ans.*

$$\begin{array}{r} \$3.25 \\ 13.2 \\ \hline 650 \\ 975 \\ 325 \\ \hline \$42.90 \end{array}$$ *Ans.*

57. The length of a year is 365.242218 mean solar days. Express the length of a year in days, hours, minutes, and seconds.

$$\begin{array}{r} 365.242218 \\ 24 \\ \hline 968872 \\ 484436 \\ \hline 5.813232 \\ 60 \\ \hline 48.79392 \\ 60 \\ \hline 47.6352 \end{array}$$

365 dy. 5 hr. 48 min. 47.6352 sec. *Ans.*

[redacted]

58. The Flying Dutchman Express runs from London to Exeter, a distance of 193⅓ mi., in 4¼ hr., making one stop of 10 min., two of 5 min. each, and one of 3 min. What is its average speed per hour when in motion?

The time lost at stations = 1 × 10 min. + 2 × 5 min. + 1 × 3 min. = 23 min.

Actual running time = 4¼ hr. − 23 min. = 4 hr. 15 min. − 23 min. = 3 hr. 52 min.

$$193\tfrac{1}{3} \div 3\tfrac{52}{60} = \frac{\overset{5}{\cancel{580}}}{\cancel{3}} \times \frac{\overset{\overset{10}{\cancel{20}}}{\cancel{60}}}{\underset{\cancel{2}}{\cancel{232}}} = 50.$$

50 mi. *Ans.*

59. The Scotch Express runs from London to Edinburgh, a distance of 393⅗ mi., in 9 hr., making one stop of 30 min., three of 5 min. each, and one of 3 min. What is its average speed per hour when in motion?

The time lost at stations = 1 × 30 min. + 3 × 5 min. + 1 × 3 min. = 48 min.

Actual running time = 9 hr. − 48 min. = 8 hr. 12 min. = 8⅕ hr.

$$393\tfrac{3}{5} \div 8\tfrac{1}{5} = \frac{\overset{48}{\cancel{1968}}}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{41}} = 48.$$

48 mi. *Ans.*

60. The Empire State Express runs from New York to Buffalo, a distance of 439 mi., in 8 hr. 15 min., making two stops of 3 min. each, and two stops of 2 min. each. What is its average speed per hour when in motion?

The time lost at stations = 2 × 3 min. + 2 × 2 min. = 10 min.

Actual running time = 8 hr. 15 min. − 10 min. = 8 hr. 5 min. = $8\frac{1}{12}$ hr.

$$439 \div 8\tfrac{1}{12} = 439 \times \tfrac{12}{97} = \tfrac{5268}{97} = 54\tfrac{30}{97} = 54.31.$$

54.31 mi. *Ans.*

61. How many dollars worth 4 s. 2 d. each will pay a bill of £11 17 s. 6 d.?

s.	d.	£	s.	d.
4	2	11	17	6
12		20		
50		237		
		12		
		2850		

2850 ÷ 50 = 57.

$57. *Ans.*

[redacted]

62. The lunar month is 29.53059 days. Express the length of a lunar month in days, hours, minutes, and seconds.

$$\begin{array}{r} 29.53059 \\ 24 \\ \hline 212236 \\ 106118 \\ \hline 12.73416 \\ 60 \\ \hline 44.0496 \\ 60 \\ \hline 2.976 \end{array}$$

29 dy. 12 hr. 44 min. 2.976 sec. *Ans.*

Exercise 87. Page 179.

1. If 15 yards of silk cost \$18.75, what will be the cost of $20\frac{1}{3}$ yards?

If 15 yd. of silk cost $\$18\frac{3}{4}$, 1 yd. will cost $\frac{1}{15}$ of $\$18\frac{3}{4}$, and $20\frac{1}{3}$ yd. will cost $20\frac{1}{3} \times \frac{1}{15} \times \$18\frac{3}{4}$.

$$20\tfrac{1}{3} \times \frac{1}{15} \times \$18\tfrac{3}{4} = \frac{61}{3} \times \frac{1}{\cancel{15}} \times \$\frac{\overset{5}{\cancel{75}}}{4} = \$\frac{305}{12} = \$25.42. \textit{ Ans.}$$

2. If $3\frac{2}{5}$ pounds of tea cost \$3.80, how many pounds can be bought for \$21.89?

If $3\frac{2}{5}$ lb. of tea cost $\$3\frac{4}{5}$, 1 lb. costs $\$\frac{3\frac{4}{5}}{3\frac{2}{5}}$, and as many pounds can be bought for \$21.89 as $21.89 \div \frac{3\frac{4}{5}}{3\frac{2}{5}}$.

$$\$21.89 \div \frac{3\frac{4}{5}}{3\frac{2}{5}} = \frac{\cancel{5}}{19} \times \frac{17}{\cancel{5}} \times \$\frac{2189}{100} = \$\frac{37213}{1900} = \$19.59. \textit{ Ans.}$$

3. If $\frac{3}{14}$ of a ton of coal costs \$1.12, what is the price of $5\frac{1}{2}$ cwt.?

$$5\tfrac{1}{2} \text{ cwt.} = \frac{5\frac{1}{2}}{20} \text{ t.} = \frac{11}{40} \text{ t.}$$

If $\frac{3}{14}$ t. of coal costs \$1.12, 1 t. costs $\$1.12 \div \frac{3}{14}$ and $\frac{11}{40}$ t. costs $\frac{11}{40} \times (\$1.12 \div \frac{3}{14})$.

$$\frac{11}{40} \times \left(\$1.12 \div \frac{3}{14}\right) = \frac{11}{\underset{5}{\cancel{40}}} \times \frac{14}{3} \times \$\overset{0.14}{\cancel{1.12}} = \$\frac{21.56}{15} = \$1.44. \textit{ Ans.}$$

[redacted]

4. If $\frac{2}{11}$ of a piece of work is done in 25 days, what fraction of the work will be done in $11\frac{2}{3}$ days?

If $\frac{2}{11}$ of the work is done in 25 dy., the fraction that can be done in $11\frac{2}{3}$ dy. is $\frac{11\frac{2}{3}}{25} \times \frac{2}{11}$.

$$\frac{11\frac{2}{3}}{25} \times \frac{2}{11} = \frac{\overset{7}{\cancel{35}}}{3} \times \frac{1}{\underset{5}{\cancel{25}}} \times \frac{2}{11} = \frac{14}{165}. \quad \textit{Ans.}$$

5. A bankrupt's debts are \$2520, and the value of his property is \$1890. How much can he pay on a dollar?

He can pay on each dollar $\$\frac{1890}{2520}$.

$$\frac{\overset{3}{\cancel{1890}}}{\underset{4}{\cancel{2520}}} \text{ of } \$\overset{0.25}{\cancel{1.00}} = \$0.75. \quad \textit{Ans.}$$

6. If a bankrupt's debts are \$4264, and he pays $62\frac{1}{2}$ cents on a dollar, what are his assets?

$\$0.62\frac{1}{2} = \$\frac{5}{8}$.

$$\frac{5}{\cancel{8}} \text{ of } \$\overset{533}{\cancel{4264}} = \$2665. \quad \textit{Ans.}$$

7. If an ounce of gold is worth \$20.67, what is the value of 0.04 of a pound?

0.04 lb. troy = 0.04 of 12 oz. = 0.48 oz.

$$\begin{array}{r} \$20.67 \\ 0.48 \\ \hline 16536 \\ 8268 \\ \hline \$9.9216 \end{array} \qquad \$9.92. \ \textit{Ans.}$$

8. A man spent $\frac{4}{9}$ of his money for dry goods, $\frac{3}{4}$ of the remainder for groceries, and had \$15 left. How much had he at first?

After spending $\frac{4}{9}$ of his money he had $\frac{5}{9}$ left. After spending $\frac{3}{4}$ of $\frac{5}{9}$ of his money he had left $\frac{1}{4}$ of $\frac{5}{9} = \frac{5}{36}$. Then, $\$15 = \frac{5}{36}$ of the money he had at first.

$$\$15 \div \frac{5}{36} = \frac{36}{\cancel{5}} \times \$\overset{3}{\cancel{15}} = \$108. \quad \textit{Ans.}$$

[redacted]

9. Sampson & Reed sold $\frac{4}{7}$ of a lot of wheat to one man, $\frac{1}{2}$ of the remainder to another, and had 84 bushels left. How much had they at first?

After selling $\frac{4}{7}$ of the wheat they had $\frac{3}{7}$ left. After selling $\frac{1}{2}$ of $\frac{3}{7}$ they had left $\frac{1}{2}$ of $\frac{3}{7} = \frac{3}{14}$. Then, 84 bu. $= \frac{3}{14}$ of the lot.

$$84 \text{ bu.} \div \frac{3}{14} = \frac{14}{3} \times \overset{28}{84} \text{ bu.} = 392 \text{ bu. } Ans.$$

10. In a certain school $\frac{9}{16}$ of the scholars are girls; $\frac{4}{7}$ of the boys are over 16 years old, and 6 boys are under 16. How many girls and how many scholars are there in the school?

Since $\frac{9}{16}$ of the scholars are girls, $\frac{7}{16}$ are boys. Since $\frac{4}{7}$ of the boys are over 16 yr. old, $\frac{3}{7}$ of the boys are under 16; that is, $\frac{3}{7}$ of $\frac{7}{16} = \frac{3}{16}$ of the scholars.

Therefore, 6 is $\frac{3}{16}$ of the number of scholars.

$$6 \div \frac{3}{16} = \frac{16}{3} \times \overset{2}{6} = 32.$$

$$\frac{9}{16} \times \overset{2}{32} = 18. \qquad 18;\ 32. \text{ Ans.}$$

11. In a certain school $\frac{13}{24}$ of the scholars are boys; $\frac{9}{22}$ of the girls are under 16, and 13 girls are over 16. How many boys and how many girls are there in the school?

Since $\frac{13}{24}$ of the scholars are boys, $\frac{11}{24}$ are girls. Since $\frac{9}{22}$ of the girls are under 16, $\frac{13}{22}$ of the girls are over 16; that is,

$$\frac{13}{\underset{2}{22}} \text{ of } \frac{11}{24} = \frac{13}{48} \text{ of the scholars.}$$

Therefore, 13 is $\frac{13}{48}$ of the number of scholars.

$$13 \div \frac{13}{48} = \frac{48}{13} \times 13 = 48.$$

$$\text{Hence, the number of boys is } \frac{13}{24} \text{ of } \overset{2}{48} = 26. \text{ Ans.}$$

$$\text{Hence, the number of girls is } \frac{11}{24} \text{ of } \overset{2}{48} = 22. \text{ Ans.}$$

[redacted]

12. If from a certain number $\frac{3}{4}$ of it is subtracted, then $\frac{1}{5}$ of the remainder, then $\frac{1}{7}$ of that remainder, 6 still remains. What is the number ?

After $\frac{3}{4}$ of the number is subtracted $\frac{1}{4}$ is left.

After $\frac{1}{5}$ of $\frac{1}{4}$ is subtracted $\frac{4}{5}$ of $\frac{1}{4} = \frac{1}{5}$ is left.

After $\frac{1}{7}$ of $\frac{1}{5}$ is subtracted $\frac{6}{7}$ of $\frac{1}{5} = \frac{6}{35}$ is left.

$$\text{Therefore, the number} = 6 \div \frac{6}{35} = \frac{35}{\cancel{6}} \times \cancel{6} = 35. \quad \textit{Ans.}$$

13. A ship's cargo sold for \$45,000 belongs to three partners. A owns $\frac{7}{9}$ of $\frac{3}{5}$ of it, B's share is equal to $3\frac{3}{14}$ of $\frac{2}{9}$ of A's share, and C owns the remainder. What does each receive from the sale ?

$$\frac{7}{\underset{3}{\cancel{9}}} \text{ of } \frac{\cancel{3}}{5} = \frac{7}{15}. \quad \frac{7}{\cancel{15}} \text{ of } \$\overset{3000}{\cancel{45000}} = \$21{,}000, \text{ A's. } \textit{Ans.}$$

$$3\tfrac{3}{14} \text{ of } \frac{2}{9} \text{ of } \$21{,}000 = \frac{\overset{5}{\cancel{45}}}{\cancel{14}} \times \frac{2}{\cancel{9}} \times \$\overset{1500}{\cancel{21000}} = \$15{,}000, \text{ B's. } \textit{Ans.}$$

$$\$45{,}000 - \$21{,}000 - \$15{,}000 = \$9000, \text{ C's. } \textit{Ans.}$$

14. A man bequeathed $\frac{5}{12}$ of his property to A, $\frac{1}{4}$ of it to B, $\frac{1}{6}$ to C, $\frac{1}{8}$ to D, and the remainder, \$550, to E. What was the value of his whole property ?

$$\frac{5}{12} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} = \frac{10 + 6 + 4 + 3}{24} = \frac{23}{24}.$$

Therefore, \$550 is $\frac{1}{24}$ of the property.

$$\$550 \div \tfrac{1}{24} = 24 \times \$550 = \$13{,}200. \quad \textit{Ans.}$$

15. A farmer raised 321 bu. 3 pk. of corn from 9 acres of land. At the same rate, what would be the yield from 25 acres ?

321 bu. 3 pk. = $321\frac{3}{4}$ bu.

If 9 A. yield $321\frac{3}{4}$ bu., 1 A. will yield $\frac{1}{9}$ of $321\frac{3}{4}$ bu., and 25 A. will yield $25 \times \frac{1}{9} \times 321\frac{3}{4}$ bu.

$$25 \times \frac{1}{9} \times 321\tfrac{3}{4} \text{ bu.} = 25 \times \frac{1}{\cancel{9}} \times \frac{\overset{143}{\cancel{1287}}}{4} \text{ bu.} = \frac{3575}{4} \text{ bu.} = 893\tfrac{3}{4} \text{ bu. } \textit{Ans.}$$

[redacted]

16. If 7 horses eat 21 bushels of oats in 16 days, how many days will 99 bu. 3 pk. last them?

$$99 \text{ bu. } 3 \text{ pk.} = 99\tfrac{3}{4} \text{ bu.}$$

If 21 bu. of oats last 16 days, $99\frac{3}{4}$ bu. will last $\frac{99\frac{3}{4}}{21} \times 16$ days.

$$\frac{99\frac{3}{4}}{21} \times 16 \text{ days} = \frac{\overset{19}{\cancel{399}}}{\underset{4}{\cancel{84}}} \times \overset{4}{\cancel{16}} \text{ days} = 76 \text{ days. } \textit{Ans.}$$

17. If 12 horses can plow 96 acres in 6 days, how many horses will plow 64 acres in 8 days?

In 6 days 96 acres can be plowed by 12 horses.

In 1 day 96 acres can be plowed by 6×12 horses.

In 1 day 1 acre can be plowed by $\frac{6 \times 12}{96}$ horses.

In 8 days 1 acre can be plowed by $\frac{6 \times 12}{8 \times 96}$ horses.

In 8 days 64 acres can be plowed by $\frac{64 \times 6 \times 12}{8 \times 96}$ horses.

$$\frac{\overset{8}{\cancel{64}} \times 6 \times \cancel{12}}{\cancel{8} \times \underset{12}{\cancel{96}}} = 6. \ \textit{Ans.}$$

18. If 40 acres of grass is mowed by 8 men in 7 days, how many acres will be mowed by 24 men in 28 days?

24 men will mow *three times* as much as 8 men in the same time; the same number of men will mow *four times* as much in 28 days as in 7 days. Hence, 24 men in 28 days will mow 3×4 or 12 times as much as 8 men in 7 days.

$$12 \times 40 \text{ A.} = 480 \text{ A. } \textit{Ans.}$$

19. How many bushels of wheat will serve 72 people 8 days when 4 bushels serve 6 people 24 days?

72 people require 12 times as much wheat as 6 people for the same time; the same number of people require $\frac{1}{3}$ as much wheat for 8 days as for 24 days. Hence for 8 days 72 people require

$$\overset{4}{\cancel{12}} \times \frac{1}{\cancel{3}} \times 4 \text{ bu.} = 16 \text{ bu. } \textit{Ans.}$$

[redacted]

20. If 2 horses eat 8 bushels of oats in 16 days, how many horses will eat 3000 bushels in 24 days?

In 16 dy. 8 bu. will be eaten by 2 horses.

In 1 dy. 8 bu. will be eaten by 16×2 horses.

In 1 dy. 1 bu. will be eaten by $\frac{16 \times 2}{8}$ horses.

In 24 dy. 1 bu. will be eaten by $\frac{16 \times 2}{24 \times 8}$ horses.

In 24 dy. 3000 bu. will be eaten by $\frac{3000 \times 16 \times 2}{24 \times 8}$ horses.

$$\frac{\overset{125}{\cancel{3000}} \times \overset{2}{\cancel{16}} \times 2}{\cancel{24} \times \cancel{8}} = 500. \ \textit{Ans.}$$

21. If a man travels 150 miles in 5 days of 12 hours, in how many days of 10 hours will he travel 500 miles?

In 1 day of 12 hr. he travels $\frac{150}{5}$ mi.

In 1 day of 10 hr. he travels $\frac{10}{12} \times \frac{150}{5}$ mi.

Hence, to travel 500 mi., the number of days he will require is

$$500 \div (\tfrac{10}{12} \times \tfrac{150}{5}).$$

$$500 \div (\tfrac{10}{12} \times \tfrac{150}{5}) = \overset{50}{\cancel{500}} \times \frac{\overset{4}{\cancel{12}}}{\cancel{10}} \times \frac{5}{\underset{3}{\cancel{150}}} = 20. \ \textit{Ans.}$$

22. If 939 soldiers consume 351 bu. of wheat in 21 days, how many soldiers will consume 1404 bu. in 7 days?

1404 bu. will last the same number of men 4 times as long as 351 bu.; the same number of bushels will last three times the number of men for 7 days as for 21 days. Hence, 1404 bu. will last 3×4 times the number of men for 7 days that 351 bu. will last 939 men for 21 days.

$$3 \times 4 \times 939 = 11{,}268. \ \textit{Ans.}$$

23. If 5 men, working 16 hours a day, can reap a field of $12\frac{1}{2}$ acres in $3\frac{1}{2}$ days, in how many days can 7 men, working 12 hours a day, reap a field of 15 acres?

[redacted]

5 men can reap 12½ A. in 3½ days of 16 hr. = 56 hr.

1 man can reap 12½ A. in 5 × 56 hr.

1 man can reap 1 A. in $\frac{5 \times 56}{12\frac{1}{2}}$ hr.

1 man can reap 15 A. in $\frac{15 \times 5 \times 56}{12\frac{1}{2}}$ hr.

7 men can reap 15 A. in $\frac{15 \times 5 \times 56}{7 \times 12\frac{1}{2}}$ hr.

7 men can reap 15 A. in $\frac{15 \times 5 \times 56}{12 \times 7 \times 12\frac{1}{2}}$ days of 12 hr.

$$\frac{15 \times 5 \times 56}{12 \times 7 \times 12\frac{1}{2}} = \frac{\overset{3}{\cancel{15}} \times \cancel{5} \times \overset{\overset{2}{\cancel{8}}}{\cancel{56}} \times 2}{\underset{4}{\cancel{12}} \times \cancel{7} \times \underset{5}{\cancel{25}}} = 4. \ \textit{Ans.}$$

24. If 7 men in 8 days of 11 hours mow 22 acres, in how many days of 10 hours will 12 men mow 360 acres?

7 men can mow 22 A. in 8 days of 11 hr. = 88 hr.

1 man can mow 22 A. in 7 × 88 hr.

1 man can mow 1 A. in $\frac{7 \times 88}{22}$ hr.

12 men can mow 1 A. in $\frac{7 \times 88}{12 \times 22}$ hr.

12 men can mow 360 A. in $\frac{360 \times 7 \times 88}{12 \times 22}$ hr.

12 men can mow 360 A. in $\frac{360 \times 7 \times 88}{10 \times 12 \times 22}$ days of 10 hr.

$$\frac{\overset{\overset{3}{\cancel{36}}}{\cancel{360}} \times 7 \times \overset{4}{\cancel{88}}}{\cancel{10} \times \cancel{12} \times \cancel{22}} = 84. \ \textit{Ans.}$$

25. If 44 cannon, firing 30 rounds an hour for 3 hours a day, use 300 barrels of powder in 5 days, how many days will 400 barrels last *66 cannon, firing* 40 rounds an hour for 5 hours a day?

[redacted]

44 cannon firing 30 rounds for 3 hr. consume
300 bbl. in 5 days.

44 cannon firing 30 rounds for 1 hr. consume
300 bbl. in 3×5 days.

44 cannon firing 1 round for 1 hr. consume
300 bbl. in $30 \times 3 \times 5$ days.

1 cannon firing 1 round for 1 hr. consumes
300 bbl. in $44 \times 30 \times 3 \times 5$ days.

1 cannon firing 1 round for 1 hr. consumes
1 bbl. in $\frac{44 \times 30 \times 3 \times 5}{300}$ days.

66 cannon firing 1 round for 1 hr. consume
1 bbl. in $\frac{44 \times 30 \times 3 \times 5}{300 \times 66}$ days.

66 cannon firing 40 rounds for 1 hr. consume
1 bbl. in $\frac{44 \times 30 \times 3 \times 5}{40 \times 300 \times 66}$ days.

66 cannon firing 40 rounds for 5 hr. consume
1 bbl. in $\frac{44 \times 30 \times 3 \times 5}{5 \times 40 \times 300 \times 66}$ days.

66 cannon firing 40 rounds for 5 hr. consume
400 bbl. in $\frac{400 \times 44 \times 30 \times 3 \times 5}{5 \times 40 \times 300 \times 66}$ days.

$$\frac{\overset{10}{\cancel{400}} \times \overset{2}{\cancel{44}} \times \cancel{30} \times \cancel{3} \times \cancel{5}}{\cancel{5} \times \cancel{40} \times \underset{10}{\cancel{300}} \times \underset{22}{\cancel{66}}} = 2. \quad \textit{Ans.}$$

Exercise 88. Page 182.

1. Find the area of a floor 16 ft. 3 in. long and 12 ft. 6 in. wide.

16 ft. 3 in. = $16\frac{1}{4}$ ft. ; 12 ft. 6 in. = $12\frac{1}{2}$ ft.

$16\frac{1}{4} \times 12\frac{1}{2} = \frac{65}{4} \times \frac{25}{2} = \frac{1625}{8} = 203\frac{1}{8}$. $\quad 203\frac{1}{8}$ sq. ft. *Ans.*

[redacted]

2. A rectangle contains 672 sq. ft. 108 sq. in., and is 19 ft. 6 in. wide. Find its length.

672 sq. ft. 108 sq. in. = $672\frac{3}{4}$ sq. ft.; 19 ft. 6 in. = $19\frac{1}{2}$ ft.

$$672\tfrac{3}{4} \div 19\tfrac{1}{2} = \frac{\cancel{2}}{\cancel{39}} \times \frac{\overset{69}{\cancel{2691}}}{\underset{2}{\cancel{4}}} = \frac{69}{2} = 34\tfrac{1}{2}. \qquad 34\tfrac{1}{2}\text{ ft. } Ans.$$

3. What length of board 15 in. wide will contain 11 sq. ft. 36 sq. in.?

11 sq. ft. 36 sq. in. = $11\frac{1}{4}$ sq. ft.; 15 in. = $1\frac{1}{4}$ ft.

$$11\tfrac{1}{4} \div 1\tfrac{1}{4} = \frac{\cancel{4}}{\cancel{5}} \times \frac{\overset{9}{\cancel{45}}}{\cancel{4}} = 9. \qquad 9\text{ ft. } Ans.$$

4. What length of road 44 ft. wide will contain an acre?

1 A. = 43,560 sq. ft.

43,560 ÷ 44 = 990. 990 ft. = 60 rd. *Ans.*

5. Find the area of a rectangular field 13.12 chains long, 10.35 chains broad.

$$\begin{array}{r} 13.12 \\ \underline{10.35} \\ 6560 \\ 3936 \\ \underline{1312} \\ 135.7920 \end{array}$$

135.792 sq. ch. = 13 A. 5.792 sq. ch. *Ans.*

6. A path 216 ft. long measures 72 sq. yd. Find its breadth.

216 ft. = 72 yd. 72 ÷ 72 = 1. 1 yd. *Ans.*

7. A rectangular field of 21.66 A. is 250.8 yd. broad. Find its length.

1 A. = 4840 sq. yd.

$$\begin{array}{r} 4840 \text{ sq. yd.} \\ \underline{21.66}\phantom{\text{ sq. yd.}} \\ 29040\phantom{\text{ sq. yd.}} \\ 29040\phantom{\text{ sq. yd.}} \\ 4840\phantom{\text{ sq. yd.}} \\ \underline{9680}\phantom{\text{ sq. yd.}} \\ 104834.40 \text{ sq. yd.} \end{array} \qquad \begin{array}{r} 418 \\ 2508\overline{)1048344} \\ \underline{10032} \\ 4514 \\ \underline{2508} \\ 20064 \\ \underline{20064} \end{array}$$

418 yd. *Ans.*

[redacted]

8. What is the area of a table, if its length and breadth are 4 ft. $3\frac{3}{7}$ in. and 2 ft. $9\frac{3}{5}$ in., respectively?

$$4 \text{ ft. } 3\tfrac{3}{7} \text{ in.} = 4\tfrac{2}{7} \text{ ft.}; \quad 2 \text{ ft. } 9\tfrac{3}{5} \text{ in.} = 2\tfrac{4}{5} \text{ ft.}$$

$$4\tfrac{2}{7} \times 2\tfrac{4}{5} = \frac{\overset{6}{\cancel{30}}}{\cancel{7}} \times \frac{\overset{2}{\cancel{14}}}{\cancel{5}} = 12.$$

12 sq. ft. *Ans.*

9. From each corner of a square, each side of which is 2 ft. 5 in. long, a square measuring 5 in. on a side is cut out. Find the area of the remainder of the figure.

$2 \text{ ft. } 5 \text{ in.} = 2\frac{5}{12} \text{ ft.} \qquad 2\frac{5}{12} \times 2\frac{5}{12} = \frac{29}{12} \times \frac{29}{12} = \frac{841}{144} = 5\frac{121}{144}.$

$5\frac{121}{144}$ sq. ft. = 5 sq. ft. 121 sq. in.

$5 \times 5 = 25. \qquad 4 \times 25$ sq. in. $= 100$ sq. in.

5 sq. ft. 121 sq. in. − 100 sq. in. = 5 sq. ft. 21 sq. in. *Ans.*

10. The length and breadth of a map are $4\frac{1}{2}$ ft. and $3\frac{1}{3}$ ft. respectively. If the map represents 77,760 sq. mi. of country, how many square miles are there to a square inch?

$4\frac{1}{2}$ ft. = 54 in.; $3\frac{1}{3}$ ft. = 40 in.

$$\begin{array}{r} 54 \\ 40 \\ \hline 2160 \end{array} \qquad \begin{array}{r} 36 \\ 216\cancel{0})\overline{7776} \\ 648 \\ \hline 1296 \\ 1296 \\ \hline \end{array}$$

36 sq. mi. *Ans.*

11. In rolling a grass plot that is 24 yd. long, and contains 400 sq. yd., how many times must a roller 3 ft. 4 in. wide be drawn over it lengthwise that the whole plot may be rolled?

$400 \div 24 = 16\frac{2}{3}. \qquad 3 \text{ ft. } 4 \text{ in.} = 3\frac{1}{3} \text{ ft.} = 1\frac{1}{9} \text{ yd.}$

$$16\tfrac{2}{3} \div 1\tfrac{1}{9} = \frac{\overset{5}{\cancel{50}}}{\cancel{3}} \times \frac{\overset{3}{\cancel{9}}}{\cancel{10}} = 15. \qquad \textit{Ans.}$$

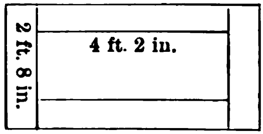
12. How many sods, each 2 ft. $3\frac{1}{2}$ in. long and $8\frac{1}{4}$ in. broad, will be required to turf an acre of ground?

$2 \text{ ft. } 3\frac{1}{2} \text{ in.} = 2\frac{7}{24} \text{ ft.}; \quad 8\frac{1}{4} \text{ in.} = \frac{11}{16} \text{ ft.} \qquad 1 \text{ A.} = 43{,}560 \text{ sq. ft.}$

$$\frac{43560}{2\frac{7}{24} \times \frac{11}{16}} = \overset{\overset{72}{\cancel{3960}}}{\cancel{43560}} \times \frac{24}{\cancel{55}} \times \frac{16}{\cancel{11}} = 27{,}648. \qquad \textit{Ans.}$$

[redacted]

13. Find the area of a picture frame $2\frac{1}{4}$ in. broad, if the outside measurement is 4 ft. $6\frac{1}{2}$ in. in length and 2 ft. 8 in. in width.



$$2 \times 2\tfrac{1}{4} \text{ in.} = 4\tfrac{1}{2} \text{ in.}$$
$$4 \text{ ft. } 6\tfrac{1}{2} \text{ in.} - 4\tfrac{1}{2} \text{ in.} = 4 \text{ ft. } 2 \text{ in.}$$
$$4 \text{ ft. } 2 \text{ in.} + 2 \text{ ft. } 8 \text{ in.} = 6 \text{ ft. } 10 \text{ in.}$$
$$2 \times (6 \text{ ft. } 10 \text{ in.}) = 13 \text{ ft. } 8 \text{ in.} = 13\tfrac{2}{3} \text{ ft.}$$
$$2\tfrac{1}{4} \text{ in.} = \tfrac{3}{16} \text{ ft.}$$

$$13\tfrac{2}{3} \times \frac{3}{16} = \frac{41}{\cancel{3}} \times \frac{\cancel{3}}{16} = \frac{41}{16} = 2\tfrac{9}{16}. \qquad 2\tfrac{9}{16} \text{ sq. ft.} = 2 \text{ sq. ft. } 81 \text{ sq. in. } \textit{Ans.}$$

14. Find the expense of glazing four windows, each containing 12 panes, if the panes are each 1 ft. long and 10 in. wide, and the price of the glass is 38 cents per square foot.

$$10 \text{ in.} = \tfrac{5}{6} \text{ ft.} \qquad 4 \times \overset{2}{\cancel{12}} \times 1 \times \frac{5}{\cancel{6}} \times \$0.38 = \$15.20. \quad \textit{Ans.}$$

15. A field 76 yd. long and 56 yd. broad, enclosed by a wall, has a border 4 ft. wide within the wall, and within this a path 5 ft. wide. If the remainder of the field is grass, find the area of the border, of the path, and of the grass.

$4 \text{ ft.} = 1\frac{1}{3} \text{ yd.}$ $\qquad 2 \times 1\frac{1}{3} \text{ yd.} = 2\frac{2}{3} \text{ yd.}$

$56 \text{ yd.} - 2\frac{2}{3} \text{ yd.} = 53\frac{1}{3} \text{ yd.}$ $\qquad 2 \times (76 \text{ yd.} + 53\frac{1}{3} \text{ yd.}) = 258\frac{2}{3} \text{ yd.}$

$$258\tfrac{2}{3} \times 1\tfrac{1}{3} = \frac{776}{3} \times \frac{4}{3} = \frac{3104}{9} = 344\tfrac{8}{9}.$$

Area of border $= 344\frac{8}{9}$ sq. yd. *Ans.*

$5 \text{ ft.} = 1\frac{2}{3} \text{ yd.}$ $\qquad 5 \text{ ft.} + 4 \text{ ft.} = 9 \text{ ft.} = 3 \text{ yd.}$

$2 \times 3 \text{ yd.} = 6 \text{ yd.}$ $\qquad 76 \text{ yd.} - 6 \text{ yd.} = 70 \text{ yd.}$

$$2 \times (70 \text{ yd.} + 53\tfrac{1}{3} \text{ yd.}) = 246\tfrac{2}{3} \text{ yd.}$$

$$246\tfrac{2}{3} \times 1\tfrac{2}{3} = \frac{740}{3} \times \frac{5}{3} = \frac{3700}{9} = 411\tfrac{1}{9}.$$

Area of path $= 411\frac{1}{9}$ sq. yd. *Ans.*

Field is 70 yd. long and 50 yd. wide.

Area of field $= (70 \times 50)$ sq. yd. $= 3500$ sq. yd. *Ans.*

16. A square plot of land 127 yd. long has a path 1 yd. wide running round the inside of it. Find the cost of graveling this path at 15 cents per square yard.

$127 \text{ yd.} - (2 \times 1 \text{ yd.}) = 125 \text{ yd.}$

$2 \times (127 \text{ yd.} + 125 \text{ yd.}) = 504 \text{ yd.}$

$(504 \times 1) \text{ sq. yd.} = 504 \text{ sq. yd.}$

$$\begin{array}{r} \$0.15 \\ \underline{504} \\ \$75.60 \end{array} \ \textit{Ans.}$$

[redacted]

17. A street $\frac{3}{4}$ of a mile long has on each side a sidewalk $7\frac{1}{2}$ ft. wide. What will it cost to pave the sidewalks with stones, each measuring 2 ft. 9 in. by 1 ft. 8 in., if the stones are worth 75 cents each?

$$\frac{3}{4}\text{ mi.} = \frac{3}{\cancel{4}} \times \overset{1320}{\cancel{5280}}\text{ ft.} = 3960\text{ ft.}$$

$$2 \times 7\tfrac{1}{2}\text{ ft.} = 15\text{ ft.}$$

$$\begin{array}{r} 3960 \\ 15 \\ \hline 19800 \\ 3960 \\ \hline 59400 \end{array}$$

$$2\text{ ft. }9\text{ in.} = 2\tfrac{3}{4}\text{ ft.};\ 1\text{ ft. }8\text{ in.} = 1\tfrac{2}{3}\text{ ft.}$$

$$2\tfrac{3}{4} \times 1\tfrac{2}{3} = \frac{11}{4} \times \frac{5}{3} = \frac{55}{12}. \qquad 59400 \div \frac{55}{12} = \frac{12}{\cancel{55}} \times \overset{1080}{\cancel{59400}} = 12{,}960.$$

$$12{,}960 \times \$0.75 = \overset{3240}{\cancel{12960}} \times \$\frac{3}{\cancel{4}} = \$9720.\ \textit{Ans.}$$

18. How many planks 11 ft. by 9 in. are needed to cover a platform 27 ft. 6 in. long and 8 yd. wide? What will be the cost at 20 cents a square foot?

$$9\text{ in.} = \tfrac{3}{4}\text{ ft.};\ 8\text{ yd.} = 24\text{ ft.};\ 27\text{ ft. }6\text{ in.} = 27\tfrac{1}{2}\text{ ft.}$$

$$\frac{27\frac{1}{2} \times 24}{11 \times \frac{3}{4}} = \frac{\overset{5}{\cancel{55}}}{\cancel{2}} \times \overset{8}{\cancel{24}} \times \frac{1}{\cancel{11}} \times \frac{\overset{2}{\cancel{4}}}{\cancel{3}} = 80.\ \textit{Ans.}$$

$$\overset{4}{\cancel{\overset{20}{\cancel{80}}}} \times 11 \times \frac{3}{\cancel{4}} \times \$\frac{1}{\cancel{5}} = \$132.\ \textit{Ans.}$$

19. How many tiles 9 in. long and 4 in. wide will be required to pave a walk 8 ft. wide that surrounds a rectangular court 60 ft. long and 36 ft. wide?

$$36\text{ ft.} + (2 \times 8\text{ ft.}) = 52\text{ ft.} \qquad 2 \times (60\text{ ft.} + 52\text{ ft.}) = 224\text{ ft.}$$

$$(8 \times 224)\text{ sq. ft.} = 1792\text{ sq. ft.} \qquad 9\text{ in.} = \tfrac{3}{4}\text{ ft.};\ 4\text{ in.} = \tfrac{1}{3}\text{ ft.}$$

$$\frac{\cancel{3}}{4} \times \frac{1}{\cancel{3}} = \frac{1}{4}$$

$$1792 \div \tfrac{1}{4} = 4 \times 1792 = 7168.\ \text{Ans.}$$

[redacted]

9/7   45.

20. How many times will a wheel $2\frac{1}{3}$ ft. in diameter turn in going a distance of 110 yards?

$$2\tfrac{1}{3}\text{ ft.} = \tfrac{7}{9}\text{ yd.}$$

7/9

$$\frac{110}{\cancel{2\frac{1}{3}} \times 3.1416} = 0.31831 \times \frac{\cancel{7}}{\cancel{3}} \times 110 = \cancel{81.7}.\ \textit{Ans.}$$

$$\begin{array}{r} 0.31831 \\ \cancel{7} \\ \hline \cancel{2.22817} \end{array}$$

2.86479   9

$$\begin{array}{r} 2.22817 \\ 110 \\ \hline 222817\,0 \\ 222817 \\ \hline 3\,)\,245.09870 \\ \cancel{81.699} \end{array}$$

21. What distance will a wheel $\frac{5}{11}$ yd. in diameter pass over in making $4\frac{1}{2}$ revolutions?

$$4\tfrac{1}{2} \times 3.1416 \times \frac{5}{11}\text{ yd.} = \frac{9}{\cancel{2}} \times \overset{\substack{0.1428 \\ \cancel{0.2856}}}{\cancel{3.1416}} \times \frac{5}{\cancel{11}}\text{ yd.} = 6.426\text{ yd. } \textit{Ans.}$$

22. Find the diameter of a wheel that makes 9 revolutions in going $7\frac{1}{5}$ yards.

$$7\tfrac{1}{5}\text{ yd.} \div 9 = \frac{1}{\cancel{9}} \times \frac{\overset{4}{\cancel{36}}}{5}\text{ yd.} = \frac{4}{5}\text{ yd.}$$

$$0.31831 \times \frac{4}{5}\text{ yd.} = \frac{1.27324}{5}\text{ yd.} = 0.25465\text{ yd.} = 9.1674\text{ in. } \textit{Ans.}$$

23. If the circumference of a wheel is $\frac{22}{7}$ of 1 yd. $1\frac{1}{8}$ ft., how many times will the wheel turn in going $3\frac{3}{7}$ miles?

$$1\text{ yd. } 1\tfrac{1}{8}\text{ ft.} = 4\tfrac{1}{8}\text{ ft.}$$

$$\frac{3\frac{3}{7} \times 5280}{\frac{22}{7} \times 4\frac{1}{8}} = \frac{\overset{8}{\cancel{24}}}{\cancel{7}} \times \overset{240}{\cancel{5280}} \times \frac{\cancel{7}}{\cancel{22}} \times \frac{8}{\underset{11}{\cancel{33}}} = \frac{15360}{11} = 1396\tfrac{4}{11}.\ \textit{Ans.}$$

24. If the wheel of a locomotive is $3\frac{1}{7}$ times 5.52 ft. in circumference, how many times does it turn in a minute when the locomotive is running at the rate of 13.34 mi. an hour?

$$13.34\text{ mi. an hour} = \frac{13.34 \times 5280}{60}\text{ ft. a minute.}$$

$$\frac{13.34 \times 5280}{60} \div (3\tfrac{1}{7} \times 5.52) = \frac{\overset{29}{\cancel{1334}}}{\cancel{100}} \times \frac{\overset{\substack{4 \\ \cancel{240}}}{\cancel{5280}}}{\cancel{60}} \times \frac{7}{\cancel{22}} \times \frac{\cancel{100}}{\underset{\substack{\cancel{12} \\ 3}}{\cancel{552}}} = \frac{203}{3} = 67\tfrac{2}{3}.\ \textit{Ans.}$$

2.86479
110
2864790

[redacted]

25. Find the area of a circle that has a radius of 3 feet.

3.1416 × (3 × 3) sq. ft. = 28.2744 sq. ft. *Ans.*

3	3.1416
3	9
9	28.2744

26. What is the area of a circular field that has a radius of 400 yards?

3.1416 × (400 × 400) sq. yd. = 502,656 sq. yd. *Ans.*

400	3.1416
400	160000
160000	1884960000
	31416
	502656.0000

27. The radius of the rotunda of the Pantheon at Rome is 71 ft. 6 in. Find the area of the floor.

$3.1416 \times 71\frac{1}{2} \times 71\frac{1}{2} = 16,060.6446.$

3.1416	224.6244
71.5	71.5
157080	11231220
31416	2246244
219912	15723708
224.62440	16060.64460

16,060.6446 sq. ft. *Ans.*

28. The diameter of a cylindrical cistern is 13 ft. What is the area of the bottom?

0.7854 × (13 × 13) sq. ft. = 132.7326 sq. ft. *Ans.*

13	0.7854
13	169
39	70686
13	47124
169	7854
	132.7326

[redacted]

29. The two dials of the clock of St. Paul's, London, are each $18\frac{1}{7}$ ft. in diameter. What is the area of each in square feet?

$$0.7854 \times (18\tfrac{1}{7} \times 18\tfrac{1}{7}) \text{ sq. ft.} = 258.5248 \text{ sq. ft. } \textit{Ans.}$$

$$\overset{0.1122}{\cancel{0.7854}} \times \frac{127}{\cancel{7}} \times \frac{127}{7} = \frac{1809.6738}{7} = 258.5248.$$

30. At 20 cents a square yard, what will it cost to gravel a walk 6 ft. wide running round a circular fish pond 70 yd. in diameter?

$$\text{Area of pond} = 0.7854 \times (70 \times 70) \text{ sq. yd.} = 3848.46 \text{ sq. yd.}$$

$$\text{Area of pond and walk} = 0.7854 \times (74 \times 74) \text{ sq. yd.} = 4300.8504 \text{ sq. yd.}$$

$$4300.85 \text{ sq. yd.} - 3848.46 \text{ sq. yd.} = 452.39 \text{ sq. yd., area of walk.}$$

$$452.39 \times \$0.20 = \$90.48. \textit{ Ans.}$$

31. How many square inches on the surface of a ball 3 inches in diameter?

$$3.1416 \times 3 \times 3 = 28.2744. \quad 28.2744 \text{ sq. in. } \textit{Ans.}$$

32. How many square inches of surface on a spherical blackboard 12 inches in diameter?

$$3.1416 \times 12 \times 12 = 452.3904.$$

```
   3.1416
      144
   ------
   125664
  125664
  31416
 --------
 452.3904
```

452.3904 sq. in. *Ans.*

33. What is the interior surface of a hemispherical vase whose interior diameter is 20 inches?

$$\tfrac{1}{2} \times 3.1416 \times (20 \times 20) \text{ sq. in.} = 628.32 \text{ sq. in.} = 4 \text{ sq. ft. } 52.32 \text{ sq. in. } \textit{Ans.}$$

```
   3.1416
      200
 --------
 628.3200
```

34. Find the external and the internal surface of a spherical shell whose external and internal diameters are 8 in. and 5 in., respectively.

$$3.1416 \times (8 \times 8) \text{ sq. in.} = 201.0624 \text{ sq. in. } \textit{Ans.}$$

$$3.1416 \times (5 \times 5) \text{ sq. in.} = 78.54 \text{ sq. in. } \textit{Ans.}$$

```
   3.1416        3.1416
       64            25
   ------        ------
   125664        157080
  188496         62832
 --------       -------
 201.0624       78.5400
```

[redacted]

35. How many square feet of tin are required to make 16 hemispherical bowls, each 2 ft. 4 in. in diameter?

$$2 \text{ ft. } 4 \text{ in.} = 2\tfrac{1}{3} \text{ ft.}$$

$$16 \times \tfrac{1}{2} \times 3.1416 \times (2\tfrac{1}{3} \times 2\tfrac{1}{3}) \text{ sq. ft.} = 136.8341 \text{ sq. ft. } \textit{Ans.}$$

$$\begin{array}{r} 2\underline{)16} \\ 8 \\ \underline{2\tfrac{1}{3}} \\ 18\tfrac{2}{3} \end{array} \qquad \begin{array}{r} 3.14\dot{1}6 \\ \underline{18\tfrac{2}{3}} \\ 251328 \\ 31416 \\ \underline{20944} \\ 58.6432 \\ \underline{2\tfrac{1}{3}} \\ 1172864 \\ \underline{195477} \\ 136.8341 \end{array}$$

36. Find the lateral surface of a right cylinder if its height is 10 in. and the radius of its base is 7 in.

$$(10 \times 2 \times 3.1416 \times 7) \text{ sq. in.} = 439.824 \text{ sq. in. } \textit{Ans.}$$

$$\begin{array}{r} 10 \\ \underline{2} \\ 20 \\ \underline{7} \\ 140 \end{array} \qquad \begin{array}{r} 3.1416 \\ \underline{140} \\ 1256640 \\ \underline{31416} \\ 439.8240 \end{array}$$

37. Find the lateral surface of a right cylinder if its height is 12 ft. and the diameter of its base is 9 ft. 4 in.

$$9 \text{ ft. } 4 \text{ in.} = 9\tfrac{1}{3} \text{ ft.}$$

$$(12 \times 3.1416 \times 9\tfrac{1}{3}) \text{ sq. ft.} = 351.8592 \text{ sq. ft. } \textit{Ans.}$$

$$\begin{array}{r} 12 \\ \underline{9\tfrac{1}{3}} \\ 112 \end{array} \qquad \begin{array}{r} 3.1416 \\ \underline{112} \\ 62832 \\ 31416 \\ \underline{31416} \\ 351.8592 \end{array}$$

38. At 32 cents a square foot, what is the cost of cementing a cylindrical cistern 20 ft. deep and 18 ft. in diameter?

Lateral area $= (20 \times 3.1416 \times 18)$ sq. ft. $= 1130.976$ sq. ft.

Area of bottom $= 3.1416 \times (9 \times 9)$ sq. ft. $= 254.4696$ sq. ft.

$$\begin{array}{r} 3.1416 \\ \underline{360} \\ 1884960 \\ \underline{94248} \\ 1130.9760 \end{array} \qquad \begin{array}{r} 3.1416 \\ \underline{81} \\ 31416 \\ \underline{251328} \\ 254.4696 \end{array}$$

$$\begin{array}{rl} 1130.976 & \text{sq. ft.} \\ \underline{254.4696} & \\ 1385.4456 & \text{sq. ft.} \end{array} \qquad \begin{array}{r} 1385.4456 \\ \underline{0.32} \\ 27708912 \\ \underline{41563368} \\ 443.342592 \end{array}$$

\$443.34. *Ans.*

39. The diameters of two right cylinders of the same height are as 6 to 1. Compare the lateral surfaces.

$$\frac{\text{Lateral area of larger}}{\text{Lateral area of smaller}} = \frac{\text{height} \times 3.1416 \times 6}{\text{height} \times 3.1416 \times 1} = \frac{6}{1}.$$

That is, the lateral areas are as 6 to 1.

Exercise 89. Page 186.

1. How many yards of carpeting 27 in. wide will be required for a floor 26 ft. long, $15\frac{3}{4}$ ft. wide, if the strips run lengthwise? How many if the strips run across the room? How much will be turned under in each case?

$$27 \text{ in.} = 2\tfrac{1}{4} \text{ ft.} \qquad 15\tfrac{3}{4} \div 2\tfrac{1}{4} = \frac{\cancel{4}}{\cancel{9}} \times \frac{\overset{7}{\cancel{63}}}{\cancel{4}} = 7.$$

Hence 7 strips will be required.

$$7 \times 26 \text{ ft.} = 182 \text{ ft.} = 60\tfrac{2}{3} \text{ yd. } \textit{Ans.}$$

$$26 \div 2\tfrac{1}{4} = \frac{4}{9} \times 26 = \frac{104}{9} = 11\tfrac{5}{9}.$$

Hence 12 strips will be required.

$$12 \times 15\tfrac{3}{4} \text{ ft.} = 12 \times 5\tfrac{1}{4} \text{ yd.} = 63 \text{ yd. } \textit{Ans.}$$

In the first case nothing will be turned under; in the second a strip $\frac{4}{9}$ of 36 in. = 16 in. wide.

7 12

[redacted]

2. How many yards of carpeting $\frac{7}{8}$ yd. wide will be required for a room $8\frac{1}{2}$ yd. by 17 ft., if the strips run lengthwise, and if there is a waste of $\frac{1}{16}$ yd. a strip?

$$17 \text{ ft.} = 5\tfrac{2}{3} \text{ yd.} \qquad 5\tfrac{2}{3} \div \frac{7}{8} = \frac{8}{7} \times \frac{17}{3} = \frac{136}{21} = 6\tfrac{10}{21}.$$

Hence 7 strips will be required.

$$8\tfrac{1}{2} \text{ yd.} + \tfrac{1}{16} \text{ yd.} = 8\tfrac{9}{16} \text{ yd.} \qquad 7 \times 8\tfrac{9}{16} \text{ yd.} = 59\tfrac{15}{16} \text{ yd. } \textit{Ans.}$$

3. How many square yards of oilcloth will be required for a hall floor $5\frac{1}{4}$ yd. long and 10 ft. wide?

$$10 \text{ ft.} = 3\tfrac{1}{3} \text{ yd.} \qquad 5\tfrac{1}{4} \times 3\tfrac{1}{3} = \frac{\overset{7}{\cancel{21}}}{\underset{2}{\cancel{4}}} \times \frac{\overset{5}{\cancel{10}}}{\cancel{3}} = \frac{35}{2} = 17\tfrac{1}{2}.$$

$17\frac{1}{2}$ sq. yd. *Ans.*

4. At \$0.92 a yard, what is the cost of a carpet 27 in. wide for a room $28\frac{1}{2}$ ft. by $18\frac{3}{4}$ ft., if the strips run lengthwise?

$$27 \text{ in.} = 2\tfrac{1}{4} \text{ ft.};\ 28\tfrac{1}{2} \text{ ft.} = 9\tfrac{1}{2} \text{ yd.} \qquad 18\tfrac{3}{4} \div 2\tfrac{1}{4} = \frac{\overset{25}{\cancel{75}}}{\cancel{4}} \times \frac{\cancel{4}}{\underset{3}{\cancel{9}}} = \frac{25}{3} = 8\tfrac{1}{3}.$$

Hence, 9 strips will be required.

$$9 \times 9\tfrac{1}{2} \text{ yd.} = 85\tfrac{1}{2} \text{ yd.}$$

$$\begin{array}{r} \$0.92 \\ \underline{85\tfrac{1}{2}} \\ 46 \\ 460 \\ \underline{736} \\ \$78.66 \end{array} \ \textit{Ans.}$$

5. Find the cost of carpet 30 in. wide, at \$1.25 per yard, for a room 18 ft. by 14 ft., if the strips run lengthwise.

$$30 \text{ in.} = 2\tfrac{1}{2} \text{ ft.};\ 18 \text{ ft.} = 6 \text{ yd.} \qquad 14 \div 2\tfrac{1}{2} = \frac{2}{5} \times 14 = \frac{28}{5} = 5\tfrac{3}{5}.$$

Hence, 6 strips will be required.

$$\overset{3}{\cancel{6}} \times \overset{3}{\cancel{6}} \times \$\frac{5}{\underset{2}{\cancel{4}}} = \$45. \ \textit{Ans.}$$

6. Find the cost of carpeting 27 in. wide, at $1.12½ per yard, for a room 29 ft. 9 in. by 23 ft. 6 in., if the strips run across the room.

$$27 \text{ in.} = 2\tfrac{1}{4} \text{ ft.};\quad 29 \text{ ft. } 9 \text{ in.} = 29\tfrac{3}{4} \text{ ft.};\quad 23 \text{ ft. } 6 \text{ in.} = 7\tfrac{5}{6} \text{ yd.}$$

$$29\tfrac{3}{4} \div 2\tfrac{1}{4} = \frac{119}{\cancel{4}} \times \frac{\cancel{4}}{9} = \frac{119}{9} = 13\tfrac{2}{9}.$$

Hence, 14 strips will be required.

$$14 \times 7\tfrac{5}{6} \times \$1\tfrac{1}{8} = \overset{7}{\cancel{14}} \times \frac{47}{\underset{\cancel{2}}{\cancel{6}}} \times \frac{\overset{3}{\cancel{9}}}{8} = \$\frac{987}{8} = \$123.38. \quad \textit{Ans.}$$

7. Find the cost of carpeting 27 in. wide, at $2.75 per yard, for a room 34 ft. 8 in. by 13 ft. 3 in., if the strips run lengthwise, and if there is a waste of ¼ yd. a strip.

$27 \text{ in.} = 2\tfrac{1}{4} \text{ ft.};\ 13 \text{ ft. } 3 \text{ in.} = 13\tfrac{1}{4} \text{ ft.}\qquad 13\tfrac{1}{4} \div 2\tfrac{1}{4} = \frac{53}{\cancel{4}} \times \frac{\cancel{4}}{9} = \frac{53}{9} = 5\tfrac{8}{9}.$

Hence, 6 strips will be required.

$34 \text{ ft. } 8 \text{ in.} = 11\tfrac{5}{9} \text{ yd.}\qquad 11\tfrac{5}{9} \text{ yd.} + \tfrac{1}{4} \text{ yd.} = 11\tfrac{29}{36} \text{ yd.}$

$$6 \times 11\tfrac{29}{36} \times \$2\tfrac{3}{4} = \cancel{6} \times \frac{425}{\underset{6}{\cancel{36}}} + \frac{11}{4} = \$\frac{4675}{24} = \$194.79. \quad \textit{Ans.}$$

8. Which way must the strips of carpet 27 in. wide run to carpet most economically a room 20½ ft. by 19½ ft.?

$27 \text{ in.} = 2\tfrac{1}{4} \text{ ft.}\qquad 20\tfrac{1}{2} \div 2\tfrac{1}{4} = \frac{41}{\cancel{2}} \times \frac{\overset{2}{\cancel{4}}}{9} = \frac{82}{9} = 9\tfrac{1}{9}.$

Hence, if the strips run across the room, 10 strips will be required.

$10 \times 19\tfrac{1}{2} \text{ ft.} = 195 \text{ ft.} = 65 \text{ yd.}\qquad 19\tfrac{1}{2} \div 2\tfrac{1}{4} = \frac{\overset{13}{\cancel{39}}}{\cancel{2}} \times \frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{9}}} = \frac{26}{3} = 8\tfrac{2}{3}.$

Hence, if the strips run lengthwise, 9 strips will be required.

$$9 \times 20\tfrac{1}{2} \text{ ft.} = 184\tfrac{1}{2} \text{ ft.} = 61\tfrac{1}{2} \text{ yd.}$$

Hence, it takes 3½ yd. less if the strips run lengthwise.

9. How many double rolls of paper will be required for a room of ordinary height, 15 ft. long and 12 ft. wide, if the room has one door and three windows, each 3½ ft. wide?

Perimeter of room = 2 × (15 + 12) ft.	= 54 ft.
Width of door and windows = 4 × 3½ ft.	= 14 ft.
Perimeter less door and windows	= 40 ft.

$40 \div 7 = 5\tfrac{5}{7}$. Hence, 6 double rolls will be required.

[redacted]

10. At $2.25 a double roll, put on, what is the cost of papering a room of ordinary height, 16 ft. by 14 ft., if the room has two doors each 4 ft. wide, and four windows each 3 ft. 6 in. wide?

$$\begin{aligned}
&\text{Perimeter of room} = 2 \times (16 + 14) \text{ ft.} && = 60 \text{ ft.}\\
&\text{Width of doors and windows}\\
&\quad = 2 \times 4 \text{ ft.} + 4 \times 3\tfrac{1}{2} \text{ ft.} = 8 \text{ ft.} + 14 \text{ ft.} && = \underline{22} \text{ ft.}\\
&\text{Perimeter less doors and windows} && = 38 \text{ ft.}
\end{aligned}$$

$38 \div 7 = 5\frac{3}{7}$. Hence, 6 double rolls will be required.

$6 \times \$2.25 = \$13.50$. *Ans.*

11. At 75 cents a single roll, put on, what is the cost of papering a room of ordinary height, 20 ft. 6 in. long and 17 ft. 4 in. wide, if the room has two doors each 3 ft. 6 in. wide, and five windows each 3 ft. 3 in. wide?

$$\begin{aligned}
&\text{Perimeter of room} = 2 \times (20\tfrac{1}{2} + 17\tfrac{1}{3}) \text{ ft.} && = 75\tfrac{2}{3} \text{ ft.}\\
&\text{Width of doors and windows}\\
&\quad = 2 \times 3\tfrac{1}{2} \text{ ft.} + 5 \times 3\tfrac{1}{4} \text{ ft.} = 7 \text{ ft.} + 16\tfrac{1}{4} \text{ ft.} && = \underline{23\tfrac{1}{4}} \text{ ft.}\\
&\text{Perimeter less doors and windows} && = 52\tfrac{5}{12} \text{ ft.}
\end{aligned}$$

$$52\tfrac{5}{12} \div 3\tfrac{1}{2} = \frac{\cancel{2}}{7} \times \frac{629}{\underset{6}{\cancel{12}}} = \frac{629}{42} = 14\tfrac{41}{42}.$$

Hence, 15 single rolls will be required. $15 \times \$0.75 = \$11.25$. *Ans.*

12. What is the cost of the border for the room of Ex. 11 at $0.45 a running yard?

Perimeter of room $= 75\frac{2}{3}$ ft. $= 25\frac{2}{9}$ yd.

$$\begin{array}{r}
\$0.45\\
\underline{25\tfrac{2}{9}}\\
10\\
225\\
\underline{90}\\
\$11.35
\end{array}\ \textit{Ans.}$$

13. At $1.75 a double roll, put on, what is the cost of papering a room of ordinary height, 18 ft. 6 in. by 14 ft. 4 in., if the room has three doors 4 ft. wide, and three windows 3 ft. 9 in. wide?

$$\begin{aligned}
&\text{Perimeter of room} = 2 \times (18\tfrac{1}{2} + 14\tfrac{1}{3}) \text{ ft.} && = 65\tfrac{2}{3} \text{ ft.}\\
&\text{Width of doors and windows}\\
&\quad = 3 \times 4 \text{ ft.} + 3 \times 3\tfrac{3}{4} \text{ ft.} = 12 \text{ ft.} + 11\tfrac{1}{4} \text{ ft.} && = \underline{23\tfrac{1}{4}} \text{ ft.}\\
&\text{Perimeter less doors and windows} && = 42\tfrac{5}{12} \text{ ft.}
\end{aligned}$$

$42\frac{5}{12} \div 7 = 6\frac{5}{84}$. Hence, 7 double rolls will be required.

$7 \times \$1.75 = \$12.25$. Ans.

14. Find at 20 cents a square yard the cost of plastering the walls and ceiling of a room 18 ft. by 16 ft. by 10 ft., if the room has two doors 7 ft. 6 in. by 4 ft., three windows 6 ft. 6 in. by 4 ft., and a base board of 10 in.

Perimeter of room = $2 \times (18 + 16)$ ft. = 68 ft.
Height of room = 10 ft. − 10 in. = $9\frac{1}{6}$ ft.
Area of walls = $9\frac{1}{6} \times 68$ sq. ft. = $623\frac{1}{3}$ sq. ft.
Area of ceiling = 18×16 sq. ft. = 288 sq. ft.
Total area = $623\frac{1}{3}$ sq. ft. + 288 sq. ft. = $911\frac{1}{3}$ sq. ft.
Area of doors and windows
$= 2 \times (6\frac{2}{3} \times 4)$ sq. ft. + $3(6\frac{1}{2} \times 4)$ sq. ft.
$= 53\frac{1}{3}$ sq. ft. + 78 sq. ft. = $131\frac{1}{3}$ sq. ft.
$\frac{1}{2}$ of $131\frac{1}{3}$ sq. ft. = $65\frac{2}{3}$ sq. ft.
Net area = $911\frac{1}{3}$ sq. ft. − $65\frac{2}{3}$ sq. ft. = $845\frac{2}{3}$ sq. ft. = $93\frac{26}{27}$ sq. yd.
$94 \times \$0.20 = \$18.80$. *Ans.*

15. Find at 25 cents a square yard the cost of plastering the walls and ceiling of a room 16 ft. by 15 ft. by 10 ft., if the room has two doors 7 ft. by 3 ft. 9 in., three windows 5 ft. 6 in. by 3 ft. 6 in., and a base board of 10 in.

Perimeter of room $= 2 \times (16 + 15)$ ft. = 62 ft.
Height above base board = 10 ft. − 10 in. = $9\frac{1}{6}$ ft.

Total wall area = $9\frac{1}{6} \times 62$ sq. ft.	$= 568\frac{1}{3}$ sq. ft.
Area of ceiling = 16×15 sq. ft.	$= 240$ sq. ft.
Total area	$= 808\frac{1}{3}$ sq. ft.

Height of doors above base board is 7 ft. − 10 in. = $6\frac{1}{6}$ ft.

Area of 2 doors = $2 \times (6\frac{1}{6} \times 3\frac{3}{4})$ sq. ft.	$= 46\frac{1}{4}$ sq. ft.
Area of 3 windows = $3 \times (5\frac{1}{2} \times 3\frac{1}{2})$ sq. ft.	$= 57\frac{3}{4}$ sq. ft.
Total area of openings	$= 104$ sq. ft.

Half area of openings	$= 52$ sq. ft.
Net area	$= 756\frac{1}{3}$ sq. ft.
	$= 84\frac{1}{27}$ sq. yd.

At $0.25 a square yard, 84 sq. yd. will cost $84 \times \$0.25 = \$21$. *Ans.*

16. Find at 20 cents a square yard the cost of plastering the walls and ceiling of a room 15 ft. by 14 ft. by 9 ft. 6 in., if the room has two doors 7 ft. 4 in. by 4 ft., two windows 5 ft. 6 in. by 3 ft. 6 in., and a *base board of* 9 in.

[redacted]

Perimeter of room $= 2 \times (15 + 14)$ ft. $= 58$ ft.
Height above base board $= 9$ ft. 6 in. $-$ 9 in. $= 8\frac{3}{4}$ ft.

Total wall area $= 8\frac{3}{4} \times 58$ sq. ft.	$= 507\frac{1}{2}$ sq. ft.
Area of ceiling $= 15 \times 14$ sq. ft.	$= 210$ sq. ft.
Total area	$= 717\frac{1}{2}$ sq. ft.

Height of doors above base board is 7 ft. 4 in. $-$ 9 in. $= 6\frac{7}{12}$ ft.

Area of 2 doors $= 2 \times (6\frac{7}{12} \times 4)$ sq. ft.	$= 52\frac{2}{3}$ sq. ft.	
Area of 3 windows $= 3 \times (5\frac{1}{2} \times 3\frac{1}{2})$ sq. ft.	$= 57\frac{3}{4}$ sq. ft.	
Total area of openings	$= 110\frac{5}{12}$ sq. ft.	
Half area of openings		$= 55\frac{5}{24}$ sq. ft.
Net area		$= 662\frac{7}{24}$ sq. ft.
		$= 73\frac{127}{216}$ sq. yd.

At \$0.20 a square yard, 74 sq. yd. will cost $74 \times$ \$0.20 $=$ \$14.80. *Ans.*

17. Find at 15 cents a square yard the cost of painting the outside of the walls of a cottage-roofed house 36 ft. by 32 ft. by 13 ft., if the house has three doors 7 ft. 6 in. by 4 ft., and eleven windows 6 ft. by 4 ft.

Perimeter of house $= 2 \times (36 + 32)$ ft. $= 136$ ft.

Total wall area $= 13 \times 136$ sq. ft.		$= 1768$ sq. ft.
Area of 3 doors $= 3 \times (7\frac{1}{2} \times 4)$ sq. ft.	$= 90$ sq. ft.	
Area of 11 windows $= 11 \times (6 \times 4)$ sq. ft.	$= 264$ sq. ft.	
Total area of openings	$= 354$ sq. ft.	
Half area of openings		$= 177$ sq. ft.
Net area		$= 1591$ sq. ft.
		$= 176\frac{7}{9}$ sq. yd.

At \$0.15 a square yard, 177 sq. yd. will cost $177 \times$ \$0.15 $=$ \$26.55. *Ans.*

18. Find at 20 cents a square yard the cost of painting the walls of a room 16 ft. by 15 ft. by 10 ft., if the room has two doors 7 ft. 6 in. by 4 ft., four windows 6 ft. by 3 ft. 9 in., and a base board of 9 in.

Perimeter of room $= 2 \times (16 + 15)$ ft. $= 62$ ft.
Height above base board $= 10$ ft. $-$ 9 in. $= 9\frac{1}{4}$ ft.

Total area $= 9\frac{1}{4} \times 62$ sq. ft.	$= 573\frac{1}{2}$ sq. ft.

Height of doors above base board is 7 ft. 6 in. $-$ 9 in. $= 6\frac{3}{4}$ ft.

Area of 2 doors $= 2 \times (6\frac{3}{4} \times 4)$ sq. ft.	$= 54$ sq. ft.	
Area of 4 windows $= 4 \times (6 \times 3\frac{3}{4})$ sq. ft.	$= 90$ sq. ft.	
Total area of openings	$= 144$ sq. ft.	
Half area of openings		$= 72$ sq. ft.
Net area		$= 501\frac{1}{2}$ sq. ft.
		$= 55\frac{13}{18}$ sq. yd.

At \$0.20 a square yard, 56 sq. yd. will cost $56 \times$ \$0.20 $=$ \$11.20. Ans.

[redacted]

19. How many bricks 8 in. long and 4 in. wide will be needed to pave a rectangular court 60 ft. by 30 ft. ?

Area of a brick $= (8 \times 4)$ sq. in.

Area of court $= (60 \times 30)$ sq. ft. $= (60 \times 30 \times 144)$ sq. in.

Hence, the number of bricks needed $= \dfrac{\overset{15}{\cancel{60}} \times 30 \times \overset{18}{\cancel{144}}}{\cancel{8} \times \cancel{4}} = 8100.$ *Ans.*

20. How many bricks 8 in. long and $2\frac{1}{2}$ in. thick, laid on edge, will be needed to pave the court of Ex. 19 ?

Area of a brick $= (8 \times 2\frac{1}{2})$ sq. in.

Hence, the number of bricks needed

$$= \frac{60 \times 30 \times 144}{8 \times 2\frac{1}{2}} = \frac{\overset{3}{\cancel{60}} \times 30 \times 144}{\cancel{20}} = 12,960. \text{ \textit{Ans.}}$$

21. How many clapboards will be required for the front of a house 40 ft. long and 20 ft. high, allowing 120 sq. ft. for doors and windows ?

Total area $= 40 \times 20$ sq. ft. $= 800$ sq. ft.

Net area $= 800$ sq. ft. $- 120$ sq. ft. $= 680$ sq. ft.

$680 \div 1\frac{1}{6} = \frac{6}{7} \times 680 = \frac{4080}{7} = 582\frac{6}{7}.$ 583. *Ans.*

22. How many clapboards will be required for a house 44 ft. long. 35 ft. wide, and 22 ft. high to the eaves, if the gables extend 14 ft. above the end walls, the two gables to be reckoned as one full wall, and 500 sq. ft. to be allowed for doors and windows ?

Perimeter $= 2 \times (44 + 35)$ ft. $= 158$ ft.

Area to eaves	$= 22 \times 158$ sq. ft.	$= 3476$ sq. ft.
Area of gables	$= 14 \times 35$ sq. ft.	$= 490$ sq. ft.
Total area		$= 3966$ sq. ft.
Area of openings		$= 500$ sq. ft.
Net area		$= 3466$ sq. ft.

$3466 \div 1\frac{1}{6} = \frac{6}{7} \times 3466 = \frac{20796}{7} = 2970\frac{6}{7}.$ 2971. *Ans.*

23. Allowing 1000 shingles for 120 sq. ft., how many thousand will be required for the pitched roof of a house 60 ft. long, if the width of each side of the roof is $24\frac{1}{2}$ ft. ?

Total area $= 2 \times (24\frac{1}{2} \times 60)$ sq. ft.

Number of thousand $= \dfrac{\cancel{2} \times 24\frac{1}{2} \times \cancel{60}}{\underset{\cancel{2}}{\cancel{120}}} = 24\frac{1}{2}.$ *Ans.*

[redacted]

24. Allowing 1000 shingles for 110 sq. ft., how many thousand will be required for the pitched roof of a barn 40 ft. long, if the width of each side of the roof is 24 ft.?

$$\text{Total area} = 2 \times (24 \times 40) \text{ sq. ft.}$$

$$\text{Number of thousand} = \frac{2 \times 24 \times \overset{4}{\cancel{40}}}{\underset{11}{\cancel{110}}} = \frac{192}{11} = 17\tfrac{5}{11}.$$

As shingles are put up in bundles of $\frac{1}{4}$ thousand, $17\frac{1}{4}$ M will be required.

25. Allowing 1000 shingles for 120 sq. ft., how many thousand will be required for the pitched roof of a house 28 ft. long, if the width of each side of the roof is 18 ft.?

$$\text{Total area} = 2 \times (18 \times 28) \text{ sq. ft.}$$

$$\text{Number of thousand} = \frac{2 \times \overset{3}{\cancel{18}} \times \overset{7}{\cancel{28}}}{\underset{\underset{5}{\cancel{20}}}{\cancel{120}}} = \frac{42}{5} = 8\tfrac{2}{5}.$$

As shingles are put up in bundles of $\frac{1}{4}$ thousand, $8\frac{1}{2}$ M will be required.

26. How many feet board measure in a board 18 ft. long, 9 in. wide, $\frac{7}{8}$ in. thick?

$$\overset{9}{\cancel{18}} \times \frac{3}{\underset{2}{\cancel{4}}} = \frac{27}{2} = 13\tfrac{1}{2}. \quad \textit{Ans.}$$

27. How many feet board measure in a board 16 ft. long, 11 in. wide, 1 in. thick?

$$\overset{4}{\cancel{16}} \times \frac{11}{\underset{3}{\cancel{12}}} \times 1 = \frac{44}{3} = 14\tfrac{2}{3}. \quad \textit{Ans.}$$

28. How many feet board measure in twenty boards averaging 14 ft. long, 10 in. wide, $1\frac{1}{2}$ in. thick?

$$\overset{10}{\cancel{20}} \times \overset{7}{\cancel{14}} \times \frac{5}{\underset{\cancel{2}}{\cancel{6}}} \times \frac{\cancel{3}}{\cancel{2}} = 350. \quad \textit{Ans.}$$

[redacted]

29. How many feet board measure in three joists 13 ft. long, 8 in. wide, 3 in. thick?

$$3 \times 13 \times \frac{2}{\cancel{3}} \times \cancel{3} = 78. \textit{ Ans.}$$

30. How many feet board measure in a stick of timber 8 in. by 9 in., and 27 ft. long?

$$27 \times \frac{3}{\cancel{4}} \times \overset{2}{\cancel{8}} = 162. \textit{ Ans.}$$

31. How many feet board measure in two beams, each 6 in. by 9 in., and 23 ft. long?

$$\cancel{2} \times 23 \times \frac{3}{\underset{\cancel{2}}{\cancel{4}}} \times \overset{3}{\cancel{6}} = 207. \textit{ Ans.}$$

32. How many feet board measure in three joists, each 3 in. by 4 in., and 11 ft. long?

$$3 \times 11 \times \frac{1}{\cancel{3}} \times \cancel{3} = 33. \textit{ Ans.}$$

33. How many feet board measure in five joists, each 6 in. by 4 in., and 14 ft. long?

$$5 \times 14 \times \frac{1}{\cancel{2}} \times \overset{2}{\cancel{4}} = 140. \textit{ Ans.}$$

34. How many feet board measure in a stick of timber 10 in. square, and 36 ft. long?

$$\overset{6}{\cancel{36}} \times \frac{5}{\cancel{6}} \times 10 = 300. \textit{ Ans.}$$

35. How many feet board measure in ten planks, each 13 ft. long, 15 in. wide, 2 in. thick?

$$\overset{5}{\cancel{10}} \times 13 \times \frac{5}{\underset{\cancel{2}}{\cancel{4}}} \times \cancel{2} = 325. \textit{ Ans.}$$

36. Find the cost of nine joists, each 15 ft. long, $3\frac{1}{2}$ in. by 5 in., at \$12 per M.

$$9 \times \overset{3}{\cancel{15}} \times \frac{\cancel{5}}{\cancel{12}} \times \frac{7}{2} \times \$\frac{\cancel{12}}{\underset{\underset{40}{\cancel{200}}}{\cancel{1000}}} = \$\frac{189}{80} = \$2.36. \textit{ Ans.}$$

[redacted]

37. Find the cost of thirty planks, each 12 ft. long, 11 in. wide, 3 in. thick, at \$15 per M.

$$\overset{3}{\cancel{30}} \times \cancel{12} \times \frac{11}{\cancel{12}} \times 3 \times \$\frac{\overset{3}{\cancel{15}}}{\underset{\underset{20}{\cancel{100}}}{\cancel{1000}}} = \$\frac{297}{20} = \$14.85. \ \textit{Ans.}$$

38. Find the cost of four sticks of timber, each 8 in. by 9 in., and 23 ft. long, at \$18 per M.

$$\cancel{4} \times 23 \times \frac{3}{\cancel{4}} \times \cancel{8} \times \$\frac{18}{\underset{125}{\cancel{1000}}} = \$\frac{1242}{125} = \$9.94. \ \textit{Ans.}$$

39. Find the cost of a board 24 ft. long, 23 in. wide at one end and 17 in. at the other, and $1\frac{1}{2}$ in. thick, at \$30 per M.

Average width = $\frac{1}{2}(23 + 17)$ in. = 20 in.

$$\overset{\overset{3}{\cancel{6}}}{\cancel{24}} \times \frac{\cancel{5}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{2}} \times \$\frac{\overset{3}{\cancel{30}}}{\underset{\underset{\underset{5}{\cancel{25}}}{\cancel{100}}}{\cancel{1000}}} = \$\frac{9}{5} = \$1.80. \ \textit{Ans.}$$

40. Find the cost of a stick of timber 29 ft. long, 10 in. by 12 in., at \$13.50 per M.

$$29 \times 1 \times \cancel{10} \times \$\frac{\overset{27}{\cancel{1350}}}{\underset{\underset{200}{\cancel{2000}}}{\cancel{100000}}} = \$\frac{783}{200} = \$3.92. \ \textit{Ans.}$$

41. Find the cost of the flooring for two floors, each 23 ft. by 17 ft., each floor double, and of boards $\frac{7}{8}$ in. thick; the under floors at \$18, and the upper at \$24, per M.

The average price is $\frac{1}{2}$ (\$18 + \$24) = \$21.

$$\cancel{4} \times 23 \times 17 \times \$\frac{21}{\underset{250}{\cancel{1000}}} = \$\frac{8211}{250} = \$32.84. \ \textit{Ans.}$$

[redacted]

42. Find the cost of the flooring timbers for a room 23 ft. by 17 ft., at \$18 per M, if they are 2 in. by 10 in., 17 ft. long, and are placed on edge, one close to each wall and the others with spaces $\frac{39}{40}$ ft. wide between them.

Since the room is 17 ft. wide, and the timbers are 17 ft. long, the timbers must run across the room. When a timber is placed against the wall the remaining distance is 23 ft. − 2 in. = $22\frac{5}{6}$ ft.

The distance taken up by a timber and its space $= \frac{1}{6}$ ft. + $\frac{39}{40}$ ft. $= \frac{137}{120}$ ft. The number of timbers required for the remaining space

$$= 22\tfrac{5}{6} \div \frac{137}{120} = \frac{\cancel{137}}{\cancel{6}} \times \frac{\overset{20}{\cancel{120}}}{\cancel{137}} = 20,$$

and the whole number of timbers is 21.

$$21 \times 17 \times \frac{5}{\cancel{6}} \times \cancel{2} \times \$\frac{\overset{3}{\cancel{18}}}{\underset{\underset{100}{\cancel{200}}}{\cancel{1000}}} = \$\frac{1071}{100} = \$10.71. \quad \textit{Ans.}$$

43. Find the number of feet board measure in a log 12 ft. long, and 20 in. in diameter at the smaller end.

$$20^2 - 2 \times 20 = 400 - 40 = 360.$$

$$\frac{\overset{6}{\cancel{12}}}{\underset{5}{\cancel{10}}} \times \frac{21}{\cancel{40}} \times \overset{9}{\cancel{360}} = \frac{1134}{5} = 227. \quad \textit{Ans.}$$

44. Find the number of feet board measure in a log 14 ft. long, smallest diameter 17 in.

$$17^2 - 2 \times 17 = 289 - 34 = 255. \qquad \frac{\overset{7}{\cancel{14}}}{\underset{\cancel{2}}{\cancel{10}}} \times \frac{21}{40} \times \overset{51}{\cancel{255}} = 187. \quad \textit{Ans.}$$

45. Find the number of feet board measure in a log 11 ft. long, smallest diameter 13 in.

$$13^2 - 2 \times 13 = 169 - 26 = 143. \qquad \frac{11}{10} \times \frac{21}{40} \times 143 = 83. \quad \textit{Ans.}$$

46. Find the number of feet board measure in a log 16 ft. long, smallest diameter 20 in.

$$20^2 - 2 \times 20 = 400 - 40 = 360. \qquad \frac{\overset{8}{\cancel{16}}}{\underset{5}{\cancel{10}}} \times \frac{21}{\cancel{40}} \times \overset{9}{\cancel{360}} = 302. \quad \textit{Ans.}$$

[redacted]

47. Find the number of feet board measure in a log 12 ft. long, smallest diameter 15 in.

$$15^2 - 2 \times 15 = 225 - 30 = 195. \qquad \frac{\overset{3}{\cancel{12}}}{10} \times \frac{21}{\underset{\underset{2}{\cancel{8}}}{\cancel{40}}} \times \overset{39}{\cancel{195}} = 123. \text{ \textit{Ans.}}$$

48. Find the value at \$9 per M of a log 15 ft. long, smallest diameter 11 in.

$$11^2 - 2 \times 11 = 121 - 22 = 99. \qquad \frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{10}}} \times \frac{21}{40} \times 99 = 78.$$

$$78 \times \$0.009 = \$0.70. \text{ \textit{Ans.}}$$

49. Find the value at \$9 per M of a log 16 ft. long, smallest diameter 13 in.

$$13^2 - 2 \times 13 = 169 - 26 = 143.$$

$$\frac{16}{10} \times \frac{21}{40} \times 143 = 120. \qquad 120 \times \$0.009 = \$1.08. \text{ \textit{Ans.}}$$

50. Find the value at \$9 per M of a log 13 ft. long, smallest diameter 16 in.

$$16^2 - 2 \times 16 = 256 - 32 = 224.$$

$$\frac{13}{10} \times \frac{21}{\underset{10}{\cancel{40}}} \times \overset{56}{\cancel{224}} = 153. \qquad 153 \times \$0.009 = \$1.38. \text{ \textit{Ans.}}$$

51. Find the value at \$9 per M of a log 14 ft. long, smallest diameter 12 in.

$$12^2 - 2 \times 12 = 144 - 24 = 120.$$

$$\frac{14}{10} \times \frac{21}{\cancel{40}} \times \overset{3}{\cancel{120}} = 88. \qquad 88 \times \$0.009 = \$0.79. \text{ \textit{Ans.}}$$

Exercise 90. Page 192.

1. Find the volume of a rectangular solid 7 ft. long, 2 ft. 6 in. wide, and 11 in. thick.

$$7 \times 2\tfrac{1}{2} \times \frac{11}{12} = 7 \times \frac{5}{2} \times \frac{11}{12} = \frac{385}{24} = 16\tfrac{1}{24}.$$

$16\frac{1}{24}$ cu. ft. = 16 cu. ft. 72 cu. in. *Ans.*

2. How many cubic feet of air in a hall 54 ft. long, 33 ft. wide, and 21 ft. 4 in. high?

$$54 \times 33 \times 21\tfrac{1}{3} = 54 \times \overset{11}{\cancel{33}} \times \frac{64}{\cancel{3}} = 38{,}016. \quad \textit{Ans.}$$

3. Find the volume of a cube whose edge is $2\frac{1}{2}$ yd.

$$2\tfrac{1}{2} \times 2\tfrac{1}{2} \times 2\tfrac{1}{2} = \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{125}{8} = 15\tfrac{5}{8}. \qquad 15\tfrac{5}{8} \text{ cu. yd. } \textit{Ans.}$$

4. A cellar was dug 21 ft. long, 17 ft. 3 in. wide, and 9 ft. deep. How many cubic yards of earth were taken out?

$$\frac{\overset{7}{\cancel{21}} \times 17\frac{1}{4} \times \cancel{9}}{\underset{\cancel{3}}{\cancel{27}}} = 120\tfrac{3}{4}. \quad \textit{Ans.}$$

5. Find the volume of a brick 8 in. long, $3\frac{1}{2}$ in. wide, and $2\frac{1}{4}$ in. thick.

$$8 \times 3\tfrac{1}{2} \times 2\tfrac{1}{4} = \overset{2}{\cancel{8}} \times \frac{7}{\cancel{2}} \times \frac{9}{\cancel{4}} = 63. \qquad 63 \text{ cu. in. } \textit{Ans.}$$

6. How many cubic feet of water will a rectangular cistern hold whose length, breadth, and height are 5 ft. 4 in., 3 ft. 6 in., and 2 ft. 10 in., respectively?

$$5\tfrac{1}{3} \times 3\tfrac{1}{2} \times 2\tfrac{5}{6} = \frac{\overset{\overset{4}{\cancel{8}}}{\cancel{16}}}{3} \times \frac{7}{\cancel{2}} \times \frac{17}{\underset{3}{\cancel{6}}} = \frac{476}{9} = 52\tfrac{8}{9}. \quad \textit{Ans.}$$

7. Find the volume in cubic inches of a bar of iron 21 ft. long, 3 in. wide, and 2 in. thick.

21 ft. = 252 in. $\qquad 252 \times 3 \times 2 = 1512.$ $\qquad$ 1512 cu. in. *Ans.*

8. What is the value at \$190 a cubic inch of a bar of gold 8 in. long and $\frac{3}{4}$ of an inch square?

$$\overset{2}{\cancel{8}} \times \frac{3}{\cancel{4}} \times \frac{3}{\underset{2}{\cancel{4}}} \times \$\overset{95}{\cancel{190}} = \$855. \quad \textit{Ans.}$$

[redacted]

9. A rectangular reservoir 15 yd. long, 12 yd. wide, holds 330 cu. yd. of water. What is its depth?

$$\frac{\overset{11}{\overset{\cancel{22}}{\cancel{330}}}}{\cancel{15}\times\underset{6}{\cancel{12}}}=\frac{11}{6}=1\tfrac{5}{6}. \qquad 1\tfrac{5}{6}\text{ yd. } \textit{Ans.}$$

10. What length must be cut off a beam 9 in. by 15 in. that the part cut off may contain $2\frac{1}{2}$ cu. ft.?

$$\frac{2\frac{1}{2}}{\frac{3}{4}\times\frac{5}{4}}=\frac{\cancel{5}}{\cancel{2}}\times\frac{\overset{2}{\cancel{4}}}{3}\times\frac{4}{\cancel{5}}=\frac{8}{3}=2\tfrac{2}{3}. \qquad 2\tfrac{2}{3}\text{ ft.} = 2\text{ ft. }8\text{ in. } \textit{Ans.}$$

11. How high is a room, if it is 31 ft. 3 in. long, 24 ft. broad, and contains 10,000 cu. ft. of air?

$$\frac{10000}{31\frac{1}{4}\times 24}=\overset{10}{\overset{\cancel{80}}{\cancel{10000}}}\times\frac{4}{\cancel{125}}\times\frac{1}{\underset{3}{\cancel{24}}}=\frac{40}{3}=13\tfrac{1}{3}.$$

$13\frac{1}{3}$ ft. = 13 ft. 4 in. *Ans.*

12. A piece of wood 5 ft. long, 1 ft. broad, and 9 in. thick, is cut up into matches $2\frac{1}{2}$ in. long and 0.1 of an inch square. How many matches will there be, if no allowance is made for waste in cutting?

Volume of the wood $=(60\times 12\times 9)$ cu. in.

Volume of a match $=(2\frac{1}{2}\times\frac{1}{10}\times\frac{1}{10})$ cu. in.

Therefore, the number of matches

$$=\frac{60\times 12\times 9}{2\frac{1}{2}\times\frac{1}{10}\times\frac{1}{10}}=60\times 12\times 9\times\frac{2}{\cancel{5}}\times\overset{2}{\cancel{10}}\times 10=259{,}200. \quad \textit{Ans.}$$

13. How long a wall 6 ft. high, $12\frac{3}{4}$ in. thick, can be built with the bricks forming a rectangular pile 17 ft. 6 in. long, 5 ft. wide, and 4 ft. 3 in. high?

$12\frac{3}{4}$ in. $= 1\frac{1}{16}$ ft.

Volume of pile of bricks $=(17\frac{1}{2}\times 5\times 4\frac{1}{4})$ cu. ft.

$$\frac{17\frac{1}{2}\times 5\times 4\frac{1}{4}}{6\times 1\frac{1}{16}}=\frac{35}{\cancel{2}}\times 5\times\frac{\cancel{17}}{\cancel{4}}\times\frac{1}{\underset{3}{\cancel{6}}}\times\frac{\overset{\cancel{2}}{\overset{\cancel{4}}{\cancel{16}}}}{\cancel{17}}=\frac{175}{3}=58\tfrac{1}{3}. \qquad 58\tfrac{1}{3}\text{ ft. } \textit{Ans.}$$

14. Find the surface of a cube whose edge is 3 ft. 5⅔ in.

The surface of the cube consists of 6 squares 3 ft. 5⅔ in., that is $3\frac{17}{36}$ ft., on a side.

$$6 \times 3\tfrac{17}{36} \times 3\tfrac{17}{36} = \cancel{6} \times \frac{125}{\underset{6}{\cancel{36}}} \times \frac{125}{36} = \frac{15625}{216} = 72\tfrac{73}{216}.$$

$72\frac{73}{216}$ sq. ft. = 72 sq. ft. 48⅔ sq. in. *Ans.*

15. Find the surface of a rectangular block of stone 4 ft. long, 2½ ft. broad, and 1¼ ft. thick.

$$2 \times 4 \times 2\tfrac{1}{2} = \cancel{2} \times 4 \times \frac{5}{\cancel{2}} = 20.$$

$$2 \times 4 \times 1\tfrac{1}{4} = 2 \times \cancel{4} \times \frac{5}{\cancel{4}} = 10.$$

$$2 \times 2\tfrac{1}{2} \times 1\tfrac{1}{4} = \cancel{2} \times \frac{5}{\cancel{2}} \times \frac{5}{4} = \frac{25}{4} = 6\tfrac{1}{4}.$$

20 sq. ft. + 10 sq. ft. + 6¼ sq. ft. = 36¼ sq. ft. = 36 sq. ft. 36 sq. in. *Ans.*

16. A lake whose area is 45 A. is covered with ice 3 in. thick. Find the weight of the ice in tons, if a cubic foot of ice weighs 920 oz.

$$45 \text{ A.} = 45 \times 43{,}560 \text{ sq. ft.} \qquad 920 \text{ oz.} = \frac{920}{16} \text{ lb.} = \frac{920}{16 \times 2000} \text{ t.}$$

$$\overset{9}{\cancel{45}} \times \overset{1089}{\cancel{43560}} \times \frac{1}{\cancel{4}} \times \frac{\overset{\overset{23}{\cancel{230}}}{\cancel{920}}}{16 \times \underset{\underset{\cancel{5}}{\cancel{50}}}{\cancel{2000}}} \text{ t.} = \frac{225423}{16} \text{ t.} = 14{,}088\tfrac{15}{16} \text{ t. } \textit{Ans.}$$

17. How many bricks will be required to build a wall 75 ft. long, 6 ft. high, and 16 in. thick, if each brick is 8 in. long, 4 in. wide, and 2¼ in. thick?

Volume of wall $= 75 \times 6 \times \frac{4}{3} \times 1728$ cu. in.

Volume of brick $= (8 \times 4 \times 2\frac{1}{4})$ cu. in.

$$\frac{75 \times 6 \times \frac{4}{3} \times 1728}{8 \times 4 \times 2\frac{1}{4}} = 75 \times \overset{2}{\cancel{6}} \times \frac{4}{\cancel{3}} \times \overset{\overset{24}{\cancel{192}}}{\cancel{1728}} \times \frac{1}{\cancel{8}} \times \frac{1}{\cancel{4}} \times \frac{\cancel{4}}{\cancel{9}} = 14{,}400. \textit{ Ans.}$$

[redacted]

18. The ceiling of a room 27 ft. long, 24 ft. broad, and 10 ft. high, is to be raised so as to increase the space by 84 cu. yd. What will then be the height of the room?

27 ft. = 9 yd.; 24 ft. = 8 yd.
$$\frac{\overset{\overset{7}{\cancel{28}}}{\cancel{84}}}{\underset{3}{\cancel{9}} \times \underset{2}{\cancel{8}}} = \frac{7}{6} = 1\tfrac{1}{6}.$$

Hence, the ceiling must be raised $1\frac{1}{6}$ yd., or $3\frac{1}{2}$ ft.

$$10 \text{ ft.} + 3\tfrac{1}{2} \text{ ft.} = 13\tfrac{1}{2} \text{ ft.} \quad \textit{Ans.}$$

19. Find the cost of making a road 110 yd. long and 18 ft. wide, if the soil is first removed to the depth of 1 ft. at a cost of 25 cents a cubic yard, rubble then laid 8 in. deep at 25 cents a cubic yard, and gravel placed on top 9 in. thick at $62\frac{1}{2}$ cents a cubic yard.

The cost of removing the soil is

$$\overset{55}{\cancel{110}} \times \overset{2}{\cancel{6}} \times \frac{1}{\cancel{3}} \times \$\frac{1}{\underset{2}{\cancel{4}}} = \$55.$$

The cost of laying the rubble is

$$110 \times \cancel{6} \times \frac{\overset{2}{\cancel{8}}}{\underset{\underset{3}{\cancel{6}}}{\cancel{36}}} \times \$\frac{1}{\cancel{4}} = \$\frac{110}{3} = \$36.67.$$

The cost of laying the gravel is

$$\overset{55}{\cancel{110}} \times \overset{3}{\cancel{6}} \times \frac{\cancel{9}}{\underset{\underset{2}{\cancel{4}}}{\cancel{36}}} \times \$\frac{5}{8} = \$\frac{825}{8} = \$103.12.$$

$$\$55 + \$36.67 + \$103.12 = \$194.79. \quad \textit{Ans.}$$

20. If a rectangular block of wood 5 ft. 4.8 in. long, 1 ft. 9 in. wide and thick, weighs 7.56 cwt., find in pounds its weight per cubic foot.

5 ft. 4.8 in. = $5\frac{2}{5}$ ft. Volume of block = $(5\frac{2}{5} \times 1\frac{3}{4} \times 1\frac{3}{4})$ cu. in.

7.56 cwt. = 756 lb.

$$\frac{756}{5\frac{2}{5} \times 1\frac{3}{4} \times 1\frac{3}{4}} = \overset{\overset{4}{\cancel{108}}}{\cancel{756}} \times \frac{5}{\cancel{27}} \times \frac{4}{\cancel{7}} \times \frac{4}{7} = \frac{320}{7} = 45\tfrac{5}{7}. \qquad 45\tfrac{5}{7} \text{ lb. } \textit{Ans.}$$

21. How many cords of wood in a pile 40 ft. long, 4 ft. wide, and 5 ft. 4 in. high?

$$\frac{40 \times 4 \times 5\frac{1}{3}}{8 \times 4 \times 4} = \frac{\overset{5}{\cancel{40}} \times \cancel{4} \times \overset{4}{\cancel{16}}}{\cancel{8} \times \cancel{4} \times \cancel{4} \times 3} = \frac{20}{3} = 6\frac{2}{3}. \quad \textit{Ans.}$$

22. A pile of wood containing $67\frac{1}{2}$ cords is 270 ft. long and 4 ft. wide. How high is it?

$$\frac{67\frac{1}{2} \times 128}{270 \times 4} = \frac{\cancel{135} \times \overset{\overset{\overset{8}{\cancel{16}}}{\cancel{32}}}{\cancel{128}}}{\cancel{2} \times \underset{2}{\cancel{270}} \times \cancel{4}} = 8.$$

8 ft. *Ans.*

23. What will be the cost of a pile of wood 25 ft. long, 4 ft. wide, and 4 ft. 8 in. high, at \$3.75 a cord?

$$\frac{25 \times 4 \times 4\frac{2}{3}}{8 \times 4 \times 4} \times \$3\frac{3}{4} = \frac{25 \times \cancel{4} \times \overset{7}{\cancel{14}}}{8 \times \cancel{4} \times \underset{2}{\cancel{4}} \times \cancel{3}} \times \$\frac{\overset{5}{\cancel{15}}}{4} = \$\frac{875}{64} = \$13.67. \quad \textit{Ans.}$$

24. What must be the length of a load of wood $3\frac{1}{2}$ ft. high and 5 ft. wide to contain a cord?

$$\frac{128}{3\frac{1}{2} \times 5} = \frac{2 \times 128}{7 \times 5} = \frac{256}{35} = 7\frac{11}{35}.$$

$7\frac{11}{35}$ ft. *Ans.*

25. How high must manure be piled in a cart 6 ft. by 4 ft., that the load may contain half a cord?

$$\frac{\frac{1}{2} \times 128}{6 \times 4} = \frac{\overset{\overset{\overset{8}{\cancel{16}}}{\cancel{32}}}{\cancel{128}}}{\cancel{2} \times \underset{3}{\cancel{6}} \times \cancel{4}} = \frac{8}{3} = 2\frac{2}{3}.$$

$2\frac{2}{3}$ ft. *Ans.*

26. How many cords of wood in a pile 32 ft. long, 8 ft. wide, and 6 ft. high?

$$\frac{\overset{\overset{2}{\cancel{8}}}{\cancel{32}} \times \cancel{8} \times 6}{\cancel{8} \times \cancel{4} \times \cancel{4}} = 12. \quad \textit{Ans.}$$

27. How many cords of wood in a pile 40 ft. long, 4 ft. wide, and 8 ft. high?

$$\frac{\overset{10}{\cancel{40}} \times \cancel{4} \times \cancel{8}}{\cancel{8} \times \cancel{4} \times \cancel{4}} = 10. \quad \textit{Ans.}$$

[redacted]

28. Find the cost of the wood at \$3.75 a cord that can be piled in a shed 18 ft. long, 16 ft. wide, and 7 ft. high.

$$\frac{\overset{9}{\cancel{18}} \times \overset{4}{\cancel{16}} \times 7}{\underset{4}{\cancel{8}} \times \cancel{4} \times \cancel{4}} \times \$\frac{15}{4} = \$\frac{945}{16} = \$59.06. \quad \textit{Ans.}$$

29. Find the number of cubic inches in a sphere 11 in. in diameter.

$0.5236 \times (11 \times 11 \times 11)$ cu. in.

$$\begin{array}{r} 11 \\ \underline{11} \\ 121 \\ \underline{11} \\ 1331 \end{array} \qquad \begin{array}{r} 0.5236 \\ \underline{1331} \\ 5236 \\ 15708 \\ 15708 \\ \underline{5236} \\ 696.9116 \end{array} \ \textit{Ans.}$$

30. How many cubic inches of water can be poured into a hollow sphere whose inner diameter is $16\frac{1}{2}$ in.?

$0.5236 \times (16\frac{1}{2} \times 16\frac{1}{2} \times 16\frac{1}{2})$ cu. in.

$$= \overset{\overset{0.1309}{\cancel{0.2618}}}{\cancel{0.5236}} \times \left(\frac{33}{\cancel{2}} \times \frac{33}{\cancel{2}} \times \frac{33}{2}\right) \text{ cu. in.}$$

$$\begin{array}{r} 33 \\ \underline{33} \\ 99 \\ \underline{99} \\ 1089 \\ \underline{33} \\ 3267 \\ \underline{3267} \\ 35937 \end{array} \qquad \begin{array}{r} 35937 \\ \underline{0.1309} \\ 323433 \\ 107811 \\ \underline{35937} \\ 2\,\big|\,4704.1533 \\ 2352.0767 \end{array} \ \textit{Ans.}$$

31. What is the volume of the ball on top of St. Paul's in London, which is 6 ft. in diameter?

$$\begin{array}{r} 6 \\ \underline{6} \\ 36 \\ \underline{6} \\ 216 \end{array} \qquad \begin{array}{r} 0.5236 \\ \underline{216} \\ 31416 \\ 5236 \\ \underline{10472} \\ 113.0976 \end{array}$$

113.0976 cu. ft. *Ans.*

32. If 30 cu. in. of powder weigh 1 lb., how many ounces of powder will just fill a shell, inner diameter 3 in.?

$$\begin{array}{r} 3 \\ \underline{3} \\ 9 \\ \underline{3} \\ 27 \end{array} \qquad \begin{array}{r} 0.5236 \\ \underline{27} \\ 36652 \\ \underline{10472} \\ 14.1372 \end{array}$$

$$\frac{\overset{4.7124}{\cancel{14.1372}}}{\underset{\underset{5}{\cancel{15}}}{\cancel{30}}} \times \overset{8}{\cancel{16}} \text{ oz.} = \frac{37.6992}{5} \text{ oz.}$$

$= 7.5398$ oz. *Ans.*

33. Find the volume of a cylinder whose height is 5 ft. and the radius of whose base is 1 ft. 2 in.

$5 \times 3.1416 \times 1\frac{1}{6} \times 1\frac{1}{6}$

$$= 5 \times \overset{\overset{0.2618}{\cancel{0.5236}}}{\cancel{3.1416}} \times \frac{7}{\cancel{6}} \times \frac{7}{\underset{3}{\cancel{6}}} = 4.2761.$$

(2)

$$\begin{array}{r} 0.2618 \\ \underline{49} \\ 23562 \\ \underline{10472} \\ 12.8282 \end{array} \qquad \begin{array}{r} 12.8282 \\ \underline{5} \\ 3\,\big|\,64.1410 \\ 21.3803 \end{array}$$

21.3803 cu. ft. Ans.

34. Find the volume of a cylinder whose height is 4 ft. 6 in. and the diameter of whose base is 8 ft. 2 in.

$$4\tfrac{1}{2} \times 0.7854 \times 8\tfrac{1}{6} \times 8\tfrac{1}{6}$$

$$= \frac{9}{2} \times \overset{0.1309}{\cancel{0.7854}} \times \frac{49}{\cancel{6}} \times \frac{49}{6} = 235.7182$$

$$\begin{array}{r} 49 \\ 49 \\ \hline 441 \\ 196 \\ \hline 2401 \\ 9 \\ \hline 21609 \end{array} \qquad \begin{array}{r} 21609 \\ 0.1309 \\ \hline 194481 \\ 64827 \\ 21609 \\ \hline 12\,\boxed{2828.6181} \\ 235.7182 \end{array}$$

235.7182 cu. ft. *Ans.*

35. How many cubic yards of earth must be excavated to make a well 3 ft. in diameter and 20 ft. deep?

$$(20 \times 0.7854 \times 3 \times 3) \text{ cu. ft.}$$

$$= \frac{20 \times \overset{0.2618}{\cancel{0.7854}} \times \cancel{3} \times \cancel{3}}{\underset{\underset{\cancel{3}}{\cancel{9}}}{\cancel{27}}} \text{ cu. yd.}$$

$= 5.236$ cu. yd. *Ans.*

36. How many cubic yards in a tunnel 800 ft. long, if a cross section is a semicircle with a radius of 10 ft.?

$$(800 \times \tfrac{1}{2} \times 3.1416 \times 10 \times 10) \text{ cu. ft.}$$

$$= \frac{800 \times \overset{\overset{0.5236}{\cancel{1.0472}}}{\cancel{3.1416}} \times 10 \times 10}{\cancel{2} \times \underset{9}{\cancel{27}}} \text{ cu. yd.}$$

$$= \frac{41888}{9} \text{ cu. yd.} = 4654\tfrac{2}{9} \text{ cu. yd.}$$

Ans.

37. Find the number of cubic feet in a bushel.

1 bu. = 2150.42 cu. in.

$$\begin{array}{r} 1.24445 \\ 1728)\overline{2150.42} \\ 1728 \\ \hline 4224 \\ 3456 \\ \hline 7682 \\ 6912 \\ \hline 7700 \\ 6912 \\ \hline 7880 \\ 6912 \\ \hline 9680 \\ 8640 \\ \hline 1040 \end{array}$$

1.24446 cu. ft. *Ans.*

38. Find the number of bushels a bin will hold that is 6 ft. long, 5 ft. wide, and 4 ft. deep.

$\frac{4}{5}$ of $6 \times 5 \times 4 = 96.$

$\frac{1}{2}$ of 0.01 of 96 = 0.48.

96.48 *Ans.*

39. Find the number of cubic feet required for 1000 bu.

$\frac{5}{4}$ of 1000 = 1250.

$\frac{1}{2}$ of 0.01 of 1250 = 6.25

1243.75 *Ans.*

40. Find the number of bushels a bin will hold that is 8 ft. long, 4 ft. wide, 3 ft. deep.

$\frac{4}{5}$ of $8 \times 4 \times 3$ = 76.8

$\frac{1}{2}$ of 0.01 of 76.8 = 0.384

77.184 *Ans.*

[redacted]

41. Find the number of bushels a bin will hold that is 9 ft. long, 6 ft. 6 in. wide, 3 ft. 4 in. deep.

$\frac{4}{5}$ of $9 \times 6\frac{1}{2} \times 3\frac{1}{3} = 156.$
$\frac{1}{2}$ of 0.01 of 156 $=$ 0.78

156.78 *Ans.*

42. Find the depth of a bin that will hold 360 bu., if its length is 12 ft. and its width 6 ft.

$\frac{5}{4}$ of 360 $= 450.$
$\frac{1}{4}$ of 0.01 of 450 $=$ 2.25

447.75

$$\frac{447.75}{12 \times 6} = \frac{\overset{\overset{199}{\cancel{597}}}{\cancel{1791}}}{4} \times \frac{1}{\underset{4}{\cancel{12}}} \times \frac{1}{\underset{2}{\cancel{6}}} = \frac{199}{32} = 6\tfrac{7}{32}.$$

$6\frac{7}{32}$ ft. $= 6$ ft. $2\frac{5}{8}$ in. *Ans.*

43. Find the length of a bin that is 6 ft. wide and 5 ft. deep, if it will hold 400 bu.

$\frac{5}{4}$ of 400 $= 500.$
$\frac{1}{2}$ of 0.01 of 500 $=$ 2.5

497.5

$$\frac{497.5}{6 \times 5} = \frac{\overset{199}{\cancel{995}}}{2} \times \frac{1}{6} \times \frac{1}{\cancel{5}} = \frac{199}{12} = 16\tfrac{7}{12}.$$

$16\frac{7}{12}$ ft. $= 16$ ft. 7 in. *Ans.*

44. Find the number of bushels that will fill a bin 8.5 ft. long, 4.5 ft. wide, 3.5 ft. deep.

$\frac{4}{5}$ of $8\frac{1}{2} \times 4\frac{1}{2} \times 3\frac{1}{2} = 107.1$
$\frac{1}{2}$ of 0.01 of 107.1 $=$ 0.5355

107.6355 *Ans.*

45. A bin 20 ft. long, 12 ft. wide, and 6 ft. deep is full of wheat. What is its value at $0.75 a bushel?

$\frac{4}{5}$ of $20 \times 12 \times 6 = 1152.$
$\frac{1}{2}$ of 0.01 of 1152 $=$ 5.76

1157.76

```
 1157.76
    0.75
 -------
  578880
 810432
 -------
868.3200
```

$868.32. *Ans.*

46. If a ton of coal occupies 40 cu. ft., how many tons of coal will fill a bin 21 ft. long, 10 ft. wide, 5 ft. deep?

$$\frac{21 \times \cancel{10} \times 5}{\underset{4}{\cancel{40}}} = \frac{105}{4} = 26\tfrac{1}{4}. \text{ *Ans.*}$$

47. If a ton of Lehigh coal occupies 35 cu. ft., how many tons of Lehigh coal will fill a bin 8 ft. long, 5 ft. 9 in. wide, 3 ft. 6 in. deep?

$$\frac{8 \times 5\frac{3}{4} \times 3\frac{1}{2}}{35} = \overset{2}{\cancel{8}} \times \frac{23}{\cancel{4}} \times \frac{\cancel{7}}{\cancel{2}} \times \frac{1}{\underset{5}{\cancel{35}}}$$

$$\frac{23}{5} = 4\tfrac{3}{5}. \text{ *Ans.*}$$

48. How many bushels will a bin hold that is 22 ft. long, 12 ft. 6 in. wide, 9 ft. 9 in. deep?

$\frac{4}{5}$ of $22 \times 12\frac{1}{2} \times 9\frac{3}{4} = 2145.$
$\frac{1}{2}$ of 0.01 of 2145 $=$ 10.725

2155.725

Ans.

49. Find the number of gallons in a cubic foot.

1 gal. = 231 cu. in.

```
         7.48051
231)1728.
    1617
     1110
      924
      1860
      1848
       1200
       1155
         450
         231
         219
```

7.48052. *Ans.*

50. Find the exact number of gallons a cistern will hold that is 5 ft. square, and 6 ft. deep.

$$\frac{5 \times 5 \times 6 \times \overset{576}{\cancel{1728}}}{\underset{77}{\cancel{231}}} = 1122.078. \textit{ Ans.}$$

```
  5      576          1122.077
  5      150     77)86400.
 25    28800        77
  6      576         94
150    86400         77
                     170
                     154
                      160
                      154
                       600
                       539
                        610
                        539
                         71
```

51. Find the exact number of gallons a cistern will hold that is 13 ft. long, 6 ft. wide, 7 ft. 4 in. deep.

$$\frac{13 \times 6 \times 7\frac{1}{3} \times 1728}{231}$$

$$= \frac{13 \times \overset{2}{\cancel{6}} \times \overset{2}{\cancel{22}} \times \overset{576}{\cancel{1728}}}{\underset{\underset{7}{\cancel{21}}}{\cancel{231}} \times \cancel{3}} = 4278.857. \textit{ Ans.}$$

```
13          576
 2           52
26         1152
 2         2880
52       7)29952
           4278.857
```

52. Find the exact number of gallons a tank will hold that is 4 ft. long, 2 ft. 8 in. wide, 1 ft. 8 in. deep.

$$\frac{4 \times 2\frac{2}{3} \times 1\frac{2}{3} \times 1728}{231} = \frac{4 \times 8 \times 5 \times \overset{\overset{\overset{64}{\cancel{192}}}{\cancel{576}}}{\cancel{1728}}}{\cancel{3} \times \cancel{3} \times \underset{77}{\cancel{231}}}$$

$= 132.987.$ *Ans.*

```
   4          132.987
   8     77)10240.
  32        77
   5        254
 160        231
  64         230
 640         154
 960          760
10240         693
               670
               616
                540
                539
                  1
```

[redacted]

53. Find the capacity in cubic feet of a cistern that will hold 200 bbl. of water.

$$200 \times 31\tfrac{1}{2} \div (1728 \div 231)$$

$$=\overset{25}{\cancel{200}}\times\frac{\overset{7}{\cancel{63}}}{2}\times\frac{\overset{77}{\cancel{231}}}{\underset{\underset{\underset{8}{\cancel{64}}}{\cancel{192}}}{\cancel{1728}}}=842.1875.\ \textit{Ans.}$$

$$\begin{array}{r} 25 \\ \underline{7} \\ 175 \\ \underline{77} \\ 1225 \\ \underline{1225} \\ 13475 \end{array} \qquad \begin{array}{r} 842.1875 \\ 16\overline{)13475.} \\ \underline{128} \\ 67 \\ \underline{64} \\ 35 \\ \underline{32} \\ 30 \\ \underline{16} \\ 140 \\ \underline{128} \\ 120 \\ \underline{112} \\ 80 \\ \underline{80} \end{array}$$

54. Find the approximate number of gallons a cylindrical cistern will hold that is 6 ft. in diameter and 7 ft. deep.

$$7 \times 0.7854 \times 6 \times 6 \times 7\tfrac{1}{2}.$$

$$\begin{array}{r} 7 \\ \underline{6} \\ 42 \\ \underline{6} \\ 252 \\ \underline{7\tfrac{1}{2}} \\ 126 \\ \underline{1764} \\ 1890 \end{array} \qquad \begin{array}{r} 0.7854 \\ \underline{1890} \\ 706860 \\ 62832 \\ \underline{7854} \\ 1484.4060 \end{array}$$

$\tfrac{1}{4}$ of 0.01 of 1484.406 = 3.711.

1484.406 − 3.711 = 1480.695. *Ans.*

55. Find the approximate number of gallons a cylindrical vessel will hold that is 12 in. in diameter and 10 in. deep.

$$\frac{5}{\cancel{6}} \times \overset{0.1309}{\cancel{0.7854}} \times 1 \times 1 \times \frac{15}{2}.$$

$$\begin{array}{r} 15 \\ \underline{5} \\ 75 \end{array} \qquad \begin{array}{r} 0.1309 \\ \underline{75} \\ 6545 \\ 9163 \\ 2\,\boxed{9.8175} \\ 4.90875 \end{array}$$

$\tfrac{1}{4}$ of 0.01 of 4.90875 = 0.01227.

4.90875 − 0.01227 = 4.89648. *Ans.*

56. How many quarts will a cylindrical vessel hold $5\tfrac{1}{6}$ in. in diameter and 6 in. deep?

$$\frac{6 \times 0.7854 \times 5\tfrac{1}{6} \times 5\tfrac{1}{6}}{\tfrac{1}{4} \times 231}$$

$$= \cancel{6} \times \overset{0.0034}{\cancel{0.7854}} \times \frac{31}{\cancel{6}} \times \frac{31}{\underset{3}{\cancel{6}}} \times \frac{\overset{2}{\cancel{4}}}{\cancel{231}}$$

= 2.1783. *Ans.*

$$\begin{array}{r} 31 \\ \underline{31} \\ 31 \\ \underline{93} \\ 961 \\ \underline{2} \\ 1922 \end{array} \qquad \begin{array}{r} 1922 \\ \underline{0.0034} \\ 7688 \\ 5766 \\ 3\,\boxed{6.5348} \\ 2.1783 \end{array}$$

57. How many quarts will a hollow sphere hold whose interior diameter is 12 in.?

$$\frac{0.5236 \times 12 \times 12 \times 12}{\frac{1}{4} \text{ of } 231}$$

$$= \overset{0.0068}{\cancel{0.5236}} \times \overset{4}{\cancel{12}} \times 12 \times 12 \times \frac{4}{\underset{\cancel{3}}{\cancel{231}}}$$

$= 15.6672.$ *Ans.*

$$\begin{array}{r} 12 \\ 12 \\ \hline 144 \\ 4 \\ \hline 576 \end{array} \qquad \begin{array}{r} 576 \\ 4 \\ \hline 2304 \\ 0.0068 \\ \hline 18432 \\ 13824 \\ \hline 15.6672 \end{array}$$

58. What part of a bushel will a hemispherical bowl hold that is 13 in. in diameter?

$$\frac{\frac{1}{2} \times 0.5236 \times 13 \times 13 \times 13}{2150.42}$$

$$= \frac{\overset{\overset{0.1309}{\cancel{0.2618}}}{\cancel{0.5236}} \times 13 \times 13 \times 13}{\cancel{2} \times \underset{1075.21}{\cancel{2150.42}}}$$

$= 0.267.$ *Ans.*

$$\begin{array}{r} 13 \\ 13 \\ \hline 39 \\ 13 \\ \hline 169 \\ 13 \\ \hline 507 \\ 169 \\ \hline 2197 \end{array} \qquad \begin{array}{r} 2197 \\ 0.1309 \\ \hline 19773 \\ 6591 \\ 2197 \\ \hline 287.5873 \end{array}$$

```
            0.267
107521)28758.73
       215042
       725453
       645126
        803270
        752647
         50623
```

59. If a cubical box 2 ft. on an edge contains a solid sphere 2 ft. in diameter, how many gallons of water can be poured into the box?

$$2 \times 2 \times 2 = 8.$$

$$0.5236 \times 2 \times 2 \times 2 = 4.1888.$$

8 cu. ft. − 4.1888 cu. ft.
= 3.8112 cu. ft.

3.8112 cu. ft. = 3.8112 × 1728 cu. in.

$$= \frac{3.8112 \times \overset{576}{\cancel{1728}}}{\underset{77}{\cancel{231}}} \text{ gal.}$$

$= 28.5098$ gal. *Ans.*

$$\begin{array}{r} 3.8112 \\ 576 \\ \hline 228672 \\ 266784 \\ 190560 \\ \hline 2195.2512 \end{array}$$

```
     28.5097
77)2195.2512
   154
    655
    616
     392
     385
      751
      693
       582
       539
        43
```

[redacted]

60. If 64 qt. of water are poured into a vessel that will hold 2 bu. of wheat, what part of the vessel will be filled?

64 qt. = 16 gal. = 16 × 231 cu. in.

2 bu. = 2 × 2150.42 cu. in.

$$\frac{\overset{\overset{4}{\cancel{8}}}{\cancel{16}} \times 231}{\cancel{2} \times \underset{1075.21}{\cancel{2150.42}}} = 0.859. \textit{ Ans.}$$

$$\begin{array}{r} 231 \\ \underline{4} \\ 924 \end{array}$$

$$\begin{array}{r} 0.859 \\ 107521\overline{)92400.} \\ \underline{860168} \\ 638320 \\ \underline{537605} \\ 1007150 \\ \underline{967689} \\ 39461 \end{array}$$

Exercise 91. Page 198.

1. Find the number of cubic inches in 1 oz. (av.) of water.

1 cu. ft. of water weighs 1000 oz. 1728 ÷ 1000 = 1.728. *Ans.*

2. Find the weight in ounces (av.) of 1 cu. in. of water.

$$1000 \text{ oz.} \div 1728 = \frac{1}{\underset{216}{\cancel{1728}}} \times \overset{125}{\cancel{1000}} \text{ oz.} = \frac{125}{216} \text{ oz. } \textit{Ans.}$$

3. Find the weight in ounces (av.) of 1 pt. of water.

$$1 \text{ pt.} = \tfrac{231}{8} \text{ cu. in.}$$

$$\frac{\overset{77}{\cancel{231}}}{8} \times \frac{125}{\underset{72}{\cancel{216}}} = \frac{9625}{576} = 16.71 \text{ oz. } \textit{Ans.}$$

4. Find the number of pints in 1 lb. of water.

By Ex. 3, 1 pt. of water weighs 16.71 oz. 1 lb. = 16 oz.

16 ÷ 16.71 = 0.9575. *Ans.*

$$\begin{array}{r} 0.9575 \\ 1671\overline{)1600.} \\ \underline{15039} \\ 9610 \\ \underline{8355} \\ 12550 \\ \underline{11697} \\ 8530 \\ \underline{8355} \\ 175 \end{array}$$

5. Find the weight in grains of 1 cu. in. of water.

1 cu. in. of water weighs $\frac{125}{216}$ oz. =

$$\frac{125}{16\times 216}\text{lb.}=\frac{125\times \overset{875}{\cancel{7000}}}{16\times \underset{27}{\cancel{216}}}\text{gr.}=\frac{109375}{432}\text{gr.}=253.183\text{ gr. } \textit{Ans.}$$

6. A bar of iron 5 in. long and 2 in. square weighs 5 lb. What is the specific gravity of the iron?

$$(5\times 2\times 2)\text{ cu. in.} = 20\text{ cu. in.}$$

If 20 cu. in. of iron weighs 5 lb., 4 cu. in. weighs 1 lb., and 1 cu. in. weighs 4 oz. But by Ex. 2, 1 cu. in. of water weighs $\frac{125}{216}$ oz.

Therefore, the specific gravity of the iron is

$$4\div\frac{125}{216}=\frac{216}{125}\times 4=\frac{864}{125}=6.912.\ \textit{Ans.}$$

7. If a bar of iron 18 in. long, $2\frac{1}{3}$ in. wide, $1\frac{3}{4}$ in. thick weighs 18 lb. 9 oz., what is the specific gravity of the iron?

$$(18\times 2\tfrac{1}{3}\times 1\tfrac{3}{4})\text{ cu. in.}=\left(\overset{\overset{3}{\cancel{6}}}{\cancel{18}}\times\frac{7}{\cancel{3}}\times\frac{7}{\underset{2}{\cancel{4}}}\right)\text{cu. in.}$$

$$=\frac{147}{2}\text{ cu. in.}=73\tfrac{1}{2}\text{ cu. in.}$$

$$18\text{ lb. }9\text{ oz.}=297\text{ oz.}$$

Therefore, 1 cu. in. of the iron weighs $\frac{297}{73\frac{1}{2}}$ oz., and the specific gravity of the iron is

$$\frac{297}{73\frac{1}{2}}\div\frac{125}{216}=\frac{216}{125}\times\overset{99}{\cancel{297}}\times\frac{2}{\underset{49}{\cancel{147}}}=\frac{42768}{6125}=6.98.\ \textit{Ans.}$$

8. If the specific gravity of iron is 7.48, find the number of cubic inches of iron to the pound.

1 cu. ft. of water weighs 62.5 lb.

Therefore, 1 lb. of water occupies $\frac{1728}{62.5}$ cu. in., and 1 lb. of iron, specific gravity 7.48, occupies

[redacted]

$$\frac{1728}{7.48 \times 62.5} \text{ cu. in} \doteq 3.696 \text{ cu. in. } \textit{Ans.}$$

$$\begin{array}{r} 7.48 \\ \underline{62.5} \\ 3740 \\ 1496 \\ \underline{4488} \\ 467.500 \end{array} \qquad \begin{array}{r} 3.696 \\ 4675)\overline{17280.} \\ \underline{14025} \\ 32550 \\ \underline{28050} \\ 45000 \\ \underline{42075} \\ 29250 \\ \underline{28050} \\ 1200 \end{array}$$

9. If the specific gravity of gold is 19.36, find the number of cubic inches in 2 lb. $6\frac{1}{2}$ oz. of gold.

2 lb. $6\frac{1}{2}$ oz. (troy) $= 2\frac{13}{24}$ lb.

1 lb. av. of water occupies $\frac{1728}{62.5}$ cu. in.

1 lb. troy of water occupies $\frac{1728}{62.5 \times \frac{7000}{5760}}$ cu. in.

1 lb. troy of gold occupies $\frac{1728}{19.36 \times 62.5 \times \frac{7000}{5760}}$ cu. in.

$2\frac{13}{24}$ lb. troy of gold occupies $\frac{2\frac{13}{24} \times 1728}{19.36 \times 62.5 \times \frac{7000}{5760}}$ cu. in.

$$\frac{2\frac{13}{24} \times 1728}{19.36 \times 62.5 \times \frac{7000}{5760}} = \frac{2\frac{13}{24} \times 1728}{19\frac{9}{25} \times 62\frac{1}{2} \times \frac{7000}{5760}}$$

$$= \frac{61}{\cancel{24}} \times \overset{432}{\cancel{1728}} \times \frac{\cancel{25}}{\underset{121}{\cancel{484}}} \times \frac{2}{\underset{5}{\cancel{125}}} \times \frac{\overset{\overset{6}{\cancel{240}}}{\cancel{5760}}}{\underset{175}{\cancel{7000}}}$$

$$= \frac{316224}{105875} = 2.987.$$

2.987 cu. in. *Ans.*

10. How many pounds does a boy lift in raising a cubic foot of stone under water, if its specific gravity is $2\frac{1}{2}$?

The boy lifts $2\frac{1}{2}$ times the weight of a cubic foot of water less the weight of a cubic foot of water; that is, he lifts $1\frac{1}{2}$ times the weight of a cubic foot of water.

$1\frac{1}{2} \times 62.5$ lb. $= 93.75$ lb. *Ans.*

11. A square-built scow 12 ft. long, $6\frac{1}{2}$ ft. wide, sinks 5 in. into the water. What does it weigh, and how many pounds will be required to sink it 7 in. deeper?

The weight of the scow is equal to the weight of the water it displaces.

$$12 \times 6\tfrac{1}{2} \times \frac{5}{12} \times 62\tfrac{1}{2} = \cancel{12} \times \frac{13}{2} \times \frac{5}{\cancel{12}} \times \frac{125}{2} = \frac{8125}{4} = 2031\tfrac{1}{4}.$$

$2031\frac{1}{4}$ lb. *Ans.*

$$12 \times 6\tfrac{1}{2} \times 1 \times 62\tfrac{1}{2} = \overset{3}{\overset{\cancel{6}}{\cancel{12}}} \times \frac{13}{\cancel{2}} \times \frac{125}{\cancel{2}} = 4875.$$

4875 lb. $- 2031\frac{1}{4}$ lb. $= 2843\frac{3}{4}$ lb. *Ans.*

12. A square-built scow 11 ft. long, $5\frac{1}{4}$ ft. wide, weighs 320 lb. and is loaded with 750 lb. of stone. How deep will it sink in the water?

The total weight of the scow is 320 lb. + 750 lb. = 1070 lb.

The volume in cubic feet of the water displaced is $\frac{1070}{62\frac{1}{2}}$.

The area in square feet of the top of the scow is $11 \times 5\frac{1}{4}$.

Therefore, the depth in feet the scow will sink is $\frac{1070}{62\frac{1}{2}} \div (11 \times 5\frac{1}{4})$,

and the depth in inches the scow will sink is $12 \times \frac{1070}{62\frac{1}{2}} \div (11 \times 5\frac{1}{4})$.

$$12 \times \frac{1070}{62\frac{1}{2}} \div (11 \times 5\tfrac{1}{4}) = \overset{4}{\cancel{12}} \times \overset{214}{\cancel{1070}} \times \frac{2}{\underset{25}{\cancel{125}}} \times \frac{1}{11} \times \frac{4}{\underset{7}{\cancel{21}}}$$

$$= \frac{6848}{1925} = 3\tfrac{1073}{1925} = 3.557. \qquad 3.557 \text{ in. } \textit{Ans.}$$

13. How many tons of ice, specific gravity 0.93, can be packed in a building 50 ft. long, 40 ft. wide, 20 ft. high?

$$\frac{50 \times 40 \times 20 \times 0.93 \times 62\frac{1}{2}}{2000} = \frac{\cancel{50} \times \cancel{40} \times \cancel{20}}{\underset{\cancel{40}}{\cancel{2000}}} \times \frac{93}{\underset{\cancel{5}}{\cancel{100}}} \times \frac{\overset{25}{\cancel{125}}}{2} = \frac{2325}{2}$$

$= 1162\frac{1}{2}$. *Ans.*

14. If the specific gravity of an iceberg is 0.9, how many cubic yards does an iceberg contain that is 40 rd. long, 6 yd. wide, and rises 160 ft. out of the sea?

[redacted]

$$40 \text{ rd.} = 220 \text{ yd.}; \quad 160 \text{ ft.} = 53\tfrac{1}{3} \text{ yd.}$$

$$220 \times 6 \times 53\tfrac{1}{3} = 220 \times \overset{2}{\cancel{6}} \times \frac{160}{\cancel{3}} = 70{,}400.$$

Now, if the specific gravity of the iceberg is 0.9, only $\frac{1}{10}$ of the iceberg is above the water.

$$10 \times 70{,}400 \text{ cu. yd.} = 704{,}000 \text{ cu. yd. } \textit{Ans.}$$

15. If a cubic foot of brick wall weighs 90 lb. and contains 22 bricks, with the mortar, what is the weight and the specific gravity of a brick and its share of mortar?

$$90 \text{ lb.} \div 22 = 4\tfrac{1}{11} \text{ lb. } \textit{Ans.}$$

$$\text{The specific gravity} = \frac{90}{62\frac{1}{2}} = \overset{18}{\cancel{90}} \times \frac{2}{\underset{25}{\cancel{125}}} = \frac{36}{25} = 1.44. \ \textit{Ans.}$$

16. What is the weight of a brick wall 40 ft. long, 20 ft. high, and 1 ft. thick, if the specific gravity of a brick with its mortar is 1.46? How many thousand bricks will be required for the wall, allowing 22 for a cubic foot?

$$40 \times 20 \times 1 \times 1.46 \times 62\tfrac{1}{2} = \overset{20}{\cancel{40}} \times \cancel{20} \times \frac{146}{\underset{5}{\cancel{100}}} \times \frac{\overset{25}{\cancel{125}}}{\cancel{2}} = 73{,}000.$$

$$73{,}000 \text{ lb.} = 36\tfrac{1}{2} \text{ t. } \textit{Ans.} \qquad 40 \times 20 \times 1 \times 22 = 17{,}600. \ \textit{Ans.}$$

17. If the specific gravity of iron is 7.48, what is the weight of a cylindrical iron shell 1 in. thick and 2 ft. long, whose inner radius is 7 in.?

The outer radius is 8 in. = $\frac{2}{3}$ ft.

$$\overset{1.0472}{\cancel{3.1416}} \times \frac{2}{\cancel{3}} \times \frac{2}{3} \times 2 = \frac{8.3776}{3} = 2.7925.$$

The inner radius is 7 in. = $\frac{7}{12}$ ft.

$$\overset{\overset{0.1309}{\cancel{0.2618}}}{\cancel{3.1416}} \times \frac{7}{\cancel{12}} \times \frac{7}{\underset{\underset{3}{\cancel{6}}}{\cancel{12}}} \times \cancel{2} = \frac{6.4141}{3} = 2.1380.$$

[redacted]

The volume of the shell $=2.7925$ cu. ft. -2.1380 cu. ft. $=0.6545$ cu. ft.

The weight of the shell $=0.6545\times7.48\times62.5$ lb. $=305.97875$ lb. *Ans.*

$$\begin{array}{r} 62.5 \\ \underline{7.48} \\ 5000 \\ 2500 \\ \underline{4375} \\ 467.500 \end{array} \qquad\qquad \begin{array}{r} 467.5 \\ \underline{0.6545} \\ 23375 \\ 18700 \\ 23375 \\ \underline{28050} \\ 305.97875 \end{array}$$

18. If a piece of marble weighs 37.78 oz. in air, and 23.89 oz. in water, what is its volume and its specific gravity?

The weight of the water displaced by the marble is

$$37.78 \text{ oz.} - 23.89 \text{ oz.} = 13.89 \text{ oz.}$$

By Ex. 1, the volume of 1 oz. of water is 1.728 cu. in.

Therefore, the volume of the marble is

$$13.89 \times 1.728 \text{ cu. in.} = 24.00192 \text{ cu. in.} \quad \textit{Ans.}$$

The specific gravity of the marble

$$= 37.78 \div 13.89 = 2.72. \quad \textit{Ans.}$$

$$\begin{array}{r} 1.728 \\ \underline{13.89} \\ 15552 \\ 13824 \\ 5184 \\ \underline{1728} \\ 24.00192 \end{array} \qquad\qquad \begin{array}{r} 2.72 \\ 1389\overline{)3778.} \\ \underline{2778} \\ 10000 \\ \underline{9723} \\ 2770 \end{array}$$

19. If a mass of lead weighs $1986\frac{1}{4}$ lb. in air, and $1811\frac{1}{4}$ lb. in water, what is its volume and its specific gravity?

The weight of the water displaced by the lead is $1986\frac{1}{4}$ lb. $-1811\frac{1}{4}$ lb. $=175$ lb.

Since 1 cu. ft. of water weighs $62\frac{1}{2}$ lb., the volume of the lead is

$$\frac{175}{62\frac{1}{2}} \text{ cu. ft.} = \overset{7}{\cancel{175}} \times \frac{2}{\underset{5}{\cancel{125}}} \text{ cu. ft.} = \frac{14}{5} \text{ cu. ft.} = 2\frac{4}{5} \text{ cu. ft.} \quad \textit{Ans.}$$

The specific gravity of the lead

$$= 1986\frac{1}{4} \div 175 = \frac{\overset{227}{\cancel{7945}}}{4} \times \frac{1}{\underset{5}{\cancel{175}}} = \frac{227}{20} = 11\frac{7}{20} = 11.35. \quad \textit{Ans.}$$

[redacted]

Exercise 92. Page 201.

Express :

1. 59° F. in Centigrade scale ; in Réaumur's scale.

$59° - 32° = 27°.$

$\frac{5}{9}$ of $27° = 15°.$

$\therefore 59°$ F. $= 15°$ C.

$180°$ F. $= 80°$ R.

$\therefore 1°$ F. $= \frac{80}{180}°$ R. $= \frac{4}{9}°$ R.

$\frac{4}{9}$ of $27° = 12°.$

$\therefore 59°$ F. $= 12°$ R.

2. 77° F. in Centigrade scale ; in Réaumur's scale.

$77° - 32° = 45°.$

$\frac{5}{9}$ of $45° = 25°.$

$\therefore 77°$ F. $= 25°$ C.

$\frac{4}{9}$ of $45° = 20°.$

$\therefore 77°$ F. $= 20°$ R.

3. 950° F. in Centigrade scale ; in Réaumur's scale.

$950° - 32° = 918°.$

$\frac{5}{9}$ of $918° = 510°.$

$\therefore 950°$ F. $= 510°$ C.

$\frac{4}{9}$ of $918° = 408°.$

$\therefore 950°$ F. $= 408°$ R.

4. $-40°$ F. in Centigrade scale ; in Réaumur's scale.

$-40°$ F. is 72° below the freezing point.

$\frac{5}{9}$ of $72° = 40°.$

$\therefore -40°$ F. $= -40°$ C.

$\frac{4}{9}$ of $72° = 32°.$

$\therefore -40°$ F. $= -32°$ R.

5. $-4°$ F. in Centigrade scale ; in Réaumur's scale.

$-4°$ F. is 36° below the freezing point.

$\frac{5}{9}$ of $36° = 20°.$

$\therefore -4°$ F. $= -20°$ C.

$\frac{4}{9}$ of $36° = 16°.$

$\therefore -4°$ F. $= -16°$ R.

6. 10° C. in Fahrenheit's scale ; in Réaumur's scale.

$\frac{9}{5}$ of $10° = 18°.$

$18° + 32° = 50°.$

$\therefore 10°$ C. $= 50°$ F.

$\frac{4}{5}$ of $10° = 8°.$

$\therefore 10°$ C. $= 8°$ R.

7. 22° C. in Fahrenheit's scale ; in Réaumur's scale.

$\frac{9}{5}$ of $22° = 39.6°.$

$39.6° + 32° = 71.6°.$

$\therefore 22°$ C. $= 71.6°$ F.

$\frac{4}{5}$ of $22° = 17.6°.$

$\therefore 22°$ C. $= 17.6°$ R.

[redacted]

8. $-30°$ C. in Fahrenheit's scale; in Réaumur's scale.

$$\tfrac{9}{5} \text{ of } 30° = 54°.$$

54° below the freezing point in Fahrenheit's scale is $-22°$.

$$\therefore\ -30° \text{ C.} = -22° \text{ F.}$$

$$\tfrac{4}{5} \text{ of } 30° = 24°. \qquad \therefore\ -30° \text{ C.} = -24° \text{ R.}$$

9. $-11\frac{3}{7}°$ C. in Fahrenheit's scale; in Réaumur's scale.

$$\tfrac{9}{5} \text{ of } 11\tfrac{3}{7}° = 20\tfrac{4}{7}°.$$

$20\frac{4}{7}°$ below the freezing point in Fahrenheit's scale is $11\frac{3}{7}°$.

$$\therefore\ -11\tfrac{3}{7}° \text{ C.} = 11\tfrac{3}{7}° \text{ F.}$$

$$\tfrac{4}{5} \text{ of } 11\tfrac{3}{7}° = 9\tfrac{1}{7}°. \qquad \therefore\ -11\tfrac{3}{7}° \text{ C.} = -9\tfrac{1}{7}° \text{ R.}$$

Exercise 93. Page 201.

1. If one man can do a piece of work in 9 days and another man can do the same work in 8 days, in how many days can the men working together do the work?

If one man can do the work in 9 days, in 1 day he can do $\frac{1}{9}$ of it.
If another man can do the work in 8 days, in 1 day he can do $\frac{1}{8}$ of it.
Both together can do $\frac{1}{9} + \frac{1}{8} = \frac{17}{72}$ of the work in 1 day.
Therefore, both together can do the work in $\frac{72}{17}$ days, or $4\frac{4}{17}$ days. *Ans.*

2. A cistern can be filled by a water-pipe in 30 min., and emptied by a waste-pipe in 20 min. If the cistern is full and both pipes are opened, in how many minutes will the cistern be emptied?

If the water-pipe will fill the cistern in 30 min., in 1 min. it will fill $\frac{1}{30}$ of the cistern.
If the waste-pipe will empty the cistern in 20 min., in 1 min. it will empty $\frac{1}{20}$ of the cistern.
When both are opened $\frac{1}{20} - \frac{1}{30} = \frac{1}{60}$ will be emptied each minute.
Therefore, when both are opened, the cistern will be emptied in $\frac{60}{1}$ min., or 60 min. *Ans.*

3. If A can mow a certain meadow in 4 days, and B in 3 days, how long will it take both together?

If A can mow the meadow in 4 days, in 1 day he can mow $\frac{1}{4}$ of it.
If B can mow the meadow in 3 days, in 1 day he can mow $\frac{1}{3}$ of it.
Both together can mow $\frac{1}{4} + \frac{1}{3} = \frac{7}{12}$ of the meadow in 1 day.
Therefore, both together can mow the meadow in $\frac{12}{7}$ days, or $1\frac{5}{7}$ days. *Ans.*

[redacted]

4. If A can lay a certain wall in $4\frac{1}{2}$ days, and B in $5\frac{1}{4}$ days, how long will it take both together?

If A can lay the wall in $4\frac{1}{2}$ days, in 1 day he can lay $\frac{1}{4\frac{1}{2}} = \frac{2}{9}$ of it.

If B can lay the wall in $5\frac{1}{4}$ days, in 1 day he can lay $\frac{1}{5\frac{1}{4}} = \frac{4}{21}$ of it.

Both together can lay $\frac{2}{9} + \frac{4}{21} = \frac{26}{63}$ of it in 1 day.
Therefore, both together can lay the wall in $\frac{63}{26}$ days, or $2\frac{11}{26}$ days. *Ans.*

5. If one pipe will fill a cistern in $4\frac{1}{2}$ hr., and another pipe in $3\frac{1}{2}$ hr., how long will it take both together to fill the cistern?

If one pipe will fill the cistern in $4\frac{1}{2}$ hr., in 1 hr. it will fill $\frac{1}{4\frac{1}{2}} = \frac{2}{9}$ of it.

If another pipe will fill the cistern in $3\frac{1}{2}$ hr., in 1 hr. it will fill $\frac{1}{3\frac{1}{2}} = \frac{2}{7}$ of it.

Both pipes together will fill $\frac{2}{9} + \frac{2}{7} = \frac{32}{63}$ of the cistern in 1 hr.
Therefore, both pipes together will fill the cistern in $\frac{63}{32} = 1\frac{31}{32}$ hr. *Ans.*

6. If A can go from Boston to Albany in $9\frac{1}{4}$ hr., and B from Albany to Boston in $11\frac{1}{3}$ hr., and they start at the same time, in how many hours will they meet?

If A can go in $9\frac{1}{4}$ hr., in 1 hr. he can go $\frac{1}{9\frac{1}{4}} = \frac{4}{37}$ of the distance.

If B can go in $11\frac{1}{3}$ hr., in 1 hr. he can go $\frac{1}{11\frac{1}{3}} = \frac{3}{34}$ of the distance.

Both together can go $\frac{4}{37} + \frac{3}{34} = \frac{247}{1258}$ of the distance in 1 hr.
Therefore, they will meet in $\frac{1258}{247}$ hr. $= 5\frac{23}{247}$ hr. *Ans.*

7. If it takes A working alone 4 days, B 3 days, and C $4\frac{1}{2}$ days to do a piece of work, how long will it take to do the work if all three work together?

If A can do the work in 4 days, in 1 day he can do $\frac{1}{4}$ of it.
If B can do the work in 3 days, in 1 day he can do $\frac{1}{3}$ of it.

If C can do the work in $4\frac{1}{2}$ days, in 1 day he can do $\frac{1}{4\frac{1}{2}} = \frac{2}{9}$ of it.

All together can do $\frac{1}{4} + \frac{1}{3} + \frac{2}{9} = \frac{29}{36}$ of the work in 1 day.
Therefore, it will take them, all working together, $\frac{36}{29}$ days $= 1\frac{7}{29}$ days. *Ans.*

8. A can mow $\frac{5}{9}$ of a field in 3 days; B can mow $\frac{2}{3}$ of it in 4 days. How long will it take both together to mow the field?

$$3 \text{ days} \div \tfrac{5}{9} = 5\tfrac{2}{5} \text{ days}; \ 4 \text{ days} \div \tfrac{2}{3} = 6 \text{ days}.$$

If A can mow the field in $5\frac{2}{5}$ days, in 1 day he can mow $\dfrac{1}{5\frac{2}{5}} = \frac{5}{27}$ of it.

If B can mow the field in 6 days, in 1 day he can mow $\frac{1}{6}$ of it.

Both together can mow $\frac{5}{27} + \frac{1}{6} = \frac{19}{54}$ of the field in 1 day.

Therefore, both together can mow the field in $\frac{54}{19}$ days $= 2\frac{16}{19}$ days. *Ans.*

9. One pipe can fill a cistern half full in $\frac{3}{4}$ of an hour, and another can fill it three quarters full in $\frac{1}{2}$ an hour. How long will it take both pipes together to fill the cistern?

$$\tfrac{3}{4} \text{ hr.} \div \tfrac{1}{2} = 1\tfrac{1}{2} \text{ hr.}; \ \tfrac{1}{2} \text{ hr.} \div \tfrac{3}{4} = \tfrac{2}{3} \text{ hr.}$$

If one pipe can fill the cistern in $1\frac{1}{2}$ hr., in 1 hr. it can fill $\dfrac{1}{1\frac{1}{2}} = \frac{2}{3}$ of it.

If another pipe can fill the cistern in $\frac{2}{3}$ hr., in 1 hr. it can fill $\dfrac{1}{\frac{2}{3}} = \frac{3}{2}$ of it.

Both together can fill $\frac{2}{3} + \frac{3}{2} = \frac{13}{6}$ of the cistern in 1 hr.

Therefore, both together can fill the cistern in $6 \div 13 = \frac{6}{13}$ hr. *Ans.*

10. A pipe can fill a cistern one third full in $\frac{1}{4}$ of an hour; a waste-pipe can empty one fourth of the cistern in 20 minutes. If both pipes are opened, in what time will the cistern be filled?

$$3 \times \tfrac{1}{4} \text{ hr.} = \tfrac{3}{4} \text{ hr.} = 45 \text{ min.}; \ 4 \times 20 = 80 \text{ min.}$$

The water-pipe can fill $\frac{1}{45}$ every minute.

The waste-pipe can empty $\frac{1}{80}$ every minute.

When both are open, $\frac{1}{45} - \frac{1}{80} = \frac{7}{720}$ is gained every minute.

Therefore, the whole will be filled in $\frac{720}{7}$ min. $= 102\frac{6}{7}$ min. *Ans.*

11. A cistern that will hold 100 gallons can be filled by a pipe in 25 minutes, and emptied by a waste-pipe in 45 minutes. If the cistern is empty and both pipes are opened, how long will it take to fill the cistern, and how much water will be wasted?

The water-pipe fills $\frac{1}{25}$ every minute.

The waste-pipe empties $\frac{1}{45}$ every minute.

When both are open, $\frac{1}{25} - \frac{1}{45} = \frac{4}{225}$ is gained every minute.

Therefore, the whole will be filled in $\frac{225}{4}$ min. $= 56\frac{1}{4}$ min. *Ans.*

If $\frac{1}{45}$ of the cistern is wasted every minute, the number of gallons wasted =

$$56\tfrac{1}{4} \times \frac{1}{45} \text{ of } 100 = \frac{\overset{5}{\cancel{225}}}{\cancel{4}} \times \frac{1}{\cancel{45}} \times \overset{25}{\cancel{100}} = 125.$$

125 gal. *Ans.*

[redacted]

12. If water runs into a cistern by one pipe at the rate of 2 gal. in 3 min., by another at the rate of 5 gal. in 4 min., and runs out by a third at the rate of 4 gal. in 5 min., how long will it take to gain 71 gal. in the cistern?

2 gal. $\div 3 = \frac{2}{3}$ gal.; 5 gal. $\div 4 = \frac{5}{4}$ gal.; 4 gal. $\div 5 = \frac{4}{5}$ gal.

If one pipe pours in $\frac{2}{3}$ gal. per minute, another pours in $\frac{5}{4}$ gal. per minute, and another empties $\frac{4}{5}$ gal. per minute, the cistern gains $\frac{2}{3} + \frac{5}{4} - \frac{4}{5} = \frac{67}{60}$ gal. per minute.

Therefore, it will take as many minutes to gain 71 gal. as $71 \div \frac{67}{60} = 63\frac{39}{67}$. $63\frac{39}{67}$ min. *Ans.*

13. A can do a piece of work in 6 days, and B can do it in 7 days. If they work together 2 days, and A then leaves, how long will it take B to finish the work?

If A can do the work in 6 days, in 1 day he can do $\frac{1}{6}$ of it.

If B can do the work in 7 days, in 1 day he can do $\frac{1}{7}$ of it.

A and B together can do $\frac{1}{6} + \frac{1}{7} = \frac{13}{42}$ of the work in 1 day, and in 2 days can do $2 \times \frac{13}{42} = \frac{13}{21}$ of the work.

The part of the work then not done is $\frac{21}{21} - \frac{13}{21} = \frac{8}{21}$.

To do $\frac{8}{21}$ of the work, it will take B $(\frac{8}{21} \div \frac{1}{7})$ days $= 2\frac{2}{3}$ days. *Ans.*

14. A cistern that will hold 200 gal. has two pipes; one will supply 0.15 gal. a second, the other $1\frac{3}{5}$ qt. a second. If the first is turned on for 10 minutes, and afterwards both run together, in what time will the cistern be filled?

0.15 gal. per sec. is 60×0.15 gal. $= 9$ gal. per min.

$1\frac{3}{5}$ qt. per sec. is $60 \times 1\frac{3}{5}$ qt. $= 24$ gal. per min.

In 10 min. the first pipe will supply 10×9 gal. $= 90$ gal.

There remains to be filled 200 gal. $-$ 90 gal. $=$ 110 gal.

Since the two pipes together supply 9 gal. $+$ 24 gal. $=$ 33 gal. per minute, to supply 110 gal. will require $\frac{110}{33}$ min. $= 3\frac{1}{3}$ min. *Ans.*

15. A and B together can do a piece of work in 15 days. After working together 6 days, A leaves and B finishes the work in 30 days more. In how many days can each alone do the work?

A and B together can do $\frac{1}{15}$ of the work in 1 day, and in 6 days can do $6 \times \frac{1}{15} = \frac{2}{5}$ of the work.

There remains $\frac{5}{5} - \frac{2}{5} = \frac{3}{5}$ of the work to be done.

If B can do $\frac{3}{5}$ of the work in 30 days, B alone can do the work in $\frac{5}{3}$ of 30 days $= 50$ days. *Ans.*

B alone can do $\frac{1}{50}$ of the work in 1 day.

Therefore, A alone can do $\frac{1}{15} - \frac{1}{50} = \frac{7}{150}$ of the work in 1 day.

Therefore, A alone can do the work in $\frac{150}{7}$ days $= 21\frac{3}{7}$ days. *Ans.*

16. A and B together can do a piece of work in 12 days. After working together 9 days, however, they call in C to help them, and the three finish the work in 2 days. In how many days can C alone do the work?

A and B together can do $\frac{1}{12}$ of the work in 1 day, and in 9 days can do $9 \times \frac{1}{12} = \frac{3}{4}$ of the work.

There remains $\frac{4}{4} - \frac{3}{4} = \frac{1}{4}$ of the work to be done.

If A, B, and C together can do $\frac{1}{4}$ of the work in 2 days, to do the whole work would require them 4×2 days $= 8$ days.

Therefore, C alone in 1 day can do $\frac{1}{8} - \frac{1}{12} = \frac{1}{24}$ of the work.

Therefore, C alone can do the work in 24 days. *Ans.*

17. A and B can do a piece of work in $2\frac{1}{2}$ days; A and C in $3\frac{1}{3}$ days; B and C in $3\frac{3}{4}$ days. How long will it take the three working together to do the work, and how long will it take each alone?

If A and B can do the work in $2\frac{1}{2}$ days, they can do $\frac{1}{2\frac{1}{2}} = \frac{2}{5}$ of it in 1 day.

If A and C can do the work in $3\frac{1}{3}$ days, they can do $\frac{1}{3\frac{1}{3}} = \frac{3}{10}$ of it in 1 day.

If B and C can do the work in $3\frac{3}{4}$ days, they can do $\frac{1}{3\frac{3}{4}} = \frac{4}{15}$ of it in 1 day.

All together in 2 days can do $\frac{2}{5} + \frac{3}{10} + \frac{4}{15} = \frac{29}{30}$ of the work.

Hence, by working 1 day each they can do $\frac{1}{2}$ of $\frac{29}{30}$ or $\frac{29}{60}$ of the work.

Therefore, all together can do the work in $\frac{60}{29}$ days $= 2\frac{2}{29}$ days. *Ans.*

In 1 day A can do $\frac{29}{60} - \frac{4}{15} = \frac{13}{60}$ of the work.

Therefore, A can do the work in $\frac{60}{13}$ days $= 4\frac{8}{13}$ days. *Ans.*

In 1 day B can do $\frac{29}{60} - \frac{3}{10} = \frac{11}{60}$ of the work.

Therefore, B can do the work in $\frac{60}{11}$ days $= 5\frac{5}{11}$ days. *Ans.*

In 1 day C can do $\frac{29}{60} - \frac{2}{5} = \frac{1}{12}$ of the work.

Therefore, C can do the work in 12 days. *Ans.*

[redacted]

18. A and B together can do a piece of work in 48 days; A and C together in 30 days; B and C together in $26\frac{2}{3}$ days. How long will it take each alone to do the work?

If A and B can do the work in 48 days, they can do $\frac{1}{48}$ of it in 1 day.

If A and C can do the work in 30 days, they can do $\frac{1}{30}$ of it in 1 day.

If B and C can do the work in $26\frac{2}{3}$ days, they can do $\frac{1}{26\frac{2}{3}} = \frac{3}{80}$ of it in 1 day.

All together in 2 days can do $\frac{1}{48} + \frac{1}{30} + \frac{3}{80} = \frac{11}{120}$ of the work.

Hence, by working 1 day each, they can do $\frac{1}{2}$ of $\frac{11}{120}$ or $\frac{11}{240}$ of the work.

In 1 day A can do $\frac{11}{240} - \frac{3}{80} = \frac{1}{120}$ of the work.

Therefore, A can do the work in 120 days. *Ans.*

In 1 day B can do $\frac{11}{240} - \frac{1}{30} = \frac{1}{80}$ of the work.

Therefore, B can do the work in 80 days. *Ans.*

In 1 day C can do $\frac{11}{240} - \frac{1}{48} = \frac{1}{40}$ of the work.

Therefore, C can do the work in 40 days. *Ans.*

19. A cistern has three pipes. The first and second will fill it in 1 hr. 10 min.; the first and third in 1 hr. 24 min.; the second and third in 2 hr. 20 min. How long will it take each alone to fill the cistern?

1 hr. 10 min. = $1\frac{1}{6}$ hr.; 1 hr. 24 min. = $1\frac{2}{5}$ hr.; 2 hr. 20 min. = $2\frac{1}{3}$ hr.

If the 1st and 2d will fill the cistern in $1\frac{1}{6}$ hr., they will fill $\frac{1}{1\frac{1}{6}} = \frac{6}{7}$ of it in 1 hr.

If the 1st and 3d will fill the cistern in $1\frac{2}{5}$ hr., they will fill $\frac{1}{1\frac{2}{5}} = \frac{5}{7}$ of it in 1 hr.

If the 2d and 3d will fill the cistern in $2\frac{1}{3}$ hr., they will fill $\frac{1}{2\frac{1}{3}} = \frac{3}{7}$ of it in 1 hr.

All together in 2 hr. will fill $\frac{6}{7} + \frac{5}{7} + \frac{3}{7} = \frac{14}{7}$ of the cistern.

Hence, in 1 hr. the three together will fill $\frac{1}{2}$ of $\frac{14}{7} = \frac{7}{7}$ or the whole cistern.

In 1 hr., the 1st will fill $\frac{7}{7} - \frac{3}{7} = \frac{4}{7}$ of the cistern.

Therefore, 1st will fill the cistern in $\frac{7}{4}$ hr. = $1\frac{3}{4}$ hr. = 1 hr. 45 min. *Ans.*

In 1 hr., the 2d will fill $\frac{7}{7} - \frac{5}{7} = \frac{2}{7}$ of the cistern.

Therefore, 2d will fill the cistern in $\frac{7}{2}$ hr. = $3\frac{1}{2}$ hr. = 3 hr. 30 min. *Ans.*

In 1 hr., the 3d will fill $\frac{7}{7} - \frac{6}{7} = \frac{1}{7}$ of the cistern.

Therefore, the 3d will fill the cistern in 7 hr. *Ans.*

[redacted]

20. A, B, and C together can do a piece of work in 10 days; A and B together in 12 days; B and C together in 20 days. How long will it take each alone to do the work?

If A, B, and C can do the work in 10 days, they can do $\frac{1}{10}$ of it in 1 day.

If A and B can do the work in 12 days, they can do $\frac{1}{12}$ of it in 1 day.

If B and C can do the work in 20 days, they can do $\frac{1}{20}$ of it in 1 day.

In 1 day C can do $\frac{1}{10} - \frac{1}{12} = \frac{1}{60}$ of the work.

Therefore, C can do the work in 60 days. *Ans.*

In 1 day B can do $\frac{1}{20} - \frac{1}{60} = \frac{1}{30}$ of the work.

Therefore, B can do the work in 30 days. *Ans.*

In 1 day A can do $\frac{1}{12} - \frac{1}{30} = \frac{1}{20}$ of the work.

Therefore, A can do the work in 20 days. *Ans.*

Exercise 94. Page 204.

1. A train travels 24 miles in 0.8 of an hour. Find its rate per hour.

$$24 \text{ mi.} \div 0.8 = 30 \text{ mi. } \textit{Ans.}$$

2. A train runs from New York to Philadelphia, 90 miles, in 1 hr. 33 min. What is its rate per hour?

$$1 \text{ hr. } 33 \text{ min.} = 1\tfrac{11}{20} \text{ hr.}$$

$$90 \text{ mi.} \div 1\tfrac{11}{20} = \frac{20}{31} \text{ of } 90 \text{ mi.} = \frac{1800}{31} \text{ mi.} = 58\tfrac{2}{31} \text{ mi. } \textit{Ans.}$$

3. A train runs from New York to Philadelphia, 90 miles, in 2 hr. 5 min. What is its rate per hour?

$$2 \text{ hr. } 5 \text{ min.} = 2\tfrac{1}{12} \text{ hr.}$$

$$90 \text{ mi.} \div 2\tfrac{1}{12} = \frac{12}{\underset{5}{\cancel{25}}} \text{ of } \overset{18}{\cancel{90}} \text{ mi.} = \frac{216}{5} \text{ mi.} = 43\tfrac{1}{5} \text{ mi. } \textit{Ans.}$$

4. Winlock, in 1869, found that electricity went through 7200 miles of wire in $\frac{2}{3}$ of a second. What was its rate per second?

$$7200 \text{ mi.} \div \frac{2}{3} = \frac{3}{\cancel{2}} \text{ of } \overset{3600}{\cancel{7200}} \text{ mi.} = 10{,}800 \text{ mi. } \textit{Ans.}$$

[redacted]

5. If the time required for a signal to pass through the cable from Brest to Duxbury, 3799 miles, is 0.816 of a second, what is the rate per second?

$$3799 \text{ mi.} \div 0.816 = 4655.637 \text{ mi. } \textit{Ans.}$$

```
        4655.637
816)3799000.
    3264
     5350
     4896
      4540
      4080
       4600
       4080
        5200
        4896
         3040
         2448
          5920
          5712
           208
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6. If the report of a gun $1\frac{1}{4}$ miles distant is heard in $5\frac{5}{8}$ seconds after the flash is seen, what is the velocity of sound in feet per second?

$$\frac{1\frac{1}{4} \times 5280}{5\frac{5}{8}} = \frac{\cancel{5}}{\cancel{4}} \times \overset{1760}{\cancel{5280}} \times \frac{\overset{2}{\cancel{8}}}{\underset{\underset{3}{\cancel{9}}}{\cancel{45}}} = \frac{3520}{3} = 1173\tfrac{1}{3}. \qquad 1173\tfrac{1}{3} \text{ ft. } \textit{Ans.}$$

7. If a man walks $3\frac{1}{5}$ miles in 46 minutes, what is his rate per hour?

$$3\tfrac{1}{5} \text{ mi.} \div \frac{46}{60} = \frac{\overset{12}{\cancel{60}}}{\underset{23}{\cancel{46}}} \times \frac{\overset{8}{\cancel{16}}}{\cancel{5}} \text{ mi.} = \frac{96}{23} \text{ mi.} = 4\tfrac{4}{23} \text{ mi. } \textit{Ans.}$$

8. If a horse goes 48 miles in 10 hr. 40 min., what is his average rate per hour?

$$10 \text{ hr. } 40 \text{ min.} = 10\tfrac{2}{3} \text{ hr.}$$

$$48 \text{ mi.} \div 10\tfrac{2}{3} = \frac{3}{\underset{2}{\cancel{32}}} \times \overset{3}{\cancel{48}} \text{ mi.} = \frac{9}{2} \text{ mi.} = 4\tfrac{1}{2} \text{ mi. } \textit{Ans.}$$

9. If a stone on a glacier is carried $95\frac{1}{2}$ feet in 188 days, what is its rate in inches per day?

$$\frac{95\frac{1}{2}\times 12}{188}=\frac{191}{2}\times\frac{\overset{3}{\cancel{12}}}{\underset{47}{\cancel{188}}}=\frac{573}{94}=6\tfrac{9}{94}. \qquad 6\tfrac{9}{94}\text{ in. } \textit{Ans.}$$

10. If a horse went $5\frac{1}{2}$ miles in 33 minutes, how long did it take him to go a mile?

$$33\text{ min.}\div 5\tfrac{1}{2}=\frac{2}{\cancel{11}}\text{ of }\overset{3}{\cancel{33}}\text{ min.}=6\text{ min. } \textit{Ans.}$$

11. If a horse can trot $\frac{5}{6}$ of a mile in $2\frac{1}{3}$ minutes, in what time can he trot a mile?

$$2\tfrac{1}{3}\text{ min.}\div\frac{5}{6}=\frac{\overset{2}{\cancel{6}}}{5}\times\frac{7}{\cancel{3}}\text{ min.}=\frac{14}{5}\text{ min.}=2\tfrac{4}{5}\text{ min. } \textit{Ans.}$$

12. If a train runs 18 miles in 39 minutes, how long does it take to run one mile?

$$39\text{ min.}\div 18=2\tfrac{1}{6}\text{ min. } \textit{Ans.}$$

13. If sound travels 1125 feet a second, how long will it take to travel one mile?

$$\begin{array}{r} 4.7 \\ 1125\overline{)5280.} \\ \underline{4500} \\ 7800 \end{array} \qquad 4.7\text{ sec. } \textit{Ans.}$$

14. If a train requires 3 hours to run $104\frac{1}{4}$ miles, find its average time for running a mile.

$$3\text{ hr.}=180\text{ min.}$$

$$180\text{ min.}\div 104\tfrac{1}{4}=\frac{4}{\underset{139}{\cancel{417}}}\times\overset{60}{\cancel{180}}\text{ min.}=\frac{240}{139}\text{ min.}=1\tfrac{101}{139}\text{ min.}$$

$$=1\text{ min. }43.6\text{ sec. } \textit{Ans.}$$

15. If a man cuts $7\frac{1}{2}$ A. of grass in $3\frac{1}{2}$ days, what part of a day will it take him to cut an acre? If 10 hr. makes a day, what part of an acre will he cut in an hour?

$$\frac{3\frac{1}{2}}{7\frac{1}{2}}=\frac{7}{15}. \qquad \tfrac{7}{15}\text{ dy. } \textit{Ans.}$$

$$\frac{7\frac{1}{2}}{3\frac{1}{2}\times 10}=\frac{\overset{3}{\cancel{15}}}{\cancel{2}}\times\frac{\cancel{2}}{7}\times\frac{1}{\underset{2}{\cancel{10}}}=\frac{3}{14}. \qquad \tfrac{3}{14}\text{ A. } \textit{Ans.}$$

[redacted]

16. If a mower cuts $3\frac{1}{2}$ square rods in $\frac{1}{8}$ of an hour, how many acres will he cut in a day of 10 hours?

$$3\tfrac{1}{2} \text{ sq. rd.} \div \tfrac{1}{8} = 28 \text{ sq. rd.}$$

$$10 \times 28 \text{ sq. rd.} = 280 \text{ sq. rd.} = 1\tfrac{3}{4} \text{ A. } \textit{Ans.}$$

17. If a fountain yields $117\frac{1}{2}$ gallons of water in $\frac{3}{4}$ of an hour, at what rate per hour is the water flowing?

$$117\tfrac{1}{2} \text{ gal.} \div \frac{3}{4} = \frac{\overset{2}{\cancel{4}}}{3} \times \frac{235}{\cancel{2}} \text{ gal.} = \frac{470}{3} \text{ gal.} = 156\tfrac{2}{3} \text{ gal. } \textit{Ans.}$$

18. If a merchant's profits are \$3147 in $7\frac{1}{2}$ months, what will be his profits at the same rate for a year?

$$12 \times \frac{\$3147}{7\frac{1}{2}} = \overset{4}{\cancel{12}} \times \frac{2}{\underset{5}{\cancel{15}}} \times \$3147 = \$\frac{25176}{5} = \$5035.20. \textit{ Ans.}$$

19. If a wheel turns 17° 30′ in 35 minutes, in how many hours does it make a complete revolution?

$$17^\circ\, 30' \div 35 = \tfrac{1}{2}^\circ.$$

$$360 \div \tfrac{1}{2} = 720. \qquad 720 \text{ min.} = 12 \text{ hr. } \textit{Ans.}$$

20. If a man's expenditures are \$4358 in $13\frac{1}{3}$ months, what is his yearly rate of expenditure?

$$12 \times \frac{\$4358}{13\frac{1}{3}} = \overset{3}{\cancel{12}} \times \frac{3}{\underset{\underset{5}{\cancel{10}}}{\cancel{40}}} \times \$\overset{2179}{\cancel{4358}} = \$\frac{19611}{5} = \$3922.20. \textit{ Ans.}$$

21. If a cistern loses by leakage 7 gal. 1 pt. in 49 hr. 40 min., what is its hourly rate of loss?

$$49 \text{ hr. } 40 \text{ min.} = 49\tfrac{2}{3} \text{ hr.}$$

$$7 \text{ gal. } 1 \text{ pt.} = 57 \text{ pt.}$$

$$57 \text{ pt.} \div 49\tfrac{2}{3} = \tfrac{3}{149} \times 57 \text{ pt.} = \tfrac{171}{149} \text{ pt.} = 1\tfrac{22}{149} \text{ pt. } \textit{Ans.}$$

[redacted]

22. If a man travels $3\frac{3}{5}$ miles in $7\frac{1}{2}$ minutes, how many miles will he travel in 50 minutes? How long will it take him to travel 50 miles?

$$7\tfrac{1}{2}\text{ min.} \div 3\tfrac{3}{5} = \frac{5}{\underset{6}{\cancel{18}}} \times \frac{\overset{5}{\cancel{15}}}{2}\text{ min.} = \frac{25}{12}\text{ min.} = 2\tfrac{1}{12}\text{ min.}$$

$$50 \div 2\tfrac{1}{12} = \frac{12}{\cancel{25}} \times \overset{2}{\cancel{50}} = 24. \qquad 24\text{ mi. } \textit{Ans.}$$

$$3\tfrac{3}{5}\text{ mi.} \div 7\tfrac{1}{2} = \frac{2}{\underset{5}{\cancel{15}}} \times \frac{\overset{6}{\cancel{18}}}{5}\text{ mi.} = \frac{12}{25}\text{ mi.}$$

$$50 \div \frac{12}{25} = \frac{25}{\underset{6}{\cancel{12}}} \times \overset{25}{\cancel{50}} = \frac{625}{6} = 104\tfrac{1}{6}.$$

$104\frac{1}{6}$ min. *Ans.*

Exercise 95. Page 206.

1. At what time between 5 and 6 o'clock do the hour and minute hands of a clock coincide?

Since in one hour the hour hand moves through 5 minute-spaces, and the minute hand through 60 minute-spaces, the minute hand moves 12 times as fast as the hour hand, and in moving through 12 minute-spaces gains 11 minute-spaces.

When the hour hand is at V, the minute hand, being at XII, is 25 minute-spaces behind. Since to gain 11 minute-spaces the minute hand must move through 12 minute-spaces, to gain 1 minute-space the minute hand must pass through $\frac{12}{11}$ of 1 minute-space, and to gain 25 minute-spaces, it must pass through $25 \times \frac{12}{11}$, or $27\frac{3}{11}$ minute-spaces.

Hence, the hands coincide when the minute hand has moved through $27\frac{3}{11}$ minute-spaces; that is, at $27\frac{3}{11}$ minutes after 5 o'clock. *Ans.*

2. At what time between 10 and 11 o'clock do the hour and minute hands of a watch coincide?

At 10 o'clock the minute hand is 50 minute-spaces behind the hour hand.

$50 \times \frac{12}{11} = \frac{600}{11} = 54\frac{6}{11}$. $54\frac{6}{11}$ minutes after 10 o'clock. *Ans.*

[redacted]

3. At what time between 1 and 2 o'clock do the hour and minute hands of a clock coincide?

At 1 o'clock the minute hand is 5 minute-spaces behind the hour hand.

$5 \times \frac{12}{11} = \frac{60}{11} = 5\frac{5}{11}$. $5\frac{5}{11}$ minutes after 1 o'clock. *Ans.*

4. At what time between 8 and 9 o'clock are the hands of a clock exactly opposite each other?

At 8 o'clock the minute hand is 40 minute-spaces behind the hour hand, and should be 30 minute-spaces behind the hour hand.

Therefore, the minute hand must gain 40 − 30, or 10 minute-spaces on the hour hand.

$10 \times \frac{12}{11} = \frac{120}{11} = 10\frac{10}{11}$. $10\frac{10}{11}$ minutes after 8 o'clock. *Ans.*

5. At what time between 11 and 12 o'clock are the hands of a clock exactly opposite each other?

At 11 o'clock the minute hand is 55 minute-spaces behind the hour hand, and should be 30 minute-spaces behind the hour hand.

Therefore, the minute hand must gain 55 − 30, or 25 minute-spaces on the hour hand.

$25 \times \frac{12}{11} = \frac{300}{11} = 27\frac{3}{11}$. $27\frac{3}{11}$ minutes after 11 o'clock. *Ans.*

6. At what time between 4 and 5 o'clock are the hands of a clock exactly opposite each other?

At 4 o'clock the minute hand is 20 minute-spaces behind the hour hand, and should be 30 minute-spaces ahead of the hour hand.

Therefore, the minute hand must gain 20 + 30, or 50 minute-spaces on the hour hand.

$50 \times \frac{12}{11} = \frac{600}{11} = 54\frac{6}{11}$. $54\frac{6}{11}$ minutes after 4 o'clock. *Ans.*

7. At what time between 2 and 3 o'clock do the hands of a clock make right angles with each other?

At 2 o'clock the minute hand is 10 minute-spaces behind the hour hand, and should be 15 minute-spaces ahead of the hour hand.

Therefore, the minute hand must gain 10 + 15, or 25 minute-spaces on the hour hand.

$25 \times \frac{12}{11} = \frac{300}{11} = 27\frac{3}{11}$. $27\frac{3}{11}$ minutes after 2 o'clock. *Ans.*

[redacted]

8. At what times between 6 and 7 o'clock do the hands of a watch make right angles with each other?

At 6 o'clock the minute hand is 30 minute-spaces behind the hour hand, and should be 15 minute-spaces behind, or 15 minute-spaces ahead of the hour hand.

Therefore, the minute hand must gain $30 - 15$, or 15 minute-spaces on the hour hand, or must gain $30 + 15$, or 45 minute-spaces on the hour hand.

$15 \times \frac{12}{11} = \frac{180}{11} = 16\frac{4}{11}$. $\quad 45 \times \frac{12}{11} = \frac{540}{11} = 49\frac{1}{11}$.

$16\frac{4}{11}$ minutes after 6 o'clock, or $49\frac{1}{11}$ minutes after 6 o'clock. *Ans.*

9. At what time between 7 and 8 o'clock do the hands of a watch make an angle of 120° with each other?

At 7 o'clock the minute hand is 35 minute-spaces behind the hour hand, and should be 20 minute-spaces behind.

Therefore, the minute hand must gain $35 - 20$, or 15 minute-spaces on the hour hand.

$15 \times \frac{12}{11} = \frac{180}{11} = 16\frac{4}{11}$. $\quad 16\frac{4}{11}$ minutes after 7 o'clock. *Ans.*

10. At what time between 12 and 1 o'clock do the hands of a watch make an angle of 60° with each other?

At 12 o'clock the hour and minute hands coincide, and the minute hand should be 10 minute-spaces ahead of the hour hand.

Therefore, the minute hand must gain 10 minute-spaces on the hour hand.

$10 \times \frac{12}{11} = \frac{120}{11} = 10\frac{10}{11}$. $\quad 10\frac{10}{11}$ minutes after 12 o'clock. *Ans.*

[redacted]

Exercise 96. Page 208.

Make out receipted bills for the following accounts, supplying dates:

1. James Hardy bought of C. H. Mills 275 bbl. flour, at $6.75; 324 bbl. flour, at $6.25; 300 bu. potatoes, at 48 cents; 1578 lb. butter, at 32 cents; 2000 bbl. apples, at $1.25; a car-load (20,000 lb.) of oats, at 42 cents a bushel; a car-load (28,575 lb.) of corn, at 55 cents a bushel.

Boston, Mass., March 1, 1898.

Mr. James Hardy,

To C. H. MILLS, Dr.

1898					
Jan.	*5*	*To 275 bbl. Flour*	*@ $6.75*	*$1856*	*25*
	12	*To 324 bbl. Flour*	*@ 6.25*	*2025*	*00*
	19	*To 300 bu. Potatoes*	*@ 0.48*	*144*	*00*
	26	*To 1578 lb. Butter*	*@ 0.32*	*504*	*96*
Feb.	*2*	*To 2000 bbl. Apples*	*@ 1.25*	*2500*	*00*
	"	*To 20,000 lb. Oats, 625 bu.*	*@ 0.42*	*262*	*50*
	9	*To 28,575 lb. Corn, 510.27 bu.*	*@ 0.55*	*280*	*65*
				$7573	*36*

1898, March 10. *Received Payment,*

C. H. Mills.

2. James Harlow bought of John Pike 12 bales, 480 lb. each, Texas cotton, at 9¼ cents; 7 bales, 502 lb. each, upland, at 10¼ cents; 3 bales, 492 lb. each, low middling, at 9¾ cents; 18 bales, 490 lb. each, good ordinary, at 9 cents.

Boston, Mass., March 1, 1898.

Mr. James Harlow,

To JOHN PIKE, Dr.

1898					
Jan.	*7*	*To 12 bales Texas Cotton, 5760 lb.*	*@ 9¼¢*	*$532*	*80*
	14	*To 7 bales Upland, 3514 lb.*	*@ 10¼¢*	*360*	*19*
	21	*To 3 bales Low middling, 1476 lb.*	*@ 9¾¢*	*143*	*91*
Feb.	*14*	*To 18 bales Good ordinary, 8820 lb.*	*@ 9¢*	*793*	*80*
				$1830	*70*

1898, March 10. *Received Payment,*

John Pike.

3. Richard Rowe bought of John Doe 125 lb. sugar, at 5 cents; 1 bag coffee, 115 lb., at 32 cents a pound; 25 gal. molasses, at 38 cents; 8 lb. Japan tea, at 92 cents; 28 lb. crackers, at 8 cents; 2 bbl. flour, at $7.50.

Boston, Mass., March 1, 1898.

Mr. Richard Rowe,

To JOHN DOE, Dr.

1898					
Feb.	*3*	*To 125 lb. Sugar*	*@ 5¢*	*$6*	*25*
		To 1 bag Coffee, 115 lb.	*@ 32¢*	*36*	*80*
		To 25 gal. Molasses	*@ 38¢*	*9*	*50*
	10	*To 8 lb. Japan Tea*	*@ 92¢*	*7*	*36*
		To 28 lb. Crackers	*@ 8¢*	*2*	*24*
		To 2 bbl. Flour	*@ $7.50*	*15*	*00*
				$77	*15*

1898, March 10. *Received Payment,*

John Doe.

4. William Litchfield bought of John Garvin 8 bags cracked corn, at 75 cents; 4 bags oats, at 80 cents; 16 lb. sweet potatoes, at $3\frac{1}{4}$ cents; 2 bu. potatoes, at $1.10; 100 lb. wire nails, at $2\frac{1}{4}$ cents; 5 lb. coffee, at 35 cents.

Boston, Mass., March 1, 1898.

Mr. William Litchfield,

To JOHN GARVIN, Dr.

1898					
Feb.	*8*	*To 8 bags Cracked Corn*	*@ 75¢*	*$6*	*00*
		To 4 bags Oats	*@ 80¢*	*3*	*20*
		To 16 lb. Sweet Potatoes	*@ $3\frac{1}{4}$¢*	*0*	*52*
	18	*To 2 bu. Potatoes*	*@ $1.10*	*2*	*20*
		To 100 lb. Wire Nails	*@ $2\frac{1}{4}$¢*	*2*	*25*
		To 5 lb. Coffee	*@ 35¢*	*1*	*75*
				$15	*92*

1898, March 10. *Received Payment,*

John Garvin.

[redacted]

5. Amos Tuck sold to Aaron Young 11 lb. ham, at 15 cents; 22 lb. beefsteak, at 24 cents; 18 lb. mutton, at 13 cents; 14 lb. veal, at 11 cents; and took in exchange 5 doz. eggs, at 18 cents; 15 lb. butter, at 26 cents; 9 bu. potatoes, at 40 cents; and 2 bbl. apples, at $ 1.35.

Boston, Mass., March 1, 1898.

Mr. Amos Tuck,

To AARON YOUNG, Dr.

1898							
Jan.	*25*	*To 5 doz. Eggs*	*@ 18¢*	*$ 0*	*90*		
Feb.	*4*	*To 15 lb. Butter*	*@ 26¢*	*3*	*90*		
	7	*To 9 bu. Potatoes*	*@ 40¢*	*3*	*60*		
	15	*To 2 bbl. Apples*	*@ $ 1.35*	*2*	*70*	*$ 11*	*10*
		Cr.					
Jan.	*25*	*By 11 lb. Ham*	*@ 15¢*	*$ 1*	*65*		
Feb.	*1*	*By 22 lb. Beefsteak*	*@ 24¢*	*5*	*28*		
	8	*By 18 lb. Mutton*	*@ 13¢*	*2*	*34*		
	22	*By 14 lb. Veal*	*@ 11¢*	*1*	*54*	*10*	*81*
		Balance due				*$ 0*	*29*

1898, March 10. *Received Payment,*

Aaron Young.

[redacted]

6. W. G. Fernald sold to John Waldron 35 lb. sugar, at 5 cents; 18 lb. coffee, at 35 cents; 20 lb. rice, at 8 cents; 4 tons hay, at $15.75; 3 cords pine wood, at $2.75; 4 cords hard wood, at $3.50; 8 tons furnace coal, at $6.75; 5 tons stove coal, at $7.25; 8 rolls wall paper, at 35 cents; and took in exchange 25 bbl. apples, at $1.15; 32 bu. pears, at 60 cents; and 42 bu. blueberries, at 8 cents a quart.

Boston, Mass., Aug. 1, 1898.

Mr. John Waldron,

To W. G. FERNALD, Dr.

1898							
Jan.	*10*	*To 35 lb. Sugar*	@ 5¢	$1	75		
	18	*To 18 lb. Coffee*	@ 35¢	6	30		
Feb.	*26*	*To 20 lb. Rice*	@ 8¢	1	60		
		To 4 t. Hay	@ $15.75	63	00		
Mar.	*7*	*To 3 cd. Pine Wood*	@ 2.75	8	25		
		To 4 cd. Hard Wood	@ 3.50	14	00		
	14	*To 8 t. Furnace Coal*	@ 6.75	54	00		
		To 5 t. Stove Coal	@ 7.25	36	25		
June	*20*	*To 8 rolls Wall Paper*	@ 35¢	2	80	$187	95
		Cr.					
1897							
Oct.	*13*	*By 25 bbl. Apples*	@ $1.15	$28	75		
		By 32 bu. Pears	@ 60¢	19	20		
1898							
July	*23*	*By 42 bu. Blueberries, 1344 qt.*	@ 8¢	107	52	155	47
		Balance due				$32	48

1898, August 5. *Received Payment,*

W. G. Fernald.

[redacted]

7. C. A. Colton bought of Green, Fisk & Co. 4 doz. No. 7 teakettles, at 85 cents each; 2 safety ash barrels, at $2.50; 3 doz. common scrapers, at 50 cents a dozen; 8 eagle shovels, at 10 cents; ½ doz. 8 by 12 black registers, at $1.50 each; ½ doz. spice boxes, at 55 cents each; ½ doz. 14-qt. dish pans, at $6.00 a dozen; 2 doz. common stove lifters, at 50 cents a dozen; ½ doz. 12 by 14 drip pans, at $4.00 a dozen; ½ gross retinned teaspoons, at 25 cents a dozen; 1 doz. ash sifters, at $1.00 each.

Boston, Mass., March 1, 1898.

Mr. C. A. Colton,

To GREEN, FISK & CO., DR.

1898					
Mar.	*1*	*To 4 doz. No. 7 Teakettles*	*@ 85¢*	*$40*	*80*
		To 2 Safety Ash Barrels	*@ $2.50*	*5*	*00*
		To 3 doz. Common Scrapers	*@ 50¢*	*1*	*50*
		To 8 Eagle Shovels	*@ 10¢*	*0*	*80*
		To ½ doz. 8 × 12 Black Registers	*@ $1.50*	*9*	*00*
		To ½ doz. Spice Boxes	*@ 55¢*	*3*	*30*
		To ½ doz. 14-qt. Dish Pans	*@ $6.00*	*3*	*00*
		To 2 doz. Common Stove Lifters	*@ 50¢*	*1*	*00*
		To ½ doz. 12 × 14 Drip Pans	*@ $4.00*	*2*	*00*
		To ½ gr. Retinned Teaspoons	*@ 25¢*	*1*	*50*
		To 1 doz. Ash Sifters	*@ $1.00*	*12*	*00*
				$79	*90*

1898, March 7. *Received Payment,*

Green, Fisk & Co.

8. R. M. Hanson bought of W. F. Fox & Co. 2 bbl. flour, at $5.75; ½ bbl. fine sugar, 153 lb., at $4.81 a cwt.; 25 lb. coffee, at 33 cents; 3 lb. Oolong tea, at 50 cents; 15 pint bottles olives, at 25 cents; 2 boxes graham wafers, at 40 cents; ½ doz. cans tomatoes, at $1.20 a dozen; ½ doz. cans J. H. F. peaches, at $3.50 a dozen; 4 Ferris hams, 48 lb., at 12½ cents a pound; 6 strips Ferris bacon, 19 lb. 9 oz., at 13 cents a pound; 3 lb. rice, at 9 cents; 3 lb. tapioca, at 5 cents; 40 lb. rye meal, at 2½ cents; 5 lb. boneless codfish, at 14 cents; ½ doz. cans plums, at $2.90 a dozen.

Boston, Mass., July 14, 1898.

Mr. R. M. Hanson,

To W. F. FOX & CO., Dr.

1898					
July	*14*	*To 2 bbl. Flour*	*@ $5.75*	*$11*	*50*
		To ½ bbl. Fine Sugar, 153 lb.	*@ 4.81*	*7*	*36*
		To 25 lb. Coffee	*@ 33¢*	*8*	*25*
		To 3 lb. Oolong Tea	*@ 50¢*	*1*	*50*
		To 15 pint bottles Olives	*@ 25¢*	*3*	*75*
		To 2 boxes Graham Wafers	*@ 40¢*	*0*	*80*
		To ½ doz. cans Tomatoes	*@ $1.20*	*0*	*60*
		To ½ doz. cans J. H. F. Peaches	*@ 3.50*	*1*	*75*
		To 4 Ferris Hams, 48 lb.	*@ 12½¢*	*6*	*00*
		To 6 strips Ferris Bacon, $19\frac{9}{16}$ lb.	*@ 13¢*	*2*	*54*
		To 3 lb. Rice	*@ 9¢*	*0*	*27*
		To 3 lb. Tapioca	*@ 5¢*	*0*	*15*
		To 40 lb. Rye Meal	*@ 2½¢*	*1*	*00*
		To 5 lb. Boneless Codfish	*@ 14¢*	*0*	*70*
		To ½ doz. cans Plums	*@ $2.90*	*1*	*45*
				$47	*62*

1898, July 23. *Received Payment,*

W. F. Fox & Co.

[redacted]

9. G. B. Cook bought of Gray, Higginson & Co. 1 No. 8-20 Glenwood B range, at $35.00; 1 No. 12 Rockford heater, at $20.00; 4 lb. Eng. stovepipe, at 15 cents; 3 lb. Rus. stovepipe, at 25 cents; 8 lb. sheet zinc, at 8 cents; 1 stove board, at $2.00; 1 set kitchen knives and forks, at $1.50; 2 washtubs, at 85 cents; 1 washboard, at 25 cents; 1 set Mrs. Potts' nickel sad-irons, at 75 cents; 2 milk cans, at 35 cents; 1 hand lamp complete, at 30 cents; 1 stand lamp, at $3.50; 1 granite iron washbowl, at 50 cents; 1 tea canister and 1 coffee canister, at 20 cents each; 1 carving knife and fork, at $2.00; 1 corn popper, at 25 cents; 1 rolling-pin, at 20 cents; 2 8-qt. porcelain kettles, at 70 cents; 1 granite iron coffee-pot, at 75 cents.

Boston, Mass., March 1, 1898.

Mr. G. B. Cook,

To GRAY, HIGGINSON & CO., Dr.

1898					
Mar.	*1*	*To 1 No. 8-20 Glenwood B Range*		*$35*	*00*
		To 1 No. 12 Rockford Heater		*20*	*00*
		To 4 lb Eng. Stovepipe	*@ 15¢*	*0*	*60*
		To 3 lb. Rus. Stovepipe	*@ 25¢*	*0*	*75*
		To 8 lb. Sheet Zinc	*@ 8¢*	*0*	*64*
		To 1 Stove Board		*2*	*00*
		To 1 set Kitchen Knives and Forks		*1*	*50*
		To 2 Washtubs	*@ 85¢*	*1*	*70*
		To 1 Washboard		*0*	*25*
		To 1 set Mrs. Potts' Nickel Sad-irons		*0*	*75*
		To 2 Milk Cans	*@ 35¢*	*0*	*70*
		To 1 Hand Lamp complete		*0*	*30*
		To 1 Stand Lamp		*3*	*50*
		To 1 Granite Washbowl		*0*	*50*
		To 1 Tea Canister		*0*	*20*
		To 1 Coffee Canister		*0*	*20*
		To 1 Carving Knife and Fork		*2*	*00*
		To 1 Corn Popper		*0*	*25*
		To 1 Rolling-pin		*0*	*20*
		To 2 8-qt. Porcelain Kettles	*@ 70¢*	*1*	*40*
		To 1 Granite Coffee-pot		*0*	*75*
				$73	*10*

1898, March 10. *Received Payment,*

Gray, Higginson & Co.

[redacted]

Exercise 97. Page 211.

1. Reduce 25.55^{kg} to pounds avoirdupois.

$$\begin{array}{r} 25.55 \\ \underline{2.205} \\ 12775 \\ 5110 \\ \underline{5110} \\ 56.33775 \end{array}$$

56.338 lb. *Ans.*

2. Reduce 5 sq. yd. 6 sq. ft. 108 sq. in. to square meters.

5 sq. yd. 6 sq. ft. 108 sq. in.
$=5$ sq. yd. $6\frac{3}{4}$ sq. ft. $=5.75$ sq. yd.

$$\begin{array}{r} 0.836^{qm} \\ \underline{5.75} \\ 4180 \\ 5852 \\ \underline{4180} \\ 4.807^{qm} \end{array}$$ *Ans.*

3. Reduce 24 gal. to liters.

24 gal. $=$ 96 qt.

$$\begin{array}{r} 0.946^{l} \\ \underline{96} \\ 5676 \\ \underline{8514} \\ 90.816^{l} \end{array}$$ *Ans.*

4. Reduce 10 lb. troy to kilograms.

10 lb. $=$ 120 oz.

$$\begin{array}{r} 31.104^{g} \\ \underline{120} \\ 622080 \\ \underline{31104} \\ 3732.480^{g} \end{array}$$

$3732.48^{g} = 3.732^{kg}$. *Ans.*

5. Reduce 50.5 cu. yd. to cubic meters.

$$\begin{array}{r} 0.765^{cbm} \\ \underline{50.5} \\ 3825 \\ \underline{3825} \\ 38.6325^{cbm} \end{array}$$ *Ans.*

6. Reduce $69\frac{17}{100}$ mi. to kilometers.

$$\begin{array}{r} 1.609^{km} \\ \underline{69.17} \\ 11263 \\ 1609 \\ 14481 \\ \underline{9654} \\ 111.29453^{km} \end{array}$$

111.295^{km}. *Ans.*

7. Reduce 12 A. 12 sq. rd. to hektars.

12 A. 12 sq. rd. $=12\frac{3}{40}$ A.
$=12.075$ A.

$$\begin{array}{r} 12.075 \\ \underline{0.405} \\ 60375 \\ \underline{48300} \\ 4.890375 \end{array}$$

4.890^{ha}. *Ans.*

8. Reduce 10 cd. to sters.

$10 \times 3.624^{st} = 36.24^{st}$. *Ans.*

9. Reduce 4 cwt. 24 lb. to kilograms.

4 cwt. 24 lb. $=$ 424 lb.

$$\begin{array}{r} 0.454^{kg} \\ \underline{424} \\ 1816 \\ 908 \\ \underline{1816} \\ 192.496^{kg} \end{array}$$ *Ans.*

[redacted]

10. Reduce 25 bu. 2 pk. to hektoliters.

25 bu. 2 pk. = 102 pk. = 816 qt.

1 qt. = 1.101^{l} = 0.0110^{hl}.

```
  816
0.011
-----
  816
 816
-----
8.976   8.976hl. Ans.
```

11. Express 15^{km} in the common system.

```
 0.621
    15
 -----
  3105
  621
 -----
 9.315
   320
 -----
  6300
  945
 -----
 100.8
   16½
 -----
  13.2
```

9 mi. 100 rd. 13.2 ft. *Ans.*

12. Express 3^{ha} in the common system.

```
 2.471
     3
 -----
 7.413
   160
 -----
 24780
 413
 -----
 66.08
   30¼
 -----
  2.42
```

7 A. 66 sq. rd. 2.4 sq. yd. *Ans.*

13. Express 12.125^{cbm} in the common system.

```
   12.125
    1.308
  -------
    97000
   36375
  12125
  -------
  15.8595
       27
  -------
   60165
  17190
  -------
  23.2065
```

15 cu. yd. 23.2 cu. ft. *Ans.*

14. Express 101.25^{l} in the common system.

```
   101.25          101.25
    1.057           0.908
   ------          ------
    70875           81000
   50625           91125
  10125            ------
  ---------        91.935
  107.02125             2
                   ------
                     1.87
```

```
4|107 qt.        8|91 qt.
  26 gal. 3 qt.  4|11 pk. . . . 3 qt.
                    2 bu. . . . 3 pk.
```

26 gal. 3 qt.; 2 bu. 3 pk. 3 qt. 1.9 pt. *Ans.*

15. Reduce 20.25^{hl} to liquid quarts; to dry quarts.

1^{hl} = 105.671 liquid quarts

= 90.810 dry quarts.

```
    105.671          90.81
      20.25          20.25
   --------        -------
     528355          45405
    211342          18162
   211342          18162
 ----------      ---------
 2139.83775      1838.9025
```

2139.838 liquid quarts;

1838.903 dry quarts. *Ans.*

16. Express 5^{kg} in troy weight.

$1^{kg} = 15{,}432.35$ gr.

```
     15432.35 gr.
            5
24 | 77161.75 gr.
20 |   3215     dwt. . . . 2 gr.
12 |    160     oz. . . . . 15 dwt.
         13     lb. . . . . 4 oz.
```

13 lb. 4 oz. 15 dwt. 2 gr. *Ans.*

17. Express 24^{st} in the common system.

```
 0.276
    24
  1104
  552
 6.624
   128
  4992
 1248
 624
79.872
```

6 cd. 80 cu. ft. nearly. *Ans.*

18. Express 62.5^{qm} in the common system.

```
 1.196
  62.5
  5980
 2392
7176
74.75
```

74¾ sq. yd. *Ans.*

19. Express 1001^{kg} in avoirdupois weight.

```
   2.205 lb.
   1001
   2205
2205
2207.205 lb.
```

2207.2 lb. = 1 t. 207.2 lb. *Ans.*

20. Express 42 A. 100 sq. rd. in the metric system.

42 A. 100 sq. rd. = 42.625 A.

```
   42.625
    0.405
   213125
 170500
17.263125
```

17.263^{ha}. *Ans.*

21. Find in acres, etc., the area of a rectangular field if it is 100^{m} long and 75^{m} broad.

$100 \times 75 = 7500.$

$7500^{qm} = 0.75^{ha}.$

```
   2.471
    0.75
   12355
  17297
 1.85325
     160
 5119500
 85325
136.52
    30¼
    13
  1560
 15.73
```

1 A. 136 sq. rd. 16 sq. yd. nearly. *Ans.*

22. Find the number of cubic meters in a rectangular box 2 yd. long, 3 ft. wide, and 2½ ft. deep.

3 ft. = 1 yd.; 2½ ft. = ⅚ yd.

$2 \times 1 \times \frac{5}{6} = 1\frac{2}{3}.$

```
0.765^cbm
   1⅔
 510
765
1.275^cbm
```

1.275^{cbm} *Ans.*

[redacted]

23. Find the number of cubic yards in a rectangular box 2^{m} long, 75^{cm} wide, and 50^{cm} deep.

$$75^{cm} = \tfrac{3}{4}^{m};\ 50^{cm} = \tfrac{1}{2}^{m}.$$

$$2 \times \tfrac{3}{4} \times \tfrac{1}{2} = \tfrac{3}{4} = 0.75.$$

1.308 cu. yd.
0.75
6540
9156
0.981 cu. yd. *Ans.*

24. If a man walks 75^{m} a minute, what is his rate in miles per hour?

75^{m} a minute $= 60 \times 75^{m}$ per hour
$= 4500^{m}$ per hour
$= 4.5^{km}$ per hour.

0.621 mi.
4.5
3105
2484
2.7945 mi. *Ans.*

25. If a cubic centimeter of cast iron weighs 7.113^{g}, how many pounds does a cubic foot weigh?

1 cu. ft. $= \frac{1}{27}$ of 0.76453^{cbm}
$= 0.0283^{cbm} = 28{,}300^{ccm}$.

7.113^{g}
28300
2133900
56904
14226
$201297.9^{g} = 201.2979^{kg}$

201.2979
2.205
10064895
4025958
4025958
443.8618695
443.86 lb. *Ans.*

26. How many steps 2 ft. 6 in. long will a man take in walking a kilometer?

$$1^{km} = 0.621 \text{ mi.}$$

$$\frac{0.621 \times 5280}{2\frac{1}{2}} = 0.621 \times \overset{1056}{\cancel{5280}} \times \frac{2}{\cancel{5}}$$

$= 1312$ nearly.

1312 steps. *Ans.*

27. Find the value of a carboy (17 qt.) of sulphuric acid, specific gravity 1.841, at $4\frac{3}{4}$ cents a kilogram.

0.946^{l}
17
6622
946
16.082^{l}

16.082^{kg}
1.841
16082
64328
128656
16082
29.606962^{kg}

\$0.0475
29.6
2850
4275
950
\$1.406

\$1.41. *Ans.*

28. Find the value of a carboy ($17\frac{1}{2}^{l}$) of nitric acid, specific gravity 1.451, at 15 cents a pound.

$17\frac{1}{2}^{l}$ of water weighs 17.5^{kg}.

2.205 lb.
17.5
11025
15435
2205
38.5875 lb.
1.451
385875
1929375
1543500
385875
55.9904625 lb.

\$0.15
56
90
75
\$8.40 *Ans.*

29. If the specific gravity of sea water is 1.026, and that of olive oil is 0.915, what is the weight of a hektoliter of each in pounds and in kilograms?

$1^{hl} = 100^{l}$.

100^{l} of water weighs 100^{kg}.

$1.026 \times 100^{kg} = 102.6^{kg}$.

2.205 lb.
102.6
13230
4410
2205
226.233 lb.

$0.915 \times 100^{kg} = 91.5^{kg}$.

2.205 lb.
91.5
11025
2205
19845
201.7575 lb.

Therefore, 1^{hl} of sea water weighs 226.23 lb., or 102.6^{kg}; 1^{hl} of olive oil weighs 201.76 lb., or 91.5^{kg}. *Ans.*

30. Find the weight in pounds and in kilograms of $31\frac{1}{8}$ gal. of the best alcohol, specific gravity 0.792.

$31\frac{1}{8}$ gal. $= 124\frac{1}{2}$ qt.

$124\frac{1}{2}$
0.946
631
744
496
1116
117.935

117.935^{kg}
0.792
235870
1061415
825545
93.404520^{kg}

93.405^{kg}. *Ans.*

93.405
2.205
467025
186810
186810
205.958025

205.958 lb. *Ans.*

31. Find the weight in pounds and in kilograms of the air, specific gravity 0.00129206, in a room 7^{m} long, 5^{m} wide, and 3.5^{m} high.

$7 \times 5 \times 3.5 = 122.5$.

122.5^{cbm} of water weighs $122{,}500^{kg}$.

0.00129206
122500
64603000
258412
258412
129206
158.27735

158.277^{kg}. *Ans.*

158.277
2.205
791385
316554
316554
349.000785

349 lb. *Ans.*

[redacted]

32. Find the weight in pounds and in kilograms of the air, specific gravity 0.00129206, in a room 23 ft. long, 16 ft. wide, and 10 ft. high.

$23 \times 16 \times 10 \times 62.5$ lb. $= 230,000$ lb.

```
   23
  160
 1380
 23
 3680
  62.5
18400
7360
22080
230000.
```

```
0.00129206
    230000
3876180000
 258412
297.1738
```

297.17 lb. *Ans.*

```
   297.1738
      0.454
   11886952
  14858690
 11886952
134.9169052
```

134.92^{kg}. *Ans.*

33. What is the lifting force in kilograms and in pounds of a balloon that weighs 2^{kg}, and contains 10,000^{l} of hydrogen gas, specific gravity 0.00008929?

The difference in weight between 10,000^{l} of air and 10,000^{l} of hydrogen is $(0.00129206 - 0.00008929) \times 10{,}000^{kg}$
$= 0.00120277 \times 10{,}000^{kg}$
$= 12.0277^{kg}$.

The lifting force of the balloon is $12.0277^{kg} - 2^{kg} = 10.0277^{kg}$. *Ans.*

```
  10.0277
   2.205
  501385
 200554
200554
22.1110785
```

22.111 lb. *Ans.*

34. What is the value at $4.50 a cord of a pile of wood 1.2^{m} wide, 7^{m} long, and 2^{m} high?

$1.2 \times 7 \times 2 = 16.8$. $16.8^{cbm} = 16.8^{st}$.

```
 0.276 cd.
 16.8
 2208
1656
276
4.6368 cd.
```

```
  4.6368
    4.50
 2318400
185472
20.8656
```

$20.87. *Ans.*

35. How many miles will a train run in 1 hr. 28 min. 21 sec., at the rate of 50^{km} an hour?

```
60 | 21.      sec.
60 | 28.35    min.
     1.4725   hr.
```

$1.4725 \times 50^{km} = 73.625^{km}$.

```
  73.625
   0.621
   73625
 147250
441750
45.721125
```

45.721 mi. *Ans.*

36. Find the time it takes a train to run 31 mi. 180 yd. at the rate of 1 min. 25 sec. per kilometer.

31 mi. 180 yd. $= 31\frac{9}{88}$ mi.
$= 31\frac{9}{88} \times 1.609^{km} = 50.044^{km}$.

```
 1.609
  31 9/88
  165
 1609
4827
50.044
```

1 min. 25 sec. $= 1\frac{5}{12}$ min.
$50.044 \times 1\frac{5}{12}$ min. $= 70.896$ min. *Ans.*

37. What is the weight of 12 cu. yd. 16 cu. ft. 720 cu. in. of earth, if a cubic meter weighs 1 t. 17 cwt.?

$$\begin{array}{r|ll} 1728 & 720. & \text{cu. in.} \\ 27 & 16.417 & \text{cu. ft.} \\ & 12.608 & \text{cu. yd.} \end{array}$$

1 t. 17 cwt. = 37 cwt.

$$\begin{array}{r} 12.608 \\ 0.765 \\ \hline 63040 \\ 75648 \\ 88256 \\ \hline 9.64512 \end{array} \qquad \begin{array}{r} 9.64512 \\ 37 \\ \hline 6751584 \\ 2893536 \\ \hline 356.86944 \end{array}$$

356.87 cwt. = 17 t. 16 cwt. 87 lb. *Ans.*

38. Find the weight in grams of a liter of mercury, if a cubic inch weighs 0.4925 of a pound avoirdupois.

$1^{l} = 1000^{ccm} = 61.03$ cu. in.

1^{l} weighs 61.03×0.4925 lb.

$= 61.03 \times 0.4925 \times 453.59^{g}$.

$$\begin{array}{r} 0.4925 \\ 61.03 \\ \hline 14775 \\ 4925 \\ 29550 \\ \hline 30.057275 \end{array} \qquad \begin{array}{r} 453.59^{g} \\ 30.057 \\ \hline 317513 \\ 226795 \\ 136077 \\ \hline 13633.55463 \end{array}$$

$13,633.55^{g}$. *Ans.*

39. How many yards of cloth, at \$3.12½ a meter, should be given in exchange for 15^{m} at \$2.75 a yard?

\$3.12½ a meter

$= 0.914 \times \$3.12\tfrac{1}{2}$ a yard.

$15^{m} = 15 \times 1.094$ yd.

$$\frac{15 \times 1.094 \times 2.75}{0.914 \times 3.125}$$

$$= \overset{3}{\cancel{15}} \times \frac{\overset{547}{\cancel{1094}}}{\cancel{1000}} \times \frac{11}{\cancel{4}} \times \frac{\cancel{1000}}{\underset{457}{\cancel{914}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{5}{\cancel{25}}}$$

$$= \frac{36102}{2285} = 15\tfrac{1827}{2285} = 15.8.$$

15.8 yd. *Ans.*

40. If a wine merchant buys 3^{hl} of wine for 1600 francs, what does a gallon cost him in United States money, if 25 francs are equivalent to \$4.825?

$3^{hl} = 300^{l}$.

$$1 \text{ fr.} = \frac{\$4.825}{25} = \$0.193.$$

$$1^{l} \text{ costs } \frac{1600 \times \$0.193}{300}.$$

1 gal. = 4 qt. = 4×0.946^{l}.
Therefore, 1 gal. costs

$$\frac{4 \times 0.946 \times 1600 \times \$0.193}{300}.$$

$$\begin{array}{r} 0.946 \\ 4 \\ \hline 3.784 \\ 1600 \\ \hline 2270400 \\ 3784 \\ \hline 6054.4 \end{array} \qquad \begin{array}{r} 6054.4 \\ 0.193 \\ \hline 181632 \\ 544896 \\ 60544 \\ \hline 1168.4992 \end{array}$$

$$3\cancel{00})\underline{11.684992}$$
$$3.894997$$

\$3.89. *Ans.*

[redacted]

41. A mill wheel is turned by a stream of water running at the rate of a yard a second in a channel 5 ft. wide and 9 in. deep. Find the weight in metric tons and in tons avoirdupois of the water supplied in 12 hr., if a cubic foot of water weighs 1000 oz.

Volume of water each second $= (3 \times 5 \times \frac{3}{4})$ cu. ft.

Volume of water for 12 hr. $= (12 \times 60 \times 60 \times 3 \times 5 \times \frac{3}{4})$ cu. ft.

Weight of water for 12 hr. $= \left(12 \times 60 \times 60 \times 3 \times 5 \times \frac{3}{4} \times \frac{1000}{2000 \times 16}\right)$ t.

$$\overset{3}{\cancel{12}} \times \overset{15}{\cancel{60}} \times \overset{15}{\cancel{60}} \times 3 \times 5 \times \frac{3}{\cancel{4}} \times \frac{\cancel{1000}}{\underset{2}{\cancel{2000}} \times \underset{\cancel{4}}{\cancel{16}}} = \frac{30375}{2} = 15{,}187.5.$$

15,187.5 t. *Ans.*

1 lb. $= 0.45359^{kg}$.

1 t. $= 2000$ lb. $= 2000 \times 0.45359^{kg} = 907^{kg} = 0.907^{t}$.

$$\begin{array}{r} 15187.5 \\ 0.907 \\ \hline 1063125 \\ 1366875 \\ \hline 13775.0625 \end{array}$$

13,775.06^{t}. *Ans.*

Exercise 98. Page 214.

1. When water is heated from the freezing point to the boiling point, it expands $\frac{1}{24}$ in volume. Find in kilograms the weight of a cubic foot of water at the freezing point and at the boiling point.

At the freezing point 1 cu. ft. of water weighs $62\frac{1}{2}$ lb.; at the boiling point $\frac{24}{25} \times 62\frac{1}{2}$ lb. $= 60$ lb. 1 lb. $= \frac{5}{11}$ kg.

$$62\tfrac{1}{2} \times \tfrac{5}{11}^{kg} = \tfrac{125}{2} \times \tfrac{5}{11}^{kg} = \tfrac{625}{22}^{kg} = 28.41^{kg}.$$

$$60 \times \tfrac{5}{11}^{kg} = \tfrac{300}{11}^{kg} = 27.27^{kg}.$$

At the freezing point 28.41^{kg}; at the boiling point 27.27^{kg}. *Ans.*

2. A circular plate of lead 8 in. in diameter and 2 in. thick is changed without loss into spherical shot, each 1.25^{mm} in radius. How many shot does it make?

The volume of the plate $= (2 \times 3.1416 \times 4 \times 4)$ cu. in.

$= (2 \times 3.1416 \times 4 \times 4 \times 16)^{ccm}$.

[redacted]

$$\text{The volume of 1 shot} = \left(\frac{1}{6} \times 3.1416 \times \frac{1}{4^3}\right)^{\text{ccm}}.$$

$$\therefore \text{ the number of shot} = \frac{2 \times 3.1416 \times 4 \times 4 \times 16}{\frac{1}{6} \times 3.1416 \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}}$$

$$= \frac{6 \times 2 \times \cancel{3.1416} \times 4 \times 4 \times 16 \times 4 \times 4 \times 4}{\cancel{3.1416}}$$

$$= 196{,}608. \quad \textit{Ans.}$$

3. If $\frac{3}{4}$ of a yard of velvet costs \$3, how many francs will $\frac{5}{7}$ of a meter cost?

$$\tfrac{3}{4}\text{ yd.} = \tfrac{3}{4} \text{ of } \tfrac{9}{10}^{\text{m}} = \tfrac{27}{40}^{\text{m}}. \quad 1 \text{ fr.} = \$0.193.$$

$$\therefore \ \$3 = \frac{3}{0.193} \text{ fr.} = \frac{3000}{193} \text{fr.}$$

If $\frac{27}{40}^{\text{m}}$ cost $\frac{3000}{193}$ fr., $\frac{5}{7}^{\text{m}}$ will cost $\frac{5}{7} \times (\frac{3000}{193} \div \frac{27}{40})$ fr.

$$= \frac{5}{7} \times \frac{40}{\underset{9}{\cancel{27}}} \times \frac{\overset{1000}{\cancel{3000}}}{193} \text{ fr.} = \frac{200000}{12159} \text{ fr.} = 16.4 \text{ fr.} \quad \textit{Ans.}$$

4. Water expands $\frac{1}{10}$ in freezing, and a floating body displaces an amount of water equal in weight to the body. What is the volume in cubic meters, and the weight in metric tons, of an iceberg floating in the ocean, if the specific gravity of sea water is 1.026, and the part of the iceberg above the water is a rectangular solid 200 ft. long, 60 ft. wide, and 12 ft. high?

If water expands $\frac{1}{10}$ in freezing, the volume of ice is $\frac{11}{10}$ of the volume of the water. Hence, the specific gravity of ice is $\frac{10}{11}$. The difference between the specific gravity of sea water and that of the iceberg is $1.026 - \frac{10}{11} = 1.026 - 0.909 = 0.117$.

Volume of iceberg above water

$$= (200 \times 60 \times 12) \text{ cu. ft.} = \frac{200 \times 60 \times 12}{27} \text{ cu. yd.} = \frac{200 \times 60 \times 12}{27} \times \frac{10^{\text{cbm}}}{13}.$$

Volume of whole iceberg

$$= 1.026 \times \left(\frac{\frac{200 \times 60 \times 12}{27} \times \frac{10}{13}}{0.117}\right)^{\text{cbm}}$$

$$= \frac{\overset{38}{\underset{}{\cancel{114}}}\!\!\!\!\!\!\!\!\!\!\!\!\cancel{1026}}{\cancel{1000}} \times \frac{\overset{20\ \ \ 4}{200 \times \cancel{60} \times \cancel{12}}}{\underset{\underset{3}{\cancel{9}}}{\cancel{27}}} \times \frac{\cancel{1000}}{\underset{13}{\cancel{117}}} \times \frac{10^{\text{cbm}}}{13} = \frac{6080000^{\text{cbm}}}{169} = 35{,}976.33^{\text{cbm}}. \quad \textit{Ans.}$$

$35{,}976.33^{\text{cbm}}$ of ice weighs $\frac{10}{11}$ of $35{,}976.33^{\text{t}} = \frac{359763.3^{\text{t}}}{11} = 32{,}705.75^{\text{t}}$. *Ans.*

[redacted]

5. How many hektoliters of wheat will a rectangular bin hold 14 ft. long, 10 ft. wide, and 6 ft. high?

$$\text{Volume of bin} = (14 \times 10 \times 6) \text{ cu. ft.} = \frac{14 \times 10 \times 6}{27} \text{ cu. yd.}$$

$$= \frac{14 \times 10 \times 6}{27} \times \frac{10^{\text{cbm}}}{13} = \frac{14 \times 10 \times \overset{2}{\cancel{6}}}{\underset{9}{\cancel{27}}} \times \frac{10}{13} \times 10^{\text{hl}}$$

$$= \frac{28000^{\text{hl}}}{117} = 239.3^{\text{hl}}. \quad \textit{Ans.}$$

6. How many hektoliters of water will a cylindrical stand-pipe hold 70 ft. high and 35 ft. in diameter?

Volume of stand-pipe

$$= \left(70 \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2}\right) \text{cu. ft.} = \frac{70 \times 22 \times 35 \times 35}{27 \times 7 \times 2 \times 2} \text{ cu. yd.}$$

$$= \frac{70 \times 22 \times 35 \times 35}{27 \times 7 \times 2 \times 2} \times \frac{10^{\text{cbm}}}{13} = \frac{\overset{10}{\cancel{70}} \times \overset{11}{\cancel{22}} \times 35 \times 35}{27 \times \cancel{7} \times \cancel{2} \times \cancel{2}} \times \frac{\overset{5}{\cancel{10}}}{13} \times 10^{\text{hl}}$$

$$= \frac{6737500^{\text{hl}}}{351} = 19{,}195.2^{\text{hl}}. \quad \textit{Ans.}$$

7. How many bushels of wheat will a rectangular bin hold 4^{m} long, 3^{m} wide, and 2.5^{m} high?

Volume of bin

$$= (4 \times 3 \times 2\tfrac{1}{2})^{\text{cbm}} = (4 \times 3 \times 2\tfrac{1}{2} \times 10)^{\text{hl}} = 4 \times 3 \times 2\tfrac{1}{2} \times 10 \times 2\tfrac{5}{6} \text{ bu.}$$

$$4 \times 3 \times 2\tfrac{1}{2} \times 10 \times 2\tfrac{5}{6} = \overset{2}{\cancel{4}} \times \cancel{3} \times \frac{5}{\cancel{2}} \times 10 \times \frac{17}{\underset{\cancel{2}}{\cancel{6}}} = 850. \qquad 850 \text{ bu. } \textit{Ans.}$$

8. How many gallons of water in a well 1.2^{m} in diameter, if the depth of the water is 2^{m}?

$$\text{Volume of the water} = (2 \times \tfrac{22}{7} \times 0.6 \times 0.6)^{\text{cbm}} = 2 \times \tfrac{22}{7} \times \tfrac{3}{5} \times \tfrac{3}{5} \times 1000^{\text{l}}$$

$$= 2 \times \tfrac{22}{7} \times \tfrac{3}{5} \times \tfrac{3}{5} \times 1000 \times \tfrac{17}{16} \text{ qt.}$$

$$= 2 \times \tfrac{22}{7} \times \tfrac{3}{5} \times \tfrac{3}{5} \times 1000 \times \tfrac{17}{16} \times \tfrac{1}{4} \text{ gal.}$$

$$\cancel{2} \times \frac{\overset{11}{\cancel{22}}}{7} \times \frac{3}{\cancel{5}} \times \frac{3}{\cancel{5}} \times \overset{\overset{\overset{5}{\cancel{40}}}{\cancel{200}}}{\cancel{1000}} \times \frac{17}{\underset{\cancel{2}}{\cancel{16}}} \times \frac{1}{\underset{2}{\cancel{4}}} = \frac{8415}{14} = 601.1.$$

601.1 gal. *Ans.*

9. If 1 lb. troy of silver is worth \$6.20, what is the value of a lump of silver weighing 2.64^{kg}?

$$2.64^{kg} = 2.64 \times \tfrac{13}{5} \text{ lb. troy} = 2\tfrac{16}{25} \times \tfrac{13}{5} \text{ lb. troy.}$$

$$2\tfrac{16}{25} \times \frac{13}{5} \times \$6.20 = 2\tfrac{16}{25} \times \frac{13}{5} \times \$6\tfrac{1}{5} = \frac{66}{25} \times \frac{13}{5} \times \$\frac{31}{5} = \$\frac{26598}{625} = \$42.56. \textit{ Ans.}$$

10. A pound of brass contains 3.3 cu. in., and a pound of antimony contains 6.27 cu. in. Find the weight in kilograms of a mass of $313\frac{1}{2}$ cu. in. that contains equal volumes of the two metals.

The volume of each metal in the mass is $\frac{1}{2}$ of $313\frac{1}{2}$ cu. in. = 156.75 cu. in.

$$\text{The weight of the mass} = \left(\frac{156.75}{3.3} + \frac{156.75}{6.27}\right) \text{lb.}$$

$$= \left(\frac{156.75}{3.3} + \frac{156.75}{6.27}\right) \times \frac{5}{11}^{kg}.$$

$$\frac{156.75}{3.3} \times \frac{5}{11} = \frac{\overset{\overset{95}{\cancel{1425}}}{\cancel{15675}}}{\underset{22}{\cancel{330}}} \times \frac{5}{\cancel{11}} = \frac{475}{22} = 21.59.$$

$$\frac{156.75}{6.27} \times \frac{5}{11} = \frac{\overset{\overset{25}{\cancel{1425}}}{\cancel{15675}}}{\underset{11}{\cancel{627}}} \times \frac{5}{\cancel{11}} = \frac{125}{11} = 11.36.$$

$$21.59^{kg} + 11.36^{kg} = 32.95^{kg}. \textit{ Ans.}$$

11. If 2 cu. in. of mercury weighs 1 lb., and 100 cu. in. of air weighs 31 gr., how many kilometers high must a column of air be to weigh as much as a column of mercury 29.388 in. high, standing on a base of the same area?

If 2 cu. in. of mercury weighs 1 lb., or 7000 gr., 1 cu. in. of mercury weighs $\frac{1}{2}$ of 7000 gr.; that is, 3500 gr.

If 100 cu. in. of air weighs 31 gr., 1 cu. in. of air weighs 0.31 gr.

Therefore, mercury weighs $\frac{3500}{0.31}$ times as much as air, and the column of air to weigh as much as a column of mercury 29.388 in. high must be $\frac{3500}{0.31} \times 29.388$ in. high.

[redacted]

$$\frac{3500}{0.31}\times 29.388\text{ in.} = \frac{3500\times 29.388}{0.31\times 36}\text{ yd.} = \frac{3500\times 29.388}{0.31\times 36}\times 0.9^{\text{m}}$$

$$= \frac{3500\times 29.388}{0.31\times 36}\times 0.0009^{\text{km}}.$$

$$\overset{7}{\cancel{3500}}\times\frac{\overset{\overset{237}{\cancel{7347}}}{\cancel{29388}}}{\underset{10}{\cancel{1000}}}\times\frac{\cancel{9}}{\underset{20}{\cancel{10000}}}\times\frac{\cancel{100}}{\cancel{31}}\times\frac{1}{\underset{\cancel{4}}{\cancel{36}}}=\frac{1659}{200}=8.295. \qquad 8.295^{\text{km}}.\ \textit{Ans.}$$

12. If a sprinter can run 0.00645 of a mile in 1.08 sec., how many meters can he run in a second? How many seconds will it take him to run 100^{m}?

The sprinter in 1 sec. can run

$$\frac{0.00645}{1.08}\text{ mi.} = \frac{0.00645}{1.08}\times 1.6^{\text{km}} = \frac{0.00645}{1.08}\times 1.6\times 1000^{\text{m}}$$

$$= \frac{0.645}{108}\times 1.6\times 1000^{\text{m}} = \frac{\overset{\overset{43}{\cancel{129}}}{\cancel{645}}}{\underset{\underset{9}{\cancel{27}}}{\cancel{108}}}\times\frac{\overset{2}{\cancel{8}}^{\text{m}}}{\cancel{5}} = \frac{86^{\text{m}}}{9} = 9\tfrac{5}{9}^{\text{m}}.\ \textit{Ans.}$$

$$100 \div 9\tfrac{5}{9} = \frac{9}{\underset{43}{\cancel{86}}}\times\overset{50}{\cancel{100}} = \frac{450}{43} = 10.465. \qquad 10.465\text{ sec. } \textit{Ans.}$$

13. Two trains going in opposite directions pass each other in $3\frac{1}{2}$ sec. If their lengths are 260 ft. and 200 ft., respectively, and the first train is going at the rate of 80^{km} an hour, what is the rate of the second train?

$$80^{\text{km}}\text{ an hour} = \frac{80}{3600}^{\text{km}}\text{ a sec.} = \frac{80\times\frac{5}{8}}{3600}\text{ mi. a sec.}$$

$$= \frac{80\times\frac{5}{8}\times 5280}{3600}\text{ ft. a sec.} = 73\tfrac{1}{3}\text{ ft. a sec.}$$

$$\frac{80\times\frac{5}{8}\times 5280}{3600} = \overset{10}{\cancel{80}}\times\frac{5}{\cancel{8}}\times\frac{\overset{44}{\cancel{5280}}}{\underset{\underset{3}{\cancel{360}}}{\cancel{3600}}} = \frac{220}{3} = 73\tfrac{1}{3}.$$

$$3\tfrac{1}{2}\times 73\tfrac{1}{3} = \frac{7}{\cancel{2}}\times\frac{\overset{110}{\cancel{220}}}{3} = \frac{770}{3} = 256\tfrac{2}{3};$$ that is, the first train in $3\frac{1}{2}$ sec. goes $256\frac{2}{3}$ ft. of the 260 ft. + 200 ft. = 460 ft.

The second train, therefore, in $3\frac{1}{2}$ sec. goes 460 ft. $-256\frac{2}{3}$ ft. $=203\frac{1}{3}$ ft.

The rate of the second train is $\frac{203\frac{1}{3}}{3\frac{1}{2}}$ ft. a second, $= 3600 \times \frac{203\frac{1}{3}}{3\frac{1}{2}}$ ft. an hour $= \frac{3600}{5280} \times \frac{203\frac{1}{3}}{3\frac{1}{2}}$ mi. an hour $= \frac{3600}{5280} \times \frac{203\frac{1}{3}}{3\frac{1}{2}} \times 1\frac{3}{5}^{km}$ an hour.

$$\frac{3600}{5280} \times \frac{203\frac{1}{3}}{3\frac{1}{2}} \times 1\frac{3}{5} = \frac{\overset{\overset{5}{\cancel{15}}}{\cancel{3600}}}{\underset{\underset{11}{\cancel{22}}}{\cancel{5280}}} \times \frac{610}{\cancel{3}} \times \frac{2}{7} \times \frac{8}{\cancel{5}} = \frac{4880}{77} = 63.4.$$

63.4^{km} an hour. *Ans.*

14. If a cubic inch of water converted into steam will produce mechanical force sufficient to raise a weight of 2200 lb. one foot high, how many meters high would the conversion into steam of a cubic centimeter of water raise a weight of one kilogram?

1 cu. in. $= 16^{ccm}$. 2200 lb. $= 2200 \times \frac{5}{11}^{kg}$. 1 ft. $= \frac{1}{3}$ yd. $= \frac{1}{3}$ of 0.9^m.

If 1^{ccm} of steam produces force sufficient to raise $2200 \times \frac{5}{11}^{kg}$ $\frac{\frac{1}{3} \text{ of } 0.9^m}{16}$ high, it will produce a force sufficient to raise 1^{kg} $2200 \times \frac{5}{11} \times \frac{\frac{1}{3} \text{ of } 0.9^m}{16}$ high.

$$\overset{\overset{\overset{5}{\cancel{20}}}{\cancel{200}}}{\cancel{2200}} \times \frac{5}{\cancel{11}} \times \frac{1}{\cancel{3}} \times \frac{\overset{3}{\cancel{9}}}{\cancel{10}} \times \frac{1}{\underset{4}{\cancel{16}}} = \frac{75}{4} = 18.75.$$

18.75^m. *Ans.*

15. If a man takes 100 steps of 0.7^m each in a minute, how long will it take him to walk a distance of 28^{km}?

$$\frac{28 \times 1000}{100 \times 0.7} = \frac{\overset{4}{\cancel{28}} \times \overset{10}{\cancel{1000}} \times 10}{\cancel{100} \times \cancel{7}} = 400.$$

400 min. $=$ 6 hr. 40 min. *Ans.*

16. A lot of land containing 63^a 21^{ca}, worth \$0.35 a square yard, is exchanged for a second lot containing 1^{ha} 5^a. What is the cost per ar of the second lot?

\$0.35 a sq. yd. $= \frac{\$0.35}{\frac{5}{6}}$ a centar $=$ \$0.42 a centar. 63^a $21^{ca} = 6321^{ca}$.

The first lot cost $6321 \times \$0.42$. 1^{ha} $5^a = 105^a$.

Therefore, the second lot cost per ar

$$\frac{\overset{301}{\cancel{6321}} \times \$0.42}{\underset{5}{\cancel{105}}} = \$\frac{126.42}{5} = \$25.284.$$ *Ans.*

[redacted]

17. Light travels in 8 min. 13 sec. from the sun to the earth, $153,624,000^{km}$. What is the velocity of light in miles per second?

$$8 \text{ min. } 13 \text{ sec.} = 493 \text{ sec.}$$

The velocity of light per second is $\dfrac{153624000^{km}}{493} = \dfrac{153624000 \times \frac{5}{8}}{493}$ mi.

$$\frac{153624000 \times \frac{5}{8}}{493} = \frac{\overset{19203000}{\cancel{153624000}} \times 5}{493 \times \cancel{8}} = \frac{96015000}{493} = 194,756.6.$$

194,756.6 mi. *Ans.*

18. How many square feet of surface has a rectangular table that is 1.1^m long and 0.85^m wide?

The area of the table $= (1.1 \times 0.85)^{qm} = 1.1 \times 0.85 \times 1.1$ sq. yd.
$= 1.1 \times 0.85 \times 1.1 \times 9$ sq. ft.
$= 9.2565$ sq. ft. *Ans.*

$$\begin{array}{r} 1.1 \\ \underline{1.1} \\ 11 \\ \underline{11} \\ 1.21 \\ \underline{9} \\ 10.89 \end{array} \qquad \begin{array}{r} 10.89 \\ \underline{0.85} \\ 5445 \\ \underline{8712} \\ 9.2565 \end{array}$$

19. How many square meters of surface has a circular table that is $3\frac{1}{2}$ ft. in diameter?

Radius $= \frac{1}{2}$ of $3\frac{1}{2}$ ft. $= 1\frac{3}{4}$ ft.

The area of the table $= (\frac{22}{7} \times 1\frac{3}{4} \times 1\frac{3}{4})$ sq. ft.

$$= \frac{\frac{22}{7} \times 1\frac{3}{4} \times 1\frac{3}{4}}{9} \text{ sq. yd.} = \frac{\frac{22}{7} \times 1\frac{3}{4} \times 1\frac{3}{4}}{9} \times \frac{5^{qm}}{6}.$$

$$\frac{\frac{22}{7} \times 1\frac{3}{4} \times 1\frac{3}{4}}{9} \times \frac{5}{6} = \frac{\overset{11}{\cancel{22}}}{\cancel{7}} \times \frac{\cancel{7}}{\underset{2}{\cancel{4}}} \times \frac{7}{4} \times \frac{1}{9} \times \frac{5}{6} = \frac{385}{432} = 0.89. \qquad 0.89^{qm}. \ \textit{Ans.}$$

20. If sound travels 340^m a second, how many feet distant is a cannon from a man who hears the report 13 sec. after he sees the flash?

$13 \times 340^m = 13 \times 340 \times 1.1$ yd. $= 13 \times 340 \times 1.1 \times 3$ ft. $= 14,586$ ft. *Ans.*

$$\begin{array}{r} 340 \\ \underline{13} \\ 1020 \\ \underline{340} \\ 4420 \end{array} \qquad \begin{array}{r} 4420 \\ \underline{3.3} \\ 13260 \\ \underline{13260} \\ 14586 \end{array}$$

21. How many square meters of zinc will be required to line a rectangular cistern open at the top, 12 ft. long, 10 ft. wide, and 8 ft. deep?

The perimeter of the bottom $= 2 \times (12 + 10)$ ft. $= 44$ ft.

The area of the four sides $= (8 \times 44)$ sq. ft. $= 352$ sq. ft.

The area of the bottom $= (12 \times 10)$ sq. ft. $= 120$ sq. ft.

352 sq. ft. + 120 sq. ft. = 472 sq. ft. $= \frac{472}{9}$ sq. yd. $= \frac{472}{9} \times \frac{5}{6}^{\text{qm}}$.

$$\frac{\overset{236}{\cancel{472}}}{9} \times \frac{5}{\underset{3}{\cancel{6}}} = \frac{1180}{27} = 43.7. \qquad 43.7^{\text{qm}}.\ \textit{Ans.}$$

22. A rectangular tank is 3^{m} long, $2\frac{1}{2}^{\text{m}}$ wide, and $1\frac{1}{2}^{\text{m}}$ high, external measurement. If its sides are 0.1^{m} thick, how many gallons of water will the tank hold?

The internal measurements are: length $3^{\text{m}} - 2 \times 0.1^{\text{m}} = 2.8^{\text{m}}$; breadth $2.5^{\text{m}} - 2 \times 0.1^{\text{m}} = 2.3^{\text{m}}$; height $1.5^{\text{m}} - 0.1^{\text{m}} = 1.4^{\text{m}}$.

Volume $= (2.8 \times 2.3 \times 1.4)^{\text{cbm}} = 2.8 \times 2.3 \times 1.4 \times 1000^{\text{l}}$
$= 2.8 \times 2.3 \times 1.4 \times 1000 \times \frac{17}{16}$ qt. $= 2.8 \times 2.3 \times 1.4 \times 1000 \times \frac{17}{16} \times \frac{1}{4}$ gal.

$$2.8 \times 2.3 \times 1.4 \times 1000 \times \frac{17}{16} \times \frac{1}{4} = \overset{7}{\cancel{28}} \times 23 \times \overset{7}{\cancel{14}} \times \frac{17}{\underset{8}{\cancel{16}}} \times \frac{1}{\cancel{4}} = \frac{19159}{8} = 2394\tfrac{7}{8}.$$

$2394\frac{7}{8}$ gal. *Ans.*

23. If a cube of pine wood 11.2^{cm} on an edge weighs 2 lb., what is the specific gravity of the pine?

$$2 \text{ lb.} = 2 \times \tfrac{5}{11}^{\text{kg}} = \tfrac{10}{11}^{\text{kg}}.$$

$$\text{Volume} = (1.12 \times 1.12 \times 1.12)^{\text{cdm}}.$$

$$(1.12 \times 1.12 \times 1.12)^{\text{cdm}} \text{ of water weighs } (1.12 \times 1.12 \times 1.12)^{\text{kg}}.$$

$$\frac{\frac{10}{11}}{1.12 \times 1.12 \times 1.12} = \frac{\overset{5}{\cancel{10}}}{11} \times \frac{\overset{25}{\cancel{100}}}{\underset{\underset{14}{\cancel{28}}}{\cancel{112}}} \times \frac{\overset{25}{\cancel{100}}}{\underset{28}{\cancel{112}}} \times \frac{\overset{25}{\cancel{100}}}{\underset{28}{\cancel{112}}} = \frac{78125}{120736} = 0.647. \quad \textit{Ans.}$$

24. Find in kilograms the weight of water a cubical cistern will hold, 6 ft. on an edge.

The weight of the water is $6 \times 6 \times 6 \times 62\frac{1}{2}$ lb. $= 6 \times 6 \times 6 \times 62\frac{1}{2} \times \frac{5}{11}^{\text{kg}}$.

$$6 \times 6 \times \overset{3}{\cancel{6}} \times \frac{125}{\cancel{2}} \times \frac{5}{11} = \frac{67500}{11} = 6136.4. \qquad 6136.4^{\text{kg}}.\ \textit{Ans.}$$

[redacted]

25. Rain has fallen to the depth of half an inch. How many cubic meters of water has fallen on an acre of land?

$$1 \text{ A.} = 43{,}560 \text{ sq. ft.}; \ \tfrac{1}{2} \text{ in.} = \tfrac{1}{24} \text{ ft.}$$

$$\text{Volume of water} = (43{,}560 \times \tfrac{1}{24}) \text{ cu. ft.} = \frac{43560 \times \frac{1}{24}}{27} \text{ cu. yd.}$$

$$= \frac{43560 \times \frac{1}{24}}{27} \times \frac{10^{\text{cbm}}}{13}.$$

$$\overset{605}{\cancel{\overset{1815}{\cancel{43560}}}} \times \frac{1}{\cancel{24}} \times \frac{1}{\underset{9}{\cancel{27}}} \times \frac{10}{13} = \frac{6050}{117} = 51.7. \qquad 51.7^{\text{cbm}}. \ \textit{Ans.}$$

26. How many centimeters will the water sink in a cylindrical cistern 7 ft. in diameter, if 310 gallons of water is pumped out?

The radius $= \frac{1}{2}$ of 84 in. $= 42$ in.

Volume of the water $= 310 \times 231$ cu. in.

Area of bottom of cistern $= (\frac{22}{7} \times 42 \times 42)$ sq. in.

Therefore, the water will sink $\dfrac{310 \times 231}{\frac{22}{7} \times 42 \times 42}$ in. $= \dfrac{310 \times 231}{\frac{22}{7} \times 42 \times 42} \times 2\frac{11}{20}^{\text{cm}}$.

$$\overset{31}{\cancel{310}} \times \overset{21}{\cancel{231}} \times \frac{\cancel{7}}{\underset{2}{\cancel{22}}} \times \frac{1}{\underset{\underset{2}{\cancel{6}}}{\cancel{42}}} \times \frac{1}{\underset{2}{\cancel{42}}} \times \frac{\overset{17}{\cancel{51}}}{\underset{2}{\cancel{20}}} = \frac{527}{16} = 33. \qquad 33^{\text{cm}}. \ \textit{Ans.}$$

27. How many square yards of tin are required to cover the roof of a hemispherical dome 12^{m} in diameter?

$$\text{Area} = \tfrac{1}{2} \text{ of } (\tfrac{22}{7} \times 12 \times 12)^{\text{qm}} = \tfrac{1}{2} \times \tfrac{22}{7} \times 12 \times 12 \times \tfrac{6}{5} \text{ sq. yd.}$$

$$\frac{1}{\cancel{2}} \times \frac{\overset{11}{\cancel{22}}}{7} \times 12 \times 12 \times \frac{6}{5} = \frac{9504}{35} = 271.5. \qquad 271.5 \text{ sq. yd. } \textit{Ans.}$$

28. If a cubic inch of iron weighs $4\frac{1}{2}$ oz., what is the weight in kilograms of an iron ball 10^{cm} in diameter?

1 cu. in. weighs $4\frac{1}{2}$ oz.; that is, 16^{ccm} weighs $\dfrac{4\frac{1}{2}}{16}$ lb., or $\dfrac{4\frac{1}{2}}{16} \times \dfrac{5}{11}^{\text{kg}}$.

Therefore, 1^{ccm} weighs $\dfrac{1}{16} \times \dfrac{4\frac{1}{2}}{16} \times \dfrac{5}{11}^{\text{kg}}$.

Volume $= (\frac{1}{6} \times \frac{22}{7} \times 10 \times 10 \times 10)^{\text{ccm}}$.

Therefore, the iron ball weighs $\dfrac{1}{6} \times \dfrac{22}{7} \times 10 \times 10 \times 10 \times \dfrac{1}{16} \times \dfrac{4\frac{1}{2}}{16} \times \dfrac{5}{11}^{\text{kg}}$.

$$\frac{1}{\underset{2}{\cancel{6}}} \times \frac{\overset{2}{\cancel{22}}}{7} \times \overset{5}{\cancel{10}} \times \overset{5}{\cancel{10}} \times \overset{5}{\cancel{10}} \times \frac{1}{\underset{\underset{4}{\cancel{8}}}{\cancel{16}}} \times \frac{\overset{3}{\cancel{9}}}{\cancel{2}} \times \frac{1}{\underset{8}{\cancel{16}}} \times \frac{5}{\cancel{11}} = \frac{1875}{448} = 4.2.$$

4.2^{kg}. *Ans.*

29. If a cubic inch of lead weighs 7 oz., what is the weight in kilograms of a lead pipe 3^{m} long, 6^{cm} in external diameter, if the pipe is 1^{cm} thick?

1 cu. in. weighs 7 oz.; that is, 16^{ccm} weighs $\frac{7}{16}$ lb., or $\frac{7}{16} \times \frac{5}{11}^{kg}$.
Therefore, 1^{ccm} weighs $\frac{1}{16} \times \frac{7}{16} \times \frac{5}{11}^{kg}$.
External contents $= (300 \times \frac{22}{7} \times 3 \times 3)^{ccm}$.
Internal contents $= (300 \times \frac{22}{7} \times 2 \times 2)^{ccm}$.

$$300 \times \frac{22}{7} \times 3 \times 3 = \frac{59400}{7};\quad 300 \times \frac{22}{7} \times 2 \times 2 = \frac{26400}{7}.$$

$$\frac{59400^{ccm}}{7} - \frac{26400^{ccm}}{7} = \frac{33000^{ccm}}{7}.$$

$$\frac{33000^{ccm}}{7} \text{ weighs } \frac{\overset{\overset{375}{\cancel{3000}}}{\cancel{33000}}}{\cancel{7}} \times \frac{1}{16} \times \frac{\cancel{7}}{\underset{2}{\cancel{16}}} \times \frac{5}{\cancel{11}}^{kg} = \frac{1875}{32}^{kg} = 58.6^{kg}.\ \textit{Ans.}$$

30. Find the cost at \$7.25 per meter of building a wall around a rectangular garden 90 ft. long and 55 ft. wide.

Length of wall $= 2 \times (90 + 55)$ ft. $= 290$ ft. $= \frac{290}{3}$ yd. $= \frac{290}{3} \times \frac{9}{10}^{m}$.

$$\frac{290}{3} \times \frac{9}{10} \times \$7\tfrac{1}{4} = \frac{\overset{29}{\cancel{290}}}{\cancel{3}} \times \frac{\overset{3}{\cancel{9}}}{\cancel{10}} \times \$\frac{29}{4} = \$\frac{2523}{4} = \$630.75.\ \textit{Ans.}$$

31. The minute hand of a clock is 0.5^{m} long. How many feet does its point move in an hour?

The point moves $\frac{22}{7} \times 2 \times 0.5^{m} = \frac{22}{7} \times 2 \times \frac{1}{2} \times \frac{11}{10}$ yd.

$$= \frac{\overset{11}{\cancel{22}}}{7} \times \cancel{2} \times \frac{1}{\cancel{2}} \times \frac{11}{\underset{5}{\cancel{10}}} \times 3 \text{ ft.} = \frac{363}{35} \text{ ft.} = 10.4 \text{ ft.}\ \textit{Ans.}$$

32. A spherical shot 3 in. in diameter is melted and then cast into a cylinder 9^{cm} in diameter. What is the height in centimeters of this cylinder?

Volume of shot $= (\frac{1}{6} \times \frac{22}{7} \times 3^{3})$ cu. in. $= \frac{1}{6} \times \frac{22}{7} \times 27 \times 16^{ccm}$.
Area of base of cylinder $= (\frac{22}{7} \times \frac{9}{2} \times \frac{9}{2})^{qcm}$.

Therefore, height of cylinder $= \left(\frac{\frac{1}{6} \times \frac{22}{7} \times 27 \times 16}{\frac{22}{7} \times \frac{9}{2} \times \frac{9}{2}}\right)^{cm}$.

$$\frac{1}{\underset{2}{\cancel{6}}} \times \frac{\cancel{22}}{\cancel{7}} \times \overset{3}{\cancel{27}} \times 16 \times \frac{\cancel{7}}{\cancel{22}} \times \frac{2}{\cancel{9}} \times \frac{2}{9} = \frac{32}{9} = 3\tfrac{5}{9}. \qquad 3\tfrac{5}{9}^{cm}.\ \textit{Ans.}$$

[redacted]

33. What is the cost at \$18 per 1000 ft. board measure of 4 beams, each 4.5^{m} long, 7.5^{cm} wide, and 5^{cm} thick?

$4.5^{m} = 4.5 \times 1.1$ yd. $= 4.5 \times 1.1 \times 3$ ft.

$7.5^{cm} = 0.075^{m} = 0.075 \times 1.1$ yd. $= 0.075 \times 1.1 \times 3$ ft.

$5^{cm} = 0.05^{m} = 0.05 \times 1.1$ yd. $= 0.05 \times 1.1 \times 36$ in.

The number of feet board measure in the 4 beams

$= 4 \times 4.5 \times 1.1 \times 3 \times 0.075 \times 1.1 \times 3 \times 0.05 \times 1.1 \times 36$

$$= \cancel{4} \times \frac{9}{2} \times \frac{11}{10} \times 3 \times \frac{\overset{3}{\cancel{75}}}{\underset{10}{\underset{\cancel{40}}{\cancel{1000}}}} \times \frac{11}{10} \times 3 \times \frac{\cancel{5}}{\underset{5}{\underset{\cancel{20}}{\cancel{100}}}} \times \frac{11}{10} \times \overset{9}{\cancel{36}} = \frac{2910897}{100000} = 29.1.$$

$$29.1 \times \$\frac{18}{1000} = \$\frac{523.8}{1000} = \$0.52. \quad \textit{Ans.}$$

34. The radius of a cylindrical roller is 0.4^{m} and its length is 2.15^{m}. Find its volume in cubic feet.

$$\begin{aligned} \text{Volume} &= (2.15 \times \tfrac{22}{7} \times 0.4 \times 0.4)^{cbm} \\ &= (2.15 \times \tfrac{22}{7} \times \tfrac{2}{5} \times \tfrac{2}{5} \times \tfrac{13}{10}) \text{ cu. yd.} \\ &= (2.15 \times \tfrac{22}{7} \times \tfrac{2}{5} \times \tfrac{2}{5} \times \tfrac{13}{10} \times 27) \text{ cu. ft.} \\ &= 37.95 \text{ cu. ft. } \textit{Ans.} \end{aligned}$$

$$\frac{\overset{43}{\cancel{215}}}{\underset{25}{\underset{\cancel{50}}{\cancel{100}}}} \times \frac{\overset{11}{\cancel{22}}}{7} \times \frac{2}{\cancel{5}} \times \frac{2}{5} \times \frac{13}{\underset{5}{\cancel{10}}} \times 27 = \frac{166023}{4375} = 37.95.$$

35. A cylindrical cistern, the circumference of whose base is 2.2^{m}, and whose depth is 2.1^{m}, is four fifths filled with water. Find in gallons the volume of the water, and in pounds the weight of the water.

$$\text{Radius of base} = \frac{2.2^{m}}{2 \times \frac{22}{7}} = \frac{7 \times 2.2^{m}}{2 \times 22} = \frac{7^{m}}{20}.$$

$$\text{Volume of the water} = \left(\frac{4}{5} \times 2.1 \times \frac{22}{7} \times \frac{7}{20} \times \frac{7}{20}\right)^{cbm} = 0.6468^{cbm}.$$

$$\frac{4}{5} \times \frac{21}{10} \times \frac{\overset{11}{\cancel{22}}}{\cancel{7}} \times \frac{\cancel{7}}{20} \times \frac{7}{\underset{10}{\cancel{20}}} = \frac{6468}{10000} = 0.6468.$$

0.6468^{cbm} of water weighs $646.8^{kg} = 646.8 \times 2.2$ lb. $= 1422.96$ lb. *Ans.*

[redacted]

$$\begin{array}{r} 646.8 \\ 2.\dot{2} \\ \hline 12936 \\ 12936 \\ \hline 1422.96 \end{array}$$

$6468^{cbm} = 646.8^{l} = 646.8 \times \frac{17}{16}$ qt. $= 646.8 \times \frac{17}{16} \times \frac{1}{4}$ gal. $= 171.8$ gal. *Ans.*

$$\frac{\overset{1617}{\cancel{6468}}}{10} \times \frac{17}{16} \times \frac{1}{\cancel{4}} = \frac{27489}{160} = 171.8.$$

Exercise 99. Page 218.

1. Which is the greater ratio, $5:8$ or $6:9$?

$5:8 = \frac{5}{8} = \frac{15}{24}$.
$6:9 = \frac{6}{9} = \frac{2}{3} = \frac{16}{24}$.
$\therefore$ $6:9$ is the greater.

2. Which is the greater ratio, $7:10$ or $9:12$?

$7:10 = \frac{7}{10} = \frac{14}{20}$.
$9:12 = \frac{9}{12} = \frac{3}{4} = \frac{15}{20}$.
$\therefore$ $9:12$ is the greater.

3. Which is the greater ratio, $8:9$ or $10:12$?

$8:9 = \frac{8}{9} = \frac{16}{18}$.
$10:12 = \frac{10}{12} = \frac{5}{6} = \frac{15}{18}$.
$\therefore$ $8:9$ is the greater.

4. Which is the greater ratio, $6:12$ or $8:14$?

$6:12 = \frac{6}{12} = \frac{1}{2} = \frac{7}{14}$.
$8:14 = \frac{8}{14}$.
$\therefore$ $8:14$ is the greater.

5. Which is the greater ratio, 10 cwt. : 15 cwt. or \$7 : \$9?

$10 \text{ cwt.} : 15 \text{ cwt.} = \dfrac{10 \text{ cwt.}}{15 \text{ cwt.}} = \frac{2}{3} = \frac{6}{9}$.

$\$7 : \$9 = \dfrac{\$7}{\$9} = \frac{7}{9}$.

$\therefore$ \$7 : \$9 is the greater.

6. Which is the greater ratio, 5 dy. : 7 dy. or 8 ft. : 11 ft.?

$5 \text{ dy.} : 7 \text{ dy.} = \dfrac{5 \text{ dy.}}{7 \text{ dy.}} = \frac{5}{7} = \frac{55}{77}$.

$8 \text{ ft.} : 11 \text{ ft.} = \dfrac{8 \text{ ft.}}{11 \text{ ft.}} = \frac{8}{11} = \frac{56}{77}$.

$\therefore$ 8 ft. : 11 ft. is the greater.

7. Which is the greater ratio, 9 yd. : 6 yd. or $5:3$?

$9 \text{ yd.} : 6 \text{ yd.} = \dfrac{9 \text{ yd.}}{6 \text{ yd.}} = \frac{3}{2}$.

$5:3 = \frac{5}{3} = \frac{10}{6}$.

$\therefore$ $5:3$ is the greater.

8. Which is the greater ratio, $\frac{2}{3}$ lb. : $\frac{1}{2}$ lb. or $\frac{5}{8}$ yd. : $\frac{3}{8}$ yd.?

$\frac{2}{3} \text{ lb.} : \frac{1}{2} \text{ lb.} = \dfrac{\frac{2}{3} \text{ lb.}}{\frac{1}{2} \text{ lb.}} = \frac{4}{3}$. $\frac{5}{8} \text{ yd.} : \frac{3}{8} \text{ yd.} = \dfrac{\frac{5}{8} \text{ yd.}}{\frac{3}{8} \text{ yd.}} = \frac{5}{3}$.

$\therefore$ $\frac{5}{8}$ yd. : $\frac{3}{8}$ yd. is the greater.

[redacted]

9. Find the ratio of 3 dry quarts to 2 pecks.

$\therefore$ 3 dry qt. : 2 pk. = 3 qt. : 16 qt. = 3 : 16. *Ans.*

10. Find the ratio of 2500 lb. to 1 ton.

$\therefore$ 2500 lb. : 1 t. = 2500 lb. : 2000 lb. = 5 : 4. *Ans.*

11. Find the ratio of a rectangular field 16 rd. long, 14 rd. wide to a rectangular field 14 rd. long, 12 rd. wide.

The ratio of the fields = 16 × 14 : 14 × 12.

$$=\frac{\overset{4}{\cancel{16}}\times\cancel{14}}{\cancel{14}\times\underset{3}{\cancel{12}}}=\frac{4}{3}=4:3.\quad \textit{Ans.}$$

12. Find the ratio of a circle 1 in. in diameter to a circle 1 in. in radius.

The ratio of the circles $=\frac{1}{4}\times 3.1416\times 1^2 : 3.1416\times 1^2$

$$=\frac{\frac{1}{4}\times 3.1416\times 1^2}{3.1416\times 1^2}=\frac{1}{4}=1:4.\quad \textit{Ans.}$$

Exercise 100. Page 220.

1. Find the missing term of 24 : 18 : : 16 : ?.

$$\frac{\overset{3}{\cancel{18}}\times\overset{4}{\cancel{16}}}{\underset{\cancel{4}}{\cancel{24}}}=12.\quad \textit{Ans.}$$

2. Find the missing term of 35 : ? : : 15 : 21.

$$\frac{\overset{7}{\cancel{35}}\times\overset{7}{\cancel{21}}}{\underset{\cancel{3}}{\cancel{15}}}=49.\quad \textit{Ans.}$$

3. Find the missing term of 45 : 40 : : ? : 32.

$$\frac{\overset{9}{\cancel{45}}\times\overset{4}{\cancel{32}}}{\underset{\cancel{8}}{\cancel{40}}}=36.\quad \textit{Ans.}$$

4. Find the missing term of 30 : 27 : : 40 : ?.

$$\frac{\overset{9}{\cancel{27}}\times\overset{4}{\cancel{40}}}{\underset{\cancel{3}}{\cancel{30}}}=36.\quad \textit{Ans.}$$

5. Find the missing term of ? : 36 : : 4 : 3.

$$\frac{\overset{12}{\cancel{36}}\times 4}{\cancel{3}}=48.\quad \textit{Ans.}$$

6. Find the missing term of 18 : ? : : 32 : 45.

$$\frac{\overset{9}{\cancel{18}}\times 45}{\underset{16}{\cancel{32}}}=\frac{405}{16}=25\tfrac{5}{16}.\quad \textit{Ans.}$$

7. Find the missing term of

? : 12 : : 5 : 18.

$$\frac{\overset{2}{\cancel{12}} \times 5}{\underset{3}{\cancel{18}}} = \frac{10}{3} = 3\frac{1}{3}. \text{ } \textit{Ans.}$$

8. Find the missing term of

8 : 17 : : ? : 119.

$$\frac{8 \times \overset{7}{\cancel{119}}}{\cancel{17}} = 56. \text{ } \textit{Ans.}$$

9. Find the missing term of

9 : 16 : : 12 : ?.

$$\frac{16 \times \overset{4}{\cancel{12}}}{\underset{3}{\cancel{9}}} = \frac{64}{3} = 21\frac{1}{3}. \text{ } \textit{Ans.}$$

10. Find the missing term of

17 : 3 : : ? : 12.

$$\frac{17 \times \overset{4}{\cancel{12}}}{\cancel{3}} = 68. \text{ } \textit{Ans.}$$

Exercise 101. Page 221.

1. If 24 men can do a piece of work in 14 days, how long will it take 21 men to do it?

21 : 24 : : 14 dy. : what?

$$\frac{\overset{8}{\cancel{24}} \times \overset{2}{\cancel{14}} \text{ dy.}}{\underset{\cancel{3}}{\cancel{21}}} = 16 \text{ dy. } \textit{Ans.}$$

2. A well is dug in 13 days of 9 hours each. How many days of 10 hours each would it have taken?

10 : 9 : : 13 dy. : what?

$$\frac{9 \times 13 \text{ dy.}}{10} = \frac{117}{10} \text{ dy.} = 11\tfrac{7}{10} \text{ dy. } \textit{Ans.}$$

3. A man who steps 2 ft. 5 in. takes 2480 steps in walking a certain distance. How many steps of 2 ft. 7 in. will be required for the same distance?

2 ft. 5 in. = 29 in. 2 ft. 7 in. = 31 in.

31 : 29 : : 2480 : what?

$$\frac{29 \times \overset{80}{\cancel{2480}}}{\cancel{31}} = 2320. \text{ } \textit{Ans.}$$

[redacted]

4. If $\frac{5}{13}$ of a ton of hay costs \$6, what will $7\frac{5}{9}$ cwt. cost, at the same rate?

$$7\tfrac{5}{9}\text{ cwt.} = \frac{7\frac{5}{9}}{20}\text{ t.} = \tfrac{17}{45}\text{ t.}$$

$$\tfrac{5}{13} : \tfrac{17}{45} :: \$6 : \text{what?}$$

$$\frac{\frac{17}{45} \times \$6}{\frac{5}{13}} = \frac{17 \times 13 \times \overset{2}{\cancel{\$6}}}{\underset{15}{\cancel{45}} \times 5} = \$\frac{442}{75} = \$5.89. \quad \textit{Ans.}$$

5. If 42 yd. of carpet 2 ft. 3 in. wide are required for a room, how many yards of carpet 2 ft. 4 in. wide will be required?

$$2\text{ ft. }3\text{ in.} = 27\text{ in.} \qquad 2\text{ ft. }4\text{ in.} = 28\text{ in.}$$

$$28 : 27 :: 42\text{ yd.} : \text{what?}$$

$$\frac{27 \times \overset{3}{\cancel{42}}\text{ yd.}}{\underset{2}{\cancel{28}}} = \frac{81}{2}\text{ yd.} = 40\tfrac{1}{2}\text{ yd.} \quad \textit{Ans.}$$

6. A court was paved with 950 stones, each containing $1\frac{5}{6}$ sq. ft., and is repaved with 836 stones of a uniform size. Find the surface of each.

$$836 : 950 :: 1\tfrac{5}{6}\text{ sq. ft.} : \text{what?}$$

$$\frac{950 \times 1\frac{5}{6}\text{ sq. ft.}}{836} = \frac{\overset{25}{\cancel{950}} \times \cancel{11}}{\underset{\underset{2}{\cancel{22}}}{\cancel{836}} \times 6}\text{ sq. ft.} = 2\tfrac{1}{12}\text{ sq. ft.} \quad \textit{Ans.}$$

7. If a train, at the rate of $\frac{5}{13}$ of a mile per minute, requires $3\frac{1}{4}$ hours to make a certain distance, how long will it require at the rate of $\frac{7}{15}$ of a mile a minute?

$$\tfrac{7}{15} : \tfrac{5}{13} :: 3\tfrac{1}{4}\text{ hr.} : \text{what?}$$

$$\frac{\frac{5}{13} \times 3\frac{1}{4}\text{ hr.}}{\frac{7}{15}} = \frac{15}{7} \times \frac{5}{\cancel{13}} \times \frac{\cancel{13}}{4}\text{ hr.} = \frac{75}{28}\text{ hr.} = 2\tfrac{19}{28}\text{ hr.} \quad \textit{Ans.}$$

8. When a post 4 ft. 8 in. high casts a shadow 7 ft. 3 in. long, how long a shadow will a post 11 ft. high cast?

$$4\text{ ft. }8\text{ in.} = 4\tfrac{2}{3}\text{ ft.} \qquad 7\text{ ft. }3\text{ in.} = 7\tfrac{1}{4}\text{ ft.}$$

$$4\tfrac{2}{3} : 11 :: 7\tfrac{1}{4}\text{ ft.} : \text{what?}$$

$$\frac{11 \times 7\frac{1}{4}\text{ ft.}}{4\frac{2}{3}} = \frac{3 \times 11 \times 29}{14 \times 4}\text{ ft.} = 17\tfrac{5}{56}\text{ ft.} = 17\text{ ft. }1\tfrac{1}{14}\text{ in.} \quad \textit{Ans.}$$

9. When a post 5 ft. 7 in. high casts a shadow 8 ft. 5 in. long, how high is a steeple that casts a shadow of 202 ft.?

$$8\tfrac{5}{12} : 202 :: 5\tfrac{7}{12}\text{ ft.} : \text{what?} \qquad \frac{202 \times 5\frac{7}{12}\text{ ft.}}{8\frac{5}{12}} = \frac{\cancel{12} \times \overset{2}{\cancel{202}} \times 67}{\cancel{101} \times \cancel{12}}\text{ ft.}$$
$$= 134\text{ ft. } \textit{Ans.}$$

10. If 4 men can mow a certain field in 10 hours, how many men will it take to mow it in 5 hours?

$$5 : 10 :: 4\text{ men} : \text{what?} \qquad \frac{\overset{2}{\cancel{10}} \times 4\text{ men}}{\cancel{5}} = 8\text{ men. } \textit{Ans.}$$

11. If a tap discharging 4 gal. a minute empties a cistern in 3 hours, how long will it take a tap discharging 7 gal. a minute to empty it?

$$7 : 4 :: 3\text{ hr.} : \text{what?} \qquad \frac{4 \times 3\text{ hr.}}{7} = 1\tfrac{5}{7}\text{ hr. } \textit{Ans.}$$

12. If a pipe discharging 3 gal. 1 pt. a minute fills a tub in 4 min. 20 sec., how long will it take a pipe discharging 83 qt. a minute to fill it?

$$3\text{ gal. }1\text{ pt.} = 25\text{ pt.} \qquad 83\text{ qt.} = 166\text{ pt.}$$

$$4\text{ min. }20\text{ sec.} = 260\text{ sec.}$$

$$166 : 25 :: 260\text{ sec.} : \text{what?} \qquad \frac{25 \times \overset{130}{\cancel{260}}\text{ sec.}}{\underset{83}{\cancel{166}}} = 39\tfrac{13}{83}\text{ sec. } \textit{Ans.}$$

13. If both pipes of Ex. 12 discharge at the same time into the tub, how long will it take to fill it?

$$25\text{ pt.} + 166\text{ pt.} = 191\text{ pt.} \qquad 191 : 25 :: 260\text{ sec.} : \text{what?}$$

$$\frac{25 \times 260\text{ sec.}}{191} = \frac{6500}{191}\text{ sec.} = 34\tfrac{6}{191}\text{ sec. } \textit{Ans.}$$

14. How long will it take to fill a cistern of 165 gal. by a pipe that fills one of 120 gal. in 7 min. 16 sec.?

$$16\text{ sec.} = \tfrac{4}{15}\text{ min.} \qquad 120 : 165 :: 7\tfrac{4}{15}\text{ min.} : \text{what?}$$

$$\frac{165 \times 7\frac{4}{15}\text{ min.}}{120} = \frac{\overset{11}{\cancel{165}} \times 109}{120 \times \cancel{15}}\text{ min.} = 9\tfrac{119}{120}\text{ min.} = 9\text{ min. }59\tfrac{1}{2}\text{ sec. } \textit{Ans.}$$

[redacted]

15. If a ship sails 1800 mi. in a fortnight, how long will it take to make a voyage of 5000 mi.?

$$1800 : 5000 : : 2 \text{ wk.} : \text{what?}$$

$$\frac{\overset{25}{\cancel{5000}} \times 2 \text{ wk.}}{\underset{9}{\cancel{1800}}} = \frac{50}{9} \text{ wk.} = 5\tfrac{5}{9} \text{ wk.}$$

$5\tfrac{5}{9}$ wk. = 5 wk. 4 dy. nearly. *Ans.*

16. The wheels of a carriage are 6 ft. 9 in. and 9 ft. 6 in., respectively, in circumference. How many times will the larger turn while the smaller turns 3762 times?

$$6 \text{ ft. } 9 \text{ in.} = 6\tfrac{3}{4} \text{ ft.} \qquad 9 \text{ ft. } 6 \text{ in.} = 9\tfrac{1}{2} \text{ ft.}$$

$$9\tfrac{1}{2} : 6\tfrac{3}{4} : : 3762 : \text{what?}$$

$$\frac{6\tfrac{3}{4} \times 3762}{9\tfrac{1}{2}} = \frac{\cancel{2} \times 27 \times \overset{\overset{99}{\cancel{198}}}{\cancel{3762}}}{\cancel{19} \times \underset{\cancel{2}}{\cancel{4}}} = 2673. \text{ } \textit{Ans.}$$

17. If $\frac{3}{25}$ of a ship is worth \$2167, what is $\frac{7}{17}$ of it worth?

$$\tfrac{3}{25} : \tfrac{7}{17} : : \$2167 : \text{what?}$$

$$\frac{\frac{7}{17} \times \$2167}{\frac{3}{25}} = \frac{25 \times 7 \times \$2167}{3 \times 17} = \$\frac{379225}{51} = \$7435.78. \text{ } \textit{Ans.}$$

18. What is the weight of 18 cu. ft. 432 cu. in. of stone, if 10 cu. ft. 864 cu. in. of the stone weighs 14 cwt. 7 lb.?

$$10\tfrac{1}{2} : 18\tfrac{1}{4} : : 1407 \text{ lb.} : \text{what?}$$

$$\frac{18\tfrac{1}{4} \times 1407 \text{ lb.}}{10\tfrac{1}{2}} = \frac{2 \times 73 \times \overset{67}{\cancel{1407}} \text{ lb.}}{\cancel{21} \times \underset{2}{\cancel{4}}} = \frac{4891}{2} \text{ lb.} = 2445\tfrac{1}{2} \text{ lb.}$$

$= 1$ t. 4 cwt. $45\tfrac{1}{2}$ lb. *Ans.*

19. If 280 lb. of flour makes 360 lb. of bread, how many four-pound loaves can be made from 1 cwt. of flour?

$$280 : 100 : : 360 \text{ lb.} : \text{what?}$$

$$\frac{100 \times \overset{9}{\cancel{360}} \text{ lb.}}{\underset{7}{\cancel{280}}} = \frac{900}{7} \text{ lb.} = 128\tfrac{4}{7} \text{ lb.} \qquad 128\tfrac{4}{7} \div 4 = 32\tfrac{1}{7}. \text{ } \textit{Ans.}$$

20. If a column of mercury 27.93 in. high weighs 0.76 of a pound, what is the weight of a column of mercury of the same diameter 29.4 in. high?

$$27.93 : 29.4 :: 0.76 \text{ lb.} : \text{what?}$$

$$\frac{\overset{0.2}{\cancel{29.4}} \times \overset{4}{\cancel{0.76}} \text{ lb.}}{\underset{\cancel{0.19}}{\cancel{27.93}}} = 0.8 \text{ lb. } \textit{Ans.}$$

21. How many francs will pay a bill of £100, when £42 10s. 8d. is equivalent to 1090.98 francs?

$$£42\ 10s.\ 8d. = £42\tfrac{8}{15}.$$

$$42\tfrac{8}{15} : 100 :: 1090.98 \text{ fr.} : \text{what?}$$

$$\frac{100 \times 1090.98 \text{ fr.}}{42\frac{8}{15}} = \frac{15}{\cancel{638}} \times \overset{171}{\cancel{109098}} \text{ fr.} = 2565 \text{ fr. } \textit{Ans.}$$

22. What is the weight of a cube of stone 2 ft. 2 in. on an edge, if a cube 1 ft. 4 in. on an edge weighs 537.6 lb.?

$$2 \text{ ft. } 2 \text{ in.} = 2\tfrac{1}{6} \text{ ft.} \qquad 1 \text{ ft. } 4 \text{ in.} = 1\tfrac{1}{3} \text{ ft.}$$

$$(1\tfrac{1}{3})^3 : (2\tfrac{1}{6})^3 :: 537.6 \text{ lb.} : \text{what?}$$

$$\tfrac{64}{27} : \tfrac{2197}{216} :: 537.6 \text{ lb.} : \text{what?}$$

$$\frac{\cancel{27}}{\underset{2}{\cancel{64}}} \times \frac{2197}{\underset{\cancel{8}}{\cancel{216}}} \times \frac{\overset{\overset{21}{\cancel{672}}}{\cancel{5376}}}{10} \text{ lb.} = \frac{46137}{20} \text{ lb.} = 2306.85 \text{ lb. } \textit{Ans.}$$

23. If a square field 50 yd. $10\frac{2}{7}$ in. on a side is worth $\$2710\frac{10}{17}$, what is a square field 62 yd. 1 ft. on a side worth?

$$50 \text{ yd. } 10\tfrac{2}{7} \text{ in.} = 50\tfrac{2}{7} \text{ yd.} \qquad 62 \text{ yd. } 1 \text{ ft.} = 62\tfrac{1}{3} \text{ yd.}$$

$$(50\tfrac{2}{7})^2 : (62\tfrac{1}{3})^2 :: \$2710\tfrac{10}{17} : \text{what?}$$

$$\frac{123904}{49} : \frac{34969}{9} :: \$\frac{46080}{17} : \text{what?}$$

$$\frac{49}{\underset{\cancel{121}}{\cancel{123904}}} \times \frac{\overset{\overset{17}{\cancel{2057}}}{\cancel{34969}}}{\cancel{9}} \times \$\frac{\overset{\overset{5}{\cancel{5120}}}{\cancel{46080}}}{\cancel{17}} = \$4165. \textit{ Ans.}$$

[redacted]

24. A gains 4 yd. on B in running 30 yd. How many yards will he gain while B is running $97\frac{1}{2}$ yd.?

B runs 26 yd. while A is running 30 yd.

$$26 : 97\tfrac{1}{2} : : 4 \text{ yd.} : \text{what?}$$

$$\frac{\overset{15}{\cancel{195}} \times \overset{2}{\cancel{4}} \text{ yd.}}{\cancel{2} \times \underset{2}{\cancel{26}}} = 15 \text{ yd. } \textit{Ans.}$$

25. If 10 cu. in. of gold weighs as much as 193 cu. in. of water, how many cubic inches are there in a nugget of gold that weighs as much as a cubic foot of water?

$$193 : 1728 : : 10 \text{ cu. in.} : \text{what?}$$

$$\frac{1728 \times 10 \text{ cu. in.}}{193} = \frac{17280}{193} \text{ cu. in.} = 89\tfrac{103}{193} \text{ cu. in. } \textit{Ans.}$$

26. If a garrison of 1500 men has provisions for 13 months, how long will the provisions last if the garrison is reënforced by 700 men?

$$1500 + 700 = 2200.$$

$$2200 : 1500 : : 13 \text{ mo.} : \text{what?}$$

$$\frac{\overset{15}{\cancel{1500}} \times 13 \text{ mo.}}{\underset{22}{\cancel{2200}}} = \frac{195}{22} \text{ mo.} = 8\tfrac{19}{22} \text{ mo.} = 8 \text{ mo. } 26 \text{ dy. } \textit{Ans.}$$

27. If a tree 38 ft. high is represented by a drawing $1\frac{1}{2}$ in. high, what height on the same scale will represent a house 45 ft. high?

$$38 : 45 : : 1\tfrac{1}{2} \text{ in.} : \text{what?}$$

$$\frac{45 \times 3}{38 \times 2} \text{ in.} = \frac{135}{76} \text{ in.} = 1\tfrac{59}{76} \text{ in. } \textit{Ans.}$$

28. If a country 630 mi. long is represented on a raised map by a length of $5\frac{1}{2}$ ft., by what height ought a mountain of 15,750 ft. to be represented on the map?

630 mi. = 3,326,400 ft. $\qquad$ $5\frac{1}{2}$ ft. = 66 in.

$$3326400 : 15750 : : 66 \text{ in.} : \text{what?}$$

$$\frac{\overset{5}{\cancel{15750}} \times \cancel{66} \text{ in.}}{\underset{\underset{16}{\cancel{50400}}}{\cancel{3326400}}} = \frac{5}{16} \text{ in. } \textit{Ans.}$$

29. A train travels $\frac{1}{4}$ of a mile in 18 sec. How many miles an hour does it travel?

1 hr. = 3600 sec. 18 : 3600 : : $\frac{1}{4}$ mi. : what?

$$\frac{\overset{200}{\cancel{3600}} \times \frac{1}{4}\text{ mi.}}{\cancel{18}} = 50\text{ mi. } \textit{Ans.}$$

30. If $4\frac{1}{2}$ t. of coal fill a bin 9 ft. long, 5 ft. broad, 5 ft. high, how many cubic feet are required for the coal of a steamer that carries coal for 3 wk. at 20 t. a day?

$9 \times 5 \times 5 = 225.$ 3 wk. = 21 dy.

21×20 t. = 420 t. $4\frac{1}{2}$: 420 : : 225 cu. ft. : what?

$$\frac{2 \times 420 \times \overset{25}{\cancel{225}}\text{ cu. ft.}}{\cancel{9}} = 21{,}000\text{ cu. ft. } \textit{Ans.}$$

31. If 2 lb. of rosin are melted with 5 oz. of mutton tallow, to make a grafting wax, how many ounces of tallow will 20 oz. of the wax contain?

2 lb. + 5 oz. = 2 lb. 5 oz. = 37 oz.

37 : 20 : : 5 oz. : what?

$$\frac{20 \times 5\text{ oz.}}{37} = \frac{100}{37}\text{ oz.} = 2\tfrac{26}{37}\text{ oz. } \textit{Ans.}$$

Exercise 102. Page 225.

1. In how many days of 8 hr. will 60 men do the same work that 24 men can do in 15 dy. of 10 hr.?

$$\left.\begin{array}{r|l} 8 & 10 \\ 60 & 24 \end{array}\right. : : 15\text{ dy.} : \text{what?}$$

$$\frac{\overset{5}{\cancel{10}} \times \overset{3}{\cancel{24}} \times \cancel{15}\text{ dy.}}{\cancel{8} \times \underset{\underset{2}{\cancel{4}}}{\cancel{60}}} = \frac{15}{2}\text{ dy.} = 7\tfrac{1}{2}\text{ dy. } \textit{Ans.}$$

[redacted]

2. What is the expense of covering a room with drugget 4 ft. wide, at 91⅔ cents a yard, if carpet 2 ft. 3 in. wide for the room costs \$70.50, at \$1.37½ a yard?

$$\$0.91\tfrac{2}{3} = \$\tfrac{11}{12}. \qquad \$1.37\tfrac{1}{2} = \$1\tfrac{3}{8}.$$

$$\left.\begin{array}{r|l} 4 & 2\frac{1}{4} \\ 1\frac{3}{8} & \frac{11}{12} \end{array}\right. :: \$70\tfrac{1}{2} : \text{what?}$$

$$\frac{1}{\cancel{4}} \times \frac{\overset{2}{\cancel{8}}}{\cancel{11}} \times \frac{\overset{3}{\cancel{9}}}{4} \times \frac{\cancel{11}}{\underset{4}{\cancel{12}}} \times \$\frac{141}{\cancel{2}} = \$\frac{423}{16} = \$26.44. \quad \textit{Ans.}$$

3. If 4418 tons of iron ore produce \$36,190 worth of metal, when iron is at \$37.50 a ton, what will be the value of the iron at \$47 a ton from 2275 tons of ore?

$$\left.\begin{array}{r|l} 37\frac{1}{2} & 47 \\ 4418 & 2275 \end{array}\right. :: \$36{,}190 : \text{what?}$$

$$\frac{2 \times \cancel{47} \times \overset{91}{\cancel{2275}} \times \$\overset{385}{\cancel{36190}}}{\underset{3}{\cancel{75}} \times \underset{\cancel{94}}{\cancel{4418}}} = \$\frac{70070}{3} = \$23{,}356.67. \quad \textit{Ans.}$$

4. If a bar of iron 3⅓ ft. long, 3 in. wide, and 2¾ in. thick weighs 93 lb., what will be the weight of a bar 3⅔ ft. long, 4 in. wide, and 2½ in. thick?

$$\left.\begin{array}{r|l} 3\frac{1}{3} & 3\frac{2}{3} \\ 3 & 4 \\ 2\frac{3}{4} & 2\frac{1}{2} \end{array}\right. :: 93 \text{ lb.} : \text{what?}$$

$$\frac{\cancel{11}}{\cancel{3}} \times \frac{\overset{2}{\cancel{4}}}{\cancel{3}} \times \frac{\cancel{5}}{\cancel{2}} \times \frac{\cancel{3}}{\underset{\cancel{2}}{\cancel{10}}} \times \frac{4}{\cancel{11}} \times \overset{31}{\cancel{93}} \text{ lb.} = 124 \text{ lb.} \quad \textit{Ans.}$$

5. If 40 bu. of wheat can be grown on the same area as 48 bu. of barley, and 28 A. produce 840 bu. of wheat, how much barley will 38 A. produce?

$$\left.\begin{array}{r|l} 40 & 48 \\ 28 & 38 \end{array}\right. :: 840 \text{ bu.} : \text{what?} \qquad \frac{\overset{6}{\cancel{48}} \times 38 \times \overset{\overset{6}{\cancel{30}}}{\cancel{840}} \text{ bu.}}{\underset{\cancel{5}}{\cancel{40}} \times \cancel{28}} = 1368 \text{ bu.}$$

[redacted]

6. If 18 men can dig a trench 150 ft. long, 6 ft. broad, and 4 ft. 6 in. deep in 12 days, in how many days will 16 men dig a trench 210 ft. long, 5 ft. broad, and 4 ft. deep?

$$\left.\begin{array}{r|l} 16 & 18 \\ 150 & 210 \\ 6 & 5 \\ 4\frac{1}{2} & 4 \end{array}\right. : : 12 \text{ dy.} : \text{what?}$$

$$\frac{\overset{\cancel{4}}{\cancel{18}} \times \overset{7}{\cancel{210}} \times \cancel{5} \times \cancel{4} \times \overset{2}{\cancel{12}} \text{ dy.}}{\underset{\cancel{4}}{\cancel{16}} \times \underset{\cancel{30}}{\cancel{150}} \times \cancel{6} \times \cancel{4\frac{1}{2}}} = 14 \text{ dy.} \quad \textit{Ans.}$$

7. A book of 810 pages, 40 lines to a page, and 60 letters to a line, is reprinted in pages of 50 lines, 72 letters to a line. How many pages will the new edition contain?

$$\left.\begin{array}{r|l} 50 & 40 \\ 72 & 60 \end{array}\right. : : 810 : \text{what?} \qquad \frac{\overset{10}{\cancel{40}} \times \overset{6}{\cancel{60}} \times \overset{\overset{9}{\cancel{45}}}{\cancel{810}}}{\underset{\cancel{5}}{\cancel{50}} \times \underset{\cancel{4}}{\cancel{72}}} = 540. \quad \textit{Ans.}$$

8. If 3280 42-lb. shot cost \$3000, how many 32-lb. shot can be bought for \$4200?

$$\left.\begin{array}{r|l} 3000 & 4200 \\ 32 & 42 \end{array}\right. : : 3280 : \text{what?} \qquad \frac{\overset{7}{\cancel{4200}} \times \overset{21}{\cancel{42}} \times \overset{\overset{41}{\cancel{205}}}{\cancel{3280}}}{\underset{\cancel{5}}{\cancel{3000}} \times \underset{\cancel{2}}{\cancel{32}}} = 6027. \quad \textit{Ans.}$$

9. What is the rate of wages, if 12 men earn in 10 dy. as much as 9 men earn in 14 dy. at \$1.50 a day?

$$\left.\begin{array}{r|l} 12 & 9 \\ 10 & 14 \end{array}\right. : : \$1.50 : \text{what?}$$

$$\frac{9 \times \overset{7}{\cancel{14}} \times \$\overset{\overset{0.05}{\cancel{0.15}}}{\cancel{1.50}}}{\underset{\underset{2}{\cancel{4}}}{\cancel{12}} \times \underset{\cancel{2}}{\cancel{10}}} = \$\frac{3.15}{2} = \$1.575. \quad \textit{Ans.}$$

10. A rectangular reservoir 15 yd. long and 4 ft. deep holds 32,500 gal. What quantity of water will it hold if its length is increased by *18 ft.* and its depth by 1 ft.?

[redacted]

$$\begin{array}{r|l} 15 & 21 \\ 4 & 5 \end{array} :: 32{,}500 \text{ gal.} : \text{what?}$$

$$\frac{\overset{7}{\cancel{21}} \times \cancel{5} \times \overset{8125}{\cancel{32500}} \text{ gal.}}{\underset{\cancel{3}}{\cancel{15}} \times \cancel{4}} = 56{,}875 \text{ gal. } \textit{Ans.}$$

11. What must be the length of a bar of silver $\frac{3}{4}$ in. square to weigh the same as a bar of gold $\frac{1}{2}$ in. square and $6\frac{3}{4}$ in. long, if the weight of a cubic inch of silver to that of a cubic inch of gold is in the ratio 47 : 88 ?

$$\begin{array}{r|l} (\frac{3}{4})^2 & (\frac{1}{2})^2 \\ 47 & 88 \end{array} :: 6\tfrac{3}{4} \text{ in.} : \text{what?} \qquad \begin{array}{r|l} \frac{9}{16} & \frac{1}{4} \\ 47 & 88 \end{array} :: 6\tfrac{3}{4} \text{ in.} : \text{what?}$$

$$\frac{\overset{4}{\cancel{16}} \times \overset{22}{\cancel{88}} \times \overset{3}{\cancel{27}}}{\cancel{9} \times 47 \times \cancel{4} \times \cancel{4}} \text{ in.} = 5\tfrac{29}{47} \text{ in. } \quad \textit{Ans.}$$

12. How far can A, who takes 3.1 ft. each step, walk, while B, who takes 2.3 ft. each step, walks 220 yd., if A takes 7 steps while B takes 11 ?

$$\begin{array}{r|l} 2.3 & 3.1 \\ 11 & 7 \end{array} :: 220 \text{ yd.} : \text{what?}$$

$$\frac{3.1 \times 7 \times \overset{20}{\cancel{220}} \text{ yd.}}{2.3 \times \cancel{11}} = \frac{434}{2.3} \text{ yd.} = 188\tfrac{16}{23} \text{ yd. } \textit{Ans.}$$

13. If 6 hr. are needed to go a given distance at a given rate, how many hours are needed when the distance is diminished by one fourth and the rate increased by one half ?

$$\begin{array}{r|l} 1 & \frac{3}{4} \\ 1\frac{1}{2} & 1 \end{array} :: 6 \text{ hr.} : \text{what?} \qquad \frac{\cancel{2}}{\cancel{3}} \times \frac{3}{\underset{\cancel{2}}{\cancel{4}}} \times \overset{\cancel{3}}{\cancel{6}} \text{ hr.} = 3 \text{ hr. } \textit{Ans.}$$

14. How many hours a day must 5 men work to mow a field in 8 dy. that 7 men can mow in 6 dy. of 10 hr. ?

$$\begin{array}{r|l} 5 & 7 \\ 8 & 6 \end{array} :: 10 \text{ hr.} : \text{what?} \qquad \frac{7 \times \overset{3}{\cancel{6}} \times \overset{\cancel{2}}{\cancel{10}} \text{ hr.}}{\cancel{5} \times \underset{\underset{2}{\cancel{4}}}{\cancel{8}}} = \tfrac{21}{2} \text{ hr.} = 10\tfrac{1}{2} \text{ hr. } \textit{Ans.}$$

[redacted]

15. If a bar of iron 10 ft. $6\frac{1}{2}$ in. long, $3\frac{5}{8}$ in. broad, and $3\frac{1}{2}$ in. thick weighs 4 cwt. 20.21 lb., what is the length of a bar of iron that weighs a long ton if its breadth and thickness are $4\frac{2}{3}$ in. and $4\frac{1}{8}$ in., respectively?

1 l. t. = 2240 lb. 4 cwt. 20.21 lb. = 420.21 lb.

$$\left.\begin{array}{r|l} 42021 & 224000 \\ 4\frac{2}{3} & 3\frac{5}{8} \\ 4\frac{1}{8} & 3\frac{1}{2} \end{array}\right. :: 10\tfrac{13}{24} \text{ ft.} : \text{what?}$$

$$\frac{\overset{\overset{1000}{\cancel{7000}}\;\overset{}{\cancel{14000}}\;\cancel{28000}}{\cancel{224000}} \times \cancel{3} \times \cancel{8} \times \cancel{29} \times \cancel{7} \times \overset{\cancel{23}}{\cancel{253}}}{\underset{\underset{\underset{9}{\cancel{63}}}{\cancel{1449}}}{\cancel{42021}} \times \underset{\cancel{2}}{\cancel{14}} \times \underset{\cancel{11}}{\cancel{33}} \times \cancel{8} \times \cancel{2} \times \underset{3}{\cancel{24}}} \text{ ft.} = 37\tfrac{1}{27} \text{ ft.} \quad \textit{Ans.}$$

16. If 27 men in 28 dy. of 10 hr. dig a trench 126 yd. long, $2\frac{1}{2}$ yd. broad, $1\frac{1}{2}$ yd. deep, how long a trench $2\frac{3}{4}$ yd. broad and $1\frac{3}{4}$ yd. deep will 56 men dig in 25 dy. of $8\frac{1}{4}$ hr.?

$$\left.\begin{array}{r|l} 27 & 56 \\ 10 & 8\frac{1}{4} \\ 28 & 25 \\ 2\frac{3}{4} & 2\frac{1}{2} \\ 1\frac{3}{4} & 1\frac{1}{2} \end{array}\right. :: 126 \text{ yd.} : \text{what?}$$

$$\frac{\cancel{4} \times \overset{\cancel{2}}{\cancel{4}} \times \overset{\cancel{2}}{\cancel{56}} \times \overset{\cancel{3}}{\cancel{33}} \times 25 \times \cancel{5} \times \cancel{3} \times \overset{\overset{6}{\cancel{42}}}{\cancel{126}} \text{ yd.}}{\cancel{11} \times \cancel{7} \times \underset{\underset{\cancel{3}}{\cancel{9}}}{\cancel{27}} \times \underset{\cancel{2}}{\cancel{10}} \times \cancel{4} \times \cancel{28} \times \cancel{2} \times \cancel{2}} = 150 \text{ yd.} \quad \textit{Ans.}$$

17. If 34^{kg} of wool makes 25^{m} of cloth 0.6^{m} wide, how long a piece of cloth 0.8^{m} wide will 108.8^{kg} of wool make?

$$\left.\begin{array}{r|l} 34 & 108.8 \\ 0.8 & 0.6 \end{array}\right. :: 25^{m} : \text{what?} \qquad \frac{\overset{\overset{4}{\cancel{136}}}{\cancel{108.8}} \times 0.6 \times 25^{m}}{\cancel{34} \times \cancel{0.8}} = 60^{m}. \quad \textit{Ans.}$$

18. If an oak beam 5.40^{m} long, 0.63^{m} thick, and 0.57^{m} wide weighs 1469.25^{kg}, what is the weight of an oak beam 4.87^{m} long, 0.58^{m} thick, and 0.53^{m} wide?

[redacted]

$$\left.\begin{array}{r|l} 5.4 & 4.87 \\ 0.63 & 0.58 \\ 0.57 & 0.53 \end{array}\right. :: 1469\tfrac{1}{4}^{\text{kg}} : \text{what?}$$

$$\frac{487 \times \overset{29}{\cancel{58}} \times 53 \times \overset{653}{\cancel{5877}}^{\text{kg}}}{\underset{60}{\cancel{540}} \times 63 \times 57 \times \underset{2}{\cancel{4}}} = \frac{488782907^{\text{kg}}}{430920} = 1134.2776^{\text{kg}}. \textit{ Ans.}$$

19. A certain quantity of air has a volume of 195.5 cu. ft. at 27.8° C. What will be its volume at 100° C.?

$100° - 27.8° = 72.2°$. $\quad 72.2 \times 0.00367 = 0.264974$.

$1 : 1.264974 :: 195.5$ cu. ft. : what?

1.264974×195.5 cu. ft. $= 247.3$ cu. ft. *Ans.*

20. A quantity of air at a temperature of 15.6° C. has a volume of 4 cu. ft. under a pressure of 12 lb. per square inch. What will be its volume at 48.7° C. under a pressure of 14 lb. per square inch?

$48.7° - 15.6° = 33.1$. $\quad 33.1 \times 0.00367 = 0.121477$.

$$\left.\begin{array}{r|l} 14 & 12 \\ 1 & 1.121477 \end{array}\right. :: 4 \text{ cu. ft.} : \text{what?}$$

$$\frac{\overset{3}{\cancel{12}} \times 1121477 \times \overset{2}{\cancel{4}} \text{ cu. ft.}}{\underset{7}{\cancel{14}} \times \underset{\underset{125000}{\cancel{500000}}}{\cancel{1000000}}} = \frac{3364431}{875000} \text{ cu. ft.} = 3.8 \text{ cu. ft. } \textit{Ans.}$$

Exercise 103. Page 227.

1. If a man can mow $\frac{3}{11}$ of a field in a day, how long will it take another man to mow $\frac{5}{6}$ of a field $5\frac{1}{4}$ times as large, if the second man works $1\frac{2}{3}$ times as fast as the first, but only $\frac{7}{8}$ as many hours each day?

1st cause. 2d cause. 1st effect. 2d effect.

$$\left.\begin{array}{c} \frac{11}{3} \text{ dy.} \\ 1 \\ 1 \end{array}\right\} : \left.\begin{array}{c} ? \text{ dy.} \\ 1\frac{2}{3} \\ \frac{7}{8} \end{array}\right\} :: 1 \quad : \quad \tfrac{5}{6} \times 5\tfrac{1}{4}.$$

$$\frac{\frac{11}{3} \times \frac{5}{6} \times 5\frac{1}{4}}{1\frac{2}{3} \times \frac{7}{8}} = \frac{11}{\cancel{3}} \times \frac{\cancel{5}}{\underset{\cancel{3}}{\cancel{6}}} \times \frac{\overset{\cancel{7}}{\cancel{21}}}{\cancel{4}} \times \frac{\cancel{3}}{\cancel{5}} \times \frac{\overset{\cancel{2}}{\cancel{8}}}{\cancel{7}} = 11. \quad 11 \text{ dy. } \textit{Ans.}$$

[redacted]

2. If 4 men or 7 boys can do a piece of work in 6 days, how long will it take 6 men and 9 boys to do the work?

$$4 \text{ men} = 7 \text{ boys.}$$
$$\therefore\ 6 \text{ men} = 10\tfrac{1}{2} \text{ boys.}$$
$$10\tfrac{1}{2} + 9 = 19\tfrac{1}{2}.$$

1st cause.		2d cause.		1st effect.		2d effect.
7 boys, 6 dy.	:	19½ boys, ? dy.	: :	1	:	1.

$$\frac{7 \times 6}{19\frac{1}{2}} = \frac{7 \times \overset{2}{\cancel{6}} \times 2}{\underset{13}{\cancel{39}}} = \frac{28}{13} = 2\tfrac{2}{13}. \qquad 2\tfrac{2}{13} \text{ dy. } \textit{Ans.}$$

3. If 50 men working 9 hr. a day require 6 dy. to dig a trench 100 yd. long, 2 yd. wide, and 3 yd. deep, how many men working 10 hr. a day for 9 dy. will be required to dig a trench 50 yd. long, 6 yd. wide, and 5 yd. deep, in ground twice as hard to dig?

1st cause.		2d cause.		1st effect.		2d effect.
50 men, 9 hr., 6 dy., 2	:	? men, 10 hr., 9 dy., 1	: :	100 yd., 2 yd., 3 yd.	:	50 yd., 6 yd., 5 yd.

$$\frac{\cancel{50} \times \cancel{9} \times \overset{\cancel{3}}{\cancel{6}} \times \cancel{2} \times 50 \times \overset{3}{\cancel{6}} \times \cancel{5}}{\underset{\cancel{2}}{\cancel{10}} \times \cancel{9} \times \underset{\cancel{2}}{\cancel{100}} \times \cancel{2} \times \cancel{3}} = 150. \qquad 150 \text{ men. } \textit{Ans.}$$

4. If 12 men in 9 dy. can harvest 40 A. of wheat, how many acres can 16 men harvest in 3 dy.?

1st cause.		2d cause.		1st effect.		2d effect.
12 men, 9 dy.	:	16 men, 3 dy.	: :	40 A.	:	? A.

$$\frac{16 \times \cancel{3} \times \overset{10}{\cancel{40}}}{\underset{\cancel{4}}{\cancel{12}} \times 9} = \frac{160}{9} = 17\tfrac{7}{9}. \qquad 17\tfrac{7}{9} \text{ A. } \textit{Ans.}$$

[redacted]

5. If 120 men can make an embankment $\frac{3}{4}$ of a mile long, 30 yd. wide, and 7 yd. high, in 42 dy., how many men will it take to make an embankment 1000 yd. long, 36 yd. wide, and 22 ft. high, in 30 dy.?

$$\begin{array}{ccccccc} \text{1st cause.} & & \text{2d cause.} & & \text{1st effect.} & & \text{2d effect.} \\ \left.\begin{array}{r} 120 \text{ men} \\ 42 \text{ dy.} \end{array}\right\} & : & \left.\begin{array}{r} ? \text{ men} \\ 30 \text{ dy.} \end{array}\right\} & :: & \left\{\begin{array}{r} \frac{3}{4} \text{ mi.} \\ 30 \text{ yd.} \\ 7 \text{ yd.} \end{array}\right. & : & \left\{\begin{array}{r} 1000 \text{ yd.} \\ 36 \text{ yd.} \\ 22 \text{ ft.} \end{array}\right. \end{array}$$

$$\frac{\overset{2}{\cancel{120}} \times \overset{2}{\cancel{42}} \times \overset{\overset{10}{\cancel{100}}}{\cancel{1000}} \times \overset{\overset{4}{\cancel{12}}}{\cancel{36}} \times \cancel{22}}{\underset{3}{\cancel{30}} \times \underset{60}{\cancel{1320}} \times \underset{3}{\cancel{30}} \times \cancel{21}} = 160. \quad 160 \text{ men. } \textit{Ans.}$$

6. If 7 women in 8 dy. of 11 hr. each can make 22 dozen shirts, in how many days of 10 hr. each can 12 women make 360 dozen shirts?

$$\begin{array}{ccccccc} \text{1st cause.} & & \text{2d cause.} & & \text{1st effect.} & & \text{2d effect.} \\ \left.\begin{array}{l} 7 \text{ women} \\ 8 \text{ dy.} \\ 11 \text{ hr.} \end{array}\right\} & : & \left.\begin{array}{l} 12 \text{ women} \\ ? \text{ dy.} \\ 10 \text{ hr.} \end{array}\right\} & :: & 22 \text{ doz.} & : & 360 \text{ doz.} \end{array}$$

$$\frac{7 \times \overset{4}{\cancel{8}} \times \cancel{11} \times \overset{\overset{3}{\cancel{30}}}{\cancel{360}}}{\cancel{12} \times \cancel{10} \times \underset{2}{\cancel{22}}} = 84.$$

84 dy. *Ans.*

7. Twenty-five lamps used 5 hr. an evening for 40 dy. required a quantity of oil that cost \$4.25. How many lamps used 4 hr. an evening for 30 dy. can be furnished with oil at a cost of \$7.65?

$$\begin{array}{ccccccc} \text{1st cause.} & & \text{2d cause.} & & \text{1st effect.} & & \text{2d effect.} \\ \left.\begin{array}{l} 25 \text{ lamps} \\ 5 \text{ hr.} \\ 40 \text{ dy.} \end{array}\right\} & : & \left.\begin{array}{l} ? \text{ lamps} \\ 4 \text{ hr.} \\ 30 \text{ dy.} \end{array}\right\} & :: & \$4.25 & : & \$7.65. \end{array}$$

$$\frac{\cancel{25} \times 5 \times \overset{\overset{5}{\cancel{10}}}{\cancel{40}} \times \overset{\overset{3}{\cancel{45}}}{\cancel{765}}}{\cancel{4} \times \underset{2}{\cancel{30}} \times \underset{17}{\cancel{425}}} = 75. \quad \textit{Ans.}$$

[redacted]

8. If 8 horses can be kept 12 dy. for a certain sum when hay is worth $15 a ton, how many days can 6 horses be kept for the same sum when hay is worth $12 a ton?

$$\begin{array}{ccccccc} \text{1st cause.} & & \text{2d cause.} & & \text{1st effect.} & & \text{2d effect.} \\ \left.\begin{array}{r}8\text{ horses}\\12\text{ dy.}\end{array}\right\} & : & \left.\begin{array}{l}6\text{ horses}\\?\text{ dy.}\end{array}\right\} & :: & \$12 & : & \$15. \end{array}$$

$$\frac{\overset{4}{\cancel{8}} \times \cancel{12} \times \overset{5}{\cancel{15}}}{\underset{3}{\cancel{6}} \times \cancel{12}} = 20.$$

20 dy. *Ans.*

9. Twenty horses working 14 wk., 6 dy. a week and 8 hr. a day, transport the output of a mine to the nearest wharf. In how many weeks will 24 horses do the same work, if they work 5 dy. a week and 7 hr. a day?

$$\begin{array}{ccccccc} \text{1st cause.} & & \text{2d cause.} & & \text{1st effect.} & & \text{2d effect.} \\ \left.\begin{array}{r}20\text{ horses}\\14\text{ wk.}\\6\text{ dy.}\\8\text{ hr.}\end{array}\right\} & : & \left.\begin{array}{l}24\text{ horses}\\?\text{ wk.}\\5\text{ dy.}\\7\text{ hr.}\end{array}\right\} & :: & 1 & : & 1. \end{array}$$

$$\frac{\overset{4}{\cancel{20}} \times \overset{2}{\cancel{14}} \times \overset{2}{\cancel{6}} \times \cancel{8}}{\underset{3}{\cancel{24}} \times \cancel{5} \times \cancel{7}} = 16.$$

16 wk. *Ans.*

10. If 6 men can reap a field of rye 200 yd. long and 150 yd. wide in 4 dy. of 12 hr. each, in how many days of 10 hr. each will 8 men reap a field 300 yd. long and 250 yd. wide?

$$\begin{array}{ccccccc} \text{1st cause.} & & \text{2d cause.} & & \text{1st effect.} & & \text{2d effect.} \\ \left.\begin{array}{r}6\text{ men}\\4\text{ dy.}\\12\text{ hr.}\end{array}\right\} & : & \left.\begin{array}{l}8\text{ men}\\?\text{ dy.}\\10\text{ hr.}\end{array}\right\} & & \left\{\begin{array}{l}200\text{ yd.}\\150\text{ yd.}\end{array}\right. & : & \left\{\begin{array}{l}300\text{ yd.}\\250\text{ yd.}\end{array}\right. \end{array}$$

$$\frac{\overset{3}{\cancel{6}} \times \cancel{4} \times \overset{\overset{3}{\cancel{6}}}{\cancel{12}} \times \overset{\cancel{3}}{\cancel{300}} \times \overset{\cancel{5}}{\cancel{250}}}{\underset{\cancel{2}}{\cancel{8}} \times \underset{\cancel{2}}{\cancel{10}} \times \underset{\cancel{2}}{\cancel{200}} \times \underset{\cancel{50}}{\cancel{150}}} = 9.$$

9 dy. *Ans.*

[redacted]

11. If a boy can do only half as much work as a man, how many hours a day must 42 boys work to accomplish as much in 45 dy. as 27 men, working 10 hr. a day, would accomplish in 28 dy. ?

42 boys = 21 men.

1st cause.		2d cause.		1st effect.		2d effect.
27 men 10 hr. 28 dy.	:	21 men ? hr. 45 dy.	: :	1	:	1.

$$\frac{\overset{\cancel{3}}{\cancel{27}} \times \overset{2}{\cancel{10}} \times \overset{4}{\cancel{28}}}{\underset{\cancel{7}}{\cancel{21}} \times \underset{\cancel{5}}{\cancel{45}}} = 8.$$

8 hr. *Ans.*

Exercise 104. Page 229.

1. Divide \$12,000 proportionally to the numbers 3, 4, 5.

$3 + 4 + 5 = 12.$

$$\frac{3}{\cancel{12}} \times \$\overset{1000}{\cancel{12000}} = \$3000.$$

$$\frac{4}{\cancel{12}} \times \$\overset{1000}{\cancel{12000}} = \$4000.$$

$$\frac{5}{\cancel{12}} \times \$\overset{1000}{\cancel{12000}} = \$5000.$$

2. Divide 815 tons proportionally to $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$.

$60 \times (\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}) = 30, 40, 45, 48.$

$30 + 40 + 45 + 48 = 163.$

$$\frac{30}{\cancel{163}} \times \overset{5}{\cancel{815}}\text{ t.} = 150\text{ t.}$$

$$\frac{40}{\cancel{163}} \times \overset{5}{\cancel{815}}\text{ t.} = 200\text{ t.}$$

$$\frac{45}{\cancel{163}} \times \overset{5}{\cancel{815}}\text{ t.} = 225\text{ t.}$$

$$\frac{48}{\cancel{163}} \times \overset{5}{\cancel{815}}\text{ t.} = 240\text{ t.}$$

3. Divide 6853 lb. of wool proportionally to $1\frac{3}{4}$, $2\frac{4}{5}$, $5\frac{5}{6}$; also proportionally to the reciprocals of these numbers.

$60 \times (\frac{7}{4}, \frac{14}{5}, \frac{35}{6}) = 105, 168, 350.$

$105 + 168 + 350 = 623.$

$$\frac{105}{\cancel{623}} \times \overset{11}{\cancel{6853}}\text{ lb.} = 1155\text{ lb.}$$

$$\frac{168}{\cancel{623}} \times \overset{11}{\cancel{6853}}\text{ lb.} = 1848\text{ lb.}$$

$$\frac{350}{\cancel{623}} \times \overset{11}{\cancel{6853}}\text{ lb.} = 3850\text{ lb.}$$

The reciprocals of

$1\frac{3}{4}, 2\frac{4}{5}, 5\frac{5}{6} = \frac{4}{7}, \frac{5}{14}, \frac{6}{35}.$

$70 \times (\frac{4}{7}, \frac{5}{14}, \frac{6}{35}) = 40, 25, 12.$

$40 + 25 + 12 = 77.$

$$\frac{40}{\cancel{77}} \times \overset{89}{\cancel{6853}}\text{ lb.} = 3560\text{ lb.}$$

$$\frac{25}{\cancel{77}} \times \overset{89}{\cancel{6853}}\text{ lb.} = 2225\text{ lb.}$$

$$\frac{12}{\cancel{77}} \times \overset{89}{\cancel{6853}}\text{ lb.} = 1068\text{ lb.}$$

4. Two men purchase some property together, one paying \$1250 and the other \$1000. If the value of the property rises to \$3600, what will be the share of each?

$$\$1250 + \$1000 = \$2250.$$

$$\frac{\overset{250}{\cancel{1250}}}{\underset{5}{\cancel{2250}}} \times \$\overset{8}{\cancel{3600}} = \$2000.$$

$$\frac{\overset{200}{\cancel{1000}}}{\underset{5}{\cancel{2250}}} \times \$\overset{8}{\cancel{3600}} = \$1600.$$

5. Gun metal is composed by weight of 3 parts of tin to 100 parts of copper. What weight of each of these metals is there in a cannon weighing 721 lb.?

$$3 + 100 = 103.$$

$$\frac{3}{\cancel{103}} \times \overset{7}{\cancel{721}} \text{ lb.} = 21 \text{ lb., tin.}$$

$$\frac{100}{\cancel{103}} \times \overset{7}{\cancel{721}} \text{ lb.} = 700 \text{ lb., copper.}$$

6. Bell metal contains by weight 78 parts of copper and 22 parts of tin. What weight of each of these metals is there in a bell weighing 937 lb.?

$$78 + 22 = 100.$$

$$\begin{array}{r} 937 \text{ lb.} \\ \underline{0.78} \\ 7496 \\ \underline{6559} \\ 730.86 \text{ lb., copper.} \end{array} \qquad \begin{array}{r} 937 \text{ lb.} \\ \underline{0.22} \\ 1874 \\ \underline{1874} \\ 206.14 \text{ lb., tin.} \end{array}$$

7. It takes 75^{kg} of saltpetre, 12.5^{kg} of charcoal, and 12.5^{kg} of sulphur to make 100^{kg} of powder. How many kilograms of each will be required to make 10,000,000 cartridges, each containing 5^{g} of powder?

$$10{,}000{,}000 \times 5^{g} = 50{,}000{,}000^{g} = 50{,}000^{kg}.$$

$$75^{kg} + 12.5^{kg} + 12.5^{kg} = 100^{kg}.$$

$$\frac{75}{\cancel{100}} \times \overset{500}{\cancel{50000}}{}^{kg} = 37{,}500^{kg}, \text{ saltpetre.}$$

$$\frac{125}{\cancel{1000}} \times \overset{50}{\cancel{50000}}{}^{kg} = 6250^{kg}, \text{ charcoal.}$$

$$\frac{125}{\cancel{1000}} \times \overset{50}{\cancel{50000}}{}^{kg} = 6250^{kg}, \text{ sulphur.}$$

8. Yellow copper contains by weight 2 parts of red copper and 1 part of zinc. How many ounces of red copper in an article of yellow copper that weighs 1 lb.?

$2 + 1 = 3.$ $\frac{2}{3} \times 16$ oz. $= \frac{32}{3}$ oz. $= 10\frac{2}{3}$ oz. *Ans.*

9. Type metal is an alloy containing by weight 39 parts of lead to 11 parts of antimony. How many pounds of each are required to make 957 lb. of type?

$$39 + 11 = 50.$$

$$\tfrac{39}{50} \times 957 \text{ lb.} = \tfrac{37323}{50} \text{ lb.} = 746.46 \text{ lb., lead.}$$

$$\tfrac{11}{50} \times 957 \text{ lb.} = \tfrac{10527}{50} \text{ lb.} = 210.54 \text{ lb., antimony.}$$

10. Plumbers' solder contains by weight 2 parts of lead and 1 part of tin. How many pounds of each are required to make 100 lb. of solder?

$$2 + 1 = 3.$$

$$\tfrac{2}{3} \times 100 \text{ lb.} = \tfrac{200}{3} \text{ lb.} = 66\tfrac{2}{3} \text{ lb., lead.}$$

$$\tfrac{1}{3} \times 100 \text{ lb.} = \tfrac{100}{3} \text{ lb.} = 33\tfrac{1}{3} \text{ lb., tin.}$$

11. The air is composed of oxygen and nitrogen. In 100 volumes of air there are 21 volumes of oxygen and 79 of nitrogen. If the weight of a liter of oxygen is 1.4295^g, and that of a liter of nitrogen is 1.2577^g, how many grams of each gas does 100^g of air contain?

$$\begin{array}{r} 1.4295^g \\ 21 \\ \hline 14295 \\ 28590 \\ \hline 30.0195^g \end{array} \qquad \begin{array}{r} 1.2577^g \\ 79 \\ \hline 113193 \\ 88039 \\ \hline 99.3583^g \end{array}$$

$$30.0195^g + 99.3583^g = 129.3778^g.$$

$$\tfrac{300195}{1293778} \times 100^g = 23.203^g, \text{ oxygen. } \textit{Ans.}$$

$$100^g - 23.203^g = 76.797^g, \text{ hydrogen. } \textit{Ans.}$$

$$\begin{array}{r} 23.202 \\ 1293778\overline{)30019500.} \\ 2587556 \\ \hline 4143940 \\ 3881334 \\ \hline 2626060 \\ 2587556 \\ \hline 3850400 \\ 2587556 \\ \hline 1262844 \end{array}$$

[redacted]

12. At \$20.67 an ounce for pure gold, what is the value of the gold in a chain that weighs 3 oz. 4 dwt., if it is 18 carats fine (that is, 18 parts of pure gold out of 24)?

$$3 \text{ oz. } 4 \text{ dwt.} = 3\tfrac{1}{5} \text{ oz.} = 3.2 \text{ oz.} \qquad \tfrac{18}{24} = \tfrac{3}{4} = 0.75.$$

$$\begin{array}{r} \$20.67 \\ 3.2 \\ \hline 4134 \\ 6201 \\ \hline \$66.144 \end{array} \qquad \begin{array}{r} \$66.144 \\ 0.75 \\ \hline 330720 \\ 463008 \\ \hline 49.60800 \end{array} \qquad \$49.61. \textit{ Ans.}$$

13. Two men agree to do a piece of work for \$63. They finish the work in 18 days, but one of them was absent 5 days of this time. How should the pay be divided?

$$18 \text{ dy.} + 13 \text{ dy.} = 31 \text{ dy.}$$

$$\tfrac{18}{31} \times \$63 = \$\tfrac{1134}{31} = \$36.58. \qquad \tfrac{13}{31} \times \$63 = \$\tfrac{819}{31} = \$26.42.$$

14. Five men working together do a piece of work in 20 days, and receive as pay \$253. One of the men was absent 5 days, and another 2 days of this time. How should the pay be divided?

$$20 \text{ dy.} + 20 \text{ dy.} + 20 \text{ dy.} + 15 \text{ dy.} + 18 \text{ dy.} = 93 \text{ dy.}$$

$$\frac{20}{93} \times \$253 = \$\frac{5060}{93} = \$54.41. \qquad \frac{\overset{5}{\cancel{15}}}{\underset{31}{\cancel{93}}} \times \$253 = \$\frac{1265}{31} = \$40.80.$$

$$\frac{\overset{6}{\cancel{18}}}{\underset{31}{\cancel{93}}} \times \$253 = \$\frac{1518}{31} = \$48.97.$$

Hence, three should receive \$54.41 each, one \$40.80, and one \$48.97. *Ans.*

15. Standard silver consists of 37 parts of pure silver to 3 parts of copper. What weight of pure silver in the crown piece that weighs $\frac{10}{11}$ oz. troy?

$$37 + 3 = 40. \qquad \frac{37}{\underset{4}{\cancel{40}}} \times \frac{\cancel{10}}{11} \text{ oz.} = \frac{37}{44} \text{ oz. } \textit{Ans.}$$

[redacted]

Exercise 105. Page 232.

1. A, B, and C entered into partnership, A furnishing \$18,150; B, \$19,360; and C, \$10,890. If their profits were \$12,100, what was each man's share of the profits?

$$\$18,150 + \$19,360 + \$10,890 = \$48,400.$$

$$\frac{\overset{9075}{\cancel{18150}}}{\underset{\underset{2}{\cancel{4}}}{\cancel{48400}}} \times \$\cancel{12100} = \$\frac{9075}{2} = \$4537.50, \text{ A.}$$

$$\frac{\overset{4840}{\cancel{19360}}}{\underset{\cancel{4}}{\cancel{48400}}} \times \$\cancel{12100} = \$4840, \text{ B.}$$

$$\frac{\overset{5445}{\cancel{10890}}}{\underset{\underset{2}{\cancel{4}}}{\cancel{48400}}} \times \$\cancel{12100} = \$\frac{5445}{2} = \$2722.50, \text{ C.}$$

2. Four men engaged in business together and made a profit of \$1200. How much of it should each man receive, if the first put in \$3000, the second \$5000, the third \$4200, and the fourth \$2400?

$$\$3000 + \$5000 + \$4200 + \$2400 = \$14,600.$$

$$\frac{3000}{\underset{73}{\cancel{14600}}} \times \$\overset{6}{\cancel{1200}} = \$\frac{18000}{73} = \$246.57, \text{ 1st.}$$

$$\frac{5000}{\underset{73}{\cancel{14600}}} \times \$\overset{6}{\cancel{1200}} = \$\frac{30000}{73} = \$410.96, \text{ 2d.}$$

$$\frac{4200}{\underset{73}{\cancel{14600}}} \times \$\overset{6}{\cancel{1200}} = \$\frac{25200}{73} = \$345.21, \text{ 3d.}$$

$$\frac{2400}{\underset{73}{\cancel{14600}}} \times \$\overset{6}{\cancel{1200}} = \$\frac{14400}{73} = \$197.26, \text{ 4th.}$$

[redacted]

3. A man dies owing three creditors \$8050, \$2970, and \$7170, respectively. If his assets, after deducting expenses, are \$13,646, how much will each creditor receive?

$$\$8050 + \$2970 + \$7170 = \$18,190.$$

$$\frac{\overset{805}{805\cancel{0}}}{\underset{1819}{1819\cancel{0}}} \times \$13646 = \$6039.05.$$

$$\frac{\overset{297}{297\cancel{0}}}{\underset{1819}{1819\cancel{0}}} \times \$13646 = \$2228.07.$$

$$\frac{\overset{717}{717\cancel{0}}}{\underset{1819}{1819\cancel{0}}} \times \$13646 = \$5378.88.$$

4. Three heirs receive from an estate \$4700, \$3200, and \$12,500, respectively, on condition that they together pay a debt of \$2000. What amount will each have?

$$\$4700 + \$3200 + \$12,500 = \$20,400.$$

$$\frac{4700}{\underset{51}{\cancel{20400}}} \times \$\overset{5}{\cancel{2000}} = \$\frac{23500}{51} = \$460.78.$$

$$\frac{3200}{\underset{51}{\cancel{20400}}} \times \$\overset{5}{\cancel{2000}} = \$\frac{16000}{51} = \$313.73.$$

$$\frac{12500}{\underset{51}{\cancel{20400}}} \times \$\overset{5}{\cancel{2000}} = \$\frac{62500}{51} = \$1225.49.$$

\$4700.	\$3200.	\$12500.
460.78	313.73	1225.49
\$4239.22 *Ans.*	\$2886.27 *Ans.*	\$11274.51 *Ans.*

5. Arnold and Baker enter into partnership. Arnold puts in \$6000 for 8 mo., and Baker \$4000 for 6 mo. Their profits are \$2000. *What is each* man's share?

$$\begin{aligned} 8 \times \$6000 &= \$48000 \\ 6 \times \quad 4000 &= \underline{\quad 24000} \\ &\quad \$72000 \end{aligned}$$

$$\frac{\overset{2}{\cancel{48000}}}{\underset{3}{\cancel{72000}}} \times \$2000 = \$1333.33, \text{ Arnold's.}$$

$$\frac{\cancel{24000}}{\underset{3}{\cancel{72000}}} \times \$2000 = \$666.67, \text{ Baker's.}$$

6. Dobson furnishes the firm of Dobson & Fogg with $5000 for 13 mo.; Fogg furnishes $7000 for 9 mo. Their profits are $1700. What is the share of each?

$$\begin{aligned} 13 \times \$5000 &= \$65000 \\ 9 \times \quad 7000 &= \underline{\quad 63000} \\ &\quad \$128000 \end{aligned}$$

$$\frac{\overset{65}{\cancel{65000}}}{\underset{\underset{2}{\cancel{128}}}{\cancel{128000}}} \times \$\overset{425}{\cancel{1700}} = \$\frac{27625}{32} = \$863.28, \text{ Dobson's.}$$

$$\$1700 - \$863.28 = \$836.72, \text{ Fogg's.}$$

7. In a business partnership, A furnishes $800, and after 3 mo. $250 more; B furnishes $950, and at the end of 2 mo. withdraws $200; C furnishes $650, and at the end of 6 mo. $400 more. At the end of a year their profit is $2516. How shall it be divided among them?

A.	B.	C.
$12 \times \$800 = \$9600$	$12 \times \$950 = \$11400$	$12 \times \$650 = \$7800$
$9 \times 250 = 2250$	$10 \times 200 = 2000$	$6 \times 400 = 2400$
$\$11850$	$\$9400$	$\$10200$

$$\$11,850 + \$9400 + \$10,200 = \$31,450.$$

$$\frac{\overset{237}{\cancel{11850}}}{\underset{\cancel{629}}{\cancel{31450}}} \times \$\overset{4}{\cancel{2516}} = \$948, \text{ A's.} \qquad \frac{\overset{188}{\cancel{9400}}}{\underset{\cancel{629}}{\cancel{31450}}} \times \$\overset{4}{\cancel{2516}} = \$752, \text{ B's.}$$

$$\frac{\overset{204}{\cancel{10200}}}{\underset{\cancel{629}}{\cancel{31450}}} \times \$\overset{4}{\cancel{2516}} = \$816, \text{ C's.}$$

8. Two partners, A and B, enter into partnership with capitals of \$3500 and \$8700, respectively, and A is to have 0.12 of the profits for managing the business. How shall a profit of \$1906.25 be divided between them?

$$0.12 \text{ of } \$1906.25 = \$228.75. \qquad \$1906.25 - \$228.75 = \$1677.50.$$

$$\$3500 + \$8700 = \$12{,}200.$$

$$\frac{\overset{35}{\cancel{3500}}}{\underset{122}{\cancel{12200}}} \times \$\frac{3355}{2} = \$481.25.$$

$$\$481.25 + \$228.75 = \$710, \text{ A's.} \qquad \$1906.25 - \$710 = \$1196.25, \text{ B's.}$$

9. A puts \$2100 into a business, and B \$1750. At the end of a year each puts in \$700 more, and C joins them with \$2500. How shall a profit of \$2166.50 be divided 18 months after C enters the firm?

A.	B.	C.
$30 \times \$2100 = \$63000$	$30 \times \$1750 = \$52500$	$18 \times \$2500 = \$45000.$
$18 \times 700 = 12600$	$18 \times 700 = 12600$	
\$75600	\$65100	

$$\$75{,}600 + \$65{,}100 + \$45{,}000 = \$185{,}700.$$

$$\frac{\overset{\overset{63}{\cancel{756}}}{\cancel{75600}}}{\underset{\cancel{15475}}{\cancel{185700}}} \times \$\frac{\overset{14}{\cancel{216650}}}{\cancel{100}} = \$882, \text{ A's.} \qquad \frac{\overset{\overset{217}{\cancel{651}}}{\cancel{65100}}}{\underset{\underset{2}{\cancel{61900}}}{\cancel{185700}}} \times \$\frac{\overset{7}{\cancel{216650}}}{\cancel{100}} = \$759.50, \text{ B's.}$$

$$\frac{\overset{\overset{3}{\cancel{450}}}{\cancel{45000}}}{\underset{\cancel{1238}}{\cancel{185700}}} \times \$\frac{\overset{175}{\cancel{216650}}}{\cancel{100}} = \$525, \text{ C's.}$$

10. Three graziers hire a pasture, for which they pay \$132.50. One puts in 10 oxen for 3 months, another 12 oxen for 4 months, and the third 14 oxen for 2 months. How much of the rent ought each to pay?

[redacted]

$3 \times 10 = 30$
$4 \times 12 = 48$
$2 \times 14 = \underline{28}$
$\qquad\qquad 106$

$$\frac{\overset{15}{\cancel{30}}}{\cancel{106}} \times \$\frac{\overset{\overset{5}{\cancel{125}}}{\cancel{13250}}}{\underset{\underset{2}{\cancel{4}}}{\cancel{100}}} = \$37.50, \text{ 1st.}$$

$$\frac{\overset{12}{\cancel{48}}}{\cancel{106}} \times \$\frac{\overset{\overset{5}{\cancel{125}}}{\cancel{13250}}}{\underset{\cancel{4}}{\cancel{100}}} = \$60, \text{ 2d.}$$

$$\frac{\overset{7}{\cancel{28}}}{\cancel{106}} \times \$\frac{\overset{\overset{5}{\cancel{125}}}{\cancel{13250}}}{\underset{\cancel{4}}{\cancel{100}}} = \$35, \text{ 3d.}$$

11. A begins business, with a capital of $2400, on the 19th of March; and on the 17th of July admits B as a partner, with a capital of $1800. December 31 the profits are $943. What is the share of each?

From March 19 to Dec. 31 is 288 dy.
From July 17 to Dec. 31 is 168 dy.

$$\begin{aligned} 288 \times \$2400 &= \$691200 \\ 168 \times 1800 &= \underline{302400} \\ &\ \$993600 \end{aligned}$$

$$\frac{\overset{16}{\cancel{691200}}}{\underset{23}{\cancel{993600}}} \times \$\overset{41}{\cancel{943}} = \$656, \text{ A's.}$$

$$\frac{\overset{7}{\cancel{302400}}}{\underset{23}{\cancel{993600}}} \times \$\overset{41}{\cancel{943}} = \$287, \text{ B's.}$$

12. A and B join capitals in the ratio 7 : 11. At the end of 7 months A withdraws ½ of his, and B ⅓ of his; and, after 11 months more, they divide a profit of $5148.50. What is the share of each?

$18 \times 7 \ = 126$
$11 \times 3\frac{1}{2} = \underline{\ 38\frac{1}{2}}$
$\qquad\qquad 87\frac{1}{2} = \frac{525}{6}.$

$18 \times 11 = 198$
$11 \times 3\frac{2}{3} = \underline{\ 40\frac{1}{3}}$
$\qquad\qquad 157\frac{2}{3} = \frac{946}{6}.$

$$525 + 946 = 1471.$$

$$\frac{\overset{21}{\cancel{525}}}{\cancel{1471}} \times \$\frac{\overset{\overset{175}{\cancel{350}}}{\cancel{514850}}}{\underset{\underset{2}{\cancel{4}}}{\cancel{100}}} = \$1837.50, \text{ A's.}$$

$$\$5148.50 - \$1837.50 = \$3311, \text{ B's.}$$

[redacted]

13. Divide £65 9s. among three men, so that the first may have as many half-crowns as the second has shillings; and the second as many guineas as the third has pounds.

1st has $2\frac{1}{2}$ times as much as 2d.

2d has $\frac{21}{20}$ as much as 3d.

3d has 1 part.
2d has $\frac{21}{20}$ part.
1st has $\frac{21}{8}$ part.

3d has 40 parts.
2d has 42 parts.
1st has 105 parts.
All have 187 parts.
£65 9s. = 1309s.

$$\frac{105}{\cancel{187}} \times \overset{7}{\cancel{1309}}\text{s.} = 735\text{s.} = £36\ 15\text{s.}$$

$$\frac{42}{\cancel{187}} \times \overset{7}{\cancel{1309}}\text{s.} = 294\text{s.} = £14\ 14\text{s.}$$

$$\frac{40}{\cancel{187}} \times \overset{7}{\cancel{1309}}\text{s.} = 280\text{s.} = £14.$$

14. A and B begin business each with a capital of \$2000. A adds \$500 at the end of 2 months, and \$500 more at the end of 7 months; B adds \$800 at the end of 3 months. If the profits are \$3605.25 at the end of a year, what is the share of each?

$12 \times \$2000 = \$24000$		$12 \times \$2000 = \$24000$
$10 \times 500 = 5000$		$9 \times 800 = 7200$
$5 \times 500 = 2500$		$\$31200$
$\$31500$		

$$\$31,500 + \$31,200 = \$62,700.$$

$$\frac{\overset{\overset{21}{\cancel{315}}}{\cancel{31500}}}{\underset{\underset{4}{\cancel{4180}}}{\cancel{62700}}} \times \$\frac{\overset{345}{\cancel{360525}}}{\cancel{100}} = \$1811.25\text{, A's.}$$

$$\$3605.25 - \$1811.25 = \$1794\text{, B's.}$$

15. Three partners in a restaurant furnish respectively \$500 for 7 months, \$600 for 8 months, and \$900 for 9 months. If they lose \$410, what is each one's share of the loss?

$7 \times \$500 = \$3500$
$8 \times 600 = 4800$
$9 \times 900 = 8100$
$\$16400$

$$\frac{\overset{120}{\cancel{4800}}}{\underset{\cancel{40}}{\cancel{16400}}} \times \$\cancel{410} = \$120.$$

$$\frac{\overset{175}{\cancel{3500}}}{\underset{\underset{2}{\cancel{40}}}{\cancel{16400}}} \times \$\cancel{410} = \$\frac{175}{2} = \$87.50.$$

$$\frac{\overset{405}{\cancel{8100}}}{\underset{\underset{2}{\cancel{40}}}{\cancel{16400}}} \times \$\cancel{410} = \$\frac{405}{2} = \$202.50.$$

[redacted]

16. Two capitalists contribute, one \$10,000, the other \$12,000, to an enterprise which continues in operation for 10 years. Ten months after starting a third man becomes a partner and contributes \$15,000; and 2 years after this a fourth man contributes \$17,400. If the total profits are \$45,600, what amount does each partner receive?

$$\begin{aligned} 120 \times \$10{,}000 &= \$1200000 \\ 120 \times 12{,}000 &= 1440000 \\ 110 \times 15{,}000 &= 1650000 \\ 86 \times 17{,}400 &= \underline{1496400} \\ & \quad \$5786400 \end{aligned}$$

$$\frac{\overset{500}{\cancel{1200000}}}{\underset{2411}{\cancel{5786400}}} \times \$45600 = \$\frac{22800000}{2411} = \$9456.66.$$

$$\frac{\overset{600}{\cancel{1440000}}}{\underset{2411}{\cancel{5786400}}} \times \$45600 = \$\frac{27360000}{2411} = \$11{,}347.99.$$

$$\frac{\overset{1375}{\cancel{1650000}}}{\underset{\underset{2411}{\cancel{4822}}}{\cancel{5786400}}} + \$\overset{22800}{\cancel{45600}} = \$\frac{31350000}{2411} = \$13{,}002.90.$$

$$\frac{\overset{1247}{\cancel{1496400}}}{\underset{\underset{2411}{\cancel{4822}}}{\cancel{5786400}}} \times \$\overset{22800}{\cancel{45600}} = \$\frac{28431600}{2411} = \$11{,}792.45.$$

17. A began business with a capital of \$2500. After three years he invested \$1250 more, and took as a partner B, who invested \$5000. At the end of four years more the profits amounted to \$9562.50. What was the share of each?

A.	B.
$7 \times \$2500 = \$17500$	$4 \times \$5000 = \$20000.$
$4 \times 1250 = \underline{5000}$	
$\$22500$	

$$\$22{,}500 + \$20{,}000 = \$42{,}500.$$

$$\frac{\overset{9}{\cancel{22500}}}{\underset{\cancel{17}}{\cancel{42500}}} \times \$\frac{\overset{1125}{\cancel{19125}}}{2} = \$\frac{10125}{2} = \$5062.50.$$

$$\frac{\overset{\overset{4}{\cancel{8}}}{\cancel{20000}}}{\underset{\cancel{17}}{\cancel{42500}}} \times \$\frac{\overset{1125}{\cancel{19125}}}{\cancel{2}} = \$4500.$$

[redacted]

Exercise 106. Page 235.

1. There were 125 pupils at school on Monday, 130 on Tuesday, 128 on Wednesday, 132 on Thursday, and 125 on Friday. What was the average daily attendance?

$$\begin{array}{r} 125 \\ 130 \\ 128 \\ 132 \\ \underline{125} \\ 5\overline{)640} \\ 128 \text{ } \textit{Ans.} \end{array}$$

2. A spring of water that yields 250 gal. an hour supplies a town containing 360 families. What is the average daily supply of water for each family?

$$\frac{\cancel{24}\times\overset{50}{\cancel{250}}}{\underset{\underset{3}{\cancel{15}}}{\cancel{360}}}\text{ gal.} = \frac{50}{3}\text{ gal.} = 16\tfrac{2}{3}\text{ gal. } \textit{Ans.}$$

3. A wine merchant put into an empty cask 15 qt. of brandy costing \$1.10 a quart, 66 qt. costing \$1.20 a quart, and 43 qt. costing \$1.40 a quart. At what price per quart must he sell the brandy to gain one fifth of the cost?

$$\begin{array}{rrr} \$1.10 & \$1.20 & \$1.40 \\ \underline{15} & \underline{66} & \underline{43} \\ 550 & 720 & 420 \\ \underline{110} & \underline{720} & \underline{560} \\ \$16.50 & \$79.20 & \$60.20 \end{array}$$

$$\begin{array}{r} \$16.50 \\ 79.20 \\ \underline{60.20} \\ 5\overline{)\$155.90} \\ \underline{31.18} \\ \$187.08 \end{array} \qquad \begin{array}{r} 15\text{ qt.} \\ 66 \\ \underline{43} \\ 124\text{ qt.} \end{array} \qquad \begin{array}{r} \$1.50 \\ 124\overline{)\$187.08} \\ \underline{124} \\ 630 \\ \underline{620} \\ 108 \end{array}$$

\$1.51. *Ans.*

4. A grocer mixed 120 lb. of tea costing 50 cents a pound with 180 lb. costing 40 cents a pound. At what price per pound must he sell the mixture to make a profit of \$30 on the whole?

$$\begin{array}{rl} 120 \times \$0.50 = & \$60.00 \\ \underline{180 \times \ \ 0.40 =} & \underline{72.00} \\ 300 & \$132.00 \\ & \underline{30.} \\ & \$162. \end{array}$$

$$3\cancel{00})\underline{\$1.62}$$
$$\$0.54 \ \textit{Ans.}$$

5. A grocer buys two kinds of tea at 40 cents a pound and 56 cents a pound, respectively, and mixes them in the ratio of 5 to 3. What is his profit, if he sells 56 lb. of the mixture at 84 cents a pound?

$$5 + 3 = 8.$$

$$\frac{5}{\cancel{8}} \times \overset{7}{\cancel{56}}\text{ lb.} = 35\text{ lb.}$$

$$\frac{3}{\cancel{8}} \times \overset{7}{\cancel{56}}\text{ lb.} = 21\text{ lb.}$$

$$\begin{array}{rl} 35 \times \$0.40 = & \$14.00 \\ \underline{21 \times \ \ 0.56 =} & \underline{11.76} \\ 56 & \$25.76 \end{array}$$

$$56 \times \$0.84 = \$47.04.$$

$$\$47.04 - \$25.76 = \$21.28. \ \textit{Ans.}$$

[redacted]

6. The average length of ten sticks is 2 ft. 10½ in.; one stick is 27½ in. long, another 37½ in. long, and the remaining eight are of the same length. What is the length of one of the remaining eight?

2 ft. 10½ in. = 34½ in.

10 × 34½ in. = 345 in.

27½ in. + 37½ in. = 65 in.

$$\frac{345 \text{ in.} - 65 \text{ in.}}{8} = \frac{280 \text{ in.}}{8} = 35 \text{ in.}$$ *Ans.*

7. The average age of the boys in the four classes of a school is 18.4 yr., 17.9 yr., 16.8 yr., and 15.7 yr. The classes contain 29, 33, 34, and 33 boys, respectively. What is the average age of the boys in the school?

18.4	17.9	16.8	15.7
29	33	34	33
1656	537	672	471
368	537	504	471
533.6	590.7	571.2	518.1

533.6	29
590.7	33
571.2	34
518.1	33
2213.6	129

$$\begin{array}{r} 17.1 \\ 129\overline{)2213.6} \\ 129 \\ \hline 923 \\ 903 \\ \hline 206 \\ 129 \\ \hline 77 \end{array}$$

17.2 yr. *Ans.*

8. Seven boys weigh respectively 119.7 lb., 105 lb., 178.3 lb., 165.3 lb., 142.8 lb., 109 lb., 154.2 lb. What is their average weight?

$$\begin{array}{r} 119.7 \text{ lb.} \\ 105. \phantom{\text{ lb.}} \\ 178.3 \phantom{\text{ lb.}} \\ 165.3 \phantom{\text{ lb.}} \\ 142.8 \phantom{\text{ lb.}} \\ 109. \phantom{\text{ lb.}} \\ 154.2 \phantom{\text{ lb.}} \\ 7\overline{)974.3} \text{ lb.} \\ 139.2 \text{ lb.} \end{array}$$ *Ans.*

9. In what proportion should tea costing 60 cents a pound be mixed with tea costing 45 cents a pound that the cost of the mixture should be 54 cents a pound?

1 lb. of the 60-cent tea loses in value \$0.06, and 1 lb. of the 45-cent tea gains in value \$0.09. Hence, to make a mixture worth \$0.54 a pound, the tea must be mixed in the ratio 9 : 6; that is, 3 : 2. *Ans.*

10. A merchant has teas that cost 80 cents, 60 cents, and 40 cents a pound, respectively. How many pounds of each kind shall he take to make a mixture of 1000 lb., so that in selling it at 70 cents a pound he may make a profit of 8 cents a pound?

The cost of the mixture must be \$0.70 − \$0.08 = \$0.62 a pound.

The 80-cent tea loses in value \$0.18; the 60-cent gains in value \$0.02; and the 40-cent gains in value \$0.22. Hence, the merchant must mix the 80-cent and 60-cent in the ratio 2 : 18, that is, 1 : 9, and the 80-cent and the 40-cent in the ratio 22 : 18, that is, 11 : 9. Therefore, he takes the 80-cent, the 60-cent, and the 40-cent proportionally to 12 : 9 : 9; that is, 4 : 3 : 3.

Hence, he takes $\frac{4}{10}$ of 1000 lb. = 400 lb. of the 80-cent; $\frac{3}{10}$ of 1000 lb. = 300 lb. of the 60-cent; and $\frac{3}{10}$ of 1000 lb. = 300 lb. of the 40-cent. *Ans.*

11. A grocer mixed black tea that cost him 28 cents a pound with green tea that cost him 42 cents, and by selling the mixture at 35 cents a pound he gained $\frac{1}{6}$ of its cost. What was the actual cost of the mixture a pound? In what ratio were the teas mixed?

If by selling the tea at 35 cents the merchant gained $\frac{1}{6}$ of its cost, the actual cost of the mixture was $\frac{6}{7}$ of 35 cents; that is, 30 cents. *Ans.*

The 42-cent tea loses in value 12 cents, and the 28-cent gains in value 2 cents. Hence, the merchant mixed the 42-cent and the 28-cent teas in the ratio 2 : 12; that is, 1 : 6. *Ans.*

12. A dealer has an order for 1000 bu. of wheat at 70 cents a bushel. In what proportion shall he mix three kinds of wheat at 66, 69, and 72 cents a bushel to fill the order?

The 66-cent wheat gains in value 4 cents a bushel and the 72-cent loses in value 2 cents.

Hence, the dealer must mix the 66-cent and 72-cent in the ratio 2 : 4; that is, 1 : 2.

The 69-cent wheat gains in value 1 cent and the 72-cent loses in value 2 cents.

Hence, the dealer must mix the 69-cent and the 72-cent in the ratio 2 : 1.

Therefore, the dealer must take the 66-cent, the 69-cent, and the 72-cent proportionally to 1 : 2 : 3. *Ans.*

13. A wine merchant mixes wines that cost \$0.95, \$1.05, \$1.10, and \$1.20 a gallon to make a mixture costing \$1.00 per gallon. How many gallons of each kind of wine does he take?

[redacted]

The 95-cent wine gains in value 5 cents, and the 105-cent loses in value 5 cents.

Hence, the merchant must take the 95-cent and the 105-cent in the ratio 1 : 1.

The 95-cent wine gains in value 5 cents, and the 110-cent loses in value 10 cents.

Hence, the merchant must take the 95-cent and the 110-cent in the ratio 2 : 1.

The 95-cent wine gains in value 5 cents, and the 120-cent loses in value 20 cents.

Hence, the merchant must take the 95-cent and the 120-cent in the ratio 4 : 1.

Therefore, the merchant may take the 95-cent wine, the 105-cent, the 110-cent, and the 120-cent proportionally to 7 : 1 : 1 : 1. *Ans.*

14. A merchant wishes to fill a barrel that will hold 240 lb. of sugar with sugar costing $4\frac{1}{2}$, $4\frac{3}{4}$, and $5\frac{1}{8}$ cents a pound, respectively, so that the mixture may cost $4\frac{7}{8}$ cents a pound. How many pounds of each kind shall he take?

The $4\frac{1}{2}$-cent sugar gains in value $\frac{3}{8}$ cent, and the $5\frac{1}{8}$-cent loses in value $\frac{2}{8}$ cent.

Hence, the merchant must take the $4\frac{1}{2}$-cent and the $5\frac{1}{8}$-cent in the ratio $\frac{2}{8} : \frac{3}{8}$; that is, 2 : 3.

The $4\frac{3}{4}$-cent sugar gains in value $\frac{1}{8}$ cent, and the $5\frac{1}{8}$-cent loses in value $\frac{2}{8}$ cent.

Hence, the merchant must take the $4\frac{3}{4}$-cent and the $5\frac{1}{8}$-cent in the ratio $\frac{2}{8} : \frac{1}{8}$; that is, 2 : 1.

Therefore, the merchant may take the $4\frac{1}{2}$-cent, the $4\frac{3}{4}$-cent, and the $5\frac{1}{8}$-cent proportionally to 2 : 2 : 4; that is, 1 : 1 : 2.

Hence, the merchant may take $\frac{1}{4}$ of 240 lb. = 60 lb. of the $4\frac{1}{2}$-cent, $\frac{1}{4}$ of 240 lb. = 60 lb. of the $4\frac{3}{4}$-cent, and $\frac{1}{2}$ of 240 lb. = 120 lb. of the $5\frac{1}{8}$-cent. *Ans.*

15. A grocer wishes to mix 12 lb. of coffee at 40 cents a pound and 20 lb. at 35 cents a pound with coffee at 28 cents a pound, so that the mixture may be worth 30 cents a pound. How many pounds at 28 cents must he use?

$$
\begin{array}{lrr}
12 \times & \$0.40 = & \$4.80 \\
\underline{20} \times & 0.35 = & \underline{7.00} \\
32 & & \$11.80
\end{array}
$$

Hence, the average cost of the 32 lb. is $\$\frac{11.80}{32} = \$0.36\frac{7}{8}$.

The 28-cent gains in value 2 cents, and the $36\frac{7}{8}$-cent loses in value $6\frac{7}{8}$ cents.

Hence, the grocer must mix the 28-cent and the $36\frac{7}{8}$-cent in the ratio $6\frac{7}{8} : 2$; that is, $55 : 16$, or 110 to 32.

Hence, the grocer must use 110 lb. at 28 cents. *Ans.*

16. A grocer mixed 14 lb. of coffee costing 32 cents a pound, 18 lb. costing 35 cents a pound, 22 lb. costing 38 cents a pound, and 40 lb. costing 30 cents a pound. What is the cost of the mixture per pound, and at what price must he sell it to gain 0.25 of the cost?

$$\begin{array}{rlrr}
14 & \times & \$0.32 = & \$4.48 \\
18 & \times & 0.35 = & 6.30 \\
22 & \times & 0.38 = & 8.36 \\
40 & \times & 0.30 = & 12.00 \\
\hline
94 & & & \$31.14
\end{array}$$

Therefore, the cost of the mixture per pound is

$$\$\frac{31.14}{94} = \$0.331277. \quad \textit{Ans.}$$

$$\begin{array}{r}
4)\ \$0.331277 \\
0.082819 \\
\hline
\$0.414096
\end{array} \quad \textit{Ans.}$$

17. In what proportion may oils costing \$1.20, \$0.80, and \$0.60 a gallon be mixed that the mixture may cost \$0.70 a gallon?

The 120-cent oil loses in value 50 cents a gallon, and the 60-cent gains in value 10 cents.

Therefore, the 120-cent and the 60-cent must be mixed in the ratio $10 : 50$; that is, $1 : 5$.

The 80-cent oil loses in value 10 cents and the 60-cent gains in value 10 cents.

Therefore, the 80-cent and the 60-cent oils must be mixed in the ratio $10 : 10$; that is, $1 : 1$.

Hence, the 120-cent, the 80-cent, and the 60-cent oils may be mixed proportionally to $1 : 1 : 6$. *Ans.*

[redacted]

Exercise 107. Page 237.

Reduce to a common fraction:

1. $20\% = \frac{20}{100} = \frac{1}{5}$.
2. $80\% = \frac{80}{100} = \frac{4}{5}$.
3. $25\% = \frac{25}{100} = \frac{1}{4}$.
4. $50\% = \frac{50}{100} = \frac{1}{2}$.
5. $75\% = \frac{75}{100} = \frac{3}{4}$.
6. $5\% = \frac{5}{100} = \frac{1}{20}$.
7. $10\% = \frac{10}{100} = \frac{1}{10}$.
8. $12\frac{1}{2}\% = \frac{12\frac{1}{2}}{100} = \frac{1}{8}$.
9. $16\frac{2}{3}\% = \frac{16\frac{2}{3}}{100} = \frac{1}{6}$.
10. $11\frac{1}{9}\% = \frac{11\frac{1}{9}}{100} = \frac{1}{9}$.
11. $62\frac{1}{2}\% = \frac{62\frac{1}{2}}{100} = \frac{5}{8}$.
12. $87\frac{1}{2}\% = \frac{87\frac{1}{2}}{100} = \frac{7}{8}$.
13. $66\frac{2}{3}\% = \frac{66\frac{2}{3}}{100} = \frac{2}{3}$.
14. $37\frac{1}{2}\% = \frac{37\frac{1}{2}}{100} = \frac{3}{8}$.
15. $83\frac{1}{3}\% = \frac{83\frac{1}{3}}{100} = \frac{5}{6}$.
16. $18\frac{3}{4}\% = \frac{18\frac{3}{4}}{100} = \frac{3}{16}$.
17. $95\% = \frac{95}{100} = \frac{19}{20}$.
18. $70\% = \frac{70}{100} = \frac{7}{10}$.
19. $144\frac{4}{9}\% = \frac{144\frac{4}{9}}{100} = \frac{13}{9}$.
20. $262\frac{1}{2}\% = \frac{262\frac{1}{2}}{100} = \frac{21}{8}$.

Exercise 108. Page 238.

Express as a rate per cent:

1. $\frac{1}{2} = \frac{1}{\cancel{2}}$ of $\overset{50}{\cancel{100}}\% = 50\%$.
2. $\frac{1}{4} = \frac{1}{\cancel{4}}$ of $\overset{25}{\cancel{100}}\% = 25\%$.
3. $\frac{3}{8} = \frac{3}{\underset{2}{\cancel{8}}}$ of $\overset{25}{\cancel{100}}\% = \frac{75}{2}\% = 37\frac{1}{2}\%$.
4. $\frac{1}{3} = \frac{1}{3}$ of $100\% = \frac{100}{3}\% = 33\frac{1}{3}\%$.

5. $\frac{1}{6} = \frac{1}{\underset{3}{\cancel{6}}}$ of $\overset{50}{\cancel{100}}\% = \frac{50}{3}\% = 16\frac{2}{3}\%$.

6. $\frac{5}{6} = \frac{5}{\underset{3}{\cancel{6}}}$ of $\overset{50}{\cancel{100}}\% = \frac{250}{3}\% = 83\frac{1}{3}\%$.

7. $\frac{2}{3} = \frac{2}{3}$ of $100\% = \frac{200}{3}\% = 66\frac{2}{3}\%$.

8. $\frac{4}{5} = \frac{4}{\cancel{5}}$ of $\overset{20}{\cancel{100}}\% = 80\%$.

9. $\frac{8}{25} = \frac{8}{\cancel{25}}$ of $\overset{4}{\cancel{100}}\% = 32\%$.

10. $\frac{7}{20} = \frac{7}{\cancel{20}}$ of $\overset{5}{\cancel{100}}\% = 35\%$.

11. $\frac{2}{9} = \frac{2}{9}$ of $100\% = \frac{200}{9}\% = 22\frac{2}{9}\%$.

12. $\frac{7}{16} = \frac{7}{\underset{4}{\cancel{16}}}$ of $\overset{25}{\cancel{100}}\% = \frac{175}{4}\% = 43\frac{3}{4}\%$.

13. $\frac{4}{11} = \frac{4}{11}$ of $100\% = \frac{400}{11}\% = 36\frac{4}{11}\%$.

14. $\frac{9}{32} = \frac{9}{\underset{8}{\cancel{32}}}$ of $\overset{25}{\cancel{100}}\% = \frac{225}{8}\% = 28\frac{1}{8}\%$.

15. $0.25 = 25\%$.

16. $0.6 = 0.60 = 60\%$.

17. $0.75 = 75\%$.

18. $0.9 = 0.90 = 90\%$.

19. $0.65 = 65\%$.

20. $0.45 = 45\%$.

21. $0.2 = 0.20 = 20\%$.

22. $0.33333 = 0.33\frac{1}{3} = 33\frac{1}{3}\%$.

23. $0.16667 = 0.16\frac{2}{3} = 16\frac{2}{3}\%$.

24. $0.83333 = 0.83\frac{1}{3} = 83\frac{1}{3}\%$.

25. $0.875 = 0.87\frac{1}{2} = 87\frac{1}{2}\%$.

26. $1.375 = 1.37\frac{1}{2} = 137\frac{1}{2}\%$.

27. $2.66667 = 2.66\frac{2}{3} = 266\frac{2}{3}\%$.

28. $4.2525 = 4.25\frac{1}{4} = 425\frac{1}{4}\%$.

Exercise 109. Page 240.

Find by using decimals:

1. 23 % of 1728.

23 % = 0.23.

```
 1728
 0.23
-----
 5184
3456
-----
397.44  Ans.
```

2. 44 % of 1861.

44 % = 0.44.

```
 1861
 0.44
-----
 7444
7444
-----
818.84  Ans.
```

3. 87 % of 14.22.

87 % = 0.87.

```
 14.22
  0.87
------
  9954
11376
------
12.3714  Ans.
```

4. 63 % of 2.832.

63 % = 0.63.

```
 2.832
  0.63
------
  8496
16992
------
1.78416  Ans.
```

5. 72 % of 841.

72 % = 0.72.

```
   841
  0.72
------
  1682
 5887
------
605.52  Ans.
```

6. 2 % of 846.

2 % = 0.02.

```
  846
 0.02
-----
16.92  Ans.
```

7. 9 % of 24.87.

9 % = 0.09.

```
 24.87
  0.09
------
2.2383  Ans.
```

8. 122 % of 12.5.

122 % = 1.22.

```
  12.5
  1.22
------
   250
  250
 125
------
15.250  Ans.
```

9. 287 % of 48.2.

287 % = 2.87.

```
   48.2
   2.87
-------
   3374
  3856
 964
-------
138.334  Ans.
```

10. 1 % of 7854.

1 % = 0.01.

```
  7854
  0.01
 78.54  Ans.
```

11. 0.5 % of 144.

0.5 % = 0.005.

```
   144
 0.005
 0.720  Ans.
```

12. 8752 % of 2645.

8752 % = 87.52.

```
      2645
     87.52
      5290
    13225
   18515
  21160
 231490.40  Ans.
```

13. 0.02 % of 52.36.

0.02 % = 0.0002.

```
    52.36
   0.0002
 0.010472  Ans.
```

14. 2 % of 3.

2 % = 0.02.

```
    3
 0.02
 0.06  Ans.
```

15. 2.06 % of 312.

2.06 % = 0.0206.

```
    312
 0.0206
   1872
   624
 6.4272  Ans.
```

Find by using common fractions:

16. $33\frac{1}{3}$ % of 363.

$33\frac{1}{3}\% = \frac{1}{3}$.

$\frac{1}{\cancel{3}}$ of $\overset{121}{\cancel{363}} = 121$. *Ans.*

17. 20 % of 545.

$20\% = \frac{1}{5}$.

$\frac{1}{\cancel{5}}$ of $\overset{109}{\cancel{545}} = 109$. *Ans.*

18. 25 % of 1728.

$25\% = \frac{1}{4}$.

$\frac{1}{\cancel{4}}$ of $\overset{432}{\cancel{1728}} = 432$. *Ans.*

19. 50 % of 8642.

$50\% = \frac{1}{2}$.

$\frac{1}{\cancel{2}}$ of $\overset{4321}{\cancel{8642}} = 4321$. *Ans.*

20. 75 % of 432.

$75\% = \frac{3}{4}$.

$\frac{3}{\cancel{4}}$ of $\overset{108}{\cancel{432}} = 324$. *Ans.*

21. $62\frac{1}{2}$ % of 216.

$62\frac{1}{2}\% = \frac{5}{8}$.

$\frac{5}{\cancel{8}}$ of $\overset{27}{\cancel{216}} = 135$. *Ans.*

22. $37\frac{1}{2}$ % of 360.

$37\frac{1}{2}\% = \frac{3}{8}$.

$\frac{3}{\cancel{8}}$ of $\overset{45}{\cancel{360}} = 135$. *Ans.*

23. $83\frac{1}{3}\%$ of 486.

$83\frac{1}{3}\% = \frac{5}{6}.$

$\frac{5}{\cancel{6}}$ of $\overset{81}{\cancel{486}} = 405.$ *Ans.*

24. $66\frac{2}{3}\%$ of 456.

$66\frac{2}{3}\% = \frac{2}{3}.$

$\frac{2}{\cancel{3}}$ of $\overset{152}{\cancel{456}} = 304.$ *Ans.*

25. $12\frac{1}{2}\%$ of 2.56.

$12\frac{1}{2}\% = \frac{1}{8}.$

$\frac{1}{8}$ of $2.56 = 0.32.$ *Ans.*

26. $14\frac{2}{7}\%$ of 81.9.

$14\frac{2}{7}\% = \frac{1}{7}.$

$\frac{1}{7}$ of $81.9 = 11.7.$ *Ans.*

27. $22\frac{2}{9}\%$ of 8.19.

$22\frac{2}{9}\% = \frac{2}{9}.$

$\frac{2}{\cancel{9}}$ of $\overset{0.91}{\cancel{8.19}} = 1.82.$ *Ans.*

28. $168\frac{3}{4}\%$ of 256.

$168\frac{3}{4}\% = 1\frac{11}{16}.$

$1\frac{11}{16}$ of $256 = \frac{27}{\cancel{16}} \times \overset{16}{\cancel{256}} = 432.$ *Ans.*

29. $143\frac{3}{4}\%$ of 288.

$143\frac{3}{4}\% = 1\frac{7}{16}.$

$1\frac{7}{16}$ of $288 = \frac{23}{\cancel{16}} \times \overset{18}{\cancel{288}} = 414.$ *Ans.*

30. 70% of 8432.

$70\% = \frac{7}{10}.$

$\frac{7}{\underset{5}{\cancel{10}}}$ of $\overset{4216}{\cancel{8432}} = \frac{29512}{5} = 5902\frac{2}{5}.$ *Ans.*

31. The population of a town in 1880 was 12,275, and it increased 8 % in the next ten years. Find the population of the town in 1890.

12275	12275
0.08	982
982.00	13257 *Ans.*

32. How much metal will be obtained from 365 tons of ore, if the ore contains 7 % of metal?

365 t.
0.07
25.55 t. *Ans.*

33. If gunpowder contains 75 % of saltpetre, 10 % of sulphur, 15 % of charcoal, how many pounds of each are there in a ton of powder?

1 t. = 2000 lb.

0.75	0.10	0.15
2000	2000	2000
1500.	200.	300.

Saltpetre, 1500 lb.; sulphur, 200 lb.; charcoal, 300 lb. *Ans.*

34. Air is composed by volume of 20.0265 % of oxygen and 79.9735 % of nitrogen. How many cubic feet of oxygen in 1750 cu. ft. of air?

0.200265
1750
10013250
1401855
200265
350.463750

350.46375 cu. ft. Ans.

35. If 2 % of a regiment of 750 men are killed in an engagement, 6 % are wounded, and 4 % are missing, what is the number still available for service?

$$2\,\% + 6\,\% + 4\,\% = 12\,\%.$$
$$100\,\% - 12\,\% = 88\,\%.$$

$$\begin{array}{r} 750 \\ \underline{0.88} \\ 6000 \\ \underline{6000} \\ 660. \end{array} \quad \textit{Ans.}$$

36. A man sold a bicycle that cost him \$60, and lost $16\frac{2}{3}\,\%$ of the cost. For what price did he sell it?

$16\frac{2}{3}\,\% = \frac{1}{6}$. $\frac{1}{6}$ of \$60 = \$10.
\$60 − \$10 = \$50. *Ans.*

37. A merchant sold hats that cost him \$1.50 each, and gained $33\frac{1}{3}\,\%$. For what price did he sell them?

$33\frac{1}{3}\,\% = \frac{1}{3}$. $\frac{1}{3}$ of \$1.50 = \$0.50.
\$1.50 + \$0.50 = \$2.00. *Ans.*

38. In a school of 80 children, $17\frac{1}{2}\,\%$ are girls. Find the number of boys.

$$\begin{array}{r} 0.175 \\ \underline{80} \\ 14.000 \end{array} \qquad \begin{array}{r} 80 \\ \underline{14} \\ 66 \end{array} \ \textit{Ans.}$$

39. The lead ore from a certain mine yields 60 % of metal, and of the metal $\frac{3}{4}$ of 1 % is silver. How much silver and how much lead will be obtained from 1200 t. of ore?

$$\begin{array}{r} 1200\ \text{t.} \\ \underline{0.60\phantom{\ \text{t.}}} \\ 720.00\ \text{t.} \end{array} \qquad \begin{array}{r} 0.0075 \\ \underline{720} \\ 1500 \\ \underline{525} \\ 5.4000 \end{array} \qquad \begin{array}{r} 720.\ \ \text{t.} \\ \underline{5.4\phantom{\ \text{t.}}} \\ 714.6\ \text{t.} \end{array}$$

Silver, 5.4 t.; lead, 714.6 t. *Ans.*

40. If 13 % of a population of 27,000,000 are foreign born, how many of the population are foreign born?

$$\begin{array}{r} 27000000 \\ \underline{0.13} \\ 81000000 \\ \underline{27} \\ 3510000. \end{array} \quad \textit{Ans.}$$

41. If iron expands $\frac{1}{8}$ of 1 % when heated 185° F., what will be the expansion of iron when heated from − 20° F. to + 120° F.?

The difference in temperature between − 20° F. and + 120° F. is 140° F.

$$\frac{\overset{\overset{7}{\cancel{28}}}{\cancel{140}}}{\underset{37}{\cancel{185}}} \text{ of } \frac{1}{\underset{2}{\cancel{8}}} \text{ of } 1\,\% = \frac{7}{74} \text{ of } 1\,\%. \quad \textit{Ans.}$$

42. A tubular iron bridge 740 ft. long has one end fast to a pier. How much play must be allowed at the other end for the expansion of the iron, if the climate varies from − 30° F. in winter to + 130° F. in a July sun?

The difference in temperature between − 30° F. and + 130° F. is 160° F.

[redacted]

The expansion for 160° is

$$\frac{\overset{\overset{4}{\cancel{20}}}{\cancel{160}}}{\underset{37}{\cancel{185}}} \text{ of } \frac{1}{\cancel{8}} \text{ of } 1\% = \frac{4}{37} \text{ of } 1\%. \qquad \frac{4}{37} \text{ of } 1\% = \frac{4}{3700}.$$

$$\frac{4}{\underset{5}{\cancel{3700}}} \text{ of } \cancel{740} \text{ ft.} = \frac{4}{5} \text{ ft.} = 9\tfrac{3}{5} \text{ in. } \textit{Ans.}$$

43. How much longer is 100 miles of iron rails at 118° F. than at 20° below zero?

100 mi. = 528,000 ft.

The difference in temperature between 118° F. and – 20° F. is 138° F. The expansion for 138° is

$$\frac{\overset{69}{\cancel{138}}}{185} \text{ of } \frac{1}{\underset{4}{\cancel{8}}} \text{ of } 1\% = \frac{69}{740} \text{ of } 1\%. \qquad \frac{69}{740} \text{ of } 1\% = \frac{69}{74000}.$$

$$\frac{69}{\underset{37}{\cancel{74000}}} \text{ of } \overset{264}{\cancel{528000}} \text{ ft.} = \frac{18216}{37} \text{ ft.} = 492\tfrac{12}{37} \text{ ft. } \textit{Ans.}$$

Exercise 110. Page 242.

1. What per cent of 64 is 16?

$$\frac{16}{64} = \frac{1}{4} = 25\%. \quad \textit{Ans.}$$

2. What per cent of 16 is 64?

$$\frac{64}{16} = 4 = 400\%. \quad \textit{Ans.}$$

3. What per cent of 450 lb. is 50 lb.?

$$\frac{50 \text{ lb.}}{450 \text{ lb.}} = \frac{1}{9} = 11\tfrac{1}{9}\%. \quad \textit{Ans.}$$

4. What per cent of 50 lb. is 450 lb.?

$$\frac{450 \text{ lb.}}{50 \text{ lb.}} = 9 = 900\%. \quad \textit{Ans.}$$

5. What per cent of \$465 is \$130.20?

$$\begin{array}{r} 0.28 \\ \$465\overline{)\$130.2} \\ \underline{930} \\ 3720 \\ \underline{3720} \end{array}$$

28%. *Ans.*

6. What per cent of \$832 is \$807.04?

$$\begin{array}{r} 0.97 \\ \$832\overline{)\$807.04} \\ \underline{7488} \\ 5824 \\ \underline{5824} \end{array}$$

97%. Ans.

7. What per cent of \$987 is \$2289.84?

$$\begin{array}{r} 2.32 \\ \$987\overline{)\$2289.84} \\ \underline{1974} \\ 3158 \\ \underline{2961} \\ 1974 \\ \underline{1974} \end{array}$$

232%. *Ans.*

8. A brick kiln contained 29,800 bricks, but after burning only 29,734 were found in good condition. What per cent had been spoiled in burning?

$$\begin{array}{r} 29800 \\ \underline{29734} \\ 66 \end{array}$$

$$\begin{array}{r} 0.0022 \\ 298\cancel{0}\cancel{0})\overline{0.66} \\ \underline{596} \\ 640 \\ \underline{596} \\ 44 \end{array}$$

0.22%. *Ans.*

9. If a house worth \$4000 rents for \$360 a year, what per cent of its value is the rent?

$$\frac{\$360}{\$4000} = \frac{9}{100} = 9\%. \ \textit{Ans.}$$

10. If 75 bu. of corn are raised from 1 pk. of corn, what per cent is the increase?

75 bu. = 300 pk.

300 ÷ 1 = 300 = 30,000%. *Ans.*

11. Ten years ago the population of a city was 26,275; its present population is 31,530. What is the increase per cent?

$$\begin{array}{r} 31530 \\ \underline{26275} \\ 5255 \end{array}$$

$$\begin{array}{r} 0.2 \\ 26275\overline{)5255.} \\ \underline{52550} \end{array}$$

20%. *Ans.*

12. If $3\frac{3}{4}$ tons of sulphur are required to make $31\frac{1}{4}$ tons of gunpowder, what per cent of gunpowder is sulphur?

$$\frac{3\frac{3}{4}}{31\frac{1}{4}} \text{ of } 100\% = \frac{\overset{3}{\cancel{15}}}{\cancel{4}} \times \frac{\cancel{4}}{\underset{5}{\cancel{125}}} \times \overset{4}{\cancel{100}}\% = 12\%. \ \textit{Ans.}$$

13. If a long ton of ore in a gold mine yields 5 oz. (troy) of gold, what is the yield per cent?

$$5 \text{ oz. troy} = \frac{5}{12} \text{ lb. troy} = \frac{\cancel{5}}{\cancel{12}} \text{ of } \frac{\overset{\overset{12}{\cancel{480}}}{\cancel{5760}}}{\underset{\underset{35}{\cancel{175}}}{\cancel{7000}}} \text{ lb. av.} = \frac{12}{35} \text{ lb. av.}$$

[redacted]

1 long ton = 2240 lb. av.

$$\frac{\frac{12}{35}}{2240} \text{ of } 100\% = \frac{\overset{3}{\cancel{12}}}{\underset{7}{\cancel{35}}} \times \frac{1}{\underset{\underset{28}{\cancel{112}}}{\cancel{2240}}} \times \overset{\cancel{20}}{\cancel{100}}\% = \frac{3}{196}\%. \quad \textit{Ans.}$$

14. If $12\frac{1}{2}$ tons of iron are obtained from 235 tons of ore, what per cent of the ore is iron?

$$\frac{12\frac{1}{2}}{235} \text{ of } 100\% = \frac{\overset{5}{\cancel{25}}}{\cancel{2}} \times \frac{1}{\underset{47}{\cancel{235}}} \times \overset{50}{\cancel{100}}\% = \frac{250}{47}\% = 5\frac{15}{47}\%. \quad \textit{Ans.}$$

15. Find the gain per cent in population in New York from 1880 to 1890, if the population in 1880 was 1,206,594, and in 1890 was 1,513,601.

$$\begin{array}{r} 1513601 \\ \underline{1206594} \\ 306907 \end{array} \qquad \begin{array}{r} 0.2543 \\ 1206594\overline{)306907.} \\ \underline{2413188} \\ 6558820 \\ \underline{6032970} \\ 5258500 \\ \underline{4826376} \\ 4321240 \\ \underline{3619782} \\ 701458 \end{array}$$

25.44 %. *Ans.*

16. Find the gain per cent in population in Chicago from 1880 to 1890, if the population in 1880 was 503,304, and in 1890 was 1,099,850.

$$\begin{array}{r} 1099850 \\ \underline{503304} \\ 596546 \end{array} \qquad \begin{array}{r} 1.1852 \\ 503304\overline{)596546.} \\ \underline{503304} \\ 932420 \\ \underline{503304} \\ 4291160 \\ \underline{4026432} \\ 2647280 \\ \underline{2516520} \\ 1307600 \\ \underline{1006608} \\ 300992 \end{array}$$

118.53 %. *Ans.*

17. Find the gain per cent in population in Philadelphia from 1880 to 1890, if the population in 1880 was 846,981, and in 1890 was 1,046,964.

```
1046964                       0.2361
 846981             846981)199983.
 199983                    1693962
                            3058680
                            2540943
                             5177370
                             5081886
                              954840
                              846981
23.61 %. Ans.                 107859
```

18. Find the gain per cent in population in Brooklyn from 1880 to 1890, if the population in 1880 was 566,689, and in 1890 was 806,343.

```
806343                        0.4229
566689              566689)239654.
239654                     2266756
                            1297840
                            1133378
                             1644620
                             1133378
                              5112420
                              5100201
42.29 %. Ans.                   12219
```

19. Find the gain per cent in population in Boston from 1880 to 1890, if the population in 1880 was 362,535, and in 1890 was 448,477.

```
448477                        0.2370
362535              362535)85942.
 85942                      725070
                            1343500
                            1087605
                             2558950
                             2537745
23.71 %. Ans.                 212050
```

[redacted]

20. If 2 gal. of water are added to 25 gal. of alcohol, what per cent of the mixture is water? What per cent is alcohol?

$$2 \text{ gal.} + 25 \text{ gal.} = 27 \text{ gal.}$$

$$\frac{2}{27} \text{ of } 100\% = \frac{200}{27}\% = 7\tfrac{11}{27}\%. \textit{ Ans.}$$

$$\frac{25}{27} \text{ of } 100\% = \frac{2500}{27}\% = 92\tfrac{16}{27}\%. \textit{ Ans.}$$

21. If 5% of the present population of a town has been the increase in the preceding ten years, what per cent of the population ten years ago has been added?

The population ten years ago was 95% of the present population.

$$\frac{\cancel{5}}{\underset{19}{\cancel{95}}} \text{ of } 100\% = \frac{100}{19}\% = 5\tfrac{5}{19}\%. \textit{ Ans.}$$

22. A man gained in weight in January 3%, and in February lost 3%. What per cent of his weight the first day of January is his weight the first day of March?

Feb. 1 the man weighed 103% of his weight Jan. 1.

Mar. 1 the man weighed 97% of his weight Feb. 1.

Therefore, Mar. 1 the man weighed 97% of 103% of his weight Jan. 1.

$$\begin{array}{r} 103\% \\ \underline{0.97} \\ 721 \\ \underline{927} \\ 99.91\% \end{array} \quad \textit{Ans.}$$

23. If 7 lb. of a certain article loses 3 oz. in weight by drying, what per cent of its original weight is water?

$$7 \text{ lb.} = 112 \text{ oz.} \qquad \frac{3}{\underset{28}{\cancel{112}}} \text{ of } \overset{25}{\cancel{100}}\% = \frac{75}{28}\% = 2\tfrac{19}{28}\%. \textit{ Ans.}$$

24. If 7 lb. of a dry article has lost 3 oz. by drying, what per cent of its original weight was water?

The original weight was 7 lb. + 3 oz. = 115 oz.

$$\frac{3}{\underset{23}{\cancel{115}}} \text{ of } \overset{20}{\cancel{100}}\% = \frac{60}{23}\% = 2\tfrac{14}{23}\%. \textit{ Ans.}$$

25. If a dry article exposed to damp air absorbed 3 oz. of water, and then weighed 7 lb., what per cent of its present weight is water?

$$7 \text{ lb.} = 112 \text{ oz.} \qquad \frac{3}{\underset{28}{\cancel{112}}} \text{ of } \overset{25}{\cancel{100}}\% = \frac{75}{28}\% = 2\tfrac{19}{28}\%. \textit{ Ans.}$$

26. If rosin is melted with 20% of its weight of tallow, what per cent of tallow does the mixture contain?

$$100\% + 20\% = 120\%. \qquad \frac{20}{120} = \frac{1}{6} = 16\tfrac{2}{3}\%. \textit{ Ans.}$$

27. If 20% of a mixture of tallow and rosin is tallow, what per cent of the weight of the rosin is the weight of the tallow?

20% of the mixture is tallow and 80% of the mixture is rosin.

$$\tfrac{20}{80} = \tfrac{1}{4} = 25\%. \textit{ Ans.}$$

28. Nitrogen gas, under standard pressure and temperature, is $\frac{1}{8}$ of 1% of the weight of an equal volume of water. What is the specific gravity of nitrogen? How many gallons of nitrogen will it take to weigh as much as a pint of water?

$$\tfrac{1}{8} \text{ of } 1\% = \tfrac{1}{800}.$$

The specific gravity of nitrogen is $\frac{1}{800} = 0.00125$. *Ans.*

To weigh as much as 1 pt. of water will be required 800 pt. of nitrogen, or 100 gal. *Ans.*

29. Oxygen gas is $\frac{1}{7}$ of 1% of the weight of an equal volume of water. What is its specific gravity? How many gallons of oxygen will it take to weigh as much as a pint of water?

$$\tfrac{1}{7} \text{ of } 1\% = \tfrac{1}{700}.$$

The specific gravity of oxygen is $\frac{1}{700} = 0.00143$. *Ans.*

To weigh as much as 1 pt. of water will be required 700 pt. of oxygen, or $87\frac{1}{2}$ gal. *Ans.*

30. If common air consists of 4 volumes of oxygen to 13 of nitrogen, what is its specific gravity?

Oxygen,	$4 \times \frac{1}{700} = \frac{1}{175}$.
Nitrogen,	$13 \times \frac{1}{800} = \frac{13}{800}$.
	$4 + 13 = 17$.

[redacted]

$$\frac{\frac{1}{175} + \frac{13}{800}}{17} = \frac{1}{17} \text{ of } \frac{123}{5600} = \frac{123}{95200}.$$

```
           0.001292  Ans.
    952ØØ)1.23
           952
           2780
           1904
            8760
            8568
             1920
             1904
               16
```

31. How many gallons of air will it take to weigh as much as a pint of water?

To weigh as much as 1 pt. of water will be required

$\frac{95200}{123}$ pt. of air. $\frac{95200}{123}$ pt. $= 773\frac{121}{123}$ pt. $= 96\frac{92}{123}$ gal. *Ans.*

Exercise 111. Page 244.

1. 15 is $\frac{3}{4}$ of what number? 15 is 75 per cent of what number?

$15 \div \frac{3}{4} = \frac{4}{\cancel{3}}$ of $\overset{5}{\cancel{15}} = 20.$ *Ans.* $15 \div \frac{75}{100} = 20.$ *Ans.*

2. \$500 is 4% of what sum of money?

$\$500 \div \frac{4}{100} = 25 \times \$500 = \$12{,}500.$ *Ans.*

3. Find the number of which 324 is 27%.

$324 \div \frac{27}{100} = \frac{100}{\cancel{27}} \times \overset{12}{\cancel{324}} = 1200.$ *Ans.*

4. 288 is 20% more than what number?

$288 \div \frac{120}{100} = \frac{\overset{5}{\cancel{100}}}{\underset{6}{\cancel{120}}}$ of $\overset{48}{\cancel{288}} = 240.$ *Ans.*

5. 145 is 25% more than what number?

$$145 \div \frac{125}{100} = \frac{\overset{4}{\cancel{100}}}{\underset{5}{\cancel{125}}} \times \overset{29}{\cancel{145}} = 116. \quad \textit{Ans.}$$

6. 1240 is 55% less than what number?

$$1240 \div \frac{45}{100} = \frac{\overset{20}{\cancel{100}}}{\underset{9}{\cancel{45}}} \times 1240 = \frac{24800}{9} = 2755\tfrac{5}{9}. \quad \textit{Ans.}$$

7. 260 is $33\frac{1}{3}$% less than what number?

$$260 \div \frac{66\frac{2}{3}}{100} = \frac{3}{\cancel{2}} \times \overset{130}{\cancel{260}} = 390. \quad \textit{Ans.}$$

8. 91 is 40% more than what number?

$$91 \div \frac{140}{100} = \frac{\overset{5}{\cancel{100}}}{\underset{7}{\cancel{140}}} \times \overset{13}{\cancel{91}} = 65. \quad \textit{Ans.}$$

9. 901 is $6\frac{1}{4}$% more than what number?

$$901 \div \frac{106\frac{1}{4}}{100} = \frac{\overset{16}{\cancel{400}}}{\underset{17}{\cancel{425}}} \times \overset{53}{\cancel{901}} = 848. \quad \textit{Ans.}$$

10. If $8\frac{3}{4}$% of a number is 4140.15, what is the number?

$$4140.15 \div \frac{8\frac{3}{4}}{100} = \frac{400}{\cancel{35}} \times \overset{118.29}{\cancel{4140.15}} = 47{,}316. \quad \textit{Ans.}$$

11. If 3% of a number is $2\frac{5}{8}$, what is the number?

$$2\tfrac{5}{8} \div \frac{3}{100} = \frac{\overset{25}{\cancel{100}}}{\cancel{3}} \times \frac{\overset{7}{\cancel{21}}}{\underset{2}{\cancel{8}}} = \frac{175}{2} = 87\tfrac{1}{2}. \quad \textit{Ans.}$$

12. If 140% of a number is 630, what is the number?

$$630 \div \frac{140}{100} = \frac{\overset{50}{\cancel{100}}}{\underset{2}{\cancel{140}}} \times \overset{9}{\cancel{630}} = 450. \quad \textit{Ans.}$$

[redacted]

13. If $6\frac{1}{4}\%$ of a number is 33.25, what is the number?

$$33.25 \div \frac{6\frac{1}{4}}{100} = \frac{400}{\cancel{25}} \times \overset{1.33}{\cancel{33.25}} = 532. \quad \textit{Ans.}$$

14. A town, after decreasing 11%, has 4539 inhabitants. Find its number at first.

$$4539 \div \frac{89}{100} = \frac{100}{\cancel{89}} \text{ of } \overset{51}{\cancel{4539}} = 5100. \quad \textit{Ans.}$$

15. In a certain school there are 200 girls, and the number of girls is 40% of the whole number of pupils. How many pupils in the school?

$$200 \div \frac{40}{100} = \frac{100}{\cancel{40}} \times \overset{5}{\cancel{200}} = 500. \quad \textit{Ans.}$$

16. A manufactory uses 24 tons of coal a day, 20% of which is lost in smoke. How much coal would be needed if this waste could be prevented?

$$100\% - 20\% = 80\% = \tfrac{4}{5}.$$

$$\tfrac{4}{5} \text{ of } 24 \text{ t.} = \tfrac{96}{5} \text{ t.} = 19\tfrac{1}{5} \text{ t.} \quad \textit{Ans.}$$

17. A town, after decreasing 25%, has 4539 inhabitants. Find its number at first.

$$4539 \div \frac{75}{100} = \frac{\overset{4}{\cancel{100}}}{\underset{\cancel{3}}{\cancel{75}}} \times \overset{1513}{\cancel{4539}} = 6052. \quad \textit{Ans.}$$

18. If the ore from a mine yields $\frac{3}{80}$ of 1% of pure gold, how many long tons of ore must be taken to obtain 7 lb. (troy) of gold?

$$\tfrac{3}{80} \text{ of } 1\% = \tfrac{3}{8000}. \qquad \tfrac{3}{8000} \text{ of 1 long ton} = \tfrac{3}{8000} \text{ of 2240 lb.}$$

$$7 \text{ lb. troy} = 7 \times \frac{5760}{7000} \text{ lb. av.}$$

$$\left(7 \times \frac{5760}{7000}\right) \div \left(\frac{3}{8000} \text{ of } 2240\right)$$

$$= \cancel{7} \times \frac{\overset{6}{\overset{\cancel{18}}{\cancel{5760}}}}{\underset{\cancel{7}}{\cancel{7000}}} \times \frac{\overset{8}{\cancel{8000}}}{\cancel{3}} \times \frac{1}{\underset{7}{\cancel{2240}}}$$

$$= \frac{48}{7} = 6\tfrac{6}{7}. \qquad 6\tfrac{6}{7} \text{ long tons. } \textit{Ans.}$$

[redacted]

19. Goods were sold, at a loss of 3 %, for \$2667.50. What was the cost?

$$\$2667.50 \div \frac{97}{100} = \frac{100}{\cancel{97}} \times \$\overset{27.50}{\cancel{2667.50}} = \$2750. \ \textit{Ans.}$$

20. A tradesman, in selling goods, deducts from the marked price 5 % for cash. What was the marked price of goods for which he received \$14.25?

$$\$14.25 \div \frac{95}{100} = \frac{100}{\cancel{95}} \times \$\overset{0.15}{\cancel{14.25}} = \$15. \ \textit{Ans.}$$

21. If an ore loses $41\frac{1}{2}$ % of its weight in roasting, and $43\frac{3}{4}$ % of the remainder in smelting, how much ore will be required to yield 1000 tons of metal?

The part remaining after roasting is $100\% - 41\frac{1}{2}\% = 58\frac{1}{2}\%$.

The part remaining after smelting is

$$58\frac{1}{2}\% - 43\frac{3}{4}\% \text{ of } 58\frac{1}{2}\% = 58\frac{1}{2}\% - 25\frac{19}{32}\% = 32\frac{29}{32}\%.$$

$$1000 \div \frac{32\frac{29}{32}}{100} = 1000 \times \frac{32}{1053} \times 100 = \frac{3200000}{1053} = 3038\tfrac{986}{1053} = 3038.936.$$

3038.936 t. *Ans.*

22. How many pounds of tallow must be mixed with $8\frac{1}{2}$ pounds of rosin that the mixture may contain 15 % of tallow?

The mixture contains 15 % of tallow and 85 % of rosin.

$$8\frac{1}{2} \div \frac{85}{15} = \frac{\overset{3}{\cancel{15}}}{\underset{5}{\cancel{85}}} \times \frac{\cancel{17}}{2} = \frac{3}{2} = 1\frac{1}{2}. \ \textit{Ans.}$$

Exercise 112. Page 246.

1. Find the net amount of a bill of \$1550, if a discount of 5 % is made for cash.

20 | \$1550.
77.50
\$1472.50 *Ans.*

2. Find the net amount of a bill of \$88, if the discounts are 20 and 10.

5 | \$88.
17.60
10 | \$70.40
7.04
\$63.36 *Ans.*

[redacted]

3. Find the net cash amount of a bill of $800, if the discounts are 75, 5, and $2\frac{1}{2}$.

```
 4 | $800.
20 | $200.
   |   10.
40 | $190.
   |   4.75
   $185.25  Ans.
```

4. Find the net cash amount of a bill of $272, if the discounts are $\frac{1}{2}$, 10, and 5.

```
 2 | $272.
10 | $136.
   |  13.60
20 | $122.40
   |   6.12
   $116.28  Ans.
```

5. Find the net cash amount of a bill of $1440, if the discounts are 55, 10, and 5.

```
20 | $1440.
     $72.
       9.
10 | $648.
   |  64.80
20 | $583.20
   |  29.16
   $554.04  Ans.
```

6. Find the net cash amount of a bill of $1125, if the discounts are $\frac{1}{2}$, 10, 10, 10, and 5.

```
 2 | $1125.
10 | $562.50
   |   56.25
10 | $506.25
   |   50.62
10 | $455.63
   |   45.56
20 | $410.07
   |   20.50
   $389.57  Ans.
```

7. Find the net amount of a bill of $872.29, if the discounts are $\frac{1}{3}$, 20, and 25.

```
3 | $872.29
  |  290.76
5 | $581.53
  |  116.31
4 | $465.22
  |  116.30
  $348.92  Ans.
```

8. Find the difference between a single discount of 50 % and two successive discounts of 25 % and 25 % off a bill of $1272.36.

```
2 | $1272.36        4 | $1272.36
    $636.18           |   318.09
                    4 | $954.27
                      |   238.57
                        $715.70

$715.70
 636.18
$79.52  Ans.
```

9. An agent bought 25 sewing machines with 15, 10, and 5 off the list price of \$40 each, and sold them at a discount of 10% off the list price. What was the net amount he received for the sewing machines and his profit?

25 × \$40 = \$1000.

$$
\begin{array}{r|r}
20 & \$1000. \\
 & 50. \\
 & 17 \\ \hline
10 & \$850. \\
 & 85. \\ \hline
20 & \$765. \\
 & 38.25 \\ \hline
 & \$726.75
\end{array}
\qquad
\begin{array}{r|r}
10 & \$1000. \\
 & 100. \\ \hline
 & \$900.\ \textit{Ans.}
\end{array}
\qquad
\begin{array}{r}
\$900. \\
726.75 \\ \hline
\$173.25\ \textit{Ans.}
\end{array}
$$

10. An agent bought a bicycle with 25 and 5 off the list price of \$100. If he received an additional discount of $2\frac{1}{2}$% for cash, and sold the bicycle at a discount of $12\frac{1}{2}$% off the list price, what was the selling price and his profit?

$$
\begin{array}{r|r}
4 & \$100. \\
 & 25. \\ \hline
20 & \$75. \\
 & 3.75 \\ \hline
40 & \$71.25 \\
 & 1.78 \\ \hline
 & \$69.47
\end{array}
\qquad
\begin{array}{r|r}
8 & \$100. \\
 & 12.50 \\ \hline
 & \$87.50\ \textit{Ans.}
\end{array}
\qquad
\begin{array}{r}
\$87.50 \\
69.47 \\ \hline
\$18.03\ \textit{Ans.}
\end{array}
$$

11. A collector collects 65% of a debt of \$727, and charges 5% of the amount he collected. What was the net amount for the creditor?

$$
\begin{array}{r|r}
 & \$727 \\
 & 0.65 \\ \hline
 & 3635 \\
 & 4362 \\ \hline
20 & \$472.55 \\
 & 23.63 \\ \hline
 & \$448.92\ \textit{Ans.}
\end{array}
$$

Exercise 113. Page 248.

1. If goods are bought for \$415, and sold for \$500, what is the gain per cent?

Gain = \$500 − \$415 = \$85.

$$\frac{\overset{17}{\cancel{85}}}{\underset{83}{\cancel{415}}} \text{ of } 100\% = \frac{1700}{83}\% = 20\tfrac{40}{83}\%.\ \textit{Ans.}$$

2. If goods are bought for \$415, and sold for \$400, what is the loss per cent?

Loss = \$415 − \$400 = \$15.

$$\frac{\overset{3}{\cancel{15}}}{\underset{83}{\cancel{415}}} \text{ of } 100\% = \frac{300}{83}\% = 3\tfrac{51}{83}\%.\ \textit{Ans.}$$

[redacted]

3. A farmer buys 24 head of cattle at \$80 a head. After losing 6 head, he sells the remainder at \$105 a head. What does he gain or lose per cent?

$$24 \times \$80 = \$1920.$$

$$18 \times \$105 = \$1890.$$

$$\text{Loss} = \$1920 - \$1890 = \$30.$$

$$\frac{\cancel{30}}{\underset{\underset{16}{\cancel{64}}}{\cancel{1920}}} \text{ of } \overset{25}{\cancel{100}}\% = \frac{25}{16}\% = 1\tfrac{9}{16}\%.$$

Therefore, he loses $1\frac{9}{16}\%$. *Ans.*

4. Teas at 68 cents, 86 cents, and 96 cents a pound are mixed in equal quantities, and sold at 90 cents a pound. Find the gain per cent.

$$\text{Cost per pound} = \tfrac{1}{3}(68 + 86 + 96) \text{ cents} = 83\tfrac{1}{3} \text{ cents.}$$

$$\text{Gain} = 90 \text{ cents} - 83\tfrac{1}{3} \text{ cents} = 6\tfrac{2}{3} \text{ cents.}$$

$$\frac{6\frac{2}{3}}{83\frac{1}{3}} \text{ of } 100\% = \frac{\overset{4}{\cancel{20}}}{\cancel{3}} \times \frac{\cancel{3}}{\underset{\cancel{5}}{\cancel{250}}} \times \overset{2}{\cancel{100}}\% = 8\%. \quad \textit{Ans.}$$

5. By selling goods for \$1173.92 a merchant gains \$153.12. Find the gain per cent.

$$\text{Cost} = \$1173.92 - \$153.12 = \$1020.80.$$

```
           0.15
102080)15312.
       102080
       ------
        510400
        510400
        ------
```

15%. *Ans.*

6. What was the cost, when $17\frac{1}{2}\%$ was gained by selling goods for \$253.80?

$$\$253.80 \div \frac{117\frac{1}{2}}{100} = 100 \times \frac{2}{\cancel{235}} \times \$\overset{1.08}{\cancel{253.80}} = \$216. \quad \textit{Ans.}$$

7. A wine merchant mixes 24 gal. of wine, at \$7 a gallon, with 18 gal. at \$5 a gallon, and sells the whole at \$7 a gallon. What does he gain per cent?

[redacted]

$$\text{Cost} = 24 \times \$7 + 18 \times \$5 = \$168 + \$90 = \$258.$$

$$\text{Selling price} = (24 + 18) \times \$7 = 42 \times \$7 = \$294.$$

$$\text{Gain} = \$294 - \$258 = \$36.$$

$$\frac{\overset{6}{\cancel{36}}}{\underset{43}{\cancel{258}}} \text{ of } 100\,\% = \frac{600}{43}\,\% = 13\tfrac{41}{43}\,\%. \quad \textit{Ans.}$$

8. By selling a horse for \$200, a dealer loses $12\frac{1}{2}\,\%$. What would he have gained or lost per cent if he had sold the horse for \$250?

$$\text{Cost} = \$200 \div \frac{87\frac{1}{2}}{100} = \frac{8}{7} \text{ of } \$200 = \$\frac{1600}{7} = \$228\tfrac{4}{7}.$$

If he had sold the horse for \$250, the gain would have been

$$\$250 - \$228\tfrac{4}{7} = \$21\tfrac{3}{7}.$$

$$\frac{21\frac{3}{7}}{228\frac{4}{7}} \text{ of } 100\,\% = \frac{\overset{75}{\cancel{150}}}{\underset{\underset{8}{\cancel{16}}}{\cancel{1600}}} \text{ of } \cancel{100}\,\% = \frac{75}{8}\,\% = 9\tfrac{3}{8}\,\%.$$

Gain, $9\frac{3}{8}\,\%$. *Ans.*

9. A spirit merchant buys 75 gal. of spirits at \$3.25 a gallon, and, after drawing off 10 gal., sells the remainder so as to gain 5 % on the cost of the whole. What is the selling price per gallon?

```
  $3.25
     75
  -----
   1625
  2275
 ------
$243.75, cost.
   1.05
 ------
 121875
 24375
---------
$255.9375
75 gal. - 10 gal. = 65 gal.
```

```
       $3.9375 Ans.
    -----------
65)$255.9375
    195
    ---
     609
     585
     ---
      243
      195
      ---
       487
       455
       ---
        325
        325
        ---
```

10. A man owns two city lots worth respectively \$9845 and \$12,155. If the first gains in value 32 %, and the second loses 13 %, what is the gain or loss per cent in the value of the two lots?

[redacted]

$$\begin{array}{r}
\$\,9845 \\
1.32 \\
\hline
19690 \\
29535 \\
9845 \\
\hline
\$\,12995.40
\end{array}
\qquad
\begin{array}{r}
\$\,12155 \\
0.87 \\
\hline
85085 \\
97240 \\
\hline
\$\,10574.85
\end{array}$$

$$\begin{array}{r}
\$\,9845 \\
12155 \\
\hline
\$\,22000
\end{array}
\qquad
\begin{array}{r}
\$\,12995.40 \\
10574.85 \\
\hline
\$\,23570.25
\end{array}$$

Gain = $23,570.25 – $22,000 = $1570.25.

$$\begin{array}{r}
0.0713 \\
22\cancel{000})\overline{1.57025} \\
154 \\
\hline
30 \\
22 \\
\hline
82 \\
66 \\
\hline
16
\end{array}$$

7.14 % gain. *Ans.*

11. A tradesman marks a hat $5, but takes off 5%. If his profit is 14%, what was the cost of the hat?

Selling price = $5 – 5% of $5 = $5 – $0.25 = $4.75.

$$\$4.75 \div \frac{114}{100} = \frac{\overset{25}{\cancel{100}}}{\underset{6}{\cancel{114}}} \times \$\frac{\cancel{19}}{\cancel{4}} = \$\frac{25}{6} = \$4.16\tfrac{2}{3}. \quad \textit{Ans.}$$

12. What would a dishonest dealer gain per cent by using a false weight of 15 oz. instead of a pound?

Gain = 16 oz. – 15 oz. = 1 oz. $\quad \frac{1}{\underset{3}{\cancel{15}}}$ of $\overset{20}{\cancel{100}}\% = \frac{20}{3}\% = 6\tfrac{2}{3}\%$. *Ans.*

13. A dishonest dealer gains 12% by using false weights. What is the real weight of his pound?

His pound weighs 16 oz. $\div \frac{112}{100} = \frac{100}{\underset{7}{\cancel{112}}}$ of $\cancel{16}$ oz. $= \frac{100}{7}$ oz. $= 14\tfrac{2}{7}$ oz. *Ans.*

14. What per cent above cost must a merchant mark his goods that he may take off 20 % from the marked price, and still make 20 % on the cost?

Since the merchant is to make 20 % on the cost of the goods, the selling price is 120 % of the cost price.

Since the selling price is to be 20 % below the marked price, the selling price is 80 % of the marked price.

Therefore, the marked price will be $\frac{100}{80}$ of 120 % of the cost price, or 150 % of the cost price; that is, the goods must be marked 50 % above cost.

15. What per cent above cost must a merchant mark his goods to take off 10 %, and still gain 17 %?

Selling price = 117 % of cost price.
Selling price = 90 % of marked price.

Therefore, marked price $= \dfrac{\overset{10}{\cancel{100}}}{\underset{9}{\cancel{90}}}$ of $\overset{13}{\cancel{117}}\,\% = 130\,\%$ of cost price.

30 % above cost. *Ans.*

16. What per cent above cost must a merchant mark his goods to take off $12\frac{1}{2}$ %, and still gain $12\frac{1}{2}$ %?

Selling price = $112\frac{1}{2}$ % of cost price.
Selling price = $87\frac{1}{2}$ % of marked price.

Therefore, marked price $= \dfrac{100}{\underset{7}{\cancel{87\frac{1}{2}}}}$ of $\overset{9}{\cancel{112\frac{1}{2}}}\,\% = \dfrac{900}{7} = 128\frac{4}{7}\,\%$ of cost price.

$28\frac{4}{7}$ % above cost. *Ans.*

17. What per cent above cost must a merchant mark his goods to take off 15 %, and still gain 15 %?

Selling price = 115 % of cost price.
Selling price = 85 % of marked price.
Therefore, marked price

$$= \frac{100}{\underset{17}{\cancel{85}}} \text{ of } \overset{23}{\cancel{115}}\,\% = \frac{2300}{17}\,\% = 135\tfrac{5}{17} \text{ of cost price.}$$

$35\frac{5}{17}$ % above cost. *Ans.*

18. What per cent above cost must a merchant mark his goods to ...e off $33\frac{1}{3}$ %, and still gain $33\frac{1}{3}$ %?

[redacted]

Selling price = $133\frac{1}{3}\%$ of cost price.
Selling price = $66\frac{2}{3}\%$ of marked price.
Therefore, marked price

$$= \frac{100}{\cancel{66\frac{2}{3}}} \text{ of } \overset{2}{\cancel{133\frac{1}{3}}}\% = 200\,\% \text{ of cost price.} \quad 100\,\% \text{ above cost. } \textit{Ans.}$$

19. A man bought a horse for \$70, and sold him for \$80. What per cent did he gain? What per cent of the selling price of the horse did he gain?

$$\text{Gain} = \$80 - \$70 = \$10.$$

$$\frac{\cancel{10}}{\underset{7}{\cancel{70}}} \text{ of } 100\,\% = \frac{100}{7}\,\% = 14\tfrac{2}{7}\,\%. \ \textit{Ans.}$$

$$\frac{\cancel{10}}{\underset{\underset{2}{\cancel{8}}}{\cancel{80}}} \text{ of } \overset{25}{\cancel{100}}\,\% = \frac{25}{2}\,\% = 12\tfrac{1}{2}\,\%. \ \textit{Ans.}$$

20. If a merchant clears \$800 by selling goods for $12\frac{1}{2}\%$ profit, what was the cost of the goods, and for how much were they sold?

$$\$800 \div \frac{12\frac{1}{2}}{100} = 8 \times \$800 = \$6400, \text{ cost. } \textit{Ans.}$$

$$\$6400 + \$800 = \$7200, \text{ selling price. } \textit{Ans.}$$

21. A man selling eggs at \$0.40 a dozen gains $33\frac{1}{3}\%$; what was the cost? Another, selling at the same price, gains $33\frac{1}{3}\%$ of his receipts; what did his eggs cost?

$$\$0.40 \div \frac{133\frac{1}{3}}{100} = \tfrac{3}{4} \text{ of } \$0.40 = \$0.30. \ \textit{Ans.}$$

$33\frac{1}{3}\%$ of $\$0.40 = \$0.13\frac{1}{3}$, gain. $\quad \$0.40 - \$0.13\frac{1}{3} = \$0.26\frac{2}{3}$. *Ans.*

22. A man lost 10% by selling a carriage for \$117. At what price should he have sold it to make 10%?

$$\text{Cost} = \$117 \div \frac{90}{100} = \frac{\overset{10}{\cancel{100}}}{\underset{\cancel{9}}{\cancel{90}}} \times \$\overset{13}{\cancel{117}} = \$130.$$

$$\$130 + 10\,\% \text{ of } \$130 = \$130 + \$13 = \$143. \ \textit{Ans.}$$

23. If a real estate dealer gained \$600 by selling a farm for 20% profit, what was the cost of the farm, and for how much did he sell it?

$\$600 \div \frac{20}{100} = 5 \times \$600 = \$3000.$ *Ans.* $\quad \$3000 + \$600 = \$3600.$ *Ans.*

Exercise 114. Page 250.

1. Find the commission on \$2595, at $2\frac{1}{2}\%$.

$$\begin{array}{r} \$2595 \\ 0.025 \\ \hline 12975 \\ 5190 \\ \hline \$64.875 \end{array}$$

\$64.88. *Ans.*

2. An agent sells 200 bbl. of flour at \$6.25, and 600 gal. of molasses at 65 cents, and charges a commission of $1\frac{3}{4}\%$. What are the net proceeds?

$$\begin{array}{r} \$6.25 \\ 200 \\ \hline \$1250. \end{array} \qquad \begin{array}{r} \$1250 \\ 390 \\ \hline \$1640 \\ 0.0175 \\ \hline 8200 \\ 11480 \\ 1640 \\ \hline \$28.7000 \end{array}$$

$$\begin{array}{r} \$0.65 \\ 600 \\ \hline \$390. \end{array}$$

$$\begin{array}{r} \$1640. \\ 28.70 \\ \hline \$1611.30 \end{array} \; \textit{Ans.}$$

3. A commission merchant received \$1640 to buy corn, and charged a commission of $2\frac{1}{2}\%$. What is his commission, and how many bushels of corn at $62\frac{1}{2}$ cents a bushel can he buy?

$$\$1640 \div \frac{102\frac{1}{2}}{100} = \frac{200}{\cancel{205}} \text{ of } \$\overset{8}{\cancel{1640}}$$

$$= \$1600.$$

Commission

$= \$1640 - \$1600 = \$40.$ *Ans.*

$$\$1600 \div \$0.62\tfrac{1}{2} = \frac{8}{\cancel{5}} \text{ of } \overset{320}{\cancel{1600}}$$

$$= 2560.$$ bu. *Ans.*

4. An agent sells a consignment of cotton for \$5216. He pays \$51 for storage, and charges a commission of $2\frac{1}{4}\%$. What are the net proceeds?

$$\begin{array}{r} \$5216 \\ 0.02\frac{1}{4} \\ \hline 1304 \\ 10432 \\ \hline \$117.36 \end{array} \qquad \begin{array}{r} \$117.36 \\ 51. \\ \hline \$168.36 \end{array}$$

$$\begin{array}{r} \$5216. \\ 168.36 \\ \hline \$5047.64 \end{array} \; \textit{Ans.}$$

5. An agent sold butter for \$1570, and remitted \$1546.45. What was the rate per cent of commission?

Commission

$= \$1570 - \$1546.45 = \$23.55.$

$$\begin{array}{r} 0.015 \\ 1570\overline{)23.55} \\ 1570 \\ \hline 7850 \\ 7850 \\ \hline \end{array}$$

$1\frac{1}{2}\%$. *Ans.*

6. What are the net proceeds from the sale of 2250 bbl. of flour at \$6.25 a barrel, if the charge for freight is 50 cents a barrel, the commission for selling 2%, and the commission for guaranteeing payment $1\frac{1}{2}\%$?

[redacted]

$6.25
2250
31250
1250
1250
$14062.50
0.035
7031250
4218750
$492.18750

$0.50
2250
$1125.

$1125.
492.19
$1617.19

$14062.50
1617.19
$12445.31 *Ans.*

7. An agent sells 350 crates of peaches at $2.60. If the commission is $4\frac{1}{2}\%$, find the net proceeds.

$2.60
350
13000
780
$910.
0.045
4550
3640
$40.95

$910.
40.95
$869.05 *Ans.*

8. An agent sells 420 acres of land at $40 an acre, and charges $1\frac{1}{4}\%$ commission. What is his commission?

420 × $40 = $16,800.

$16800
$0.01\frac{1}{4}$
4200
16800
$210. *Ans.*

9. An agent, charging $4\frac{1}{2}\%$ commission, receives for his services $313. Find the amount of his sales.

$313 ÷ 0.045 = $6955.56. *Ans.*

6955.55
45)313000.
270
430
405
250
225
250
225
250
225
250
225
25

10. A merchant buys 730 yd. of carpeting at $1.25 a yard, and pays his agent $\frac{3}{4}$ of 1% commission. If the freight amounts to $23.58, at what price per yard must he sell the carpeting to gain 20%?

$1.25
730
3750
875
$912.50
0.0075
456250
638750
$6.843750

$912.50
6.84
23.58
$942.92 cost.

5 | $942.92
188.58
$1131.50

$1.55 *Ans.*
73$\cancel{0}$)$113.15
73
401
365
365
365

11. An agent sells a consignment of goods for $2100. He pays $33.50 for freight, and remits $2024.50. Find his rate of commission.

$$\begin{array}{r} \$2100. \\ 2024.50 \\ \hline \$75.50 \end{array} \qquad \begin{array}{r} \$75.50 \\ 33.50 \\ \hline \$42.00 \end{array} \text{ commission.}$$

$$21\cancel{00}\overline{)0.42}^{\;0.02} \qquad \underline{42} \qquad 2\%. \textit{ Ans.}$$

12. An agent sells 5000 lb. of cotton at 14 cents a pound, charging 2% commission. With the net proceeds he buys cotton cloth at 10 cents a yard, charging $1\frac{1}{2}\%$ commission. How many yards of cloth does he buy?

$$\begin{array}{r} \$0.14 \\ 5000 \\ \hline \$700. \\ 0.02 \\ \hline \$14. \end{array} \qquad \begin{array}{r} \$700 \\ 14 \\ \hline \$686 \end{array} \text{, net proceeds.}$$

Amount paid for cloth

$$= \$686 \div \frac{101\frac{1}{2}}{100} = \frac{200}{\cancel{203}_{29}} \times \$\cancel{686}^{98}$$

$$= \$\frac{19600}{29}$$

$$= \$675.86.$$

$675.86 ÷ $0.10 = 6758.6. *Ans.*

13. An agent sold 500 bbl. of flour at $5.50 a barrel, and charged $2\frac{1}{2}\%$ commission; the expenses for freight, etc., were $250. With the net proceeds he bought sugar at $4\frac{3}{5}$ cents a pound, charging $2\frac{1}{2}\%$ commission. How much sugar did he buy, and what was his total commission?

$$\begin{array}{r} \$5.50 \\ 500 \\ \hline \$2750. \\ 0.025 \\ \hline 13750 \\ 5500 \\ \hline \$68.75 \end{array} \qquad \begin{array}{r} \$68.75 \\ 250. \\ \hline \$318.75 \end{array} \qquad \begin{array}{r} \$2750. \\ 318.75 \\ \hline \$2431.25 \end{array} \text{, net proceeds.}$$

Amount paid for sugar
$= \$2431.25 \div 1.025 = \$2371.95.$

$$\begin{array}{r} 2371.95 \\ 1025\overline{)2431250.} \\ 2050 \\ \hline 3812 \\ 3075 \\ \hline 7375 \\ 7175 \\ \hline 2000 \\ 1025 \\ \hline 9750 \\ 9225 \\ \hline 5250 \\ 5125 \\ \hline 125 \end{array}$$

$2371.95 ÷ $0.046 = 51564.

$$\begin{array}{r} 51564. \\ 46\overline{)2371950.} \\ 230 \\ \hline 71 \\ 46 \\ \hline 259 \\ 230 \\ \hline 295 \\ 276 \\ \hline 190 \\ 184 \\ \hline 6 \end{array}$$

51,564 lb. *Ans.*

Commission for buying = $2431.25 − $2371.95 =	$59.30
Commission for selling =	$68.75
Total commission =	$128.05

Ans.

[redacted]

14. A collector's commission for collecting taxes, at $1\frac{1}{2}\%$, is \$206.55. What sum did he collect?

$$\$206.55 \div \frac{1\frac{1}{2}}{100} = \frac{200}{\cancel{3}} \times \$\overset{68.85}{\cancel{206.55}}$$

$$= \$13,770. \textit{ Ans.}$$

15. An agent received \$2961 to purchase goods, and charged 5% commission. What was his commission?

$$\$2961 \div \frac{105}{100} = \frac{\overset{20}{\cancel{100}}}{\underset{21}{\cancel{105}}} \times \$\overset{141}{\cancel{2961}}$$

$$= \$2820.$$

\$2961 − \$2820 = \$141, commission. *Ans.*

16. An agent buys 3100 bbl. of flour at \$4.50 a barrel, and charges $1\frac{1}{2}\%$ commission. What is his commission?

```
  $4.50
   3100
  -----
  45000
  1350
  -----
$13950.
  0.015
  -----
  69750
 13950
 ------
$209.25 Ans.
```

17. A broker receives \$6150 to invest in cotton, at $7\frac{3}{8}$ cents a pound. If his commission is $2\frac{1}{2}\%$, how many pounds of cotton can he buy?

Amount expended for cotton

$$= \$6150 \div 1.025 = \$6000.$$

$$\$6000 \div \$0.07\tfrac{3}{8} = 6000 \times \frac{800}{59}$$

$$= \frac{4800000}{59} = 81355.9.$$

81,355.9 lb. *Ans.*

18. An agent sells 1100 bbl. of flour at \$4.50 a barrel, and charges $2\frac{1}{2}\%$ commission. He invests the proceeds in steel at $1\frac{1}{2}$ cents a pound, charging $1\frac{1}{2}\%$ commission. What is his entire commission, and how many long tons of steel does he buy?

```
  $4.50       $4950.
   1100        123.75
  -----       -------
  45000       $4826.25, net pro-
  450                   [ceeds.
 ------
 $4950.
  0.025
 ------
  24750
  9900
 -------
$123.75
```

Amount expended for steel

$$= \$4826.25 \div 1.015 = \$4754.93.$$

```
          4754.92
    1015)4826250.
         4060
         ----
          7662
          7105
          ----
           5575
           5075
           ----
            5000
            4060
            ----
             9400
             9135
             ----
              2650
              2030
              ----
               620
```

1 long ton costs 2240 × $0.01½ = $33.60.

$$
\begin{array}{r}
141.5\\
336\cancel{0})\overline{47549.3}\\
336\\
\hline
1394\\
1344\\
\hline
509\\
336\\
\hline
1733\\
1680\\
\hline
53
\end{array}
$$

141.5 t. *Ans.*

Commission for selling
= $4826.25 − $4754.93 = $71.32.

$$
\begin{array}{r}
\$123.75\\
71.32\\
\hline
\$195.07
\end{array}
$$
$195.07, commission. *Ans.*

Exercise 115. Page 252.

1. Find the premium of the fire insurance on a house for $2650 at ½ of 1%.

$$
\begin{array}{r}
\$2650\\
0.005\\
\hline
\$13.25
\end{array}
$$
$13.25 *Ans.*

2. Find the premium for insuring a man's life for $2500, at an age for which the rate is 2¼%.

$$
\begin{array}{r}
\$2500\\
0.02\frac{1}{4}\\
\hline
625\\
5000\\
\hline
\$56.25
\end{array}
$$
$56.25 *Ans.*

3. At 6¾%, what premium will be paid on a vessel worth $36,400, insured for ¾ its value?

$$
\begin{array}{r|r}
4 & \$36400\\
 & 9100\\
\hline
 & \$27300\\
 & 0.0675\\
\hline
 & 136500\\
 & 191100\\
 & 163800\\
\hline
 & \$1842.75
\end{array}
$$
$1842.75 *Ans.*

4. A vessel worth $16,000 is insured for ¾ its value at 7¼%. What is the premium?

$$
\begin{array}{r|r}
4 & \$16000\\
 & 4000\\
\hline
 & \$12000\\
 & 0.07\frac{1}{4}\\
\hline
 & 3000\\
 & 84\\
\hline
 & \$870.
\end{array}
$$
$870. *Ans.*

5. The premium of insurance at 1¼% is $150. What is the amount insured?

$$\$150 \div \frac{1\frac{1}{4}}{100} = \frac{400}{\cancel{5}} \times \$\overset{30}{\cancel{150}} = \$12{,}000.$$

Ans.

6. A vessel valued at $128,000 is insured for ¾ its value at 3⅞%. What is the net loss to the owners, if the vessel is destroyed during the third year after it is insured?

$$\begin{array}{r|l} 4 & \$\,128000 \\ \hline & \;\;32000 \\ \end{array}$$

$$\begin{array}{r} \$\,96000 \\ \underline{0.03\tfrac{7}{8}} \\ 84000 \\ \underline{288} \\ \$\,3720. \end{array}$$

Gross loss $= \$128,000 + 3 \times \$3720$
$= \$\,128,000 + \$\,11,160$
$= \$\,139,160.$

Net loss $= \$\,139,160 - \$\,96,000$
$= \$\,43,160.$ *Ans.*

7. A building worth \$7500 is insured for $\frac{2}{3}$ its value, at $\frac{1}{8}$ of 1% per annum. What is the annual premium?

$$\begin{array}{r|l} 3 & \$\,7500 \\ \hline & \;\;2500 \\ \end{array}$$

$$\begin{array}{r} \$\,5000 \\ \underline{0.00\tfrac{1}{8}} \\ \$\,6.25 \end{array}$$ *Ans.*

8. Four companies insure a store and contents for \$60,000. One company takes \$20,000, at $\frac{3}{5}$ of 1%; a second takes \$10,000, at $\frac{3}{4}$ of 1%; a third, \$15,000, at $\frac{5}{8}$ of 1%; a fourth, the remainder, at $\frac{1}{2}$ of 1%. What is the premium?

The remainder $= \$\,60,000 - (\$\,20,000 + \$\,10,000 + \$\,15,000) = \$\,15,000.$

$$\begin{array}{r} 0.006 \\ \underline{20000} \\ 120. \end{array} \qquad \begin{array}{r} 0.0075 \\ \underline{10000} \\ 75. \end{array}$$

$$\begin{array}{r} 0.00625 \\ \underline{15000} \\ 3125000 \\ \underline{625} \\ 93.75 \end{array} \qquad \begin{array}{r} 0.005 \\ \underline{15000} \\ 75. \end{array} \qquad \begin{array}{r} \$\,120. \\ 75. \\ 93.75 \\ \underline{75.} \\ \$\,363.75 \end{array}$$ *Ans.*

9. If the store of Ex. 8 is damaged to the extent of \$4500, what amount does each company pay?

$$\frac{\cancel{20000}}{\underset{3}{\cancel{60000}}} \text{ of } \$\,\overset{1500}{\cancel{4500}} = \$\,1500, \text{ 1st Co.}$$

$$\frac{\cancel{10000}}{\underset{6}{\cancel{60000}}} \text{ of } \$\,\overset{750}{\cancel{4500}} = \$\,750, \text{ 2d Co.}$$

$$\frac{\cancel{15000}}{\underset{4}{\cancel{60000}}} \text{ of } \$\,\overset{1125}{\cancel{4500}} = \$\,1125, \text{ 3d Co. and 4th Co.}$$

[redacted]

10. A man insures his life for $10,000, paying $350 a year in advance, and dies the day before the fifth premium is due. The company pays his widow $10,000. How much has the company lost by him, if the interest gained on the premiums paid amounts to $175?

$350	$10000.
4	1575.
$1400	$8425. *Ans.*
175	
$1575	

11. A merchant shipped a cargo to London, and took a policy of $100,800 at 3½%, to cover both the cargo and the premium. Find the value of the cargo.

100% of policy = policy (cargo and premium).
3½% of policy = premium.
96½% of policy = cargo.

$$\begin{array}{r} 0.965 \\ 100800 \\ \hline 772000 \\ 965 \\ \hline 97272. \end{array}$$

$97,272. *Ans.*

12. Three companies insure, at ¾ its value, a building worth $16,000. The first company takes ⅓ the risk at ¾ of 1%; the second, ⅖ at ⅞ of 1%; and the third, the remainder at ¾ of 1%. Find the total premium.

¾ of $16,000 = $12,000.
⅓ of $12,000 = $4000.
¾% of $4000 = $30.
⅖ of $12,000 = $4800.
⅞% of $4800 = $42.
$12,000 − ($4000 + $4800) = $3200.
¾% of $3200 = $24.
$30 + $42 + $24 = $96. *Ans.*

13. S. Williams pays $18.40 premium for insuring his house for ⅔ its value at 1½%. What is the value of his house?

$$\text{Policy} = \$18.40 \div \frac{1\frac{1}{2}}{100} = \frac{200}{3} \times \$18.40 = \$1226\tfrac{2}{3}.$$

$$\text{Value of house} = \$1226\tfrac{2}{3} \div \frac{2}{3} = \frac{\cancel{3}}{\cancel{2}} \times \$\frac{\overset{1840}{\cancel{3680}}}{\cancel{3}} = \$1840. \ \textit{Ans.}$$

14. Find the annual premium for an ordinary life policy of $5000 issued to a man 30 years old, if the rate of insurance is 1.93%.

$$\begin{array}{r} 0.0193 \\ 5000 \\ \hline 96.50 \end{array}$$

$96.50. *Ans.*

15. What is the annual premium for an ordinary life policy of \$12,000 issued to a man 40 years old, if the rate of insurance is 2.661%?

$$\begin{array}{r} 0.02661 \\ 12000 \\ \hline 5322000 \\ 2661 \\ \hline 319.32 \end{array}$$

\$319.32. *Ans.*

Exercise 116. Page 255.

Make a table for a tax rate of 16 mills.

PROP.	TAX.	PROP.	TAX.	PROP.	TAX.	PROP.	TAX.
\$1	\$0.016	\$10	\$0.16	\$100	\$1.60	\$1000	\$16.00
2	0.032	20	0.32	200	3.20	2000	32.00
3	0.048	30	0.48	300	4.80	3000	48.00
4	0.064	40	0.64	400	6.40	4000	64.00
5	0.080	50	0.80	500	8.00	5000	80.00
6	0.096	60	0.96	600	9.60	6000	96.00
7	0.112	70	1.12	700	11.20	7000	112.00
8	0.128	80	1.28	800	12.80	8000	128.00
9	0.144	90	1.44	900	14.40	9000	144.00

1. Find the tax on property assessed at \$7500.

Tax on \$7000 = \$112.00
Tax on 500 = 8.00
Total tax = \$120.00 *Ans.*

2. Find the tax on property assessed at \$4825.

Tax on \$4000 = \$64.00
Tax on 800 = 12.80
Tax on 20 = 0.32
Tax on 5 = 0.08
Total tax = \$77.20 *Ans.*

3. Find the tax on property assessed at \$9685.

Tax on \$9000 = \$144.00
Tax on 600 = 9.60
Tax on 80 = 1.28
Tax on 5 = 0.08
Total tax = \$154.96 *Ans.*

4. Find the tax on property assessed at \$10,727.

Tax on \$10000 = \$160.00
Tax on 700 = 11.20
Tax on 20 = 0.32
Tax on 7 = 0.11
Total tax = \$171.63 Ans.

[redacted]

5. Find the tax on property assessed at \$12,863.

Tax on	\$10000 =	\$160.00
Tax on	2000 =	32.00
Tax on	800 =	12.80
Tax on	60 =	0.96
Tax on	3 =	0.05
	Total tax =	\$205.81 *Ans.*

6. Find the tax on property assessed at \$16,458.

Tax on	\$10000 =	\$160.00
Tax on	6000 =	96.00
Tax on	400 =	6.40
Tax on	50 =	0.80
Tax on	8 =	0.13
	Total tax =	\$263.33 *Ans.*

7. Find the tax on property assessed at \$38,249.

Tax on	\$30000 =	\$480.00
Tax on	8000 =	128.00
Tax on	200 =	3.20
Tax on	40 =	0.64
Tax on	9 =	0.14
		\$611.98 *Ans.*

8. James Brown is assessed \$2500 on his real estate and \$5200 on his personal property, and pays for two polls at \$1.50 each. If the rate is \$12.18 on \$1000, what is his total tax?

\$2500	\$0.01218
5200	7700
\$7700	852600
	8526
\$1.50	\$93.786
2	3.00
\$3.00	\$96.79 *Ans.*

9. If the tax rate of a town is \$12.25 on \$1000, and the amount of the levy \$11,788.50, what is the assessed valuation of the town?

\$12.25 on \$1000 = 1.225 %
= 0.01225.

```
            $962326.53 Ans.
1225)$ 1178850000.
       11025
        7635
        7350
         2850
         2450
          4000
          3675
           3250
           2450
            8000
            7350
             6500
             6125
              3750
              3675
                75
```

10. If the assessed valuation of a town is \$1,777,000, and the levy is \$29,231.65, what is the rate on \$1000?

```
                 $0.01645
1777000)$29.23165
         1777
         11461
         10662
           7996
           7108
            8885
            8885
```

\$0.01645 on \$1
= \$16.45 on \$1000. *Ans.*

[redacted]

Amount collected = $2700 + 0.96 of $16,337.30
= $2700 + $15,683.81 = $18,383.

11. What sum must be assessed that $15,000 may remain after paying 2% commission for collecting the taxes?

$15,000 is 98% of amount assessed.

```
          $15306.12  Ans.
98)$1500000.
    98
    520
    490
     300
     294
       600
       588
        120
         98
        220
        196
         24
```

12. For building a schoolhouse a tax of $1857.60 was levied upon a school district, assessed valuation $1,935,000. What was the tax on property assessed at $6250?

```
             $0.00096
1935000)$1.8576
          17415
           11610
           11610

$0.00096
    6250
    4800
   192
  576
$6.00000  Ans.
```

13. In a certain town there are 1350 polls. The assessed valuation of the real estate is $713,250, and of the personal property is $738,954. The poll tax is $2 per poll, and the tax on property is $1\frac{1}{8}$%. Only 96% of the property tax can be collected, and the collector is paid $2\frac{1}{2}$% of the amount collected. How much does the town receive from the taxes? How much does the collector receive for his services?

Poll tax = 1350 × $2 = $2700.

Assessed valuation of town = $713,250 + $738,954 = $1,452,204.

```
$1452204
   0.01⅛
  181525½
 1452204
$16337.29½
```

~~Total tax = $2700 +~~ $16,337.30 = $19,037.30.

~~Amount collected = 0.96 of $19,037.30 = $18,275.81.~~

383 Collector receives 0.025 of $18,~~275~~.81 = $~~456.90~~. *Ans.* 459.6

Town receives $18,~~275~~.81 − $~~456.90~~ = $~~17,818.91~~ *Ans.* 17,92

```
$19037.30          $18275.81
     0.96             0.025
 11422380           9137905
17133570           3655162
$18275.8080        $456.89525
```

$16337.30

[redacted]

Exercise 117. Page 257.

1. What is the duty at $2\frac{1}{2}$ cents a pound on 320 boxes of raisins each containing 40 pounds?

$$320 \times 40 \text{ lb.} = 12{,}800 \text{ lb.}$$

$$\begin{array}{r} 12800 \\ 0.025 \\ \hline 64000 \\ 25600 \\ \hline 320.000 \end{array}$$

$320. *Ans.*

2. What is the duty at 6 cents a gallon on 420 hhd. of best molasses of 63 gal. each?

$$\begin{array}{r} 63 \text{ gal.} \\ 420\phantom{\text{ gal.}} \\ \hline 1260\phantom{\text{ gal.}} \\ 252\phantom{0\text{ gal.}} \\ \hline 26460 \text{ gal.} \end{array} \qquad \begin{array}{r} 26460 \\ 0.06 \\ \hline 1587.60 \end{array}$$

$ 1587.60. *Ans.*

3. What is the duty at $4 a dozen bottles on 50 cases of champagne, each containing 24 pint bottles, if breakage of 5% is allowed?

Each case contains 2 doz. bottles.

50×2 doz. $= 100$ doz. 100 doz. $-$ 5 doz. $= 95$ doz.

Breakage $= 5\%$ of 100 doz. $= 5$ doz. $95 \times \$4 = \$380$. *Ans.*

4. Find the duty on 150 gross of spectacles, cost price $1.20 a dozen; specific duty 45 cents a dozen, breakage allowed $2\frac{1}{2}\%$; and 20% ad valorem.

150 gross $= 1800$ doz. Breakage $= 2\frac{1}{2}\%$ of 1800 doz. $= 45$ doz.

1800 doz. $-$ 45 doz. $= 1755$ doz.

$$\begin{array}{r} 1755 \\ 0.45 \\ \hline 8775 \\ 7020 \\ \hline 789.75 \end{array} \text{, specific.} \qquad \begin{array}{r} 1755 \\ 1.20 \\ \hline 35100 \\ 1755 \\ \hline 2106. \\ 0.20 \\ \hline 421.20 \end{array} \text{, ad valorem.}$$

$ 789.75 + $ 421.20 = $ 1210.95. *Ans.*

5. Find the duty on 100 shotguns, cost price $8.50 each; specific duty of $4 each, and 15% ad valorem.

$4	$8.50
100	100
$400, specific.	$850
	0.15
$400.	4250
127.50	850
$527.50 *Ans.*	$127.50, ad valorem.

6. Find the duty at $1 per M on 12,500 ft. of whitewood boards, planed on one side, if an additional duty of 50 cents per M is collected for each side planed.

12,500 = 12.5 M. Duty per M = $1 + $0.50 = $1.50.

12.5
1.50
6250
125
18.750 $18.75. *Ans.*

7. Find the duty on 500 boxes of cigars, gross weight 475 lb., tare 40%, costing 82½ cents per box in Havana. Specific duty $4.50 per pound; and 25% ad valorem.

475 lb.
0.40
190.00 lb.

Net weight = 475 lb. − 190 lb. = 285 lb.

285	$0.825
4.50	500
14250	$412.50
1140	0.25
1282.50, specific.	206250
	82500
$1282.50	$103.125, ad valorem.
103.13	
$1385.63 *Ans.*	

[redacted]

8. Find the duty on 400 pairs of woolen blankets, cost price $1.75 per pair; weighing 7¼ lb. per pair, tare 5 %. Specific duty 33 cents per pound, ad valorem 40 %.

400 × 7¼ lb. = 2900 lb.
Tare = 5 % of 2900 lb. = 145 lb.
Net weight = 2900 lb. − 145 lb. = 2755 lb.

2755	$1.75
0.33	400
8265	$700.
8265	0.40
909.15, specific.	$280.00, ad valorem.

$909.15
280.
$1189.15 *Ans.*

9. Find the duty on 12 boxes of skein silk, each box weighing 40 lb.; cost price $2.125 per pound, tare 10 %. Specific duty 50 cents per pound, ad valorem 15 %.

12 × 40 lb. = 480 lb.
Tare = 10 % of 480 lb. = 48 lb.
Net weight = 432 lb.

432	$2.125
0.50	432
216.00, specific.	4250
	6375
	8500
	$918.
	0.15
$216.	4590
137.70	918
$353.70 *Ans.*	$137.70, ad valorem.

10. Find the duty on 150 gross of clay tobacco pipes, cost price 55 cents a gross. Specific duty 15 cents a gross, and 25 % ad valorem.

$0.15	$0.55
150	150
750	2750
15	55
$22.50, specific.	$82.50
	0.25
$22.50	41250
20.63	16500
$43.13 *Ans.*	$20.625, ad valorem.

11. A New York merchant bought in London 400 gal. of cologne at \$1.25 a gallon, and commission and other expenses amounted to \$56.25. At what price per pint must he sell the cologne to gain 40% on the cost, if he paid a specific duty of 60 cents a gallon, and an ad valorem duty of 45%?

$400 \times \$1.25 = \$500.$

Total cost = \$500 + \$56.25 = \$556.25.

Specific duty = 400 × \$0.60 = \$240.

Ad valorem duty = 45% of \$556.25 = \$250.31.

Total cost in New York

= \$556.25 + \$240 + \$250.31 = \$1046.56.

400 gal. = 400 × 8 pt. = 3200 pt.

Total selling price must be

\$1046.56 + 40% of \$1046.56

= \$1046.56 + \$418.62 = \$1465.18.

Selling price per pint must be

\$1465.18 ÷ 3200 = \$0.458. *Ans.*

12. Find the duty on 750 lb. of glue, cost price 40 cents; specific duty of 15 cents a pound, tare 2%; and ad valorem duty of 25%.

750 × \$0.40 = \$300.

Tare = 2% of 750 lb. = 15 lb.

Net weight = 750 lb. − 15 lb. = 735 lb.

735	\$300
0.15	0.25
3675	\$75. ad valorem.
735	
110.25, specific.	

Total duty \$110.25 + \$75 = \$185.25. *Ans.*

13. A Boston merchant bought in Sheffield 50 gross of razors at a net price of \$4.25 a dozen. At what price per dozen must he sell the razors to gain $33\frac{1}{3}$% on the net cost, if he paid a specific duty of \$1.75 a dozen, and an ad valorem duty of 20%?

50 gross = 600 doz.

Net cost = 600 × \$4.25 = \$2550.

Specific duty = 600 × \$1.75 = \$1050.

Ad valorem duty = 20% of \$2550 = \$510.

Total cost = \$2550 + \$1050 + \$510 = \$4110.

Total selling price must be

\$4110 + $33\frac{1}{3}$% of \$4110 = \$4110 + \$1370 = \$5480.

Selling price per dozen must be

\$5480 ÷ 600 = \$$9.13\frac{1}{3}$. *Ans.*

[redacted]

Exercise 118. Page 259.

1. Find the interest on \$125.65 for 1 mo. at 6 %.

1 mo. = $\frac{1}{12}$ yr.

$\$125.65$
0.06
$12\,\underline{|\,\$7.5390}$
$\$0.62825$
$\$0.63.$ *Ans.*

2. Find the interest on \$1165 for 3 yr. at 5 %.

$\$1165$
0.05
$\$58.25$
3
$\$174.75$ *Ans.*

3. Find the interest on \$1296.50 for 2 mo. at $5\frac{1}{2}$ %.

2 mo. = $\frac{1}{6}$ yr.

$\$1296.50$
0.055
648250
648250
$6\,\underline{|\,\$71.30750}$
$\$11.88458$
$\$11.88.$ *Ans.*

4. Find the interest on \$630.50 for 3 yr. at 4 %.

$\$630.50$
0.04
$\$25.2200$
3
$\$75.66$ *Ans.*

5. Find the interest on \$231.50 for 3 yr. 8 mo. at $4\frac{1}{2}$ %.

3 yr. 8 mo. = $3\frac{2}{3}$ yr.

$\$231.50$
0.045
115750
92600
$\$10.4175$
$3\frac{2}{3}$
69450
312525
$\$38.1975$
$\$38.20.$ *Ans.*

6. Find the interest on \$580.40 for 2 yr. 4 mo. at 6 %.

2 yr. 4 mo. = $2\frac{1}{3}$ yr.

$\$580.40$
0.06
$\$34.8240$
$2\frac{1}{3}$
11608
69648
$\$81.256$
$\$81.26.$ *Ans.*

7. Find the interest on \$285.85 for 1 yr. 7 mo. at 4 %.

1 yr. 7 mo. = $1\frac{7}{12}$ yr.

$\$285.85$
0.04
$\$11.4340$
$1\frac{7}{12}$
$6669\frac{5}{6}$
11434
$\$18.103\frac{5}{6}$
$\$18.10.$ *Ans.*

8. Find the interest on $1275.35 for 3 yr. 2 mo. at $3\frac{1}{2}$%.

3 yr. 2 mo. = $3\frac{1}{6}$ yr.

$$\begin{array}{r} \$1275.35 \\ 0.035 \\ \hline 637675 \\ 382605 \\ \hline \$44.63725 \\ 3\frac{1}{6} \\ \hline 743954\frac{1}{6} \\ 13391175 \\ \hline \$141.35129\frac{1}{6} \end{array}$$

$141.35. *Ans.*

Exercise 119. Page 260.

1. Find the interest at 6% on $744.20 for 3 yr. 6 mo. 18 dy.

3 yr.	6 mo.	18 dy.
$0.18	0.03	0.003

$$\begin{array}{r} \$0.18 \\ 0.03 \\ 0.003 \\ \hline \$0.213 \end{array} \qquad \begin{array}{r} \$744.20 \\ 0.213 \\ \hline 223260 \\ 74420 \\ 148840 \\ \hline \$158.51460 \end{array}$$

$158.51. *Ans.*

2. Find the interest at 6% on $625.44 for 6 yr. 7 mo. 12 dy.

6 yr.	7 mo.	12 dy.
$0.36	0.035	0.002

$$\begin{array}{r} \$0.36 \\ 0.035 \\ 0.002 \\ \hline \$0.397 \end{array} \qquad \begin{array}{r} \$625.44 \\ 0.397 \\ \hline 437808 \\ 562896 \\ 187632 \\ \hline \$248.29968 \end{array}$$

$248.30. *Ans.*

3. Find the interest at 6% on $124.87 for 2 yr. 10 mo. 16 dy.

2 yr.	10 mo.	16 dy.
$0.12	0.05	$0.002\frac{2}{3}$

$$\begin{array}{r} \$0.12 \\ 0.05 \\ 0.002\frac{2}{3} \\ \hline \$0.172\frac{2}{3} \end{array} \qquad \begin{array}{r} \$124.87 \\ 0.172\frac{2}{3} \\ \hline 8324\frac{2}{3} \\ 24974 \\ 87409 \\ 12487 \\ \hline \$21.56088\frac{1}{3} \end{array}$$

$21.56. *Ans.*

[redacted]

4. Find the interest at 6% on $847.64 from Jan. 12, 1896 to Aug. 7, 1899.

yr.	mo.	dy.
1899	8	7
1896	1	12
3	6	25

3 yr.	6 mo.	25 dy.
$0.18	0.03	$0.004\frac{1}{6}$

$0.18
0.03
$0.004\frac{1}{6}$
$\$0.214\frac{1}{6}$

$847.64
$0.214\frac{1}{6}$
$14127\frac{1}{3}$
339056
84764
169528
$\$181.53623\frac{1}{3}$

$181.54. *Ans.*

5. Find the interest at 6% on $84.84 from Mar. 22, 1895 to Jan. 1, 1898.

yr.	mo.	dy.
1898	1	1
1895	3	22
2	9	9

2 yr.	9 mo.	9 dy.
$0.12	0.045	0.0015

$0.12
0.045
0.0015
$0.1665

$84.84
0.1665
42420
50904
50904
8484
$14.125860

$14.13. *Ans.*

6. Find the interest at 6% on $1248.27 from Apr. 7, 1894 to May 17, 1897.

yr.	mo.	dy.
1897	5	17
1894	4	7
3	1	10

3 yr.	1 mo.	10 dy.
$0.18	0.005	$0.001\frac{2}{3}$

$0.18
0.005
$0.001\frac{2}{3}$
$\$0.186\frac{2}{3}$

$1248.27
$0.186\frac{2}{3}$
83218
748962
998616
124827
$233.01040

$233.01. *Ans.*

[redacted]

Exercise 120. Page 261.

1. Find the interest at 6% on $1278.75 for 1 mo.; 2 mo.; 3 mo.; 4 mo.

Interest on $1278.75 for 2 mo. = $12.7875 = $12.79. *Ans.*
Interest on $1278.75 for 1 mo. = $\frac{1}{2}$ of $12.7875 = $6.39. *Ans.*
Interest on $1278.75 for 3 mo. = $\frac{3}{2}$ of $12.7875 = $19.18. *Ans.*
Interest on $1278.75 for 4 mo. = 2 × $12.7875 = $25.58. *Ans.*

2. Find the interest at 6% on $2265.50 for 1 mo.; 2 mo.; 3 mo.; 4 mo.

Interest on $2265.50 for 2 mo. = $22.655 = $22.66. *Ans.*
Interest on $2265.50 for 1 mo. = $\frac{1}{2}$ of $22.655 = $11.33. *Ans.*
Interest on $2265.50 for 3 mo. = $\frac{3}{2}$ of $22.655 = $33.98. *Ans.*
Interest on $2265.50 for 4 mo. = 2 × $22.655 = $45.31. *Ans.*

3. Find the interest at 6% on $1840.25 for 30 dy.; 60 dy.; 90 dy.

Interest on $1840.25 for 30 dy. = 5 × $1.84025 = $9.20. *Ans.*
Interest on $1840.25 for 60 dy. = 10 × $1.84025 = $18.40. *Ans.*
Interest on $1840.25 for 90 dy. = 15 × $1.84025 = $27.60. *Ans.*

4. Find the interest at 6% on $1946.75 for 30 dy.; 60 dy.; 90 dy.

Interest on $1946.75 for 30 dy. = 5 × $1.94675 = $9.73. *Ans.*
Interest on $1946.75 for 60 dy. = 10 × $1.94675 = $19.47. *Ans.*
Interest on $1946.75 for 90 dy. = 15 × $1.94675 = $29.20. *Ans.*

Exercise 121. Page 262.

1. Find the interest on $680.40 for 2 yr. 4 mo. 6 dy. at 6%.

2 yr.	4 mo.	6 dy.
$0.12	0.02	0.001

$0.12
0.02
0.001
$0.141

$680.40
0.141
68040
272160
68040
$95.93640 $95.94. *Ans.*

2. Find the interest on $25.62 for 30 dy. at 6%.

30 dy.
$0.005

$25.62
0.005
$0.12810 $0.13. *Ans.*

[redacted]

3. Find the interest on $85.85 for 1 yr. 7 mo. 21 dy., at 6 %.

1 yr.	7 mo.	21 dy.
$0.06	0.035	0.0035

$0.06
0.035
0.0035
$0.0985

$85.85
0.0985
42925
68680
77265
$8.456225

$8.46. *Ans.*

4. Find the interest on $1100 for 3 yr. 4 mo., at 5 %.

3 yr.	4 mo.
$0.18	0.02

$0.18
0.02
$0.20

$1100
0.20
6 | $220.00
36.67
$183.33 *Ans.*

5. Find the interest on $1275 for 3 yr. 2 mo. 15 dy., at 8 %.

3 yr.	2 mo.	15 dy.
$0.18	0.01	0.0025

$0.18
0.01
0.0025
$0.1925

$1275
0.1925
6375
2550
11475
1275
3 | $245.4375
81.8125
$327.25 *Ans.*

6. Find the interest on $475.16 for 27 dy., at 4½ %.

27 dy.
$0.0045

$475.16
0.0045
237580
190064
4 | $2.138220
0.534555
$1.603665

$1.60. *Ans.*

7. Find the interest on $1290.50 for 60 dy., at 6 %.

60 dy.
$0.01

$1290.50
0.01
$12.9050

$12.91. *Ans.*

[redacted]

8. Find the interest on $125 for 1 yr. 2 mo. 2 dy., at 9%.

1 yr.	2 mo.	2 dy.
$0.06	0.01	$0.000\frac{1}{3}$

$0.06
0.01
$0.000\frac{1}{3}$
$\$0.070\frac{1}{3}$

$125
$0.070\frac{1}{3}$
$41\frac{2}{3}$
8750
2 | $$8.791\frac{2}{3}$
4.395
$13.186

$13.19. *Ans.*

9. Find the interest on $250.80 for 10 mo. 10 dy., at $3\frac{1}{2}$%.

10 mo.	10 dy.
$0.05	$0.001\frac{2}{3}$

$0.05
$0.001\frac{2}{3}$
$\$0.051\frac{2}{3}$

$250.80
$0.051\frac{2}{3}$
16720
25080
125400
12 | $12.95800
$1.07983
7
$7.55881

$7.56. *Ans.*

10. Find the interest on $258.85 from Mar. 6 to June 24, at 5%.

mo.	dy.
6	24
3	6
3	18

3 mo.	18 dy.
$0.015	0.003

$0.015
0.003
$0.018

$258.85
0.018
207080
25885
6 | $4.65930
0.77655
$3.88275

$3.88. *Ans.*

11. Find the interest on $380 for 2 yr. 11 mo. 27 dy., at $4\frac{1}{2}$%.

2 yr.	11 mo.	27 dy.
$0.12	0.055	0.0045

$0.12
0.055
0.0045
$0.1795

$0.1795
380
143600
5385
4 | $68.2100
17.0525
$51.1575

$51.16. *Ans.*

[redacted]

12. [illegible]

[illegible] Ans.

13. [illegible]

[illegible] Ans.

14. Find the interest on [illegible] at [illegible] %.

[illegible]	[illegible]
	13.7234
$.21	$ 68.6171
	$68.62. Ans.

15. Find the interest on $630.50 for 90 dy., at 6 %.

90 dy	$630.50
$0.015	0.015
	315250
	63050
	$9.45750
	$9.46. Ans.

[redacted]

16. Find the interest on \$547.60 from Feb. 20 to Dec. 5, at 6½%.

mo.	dy.
12	5
2	20
9	15

9 mo.	15 dy.
\$0.045	0.0025
0.0025	
\$0.0475	

\$547.60
0.0475
273800
383320
219040
12 | \$26.011000
2.1676
\$28.1786

\$28.18. *Ans.*

17. Find the interest on \$875 from May 5, 1897 to June 21, 1898, at 5½%.

yr.	mo.	dy.
1898	6	21
1897	5	5
1	1	16

1 yr.	1 mo.	16 dy.
\$0.06	0.005	0.002⅔
0.005		
0.002⅔		
\$0.067⅔		

\$875
0.067⅔
583⅓
6125
5250
12 | \$59.208⅓
4.934
\$54.274

\$54.27. *Ans.*

18. Find the interest on \$758.50 from Jan. 5 to July 1, at 4½%.

mo.	dy.
7	1
1	5
5	26

5 mo.	26 dy.
\$0.025	0.004⅓
0.004⅓	
\$0.029⅓	

\$758.50
0.029⅓
25283⅓
682650
151700
4 | \$22.24933⅓
5.56233
\$16.68700

\$16.69. *Ans.*

[redacted]

19. Find the interest on $342.42 from Feb. 5, 1897 to Mar. 15, 1899, at 7 %.

yr.	mo.	dy.
1899	3	15
1897	2	5
2	1	10

2 yr.	1 mo.	10 dy.
$0.12	0.005	0.001⅔

$0.12
0.005
0.001⅔
$0.126⅔

$342.42
0.126⅔
22828
205452
68484
34242
6 | $43.37320
7.2288
$50.6020

$50.60. *Ans.*

20. Find the interest on $540 from Mar. 5 to Sept. 21, at 3½ %.

mo.	dy.
9	21
3	5
6	16

6 mo.	16 dy.
$0.03	0.002⅔

$0.03
0.002⅔
$0.032⅔

$540
0.032⅔
360
1080
1620
12 | $17.640
$1.47
7
$10.29 *Ans.*

21. Find the amount of $431.50 for 2 yr. 8 mo., at 4½ %.

2 yr.	8 mo.
$0.12	0.04

$0.12
0.04
$0.16

$431.50
0.16
258900
43150
4 | $69.0400
17.26
$51.78
431.50
$483.28 *Ans.*

[redacted]

22. Find the amount of $476.50 from July 5, 1897 to Feb. 9, 1898, at 4 %.

yr.	mo.	dy.
1898	2	9
1897	7	5
	7	4

7 mo.	4 dy.
$0.035	0.000⅔

$0.035
0.000⅔
$0.035⅔

$476.50
0.035⅔
31766⅔
238250
142950
3 | $16.99516⅔
5.665
$11.33
476.50
$487.83 *Ans.*

23. Find the amount of $319.20 from Apr. 7 to Aug. 31, at 3¼ %.

mo.	dy.
8	31
4	7
4	24

4 mo.	24 dy.
$0.02	0.004

$0.02
0.004
$0.024

$4.15
319.20
$323.35 *Ans.*

$319.20
0.024
127680
63840
24 | $7.66080
0.3192
13
9576
3192
$4.1496

24. Find the amount of $6460 from June 15, 1897 to May 7, 1899, at 4¼ %.

yr.	mo.	dy.
1899	5	7
1897	6	15
1	10	22

1 yr.	10 mo.	22 dy.
$0.06	0.05	0.003⅔

$0.06
0.05
0.003⅔
$0.113⅔

$520.12
6460.
$6980.12 *Ans.*

$6460
0.113⅔
4306⅔
19380
6460
6460
24 | $734.286⅔
$30.5953
17
2141671
305953
$520.1201

[redacted]

25. Find the amount of $150 from Aug. 5, 1897 to Mar. 17, 1899, at 7%.

yr.	mo.	dy.
1899	3	17
1897	8	5
1	7	12

1 yr.	7 mo.	12 dy.
$0.06	0.035	0.002
0.035		
0.002		
$0.097		

$150
0.097
1050
1350
6 | $14.550
2.425
$16.975

$16.98
150.
$166.98 *Ans.*

26. Find the amount of $527.20 from Jan. 1 to Nov. 20, at 4½%.

mo.	dy.
11	20
1	1
10	19

10 mo.	19 dy.
$0.05	0.003⅙
0.003⅙	
$0.053⅙	

$527.20
0.053⅙
8786⅔
158160
263600
4 | $28.02946⅔
7.00736
$21.02210

$21.02
527.20
$548.22 *Ans.*

27. Find the amount of $1250 from Nov. 15, 1897 to Mar. 1, 1898, at 5%.

yr.	mo.	dy.
1898	3	1
1897	11	15
	3	16

3 mo.	16 dy.
$0.015	0.002⅔
0.002⅔	
$0.017⅔	

$1250
0.017⅔
833⅓
8750
1250
6 | $22.083⅓
3.680
$18.40
1250.
$1268.40 *Ans.*

[redacted]

28. Find the amount of $624.36 from Mar. 5 to Dec. 20, at $7\frac{3}{10}$%.

mo.	dy.
12	20
3	5
9	15

9 mo.	15 dy.
$0.045	0.0025
0.0025	
$0.0475	

$36.08
624.36
$660.44 *Ans.*

$624.36
0.0475
312180
437052
249744
6 | $29.657100
$4.94285
7.3
1482855
3459995
$36.082805

29. Find the amount of $12,260 from May 6 to Oct. 24, at $3\frac{3}{4}$%.

mo.	dy.
10	24
5	6
5	18

5 mo.	18 dy.
$0.025	0.003
0.003	
$0.028	

$12260
0.028
98080
24520
8 | $343.280
$42.91
5
$214.55
12260.
$12474.55 *Ans.*

30. Find the amount of $11,216 from Oct. 20 to Dec. 31, at 1% a month.

1% a month is 12% a year.

mo.	dy.
12	31
10	20
2	11

2 mo.	11 dy.
$0.01	$0.001\frac{5}{6}$
$0.001\frac{5}{6}$	
$$0.011\frac{5}{6}$	

$11216
$0.011\frac{5}{6}$
$9346\frac{2}{3}$
11216
11216
$$132.722\frac{2}{3}$
2
$$265.445\frac{1}{3}$
11216.
$11481.45 *Ans.*

Exercise 122. Page 264.

1. Find the rate per cent when the interest on \$326 for 15 yr. is \$220.05.

$$r = \frac{i}{pt}.$$

Here $i = \$220.05$; $p = \$326$; $t = 15$ yr.

Hence, $$r = \frac{220.05}{326 \times 15} = 0.045.$$

Therefore, the rate required is $4\frac{1}{2}\%$.

2. Find the rate per cent when the interest on \$745 for 18 yr. is \$603.45.

$$r = \frac{i}{pt}.$$

Here $i = \$603.45$; $p = \$745$; $t = 18$ yr.

Hence, $$r = \frac{603.45}{745 \times 18} = 0.045.$$

Therefore, the rate required is $4\frac{1}{2}\%$.

3. Find the rate per cent when \$980 amounts to \$1016.75 in 9 mo.

$$r = \frac{i}{pt}.$$

Here $i = \$1016.75 - \$980 = \$36.75$; $p = \$980$; $t = 9$ mo. $= 0.75$ yr.

Hence, $$r = \frac{36.75}{980 \times 0.75} = 0.05.$$

Therefore, the rate required is 5%.

4. Find the rate per cent when the interest on \$470.50 is \$141.15 for 5 yr.

$$r = \frac{i}{pt}.$$

Here $i = \$141.15$; $p = \$470.50$; $t = 5$ yr.

Hence, $$r = \frac{141.15}{470.50 \times 5} = 0.06.$$

Therefore, the rate required is 6%.

5. Find the rate per cent when \$3631.25 amounts to \$3715.98 for 7 mo.

$$r = \frac{i}{pt}.$$

Here $\quad i = \$3715.98 - \$3631.25 = \$84.73;$

$$p = \$3631.25; \; t = 7 \text{ mo.} = \tfrac{7}{12} \text{ yr.}$$

Hence, $\quad r = \dfrac{84.73}{3631.25 \times \frac{7}{12}} = 0.04.$

Therefore, the rate required is 4 %.

6. Find the rate per cent when the interest on \$997.75 is \$199.55 for 5 yr. 4 mo.

$$r = \frac{i}{pt}.$$

Here $\quad i = \$199.55; \; p = \$997.75; \; t = 5 \text{ yr. } 4 \text{ mo.} = 5\tfrac{1}{3} \text{ yr.}$

Hence, $\quad r = \dfrac{199.55}{997.75 \times 5\frac{1}{3}} = 0.0375.$

Therefore, the rate required is $3\frac{3}{4}$ %.

7. Find the rate per cent when \$350 amounts to \$406.70 in 3 yr 7 mo. 6 dy.

$$r = \frac{i}{pt}.$$

Here $\quad i = \$406.70 - \$350 = \$56.70;$

$$p = \$350; \; t = 3 \text{ yr. } 7 \text{ mo. } 6 \text{ dy.} = 3.6 \text{ yr.}$$

Hence, $\quad r = \dfrac{56.70}{350 \times 3.6} = 0.045.$

Therefore, the rate required is $4\frac{1}{2}$ %.

8. Find the rate per cent when the interest on \$6875 is \$68.75 for 90 dy.

$$r = \frac{i}{pt}.$$

Here $\quad i = \$68.75; \; p = \$6875; \; t = 90 \text{ dy.} = 0.25 \text{ yr.}$

Hence, $\quad r = \dfrac{68.75}{6875 \times 0.25} = 0.04.$

Therefore, the rate required is 4 %.

[redacted]

9. Find the rate per cent when the interest on \$642 is \$10.70 for 5 mo.

$$r = \frac{i}{pt}.$$

Here $i = \$10.70$; $p = \$642$; $t = 5$ mo. $= \frac{5}{12}$ yr.

Hence, $$r = \frac{10.70}{642 \times \frac{5}{12}} = 0.04.$$

Therefore, the rate required is 4 %.

10. Find the rate per cent when the interest on \$8432 for 2 yr. 7 mo. 23 dy. is \$1339.28.

$$r = \frac{i}{pt}.$$

Here $i = \$1339.28$; $p = \$8432$; $t = 2$ yr. 7 mo. 23 dy. $= 2\frac{233}{360}$ yr.

Hence, $$r = \frac{1339.28}{8432 \times 2\frac{233}{360}} = 0.06.$$

Therefore, the rate required is 6 %.

11. Find the rate per cent when a sum of money is doubled in 14 yr.

$$r = \frac{i}{pt}.$$

Here $i = \$1$; $p = \$1$; $t = 14$ yr.

Hence, $$r = \frac{1}{1 \times 14} = 0.07\frac{1}{7}.$$

Therefore, the rate required is $7\frac{1}{7}$ %.

12. Find the rate per cent when an investment for 4 yr. 2 mo. produces a sum equal to $\frac{5}{24}$ of the capital.

$$r = \frac{i}{pt}.$$

Here $i = \$\frac{5}{24}$; $p = \$1$; $t = 4$ yr. 2 mo. $= 4\frac{1}{6}$ yr.

Hence, $$r = \frac{\frac{5}{24}}{1 \times 4\frac{1}{6}} = 0.05.$$

Therefore, the rate required is 5 %.

13. Find the rate per cent when an investment for 3 yr. 1 mo. 15 dy. produces a sum equal to $\frac{1}{8}$ of the capital.

$$r = \frac{i}{pt}.$$

Here $\quad i = \$\frac{1}{8}$; $p = \$1$; $t = 3$ yr. 1 mo. 15 dy. $= 3\frac{1}{8}$ yr.

Hence, $\quad r = \dfrac{\frac{1}{8}}{1 \times 3\frac{1}{8}} = 0.04.$

Therefore, the rate required is 4 %.

14. Find the time in which the interest on \$450 will amount to \$72, at 4 %.

$$t = \frac{i}{pr}.$$

Here $\quad i = \$72$; $p = \$450$; $r = 4\,\% = 0.04.$

Hence, $\quad t = \dfrac{72}{450 \times 0.04} = 4.$

Therefore, the time required is 4 yr.

15. Find the time in which the interest on \$487.50 will amount to \$39, at 4 %.

$$t = \frac{i}{pr}.$$

Here $\quad i = \$39$; $p = \$487.50$; $r = 4\,\% = 0.04.$

Hence, $\quad t = \dfrac{39}{487.50 \times 0.04} = 2.$

Therefore, the time required is 2 yr.

16. Find the time in which the interest on \$238.75 will amount to \$64.46, at $4\frac{1}{2}$ %.

$$t = \frac{i}{pr}.$$

Here $\quad i = \$64.46$; $p = \$238.75$; $r = 4\frac{1}{2}\,\% = 0.045.$

Hence, $\quad t = \dfrac{64.46}{238.75 \times 0.045} = 6.$

Therefore, the time required is 6 yr.

[redacted]

17. Find the time in which the sum of \$1587.75 will amount to \$1611.68, at $5\frac{1}{2}\%$.

$$t = \frac{i}{pr}.$$

Here

$i = \$1611.68 - \$1587.75 = \$23.93$; $p = \$1587.75$; $r = 5\frac{1}{2}\% = 0.055$.

Hence, $$t = \frac{23.93}{1587.75 \times 0.055} = 0.274.$$

Therefore, the time required is 0.274 yr. = 3 mo. 9 dy.

18. Find the time in which the sum of \$1 will double itself, at 4%.

$$t = \frac{i}{pr}.$$

Here $i = \$1$; $p = \$1$; $r = 4\% = 0.04$.

Hence, $$t = \frac{1}{1 \times 0.04} = 25.$$

Therefore, the time required is 25 yr.

19. Find the time in which the sum of \$10 will amount to \$17, at 6%.

$$t = \frac{i}{pr}.$$

Here $i = \$17 - \$10 = \$7$; $p = \$10$; $r = 6\% = 0.06$.

Hence, $$t = \frac{7}{10 \times 0.06} = 11\frac{2}{3}.$$

Therefore, the time required is $11\frac{2}{3}$ yr. = 11 yr. 8 mo.

20. Find the time in which the sum of \$502.67 will amount to \$578.07, at $4\frac{1}{2}\%$.

$$t = \frac{i}{pr}.$$

Here $i = \$578.07 - \$502.67 = \$75.40$; $p = \$502.67$; $r = 4\frac{1}{2}\% = 0.045$.

Hence, $$t = \frac{75.40}{502.67 \times 0.045} = 3.333.$$

Therefore, the time required is 3.333 yr. = 3 yr. 4 mo.

21. Find the time in which the interest on \$537.50 will amount to \$80.62, at 4%.

$$t = \frac{i}{pr}.$$

Here $i = \$80.62;\ p = \$537.50;\ r = 4\% = 0.04.$

Hence, $$t = \frac{80.62}{537.50 \times 0.04} = 3.750.$$

Therefore, the time required is 3.750 yr. = 3 yr. 9 mo.

22. Find the time in which the interest on \$6875 will amount to \$75.05, at $4\frac{1}{4}$%.

$$t = \frac{i}{pr}.$$

Here $i = \$75.05;\ p = \$6875;\ r = 4\tfrac{1}{4}\% = 0.0425.$

Hence, $$t = \frac{75.05}{6875 \times 0.0425} = 0.2569.$$

Therefore, the time required is 0.2569 yr. = 3 mo. 2 dy.

23. Find the time in which the interest on \$8520 will amount to \$1746.60, at 6%.

$$t = \frac{i}{pr}.$$

Here $i = \$1746.60;\ p = \$8520;\ r = 6\% = 0.06.$

Hence, $$t = \frac{1746.60}{8520 \times 0.06} = 3\tfrac{5}{12}.$$

Therefore, the time required is $3\frac{5}{12}$ yr. = 3 yr. 5 mo.

24. Find the principal that will produce \$90 interest in 3 yr., at 4%.

$$p = \frac{i}{rt}.$$

Here $i = \$90;\ r = 4\% = 0.04;\ t = 3$ yr.

Hence, $$p = \$\frac{90}{0.04 \times 3} = \$750.$$

Therefore, the principal required is \$750.

[redacted]

25. Find the principal that will produce \$63 interest in 3 yr., at $6\frac{1}{4}\%$.

$$p = \frac{i}{rt}.$$

Here $\qquad i = \$63;\ r = 6\frac{1}{4}\% = 0.0625;\ t = 3$ yr.

Hence, $\qquad p = \$\frac{63}{0.0625 \times 3} = \$336.$

Therefore, the principal required is \$336.

26. Find the principal that will produce \$100 interest in 8 yr. 6 mo., at 5%.

$$p = \frac{i}{rt}.$$

Here $\quad i = \$100;\ r = 5\% = 0.05;\ t = 8$ yr. 6 mo. $= 8.5$ yr.

Hence, $\qquad p = \$\frac{100}{0.05 \times 8.5} = \$235.29.$

Therefore, the principal required is \$235.29.

27. Find the principal that will produce \$1746.60 interest in 3 yr. 5 mo., at 6%.

$$p = \frac{i}{rt}.$$

Here $\quad i = \$1746.60;\ r = 6\% = 0.06;\ t = 3$ yr. 5 mo. $= 3\frac{5}{12}$ yr.

Hence, $\qquad p = \$\frac{1746.60}{0.06 \times 3\frac{5}{12}} = \$8520.$

Therefore, the principal required is \$8520.

28. Find the principal that will produce \$12 interest in 7 mo., at 5%.

$$p = \frac{i}{rt}.$$

Here $\qquad i = \$12;\ r = 5\% = 0.05;\ t = 7$ mo. $= \frac{7}{12}$ yr.

Hence, $\qquad p = \$\frac{12}{0.05 \times \frac{7}{12}} = \$411.43.$

Therefore, the principal required is \$411.43.

29. Find the principal that will produce \$50 interest in 228 dy., at $4\frac{1}{2}\%$.

$$p = \frac{i}{rt}.$$

Here $i = \$50$; $r = 4\frac{1}{2}\% = 0.045$; $t = 228 \text{ dy.} = \frac{19}{30}$ yr.

Hence, $$p = \$\frac{50}{0.045 \times \frac{19}{30}} = \$1754.39.$$

Therefore, the principal required is \$1754.39.

30. Find the principal that will produce \$1339.28 interest in 2 yr. 7 mo. 24 dy., at 6%.

$$p = \frac{i}{rt}.$$

Here $i = \$1339.28$; $r = 6\% = 0.06$; $t = 2 \text{ yr. } 7 \text{ mo. } 24 \text{ dy.} = 2.65$ yr.

Hence, $$p = \$\frac{1339.28}{0.06 \times 2.65} = \$8423.14.$$

Therefore, the principal required is \$8423.14.

31. Find the principal that will produce \$1312.65 interest in 2 yr. 3 mo., at 6%.

$$p = \frac{i}{rt}.$$

Here $i = \$1312.65$; $r = 6\% = 0.06$; $t = 2 \text{ yr. } 3 \text{ mo.} = 2.25$ yr.

Hence, $$p = \$\frac{1312.65}{0.06 \times 2.25} = \$9723.33.$$

Therefore, the principal required is \$9723.33.

32. Find the principal that will produce \$750 interest in 3 yr. 8 mo., at 5%.

$$p = \frac{i}{rt}.$$

Here $i = \$750$; $r = 5\% = 0.05$; $t = 3 \text{ yr. } 8 \text{ mo.} = 3\frac{2}{3}$ yr.

Hence, $$p = \$\frac{750}{0.05 \times 3\frac{2}{3}} = \$4090.91.$$

Therefore, the principal required is \$4090.91.

[redacted]

33. Find the principal that will amount to $ [illegible] in 3 [illegible] at 4 %.

$$p = \frac{a}{1 + rt}.$$

Here $a = \$$ [illegible]; $r =$ [illegible]; $t =$ [illegible]

Hence $p = \$$ [illegible] $= \$$ [illegible]

Therefore the principal required is $ [illegible]

34. Find the principal that will amount to $ [illegible] in 2 yr. 6 mo. at $4\frac{1}{4}$ %.

$$p = \frac{a}{1 + rt}.$$

Here $a = \$$ [illegible]; $r = 4\frac{1}{4}\% = 0.0425$; $t = 2$ yr. 6 mo. $= 2.5$ yr.

Hence, $p = \$ \frac{[illegible]}{1 + 0.0425 \times 2.5} = \$ \frac{[illegible]}{1.10625} = \$$ [illegible]

Therefore the principal required is $ [illegible]

35. Find the principal that will amount to $ 6000 in 21 dy., at 5 %.

$$p = \frac{a}{1 + rt}.$$

Here $a = \$6000$; $r = 5\% = 0.05$; $t = 21$ dy. $= \frac{7}{120}$ yr.

Hence, $p = \$ \frac{6000}{1 + 0.05 \times \frac{7}{120}} = \$ \frac{6000}{1\frac{7}{2400}} = \$5982.55.$

Therefore, the principal required is $ 5982.55.

36. Find the principal that will amount to $ 297.60 in 8 mo., at 6 %.

$$p = \frac{a}{1 + rt}.$$

Here $a = \$297.60$; $r = 6\% = 0.06$; $t = 8$ mo. $= \frac{2}{3}$ yr.

Hence, $p = \$ \frac{297.60}{1 + 0.06 \times \frac{2}{3}} = \$ \frac{297.60}{1.04} = \$286.15.$

Therefore, the principal required is $ 286.15.

[redacted]

37. Find the principal that will amount to \$6378.75 in 1 yr. 1 mo., at 5%.

$$p = \frac{a}{1 + rt}.$$

Here $a = \$6378.75$; $r = 5\% = 0.05$; $t = 1$ yr. 1 mo. $= 1\frac{1}{12}$ yr.

Hence, $p = \$\frac{6378.75}{1 + 0.05 \times 1\frac{1}{12}} = \$\frac{6378.75}{1\frac{13}{240}} = \$6050.99.$

Therefore, the principal required is \$6050.99.

38. Find the principal that will amount to \$21,047.95 in 1 yr. 7 mo. 21 dy., at $4\frac{1}{2}\%$.

$$p = \frac{a}{1 + rt}.$$

Here $a = \$21,047.95$; $r = 4\frac{1}{2}\% = 0.045$;

$t = 1$ yr. 7 mo. 21 dy. $= 1\frac{77}{120}$ yr.

Hence, $p = \$\frac{21047.95}{1 + 0.045 \times 1\frac{77}{120}} = \$\frac{21047.95}{1.073875} = \$19,600.$

Therefore, the principal required is \$19,600.

39. Find the principal that will amount to \$185.09 in 2 yr. 3 mo. 18 dy., at 5%.

$$p = \frac{a}{1 + rt}.$$

Here $a = \$185.09$; $r = 5\% = 0.05$; $t = 2$ yr. 3 mo. 18 dy. $= 2.3$ yr.

Hence, $p = \$\frac{185.09}{1 + 0.05 \times 2.3} = \$\frac{185.09}{1.115} = \$166.$

Therefore, the principal required is \$166.

40. Find the principal that will amount to \$659.40 in 2 yr. 11 mo. 15 dy., at 6%.

$$p = \frac{a}{1 + rt}.$$

Here $a = \$659.40$; $r = 6\% = 0.06$; $t = 2$ yr. 11 mo. 15 dy. $= 2\frac{23}{24}$ yr.

Hence, $p = \$\frac{659.40}{1 + 0.06 \times 2\frac{23}{24}} = \$\frac{659.40}{1\frac{71}{400}} = \$560.$

Therefore, the principal required is \$560.

[redacted]

[illegible]

44. At what rate per cent [illegible] interest in [illegible]

$$r = \frac{[illegible]}{pt}$$

Here [illegible]

Hence $r = \frac{[illegible]}{[illegible]} = 0.07.$

Therefore, the rate required is 7 %.

45. The principal is \$653; the interest, \$5.52; the rate, 8%. Find the time.

$$t = \frac{i}{pr}.$$

Here $\quad i = \$5.52$; $p = \$653$; $r = 8\% = 0.08.$

Hence, $$t = \frac{5.52}{653 \times 0.08} = 0.1057.$$

Therefore, the time required is 0.1057 yr. = 1 mo. 8 dy.

46. Find the amount of \$520 for 2 mo. 3 dy., at $4\frac{1}{2}$%.

2 mo.	3 dy.
\$0.01	0.0005
0.0005	
\$0.0105	

$$\begin{array}{r} \$520 \\ 0.0105 \\ \hline 2600 \\ 520 \\ \hline 4\,|\,\$5.4600 \\ 1.365 \\ \hline \$4.095 \end{array}$$

$$\begin{array}{r} \$4.10 \\ 520. \\ \hline \$524.10 \end{array}$$ *Ans.*

47. What sum bearing interest at $4\frac{1}{2}$% will yield an annual income of \$1000?

$$p = \frac{i}{rt}.$$

Here $\quad i = \$1000$; $r = 4\frac{1}{2}\% = 0.045$; $t = 1$ yr.

Hence, $$p = \$\frac{1000}{0.045 \times 1} = \$22{,}222.22.$$

Therefore, the principal required is \$22,222.22.

48. In what time will \$4000 amount to \$4625, at $5\frac{1}{2}$%?

$$t = \frac{i}{pr}.$$

Here $\quad i = \$4625 - \$4000 = \$625$; $p = \$4000$; $r = 5\frac{1}{2}\% = 0.055.$

Hence, $$t = \frac{625}{4000 \times 0.055} = 2.841.$$

Therefore, the time required is 2.841 yr. = 2 yr. 10 mo. 3 dy.

[redacted]

49. At what rate per cent will $3000 produce $250 interest in 1 yr. 10 mo. 7 dy.?

$$r = \frac{i}{pt}.$$

Here $i = \$250$; $p = \$3000$; $t = 1$ yr. 10 mo. 7 dy. $= 1\frac{307}{360}$ yr.

Hence, $$r = \frac{250}{3000 \times 1\frac{307}{360}} = 0.045.$$

Therefore, the rate required is $4\frac{1}{2}\%$.

50. Find the interest on $1721.84 from April 1 to Nov. 12, at $4\frac{1}{2}\%$.

mo.	dy.
11	12
4	1
7	11

7 mo.	11 dy.
$0.035	$0.001\frac{5}{6}$
$0.001\frac{5}{6}$	
$\$0.036\frac{5}{6}$	

$$\begin{array}{r} \$1721.84 \\ \underline{0.036\frac{5}{6}} \\ 143486\frac{2}{3} \\ 1033104 \\ \underline{516552} \\ 4\,|\,\$63.42110\frac{2}{3} \\ \underline{15.8553} \\ \$47.5658 \end{array}$$

$47.57. *Ans.*

51. How long must $3904.92 be on interest to amount to $4568.76, at 5%?

$$t = \frac{i}{pr}.$$

Here $i = \$4568.76 - \$3904.92 = \$663.84$; $p = \$3904.92$;
$r = 5\% = 0.05.$

Hence, $$t = \frac{663.84}{3904.92 \times 0.05} = 3.400.$$

Therefore, the time required = 3.400 yr. = 3 yr. 4 mo. 24 dy.

52. Find the interest on $137.60 from July 3 to Dec. 12, at $7\frac{3}{10}\%$.

mo.	dy.
12	12
7	3
5	9

5 mo.	9 dy.
$0.025	0.0015
0.0015	
$0.0265	

$$\begin{array}{r} \$137.60 \\ \underline{0.0265} \\ 68800 \\ 82560 \\ \underline{27520} \\ 6\,|\,\$3.646400 \\ \$0.607733 \\ \underline{7.3} \\ 1823199 \\ \underline{4254131} \\ \$4.4364509 \end{array}$$

$4.44. *Ans.*

[redacted]

53. Find the interest on $680.20, at $7\frac{1}{2}\%$, for 73 dy., reckoning 365 dy. for a year.

The interest on $1 for 1 yr. at $7\frac{1}{2}\%$ is $0.075.

The interest on $1 for 73 dy., that is $\frac{73}{365}$ yr., or $\frac{1}{5}$ yr., $= \frac{1}{5}$ of $0.075 = $0.015.

$$\begin{array}{r} \$\,680.20 \\ 0.015 \\ \hline 340100 \\ 68020 \\ \hline \$\,10.20300 \end{array}$$

$10.20. *Ans.*

Exercise 123. Page 268.

1. Find the day of maturity, and amount due, having given face of note, $530.25; date of note, Jan. 12, 1897; time, 60 dy.; and rate of interest, 6%.

Time to run is 19 dy. in Jan., 28 dy. in Feb., 13 dy. in Mar.

Hence, day of maturity is Mar. 13, 1897. *Ans.*

Interest on $530.25 at 6% for 60 dy. is $10 \times \$0.53025 = \$5.30$.

$\$530.25 + \$5.30 = \$535.55$. *Ans.*

2. Find the day of maturity, and amount due, having given face of note, $687.45; date of note, Mar. 22, 1897; time, 90 dy.; and rate of interest, 5%.

Time to run is 9 dy. in Mar., 30 dy. in Apr., 31 dy. in May, 20 dy. in June.

Hence, day of maturity is June 20, 1897. *Ans.*

$$\begin{array}{r} 90 \text{ dy.} \\ \hline \$\,0.015 \end{array}$$

$$\begin{array}{r} \$\,8.59 \\ 687.45 \\ \hline \$\,696.04 \end{array}$$ *Ans.*

$$\begin{array}{r|l} & \$\,687.45 \\ & 0.015 \\ \hline & 343725 \\ & 68745 \\ \hline 6 & \$\,10.31175 \\ & 1.7186 \\ \hline & \$\,8.5931 \end{array}$$

[redacted]

3. Find the day of maturity, and amount due, having given face of note, \$286.75; date of note, Aug. 5, 1897; time, 4 mo.; and rate of interest, 4 %.

Day of maturity is 4 mo. after Aug. 5, 1897; that is, Dec. 5, 1897. *Ans.*

4 mo.	\$286.75
\$0.02	0.02
	3 | \$5.7350
\$3.82	1.9117
286.75	\$3.8233
\$290.57 *Ans.*	

4. Find the day of maturity, and amount due, having given face of note, \$944.40; date of note, Oct. 20, 1897; time, 3 mo.; and rate of interest, 4½ %.

Day of maturity is 3 mo. after Oct. 20, 1897; that is, Jan. 20, 1898. *Ans.*

3 mo.	\$944.40
\$0.015	0.015
	472200
\$10.62	94440
944.40	4 | \$14.16600
\$955.02 *Ans.*	3.5415
	\$10.6245

5. Find the day of maturity, and amount due, having given face of note, \$1262.72; date of note, Oct. 5, 1897; time, 30 dy.; and rate of interest, 5½ %.

Time to run is 26 dy. in Oct., 4 dy. in Nov.
Hence, day of maturity is Nov. 4, 1897. *Ans.*

30 dy.	\$1262.72
\$0.005	0.005
	12 | \$6.31360
\$5.79	0.5261
1262.72	\$5.7875
\$1268.51 *Ans.*	

6. Find the day of maturity, and amount due, having given face of note, \$1875.44; date of note, Dec. 16, 1897; time, 6 mo.; and rate of interest, 4 %.

Day of maturity is 6 mo. after Dec. 16, 1897; that is, June 16, 1898. *Ans.*

6 mo.	$ 1875.44
$ 0.03	0.03
	3 | $ 56.2632
$ 37.51	18.7544
1875.44	$ 37.5088
$ 1912.95 *Ans.*	

7. Find the day of maturity, and amount due, having given face of note, $ 1521.87; date of note, Apr. 30, 1897; time, 1 mo.; and rate of interest, 6 %.

Day of maturity is 1 mo. after Apr. 30, 1897; that is, May 30, 1897. *Ans.*

1 mo.	$ 1521.87	$ 7.61
$ 0.005	0.005	1521.87
	$ 7.60935	$ 1529.48 *Ans.*

8. Find the day of maturity, and amount due, having given face of note, $ 2849.65; date of note, May 22, 1897; time, 2 yr.; and rate of interest, 3½ %.

Day of maturity is 2 yr. after May 22, 1897; that is, May 22, 1899. *Ans.*

Interest on $ 1 for 2 yr. at 3½ % is $ 0.07.

$ 2849.65	$ 199.48
0.07	2849.65
$ 199.4755	$ 3049.13 *Ans.*

9. Find the day of maturity, and amount due, having given face of note, $ 1968.10; date of note, July 10, 1897; time, 2 mo.; and rate of interest, 4½ %.

Day of maturity is 2 mo. after July 10, 1897; that is, Sept. 10, 1897. *Ans.*

2 mo.	$ 1968.10
$ 0.01	0.01
	4 | $ 19.6810
$ 14.76	4.92025
1968.10	$ 14.76075
$ 1982.86 *Ans.*	

[redacted]

Find the amount due Dec. 3, 1898, on the following demand notes:

10. $875.18. CONCORD, N. H., May 10, 1897.

On demand, I promise to pay George H. Chick, or order, Eight Hundred Seventy-five and $\frac{18}{100}$ Dollars, with interest at 5%. Value received.

FREDERICK D. SIBLEY.

yr.	mo.	dy.
1898	12	3
1897	5	10
1	6	23

1 yr.	6 mo.	23 dy.
$0.06	0.03	0.003⅚
0.03		
0.003⅚		
$0.093⅚		

$875.18
0.093⅚
72931⅔
262554
787662
6 | $82.12105⅔
13.68684
$68.43421

$68.43
875.18
$943.61 *Ans.*

11. $642.75. LAKEWOOD, N. J., Oct. 25, 1897.

On demand, I promise to pay Harry Jones, or order, Six Hundred Forty-two and $\frac{75}{100}$ Dollars, with interest at 4½%. Value received.

GEORGE B. ATKINS.

yr.	mo.	dy.
1898	12	3
1897	10	25
1	1	8

1 yr.	1 mo.	8 dy.
$0.06	0.005	0.001⅓
0.005		
0.001⅓		
$0.066⅓		

$642.75
0.066⅓
21425
385650
385650
4 | $42.63575
10.65894
$31.97681

$31.98
642.75
$674.73 *Ans.*

12. $1286.50. ATLANTA, GA., Apr. 22, 1897.

On demand, I promise to pay Clarence E. Garland, or order, Twelve Hundred Eighty-six and $\frac{50}{100}$ Dollars, with interest at $5\frac{1}{2}$%. Value received.

ROBERT PAGE.

yr.	mo.	dy.
1898	12	3
1897	4	22
1	7	11

1 yr.	7 mo.	11 dy.
$0.06	0.035	$0.001\frac{5}{6}$

$0.06
0.035
$0.001\frac{5}{6}$
\$$0.096\frac{5}{6}$

$1286.50
$0.096\frac{5}{6}$
$107208\frac{1}{3}$
771900
1157850
12 | \$$124.57608\frac{1}{3}$
10.38134
$114.19474

$114.19
1286.50
$1400.69 *Ans.*

13. $2548.25. ST. PAUL, MINN., June 17, 1897.

On demand, I promise to pay Fred Lacey, or order, Twenty-five Hundred Forty-eight and $\frac{25}{100}$ Dollars, with interest at 7%. Value received.

WILLIAM P. WISSMAN.

yr.	mo.	dy.
1898	12	3
1897	6	17
1	5	16

1 yr.	5 mo.	16 dy.
$0.06	0.025	$0.002\frac{2}{3}$

$0.06
0.025
$0.002\frac{2}{3}$
\$$0.087\frac{2}{3}$

$2548.25
$0.087\frac{2}{3}$
$169883\frac{1}{3}$
1783775
2038600
6 | \$$223.39658\frac{1}{3}$
37.23276
$260.62934

$260.63
2548.25
$2808.88 *Ans.*

[redacted]

14. $418.33. OAKLAND, CAL., Dec. 23, 1897.

On demand, I promise to pay Albert J. Farnham, or order, Four Hundred Eighteen and $\frac{33}{100}$ Dollars, with interest at $4\frac{1}{2}\%$. Value received.

AUSTIN C. WIGGIN.

yr.	mo.	dy.
1898	12	3
1897	12	23
	11	10

11 mo. 10 dy.

$0.055 $0.001\frac{2}{3}$

$0.001\frac{2}{3}$

$0.056\frac{2}{3}$

$418.33
$0.05\frac{2}{3}$
$27888\frac{2}{3}$
209165
4 | $$23.7053\frac{2}{3}$
5.9263
$17.7790

$17.78
418.33
$436.11 *Ans.*

15. $7486.45. WATERTOWN, IA., Apr. 16, 1898.

On demand, I promise to pay Harry D. Smith, or order, Seven Thousand Four Hundred Eighty-six and $\frac{45}{100}$ Dollars, with interest at 5%.

FRANK J. LEAVITT.

yr.	mo.	dy.
1898	12	3
1898	4	16
	7	17

7 mo. 17 dy.

$0.035 $0.002\frac{5}{6}$

$0.002\frac{5}{6}$

$0.037\frac{5}{6}$

$7486.45
$0.037\frac{5}{6}$
$623870\frac{5}{6}$
5240515
2245935
6 | $$283.23735\frac{5}{6}$
47.2062
$236.0311

$236.03
7486.45
$7722.48 *Ans.*

Exercise 124. Page 274.

Find the day of maturity, the time to run, the discount, and the proceeds of the following notes, without grace:

1. \$750. NEW YORK, Jan. 1, 1897.

Four months from date, I promise to pay to the order of James Fay Seven Hundred Fifty Dollars, value received.

Payable at the National Bank of the Republic.

Discounted at 5%, Jan. 12. JOHN PRAY.

Day of maturity is 4 mo. after Jan. 1, 1897; that is, May 1, 1897.

Time to run is 19 dy. in Jan., 28 dy. in Feb., 31 dy. in Mar., 30 dy. in Apr., 1 dy. in May = 109 dy.

Discount on \$750 for 109 dy. at 6% = $18\frac{1}{6} \times$ \$0.75.

$$\begin{array}{r|l} & \$0.75 \\ & \underline{18\frac{1}{6}} \\ & 12\frac{1}{2} \\ & 600 \\ & 75 \\ 6 & \$13.62\frac{1}{2} \\ & \underline{2.27} \\ & \$11.35 \ \textit{Ans.} \end{array} \qquad \begin{array}{r} \$750. \\ \underline{11.35} \\ \$738.65 \ \textit{Ans.} \end{array}$$

2. \$4325.50. BOSTON, Mar. 4, 1897.

Sixty days from date, I promise to pay to James Finn, or order, Four Thousand Three Hundred Twenty-five and $\frac{50}{100}$ Dollars, value received.

Payable at the Merchants National Bank.

Discounted at $5\frac{1}{2}$%, Mar. 8. GEORGE BELLOWS.

Day of maturity is 60 dy. after Mar. 4, 1897; that is, May 3, 1897.

Time to run is 23 dy. in Mar., 30 dy. in Apr., 3 dy. in May = 56 dy.

Discount on \$4325.50 for 56 dy. at 6% = $9\frac{1}{3} \times$ \$4.3255.

$$\begin{array}{r|l} & \$4.3255 \\ & \underline{9\frac{1}{3}} \\ & 14418\frac{1}{3} \\ & \underline{389295} \\ 12 & \$40.3713\frac{1}{3} \\ & \underline{3.3643} \\ & \$37.007 \quad \$37.01. \ \textit{Ans.} \end{array} \qquad \begin{array}{r} \$4325.50 \\ \underline{37.01} \\ \$4288.49 \ \textit{Ans.} \end{array}$$

[redacted]

3. $1300. RICHMOND, VA., July 14, 1897.

Ninety days from date, I promise to pay to the order of Peter Bright Thirteen Hundred Dollars, value received.

Payable at the First National Bank.

Discounted at 4 %, Aug. 3. GEORGE WRIGHT.

Day of maturity is 90 dy. after July 14, 1897; that is, Oct. 12, 1897.
Time to run is 28 dy. in Aug., 30 dy. in Sept., 12 dy. in Oct. = 70 dy.
Discount on $ 1300 for 70 dy. at 6 % = 11⅔ × $ 1.30.

$$\begin{array}{r}
\$1.30 \\
11\frac{2}{3} \\
\hline
86\frac{2}{3} \\
130 \\
130 \\
\hline
3\,)\ \$15.16\frac{2}{3} \\
5.05\frac{5}{9} \\
\hline
\$10.11\ \textit{Ans.}
\end{array}
\qquad
\begin{array}{r}
\$1300. \\
10.11 \\
\hline
\$1289.89\ \textit{Ans.}
\end{array}$$

4. $1456.30. CHARLESTON, S. C., Aug. 27, 1897.

Three months after date, I promise to pay to the order of John George Fourteen Hundred Fifty-six and $\frac{30}{100}$ Dollars, value received.

Payable at the Second National Bank.

Discounted at 5 %, Sept. 10. JOHN WALDORF.

Day of maturity is 3 mo. after Aug. 27, 1897; that is, Nov. 27, 1897.
Time to run is 20 dy. in Sept., 31 dy. in Oct., 27 dy. in Nov. = 78 dy.
Discount on $ 1456.30 for 78 dy. at 6 % = 13 × $ 1.4563.

$$\begin{array}{r}
\$1.4563 \\
13 \\
\hline
43689 \\
14563 \\
\hline
6\,)\ \$18.9319 \\
3.1553 \\
\hline
\$15.7766 \\
\$15.78.\ \textit{Ans.}
\end{array}
\qquad
\begin{array}{r}
\$1456.30 \\
15.78 \\
\hline
\$1440.52\ \textit{Ans.}
\end{array}$$

5. $4550.36. BALTIMORE, MD., Nov. 10, 1897.

Four months after date, I promise to pay to the order of John Callender Four Thousand Five Hundred Fifty and $\frac{36}{100}$ Dollars, value received.

Payable at the National Mechanics Bank.

Discounted at 5½%, Nov. 24. JAMES BARTON.

Day of maturity is 4 mo. after Nov. 10, 1897; that is, Mar. 10, 1898.

Time to run is 6 dy. in Nov., 31 dy. in Dec., 31 dy. in Jan., 28 dy. in Feb., 10 in Mar. = 106 dy.

Discount on $4550.36 for 106 dy. at 6% = 17⅔ × $4.55036.

$4.55036
17⅔
303357⅓
3185252
455036
12 | $80.38969⅓
6.69914
$73.69055
$73.69. *Ans.*

$4550.36
73.69
$4476.67 *Ans.*

6. $5000. CHICAGO, ILL., Dec. 23, 1897.

Six months after date, we jointly and severally promise to pay to John Adams, or order, Five Thousand Dollars, value received, with interest at 5 per cent.

Payable at the Metropolitan National Bank.

Discounted at 4%, Jan. 21, 1898. WILLIAM DUNN,
F. R. CROCKETT.

Day of maturity is 6 mo. after Dec. 23, 1897; that is, June 23, 1898.

6 mo.
$0.025
5000
$125.000

$5000.
125.
$5125. amount of note.

Time to run is 10 dy. in Jan., 28 dy. in Feb., 31 dy. in Mar., 30 dy. in Apr., 31 dy. in May, 23 dy. in June = 153 dy.

Discount on $5125 for 153 dy. at 6% = 25.5 × $5.125; at 4% = 17 × $5.125.

$5.125
17
35875
5125
$87.125
$87.13. *Ans.*

$5125.
87.13
$5037.87 *Ans.*

Find the day of maturity, the time to run, the discount, and the proceeds of the following notes, with grace:

7\. $4760. MILWAUKEE, WIS., Jan. 1, 1897.

Ninety days after date, I promise to pay to the order of James Pike Four Thousand Seven Hundred Sixty Dollars, value received.

Payable at the Wisconsin National Bank.

Discounted at $4\frac{1}{2}\%$, Feb. 15. WILLIAM CLEMENT.

Day of maturity is 93 dy. after Jan. 1, 1897; that is, Apr. 4, 1897.

Time to run is 13 dy. in Feb., 31 dy. in Mar., 4 dy. in Apr. = 48 dy.

Discount on $4760 for 48 dy. at $6\% = 8 \times \$4.76$; at $4\frac{1}{2}\% = 6 \times \$4.76$.

$$\begin{array}{r} \$4.76 \\ 6 \\ \hline \$28.56 \end{array} \textit{ Ans.} \qquad\qquad \begin{array}{r} \$4760. \\ 28.56 \\ \hline \$4731.44 \end{array} \textit{ Ans.}$$

8\. $2017.85. ST. PAUL, MINN., Jan. 14, 1897.

Three months after date, I promise to pay to the order of John Brown Two Thousand Seventeen and $\frac{85}{100}$ Dollars, value received.

Payable at the German-American National Bank.

Discounted at 7%, Mar. 1. TIMOTHY BRUCE.

Day of maturity is 3 mo. 3 dy. after Jan. 14, 1897; that is, Apr. 17, 1897.

Time to run is 30 dy. in Mar., 17 dy. in Apr. = 47 dy.

Discount on $2017.85 for 47 dy. at $6\% = 7\frac{5}{6} \times \$2.01785$.

$$\begin{array}{r} \$2.01785 \\ 7\frac{5}{6} \\ \hline 168154\frac{1}{6} \\ 1412495 \\ 6\,|\,\$15.80649\frac{1}{6} \\ 2.6344 \\ \hline \$18.4408 \end{array} \qquad\qquad \begin{array}{r} \$2017.85 \\ 18.44 \\ \hline \$1999.41 \end{array} \textit{ Ans.}$$

$18.44. *Ans.*

9. \$9040. GALVESTON, TEX., Jan. 19, 1897.

Sixty days from date, I promise to pay to the order of Charles Carroll Nine Thousand Forty Dollars, value received.

Payable at the First National Bank.

Discounted at $5\frac{1}{2}\%$, Feb. 16. JAMES MONROE.

Day of maturity is 63 dy. after Jan. 19, 1897; that is, Mar. 23, 1897.

Time to run is 12 dy. in Feb., 23 dy. in Mar. = 35 dy.

Discount on \$9040 for 35 dy. at $6\% = 5\frac{5}{6} \times \$9.04$.

$$\begin{array}{r|r} & \$9.04 \\ & 5\frac{5}{6} \\ \hline & 753\frac{1}{3} \\ & 4520 \\ \hline 12 & \$52.73\frac{1}{3} \\ & 4.39\frac{4}{9} \\ \hline & \$48.34 \end{array} \quad \textit{Ans.}$$

$$\begin{array}{r} \$9040. \\ 48.34 \\ \hline \$8991.66 \end{array} \quad \textit{Ans.}$$

10. \$215. AUGUSTA, ME., Jan. 28, 1897.

Thirty days after date, I promise to pay to the order of James Fogg Two Hundred Fifteen Dollars, value received.

Payable at the Maine National Bank.

Discounted at 6%, Feb. 3. JOHN MOSES.

Day of maturity is 33 dy. after Jan. 28, 1897; that is, Mar. 2, 1897.

Time to run is 25 dy. in Feb., 2 dy. in Mar. = 27 dy.

Discount on \$215 for 27 dy. at $6\% = 4.5 \times \$0.215$.

$$\begin{array}{r} \$0.215 \\ 4.5 \\ \hline 1075 \\ 860 \\ \hline \$0.9675 \end{array}$$

\$0.97. *Ans.*

$$\begin{array}{r} \$215. \\ 0.97 \\ \hline \$214.03 \end{array} \quad \textit{Ans.}$$

[redacted]

11. $2216.85. OMAHA, NEB., Dec. 15, 1897.

Ninety days after date, I promise to pay to the order of F. C. Green Two Thousand Two Hundred Sixteen and $\frac{85}{100}$ Dollars, value received.

Payable at the Omaha National Bank.

Discounted at 7%, Jan. 8, 1898. W. C. COLBURN.

Day of maturity is 93 dy. after Dec. 15, 1897; that is, Mar. 18, 1898.

Time to run is 23 dy. in Jan., 28 dy. in Feb., 18 dy. in Mar. = 69 dy.

Discount on $2216.85 for 69 dy. at 6% = 11.5 × $2.21685.

$$\begin{array}{r|l} & \$2.21685 \\ & 11.5 \\ \hline & 1108425 \\ & 221685 \\ & 221685 \\ \hline 6 & \$25.493775 \\ & 4.24896 \\ \hline & \$29.74273 \end{array} \qquad \$29.74. \textit{ Ans.}$$

$$\begin{array}{r} \$2216.85 \\ 29.74 \\ \hline \$2187.11 \end{array} \quad \textit{Ans.}$$

Find the proceeds of the following drafts, with grace:

12. Draft for $620 at 60 days; rate of discount 6%; exchange $\frac{1}{8}$%.

Discount on $620 for 63 dy. at 6% = $10\frac{1}{2}$ × $0.62 = $6.51.

Exchange = $\frac{1}{8}$% of $700 = $0.88.

Total discount = $6.51 + $0.88 = $7.39.

Proceeds = $620 − $7.39 = $612.61. *Ans.*

13. Draft for $890 at 90 days; rate of discount $4\frac{1}{2}$%; exchange $\frac{1}{4}$%.

Discount on $890 for 93 dy. at 6% = 15.5 × $0.89.

$$\begin{array}{r|l} & \$0.89 \\ & 15.5 \\ \hline & 445 \\ & 445 \\ & 89 \\ \hline 4 & \$13.795 \\ & 3.44875 \\ \hline & \$10.34625 \end{array}$$

Exchange = $\frac{1}{4}$% of $900 = $2.25.

Total discount = $10.35 + $2.25 = $12.60.

Proceeds = $890 − $12.60 = $877.40. *Ans.*

14. Draft for $12,500 at 60 days; rate of discount 5%; exchange 15 cents on $1000.

Discount on $12,500 for 63 dy. at 6% = 10.5 × $12.50.

$$
\begin{array}{r|r}
 & \$12.50 \\
 & \underline{10.5} \\
 & 6250 \\
 & 1250 \\
\hline
6 & \$131.250 \\
 & \underline{21.875} \\
 & \$109.375
\end{array}
$$

Exchange = 12½ × $0.15 = $1.88.

Total discount = $109.38 + $1.88 = $111.26.

Proceeds = $12,500 − $111.26 = $12,388.74. *Ans.*

15. Draft for $1260 at 30 days; rate of discount 5½%; exchange ⅛%.

Discount on $1260 for 33 dy. at 6% = 5.5 × $1.26.

$$
\begin{array}{r|r}
 & \$1.26 \\
 & \underline{5.5} \\
 & 630 \\
 & 630 \\
\hline
12 & \$6.930 \\
 & \underline{0.5775} \\
 & \$6.3525
\end{array}
$$

Exchange = ⅛% of $1300 = $1.63.

Total discount = $6.35 + $1.63 = $7.98.

Proceeds = $1260 − $7.98 = $1252.02. *Ans.*

16. Draft for $1430 at 3 months; rate of discount 6%; exchange ¼%.

$$
\begin{array}{ll}
\underline{3\text{ mo.}} & \underline{3\text{ dy.}} \\
\$0.015 & 0.0005 \\
\underline{0.0005} & \\
\$0.0155 &
\end{array}
\qquad\qquad
\begin{array}{r}
\$1430 \\
\underline{0.0155} \\
7150 \\
7150 \\
\underline{1430} \\
\$22.1650
\end{array}
$$

Exchange = ¼% of $1500 = $3.75.

Total discount = $22.17 + $3.75 = $25.92.

Proceeds = $1430 − $25.92 = $1404.08. Ans.

[redacted]

17. Draft for \$1875 at 4 months; rate of discount 5%; exchange $\frac{1}{8}$%.

4 mo.	3 dy.
\$0.02	0.0005
0.0005	
\$0.0205	

$$\begin{array}{r} \$1875 \\ 0.0205 \\ \hline 9375 \\ 3750 \\ \hline 6\,)\ \$38.4375 \\ 6.40625 \\ \hline \$32.03125 \end{array}$$

Exchange = $\frac{1}{8}$% of \$1900 = \$2.38.

Total discount = \$32.03 + \$2.38 = \$34.41.

Proceeds = \$1875 − \$34.41 = \$1840.59. *Ans.*

18. Draft for \$22,843 at 60 days; rate of discount $4\frac{1}{2}$%; exchange 25 cents on \$1000.

Discount on \$22,843 for 63 dy. at 6% = 10.5 × \$22.843.

$$\begin{array}{r} \$22.843 \\ 10.5 \\ \hline 114215 \\ 22843 \\ \hline 4\,)\ \$239.8515 \\ 59.9629 \\ \hline \$179.8886 \end{array}$$

Exchange = 22.9 × \$0.25 = \$5.73.

Total discount = \$179.89 + \$5.73 = \$185.62.

Proceeds = \$22,843 − \$185.62 = \$22,657.38. *Ans.*

19. Draft for \$18,000 at 2 months; rate of discount 5%; exchange $\frac{1}{8}$%.

2 mo.	3 dy.
\$0.01	0.0005
0.0005	
\$0.0105	

$$\begin{array}{r} \$0.0105 \\ 18000 \\ \hline 840000 \\ 105 \\ \hline 6\,)\ \$189. \\ 31.50 \\ \hline \$157.50 \end{array}$$

Exchange = $\frac{1}{8}$% of \$18,000 = \$22.50.

Total discount = \$157.50 + \$22.50 = \$180.

Proceeds = \$18,000 − \$180 = \$17,820. *Ans.*

20. Draft for $3437.50 at 90 days; rate of discount 5%; exchange ¼%.

Discount on $3437.50 for 93 dy. at 6% = 15.5 × $3.4375.

$$\begin{array}{r} \$3.4375 \\ 15.5 \\ \hline 171875 \\ 171875 \\ 34375 \\ \hline 6\,|\,\$53.28125 \\ 8.8802 \\ \hline \$44.4010 \end{array}$$

Exchange = ¼% of $3500 = $8.75.

Total discount = $44.40 + $8.75 = $53.15.

Proceeds = $3437.50 − $53.15 = $3384.35. *Ans.*

21. Draft for $1287.50 at 60 days; rate of discount 4½%; exchange ⅜%.

Discount on $1287.50 for 63 dy. at 6% = 10.5 × $1.2875.

$$\begin{array}{r} \$1.2875 \\ 10.5 \\ \hline 64375 \\ 12875 \\ \hline 4\,|\,\$13.51875 \\ 3.37969 \\ \hline \$10.13906 \end{array}$$

Exchange = ⅜% of $1300 = $4.88.

Total discount = $10.14 + $4.88 = $15.02.

Proceeds = $1287.50 − $15.02 = $1272.48. *Ans.*

22. Draft for $866.65 at 3 months; rate of discount 5%; exchange ⅛%.

3 mo.	3 dy.
$0.015	0.0005
0.0005	
$0.0155	

$$\begin{array}{r} \$866.65 \\ 0.0155 \\ \hline 433325 \\ 433325 \\ 86665 \\ \hline 6\,|\,\$13.433075 \\ 2.23884 \\ \hline \$11.19423 \end{array}$$

Exchange = ⅛% of $900 = $1.13.

Total discount = $11.19 + $1.13 = $12.32.

Proceeds = $866.65 − $12.32 = $854.33. *Ans.*

[redacted]

Exercise 125. Page 277.

1. Find the present worth of \$500 due in 11 mo., if money is worth 5 %.

11 mo.
6 | \$0.055
  | 0.009$\frac{1}{6}$
\$0.045$\frac{5}{6}$

$\$\frac{500}{1.045\frac{5}{6}} = \$\frac{3000}{6.275} = \$478.09.$ *Ans.*

```
         478.08
6275)3000000.
     25100
      49000
      43925
       50750
       50200
         55000
         50200
          4800
```

2. Find the present worth and discount of \$3334.62 due in 2 yr., if money is worth 4½ %.

Amount of \$1 for 2 yr. at 4½ % is \$1.09.

$\$\frac{3334.62}{1.09} = \$3059.28.$ *Ans.*

```
      3059.28
109)333462.
    327
     646
     545
     1012
      981
       310
       218
        920      $3334.62
        872       3059.28
         48      $ 275.34  Ans.
```

3. Find the present worth and discount of \$4261.33 due in 1 yr. 6 mo., if money is worth 6 %.

Amount of \$1 for 1 yr. 6 mo. at 6 % is \$1.09.

$\$\frac{4261.33}{1.09} = \$3909.48.$ *Ans.*

```
        3909.47
109)426133.
    327
     991
     981
      1033
       981
        520
        436
         840      $4261.33
         763       3909.48
          77      $ 351.85  Ans.
```

4. Find the present worth and discount of \$2416.50 due in 7 mo., if money is worth 5 %.

7 mo.
6 | \$0.035
  | 0.005$\frac{5}{6}$
\$0.029$\frac{1}{6}$

$\$\frac{2416.50}{1.029\frac{1}{6}} = \$\frac{14499}{6.175} = \$2348.02.$ *Ans.*

```
         2348.01
6175)14499000.
     12350
      21490
      18525
       29650
       24700
        49500
        49400
         10000    $2416.50
          6175     2348.02
          3825    $  68.48  Ans.
```

5. Find the present worth of \$678.40 due in 16 mo., if money is worth $4\frac{1}{2}$%.

16 mo.
4 | \$0.08
  | 0.02
\$0.06

$$\$\frac{678.40}{1.06} = \$640. \ \textit{Ans.}$$

$$\begin{array}{r} 640 \\ 106\overline{)67840} \\ \underline{636} \\ 424 \\ \underline{424} \\ 0 \end{array}$$

6. Find the present worth and discount of \$574.17 due in 2 yr. 3 mo., if money is worth $5\frac{1}{3}$%.

2 yr.	3 mo.
\$0.12	0.015

0.015
9 | \$0.135
  | 0.015
\$0.12

$$\$\frac{574.17}{1.12} = \$512.65. \ \textit{Ans.}$$

$$\begin{array}{r} 512.65 \\ 112\overline{)57417.} \\ \underline{560} \\ 141 \\ \underline{112} \\ 297 \\ \underline{224} \\ 730 \\ \underline{672} \\ 580 \\ \underline{560} \\ 20 \end{array} \qquad \begin{array}{r} \$574.17 \\ \underline{512.65} \\ \$61.52 \end{array} \ \textit{Ans.}$$

7. Find the present worth and discount of \$625.13 due in 8 mo., if money is worth 4%.

8 mo.
3 | \$0.04
  | $0.01\frac{1}{3}$
$\$0.02\frac{2}{3}$

$$\$\frac{625.13}{1.02\frac{2}{3}} = \$\frac{1875.39}{3.08} = \$608.89. \ \textit{Ans.}$$

$$\begin{array}{r} 608.89 \\ 308\overline{)187539.} \\ \underline{1848} \\ 2739 \\ \underline{2464} \\ 2750 \\ \underline{2464} \\ 2860 \\ \underline{2772} \\ 88 \end{array} \qquad \begin{array}{r} \$625.13 \\ \underline{608.89} \\ \$16.24 \end{array} \ \textit{Ans.}$$

8. Find the present worth and discount of \$715.20 due in 1 yr. 4 mo., if money is worth $3\frac{1}{2}$%.

1 yr.	4 mo.
\$0.06	0.02

0.02
12 | \$0.08
$\$0.00\frac{2}{3}$
7
$\$0.04\frac{2}{3}$

$$\$\frac{715.20}{1.04\frac{2}{3}} = \$\frac{2145.60}{3.14} = \$683.31. \ \textit{Ans.}$$

$$\begin{array}{r} 683.31 \\ 314\overline{)214560.} \\ \underline{1884} \\ 2616 \\ \underline{2512} \\ 1040 \\ \underline{942} \\ 980 \\ \underline{942} \\ 380 \\ \underline{314} \\ 66 \end{array} \qquad \begin{array}{r} \$715.20 \\ \underline{683.31} \\ \$31.89 \end{array} \ \textit{Ans.}$$

[redacted]

Exercise 126. Page 278.

1. Find the exact interest at 6% on $692.74 for 250 days.

6 | 250	$0.69274	$28.86416
41⅔	41⅔	$0.28864
	46182⅔	0.09621
	69274	0.00962
	277096	0.00096
	$28.86416⅔	$0.39543
		$28.46873
		$28.47. *Ans.*

2. Find the exact interest at 6% on $1472.38 from Jan. 7, 1897 to Oct. 4, 1897.

24	$1.47238	$66.2571
28	45	$0.6625
31	736190	0.2208
30	588952	0.0220
31	$66.25710	0.0022
30		$0.9075
31		$65.3496
31		
30		$65.35. *Ans.*
4		
6 | 270		
45		

3. Find the exact interest at 6% on $1247.75 from Mar. 4, 1897 to Dec. 22, 1897.

27	$1.24775	$60.93179
30	48⅚	$0.60931
31	103979⅙	0.20310
30	998200	0.02031
31	499100	0.00203
31	$60.93179⅙	$0.83475
30		$60.09704
31		
30		60.10. *Ans*
22		
6 | 293		
48⅚		

4. Find the exact interest at 6% on $1898.48 from Feb. 26, 1897 to Aug. 12, 1899.

2	$1.89848	$52.84102
31	27⅚	$0.52841
30	158206⅔	0.17613
31	1328936	0.01761
30	379696	0.00176
31	$52.84102⅔	$0.72391
12		$52.11711
6 | 167		
27⅚		

2 yr.	$1898.48
$0.12	0.12
	379696
	189848
	$227.8176

$227.82
52.12
$279.94 *Ans.*

[redacted]

Exercise 127. Page 279.

1. Find the amount at annual interest of $1247.75 for 3 yr. 5 mo. 10 dy., at 6%.

3 yr.	5 mo.	10 dy.
$0.18	0.025	0.001⅔

$0.18
0.025
0.001⅔
$0.206⅔

yr.	mo.	dy.
2	5	10
1	5	10
	5	10
4	4	

4 yr.	4 mo.
$0.24	0.02

$0.24
0.02
$0.26

$1247.75
0.20⅔
83183⅓
2495500
$257.8683⅓

$1247.75
0.06
$74.8650

$74.87
0.26
44922
14974
$19.4662

$257.87
19.47
$277.34
1247.75
$1525.09 *Ans.*

2. Find the interest due on $987.25 in 4 yr. 9 mo. 6 dy., interest at 4%, payable annually.

4 yr.	9 mo.	6 dy.
$0.24	0.045	0.001

$0.24
0.045
0.001
$0.286

yr.	mo.	dy.
3	9	6
2	9	6
1	9	6
	9	6
9	0	24

9 yr.	24 dy.
$0.54	0.004

$0.54
0.004
$0.544

$987.25
0.286
592350
789800
197450
3 | $282.35350
94.1178
$188.2357

$987.25
0.04
$39.4900

$39.49
0.544
15796
15796
19745
3 | $21.48256
7.16085
$14.32171

$188.24
14.32
$202.56 *Ans.*

[redacted]

3. Find the interest due on $742.60 in 5 yr. 11 mo. 27 dy., interest at 4½%, payable annually.

5 yr.	11 mo.	27 dy.
$0.30	0.055	0.0045

0.055
0.0045
$0.3595

yr.	mo.	dy.
4	11	27
3	11	27
2	11	27
1	11	27
	11	27
14	11	15

14 yr.	11 mo.	15 dy.
$0.84	0.055	0.0025

0.055
0.0025
$0.8975

$200.22
22.50
$222.72 *Ans.*

$742.60
0.3595
371300
668340
371300
222780
4 | $266.964700
66.7412
$200.2235

$742.60
0.045
371300
297040
$33.41700

$33.42
0.8975
16710
23394
30078
26736
4 | $29.994450
7.49861
$22.49584

4. Find the interest due May 19, 1898, on a note dated Dec. 26, 1894, for $1224.60, with interest payable annually, at 5%, if no interest has been paid.

yr.	mo.	dy.
1898	5	19
1894	12	26
3	4	23

3 yr.	4 mo.	23 dy.
$0.18	0.02	0.003⅚

0.02
0.003⅚
$0.203⅚

yr.	mo.	dy.
2	4	23
1	4	23
	4	23
4	2	9

$1224.60
0.203⅚
102050
367380
244920
6 | $249.61430
41.6024
$208.0119

$1224.60
0.05
$61.2300

[redacted]

4 yr.	2 mo.	9 dy.
\$0.24	0.01	0.0015

\$0.24
0.01
0.0015
\$0.2515

\$208.01
12.83
\$220.84 *Ans.*

\$61.23
0.2515
30615
6123
30615
12246
6 | \$15.399345
2.566556
\$12.83278

5. Find the amount due May 27, 1898, on a note dated Jan. 4, 1896, for \$215.50, with interest payable annually at $5\frac{1}{2}$%, if no interest has been paid.

yr.	mo.	dy.
1898	5	27
1896	1	4
2	4	23

2 yr.	4 mo.	23 dy.
\$0.12	0.02	$0.003\frac{5}{6}$

\$0.12
0.02
$0.003\frac{5}{6}$
$\$0.143\frac{5}{6}$

\$215.50
$0.143\frac{5}{6}$
$17958\frac{1}{3}$
64650
86200
21550
12 | $\$30.99608\frac{1}{3}$
2.583
\$28.413

yr.	mo.	dy.
1	4	23
	4	23
1	9	16

1 yr.	9 mo.	16 dy.
\$0.06	0.045	$0.002\frac{2}{3}$

\$0.06
0.045
$0.002\frac{2}{3}$
$\$0.107\frac{2}{3}$

\$215.50
0.055
107750
107750
\$11.85250

\$11.85
$0.107\frac{2}{3}$
790
8295
1185
12 | \$1.27585
0.10632
\$1.16953

\$28.41
1.17
\$29.58
215.50
\$245.08 *Ans.*

[redacted]

6. Find the amount due Jan. 16, 1897, on a note dated Jan. 8, 1895, for $3115.20, with interest payable annually at 5%, if no interest has been paid.

yr.	mo.	dy.
1897	1	16
1895	1	8
2	0	8

2 yr.	8 dy.
$0.12	0.001⅓
0.001⅓	
$0.121⅓	

yr.	mo.	dy.
1	0	8
		8
1	0	16

1 yr.	16 dy.
$0.06	0.002⅔
0.002⅔	
$0.062⅔	

$3115.20
0.121⅓
103840
311520
623040
311520
6 | $377.97760
62.9963
$314.9813

$3115.20
0.05
$155.7600

$155.76
0.062⅔
10384
31152
93456
6 | $9.76096
1.62682
$8.13414

$314.98
8.13
$323.11
3115.20
$3438.31 *Ans.*

Exercise 128. Page 280.

1. Find the amount of $356.25 for 4 yr., at 5 % compound interest.

```
 $356.25
    0.05
 $17.8125
  356.25
$374.06
   0.05
$18.7030
 374.06
$392.76
   0.05
$19.6380
 392.76
$412.40
   0.05
$20.6200
 412.40
$433.02 Ans.
```

2. Find the amount of $637.50 for 2 yr. 6 mo., at 4 % compound interest.

```
                 $637.50
                    0.04
                 $25.5000
                  637.50
                 $663.
                   0.04
                 $26.52
                  663.
6 mo. at 4 %.    $689.52
$0.02               0.02
                 $13.7904
                  689.52
                 $703.31 Ans.
```

3. Find the compound interest on $800 for 3 yr. 9 mo., at 6 %.

```
               $800.
                 0.06
               $48.
                800.
               $848.
                 0.06
               $50.88
                848.
               $898.88
                  0.06
               $53.9328
                898.88
9 mo.          $952.81
$0.045            0.045
                476405
               381124
              $42.87645
               952.81
              $995.69
               800.
              $195.69 Ans.
```

4. Find the compound interest on $39.35 for 4 yr. 9 mo., at 5 %.

```
    $39.35
      0.05
    $1.9675
     39.35
   $41.32        9 mo. at 5 %.
     0.05        $0.0375
   $2.0660
    41.32          $47.84
  $43.39            0.0375
    0.05            23920
  $2.1695          33488
   43.39          14352
  $45.56          $1.794000
    0.05           47.84
  $2.2780         $49.63
   45.56           39.35
  $47.84          $10.28 Ans.
```

5. Find the compound interest on \$300 for 2 yr., at 4 %, interest being compounded semi-annually.

The interest is 2 % semi-annually.

\$300.
0.02
\$6.
300.
\$306.
0.02
\$6.12
306.
\$312.12
0.02
\$6.2424
312.12
\$318.36
0.02
\$6.3672
318.36
\$324.73
300.
\$24.73 *Ans.*

6. Find the compound interest on \$525 for 1 yr. 6 mo., at 5 %, interest being compounded quarterly.

The interest is $1\frac{1}{4}$ % quarterly.

\$525.
0.0125
2625
1050
525
\$6.5625
525.
\$531.56
0.0125
265780
106312
53156
\$6.644500
531.56
\$538.20
0.0125
269100
107640
53820
\$6.727500
538.20
\$544.93
0.0125
272465
108986
54493
\$6.811625
544.93
\$551.74
0.0125
275870
110348
55174
\$6.896750
551.74
\$558.64
0.0125
279320
111728
55864
\$6.983000
558.64
\$565.62
525.
\$40.62 *Ans.*

7. Find the compound interest on \$10,000 for 6 mo., at 6 %, interest being compounded monthly.

The interest is ½ % monthly.

$$
\begin{array}{r}
\$10000. \\
0.005 \\ \hline
\$50.000 \\
10000. \\ \hline
\$10050. \\
0.005 \\ \hline
\$50.250 \\
10050. \\ \hline
\$10100.25 \\
0.005 \\ \hline
\$50.50125 \\
10100.25 \\ \hline
\$10150.75 \\
0.005 \\ \hline
\$50.75375 \\
10150.75 \\ \hline
\$10201.50 \\
0.005 \\ \hline
\$51.00750 \\
10201.50 \\ \hline
\$10252.51 \\
0.005 \\ \hline
\$51.26255 \\
10252.51 \\ \hline
\$10303.77 \\
10000. \\ \hline
\$303.77 \ \textit{Ans.}
\end{array}
$$

Exercise 129. Page 282.

1. A note of \$618.75, dated Apr. 17, 1897, payable on demand, bears the following endorsements: June 5, \$126.50; Aug. 20, \$137.25; Nov. 17, \$210. What is due Jan. 1, 1898, reckoning interest at 6 %?

yr.	mo.	dy.	yr.	mo.	dy.	yr.	mo.	dy.	yr.	mo.	dy.
1898	1	1	1898	1	1	1898	1	1	1898	1	1
1897	4	17	1897	6	5	1897	8	20	1897	11	17
	8	14		6	26		4	11		1	14

8 mo.	14 dy.	6 mo.	26 dy.	4 mo.	11 dy.	1 mo.	14 dy.
\$0.04	$0.002\frac{1}{3}$	\$0.03	$0.004\frac{1}{3}$	\$0.02	$0.001\frac{5}{6}$	\$0.005	$0.002\frac{1}{3}$
$0.002\frac{1}{3}$		$0.004\frac{1}{3}$		$0.001\frac{5}{6}$		$0.002\frac{1}{3}$	
\$$0.042\frac{1}{3}$		\$$0.034\frac{1}{3}$		\$$0.021\frac{5}{6}$		\$$0.007\frac{1}{3}$	

[redacted]

$618.75
0.042⅓
20625
123750
247500
$26.19375
618.75
$644.94

$126.50
0.034⅓
4216⅔
50600
37950
$4.34316⅔
126.50
$130.84

$137.25
0.021⅚
11437½
13725
27450
$2.99662½
137.25
$140.25

$210.
0.007⅓
70
1470
$1.540
210.
$211.54

$130.84
140.25
211.54
$482.63

$644.94
482.63
$162.31 *Ans.*

2. A note of $1000, dated Apr. 1, 1897, payable on demand, with interest at 6%, bears the following endorsements: May 6, $200; July 5, $225.37; Oct. 18, $322. What is due Jan. 1, 1898?

yr.	mo.	dy.
1898	1	1
1897	4	1
	9	0

yr.	mo.	dy.
1898	1	1
1897	5	6
	7	25

yr.	mo.	dy.
1898	1	1
1897	7	5
	5	26

yr.	mo.	dy.
1898	1	1
1897	10	18
	2	13

9 mo.
$0.045

7 mo. 25 dy.
$0.035 0.004⅙
0.004⅙
$0.039⅙

5 mo. 26 dy.
$0.025 0.004⅓
0.004⅓
$0.029⅓

2 mo. 13 dy.
$0.01 0.002⅙
0.002⅙
$0.012⅙

$0.045
1000
6 | $45.000
7.50
$37.50
1000.
$1037.50

$0.039⅙
200
33⅓
7800
6 | $7.833⅓
1.305
$6.528
200.
$206.53

$225.37
0.029⅓
7512⅓
202833
45074
6 | $6.61085⅓
1.1018
$5.5090
225.37
$230.88

$322.
0.012⅙
53⅔
644
322
6 | $3.917⅔
0.653
$3.264
322.
$325.26

$206.53
230.88
325.26
$762.67

$1037.50
762.67
$274.83 *Ans.*

3. A note of $835.25, dated July 1, 1897, payable on demand, with interest at $4\frac{1}{2}\%$, bears the following endorsements: Aug. 20, $157.50; Sept. 21, $180.25; Oct. 5, $200; Dec. 1, $80. What is due Jan. 1, 1898?

yr.	mo.	dy.	yr.	mo.	dy.	yr.	mo.	dy.	yr.	mo.	dy.	yr.	mo.	dy.
1898	1	1	1898	1	1	1898	1	1	1898	1	1	1898	1	1
1897	7	1	1897	8	20	1897	9	21	1897	10	5	1897	12	1
	6	0		4	11		3	10		2	26		1	0

6 mo.	4 mo. 11 dy.	3 mo. 10 dy.	2 mo. 26 dy.	1 mo.
$0.03	$0.02 $0.001\frac{5}{6}$	$0.015 $0.001\frac{2}{3}$	$0.01 $0.004\frac{1}{3}$	$0.005
	$0.001\frac{5}{6}$	$0.001\frac{2}{3}$	$0.004\frac{1}{3}$	
	$\$0.021\frac{5}{6}$	$\$0.016\frac{2}{3}$	$\$0.014\frac{1}{3}$	

$835.25	$157.50	$180.25	$\$0.014\frac{1}{3}$	$80.
0.03	$0.021\frac{5}{6}$	$0.01\frac{2}{3}$	200	0.005
4) $25.0575	13125	$12016\frac{2}{3}$	$66\frac{2}{3}$	4) $0.400
6.2644	15750	18025	2800	0.10
$18.7931	31500	4) $\$3.0041\frac{2}{3}$	4) $\$2.866\frac{2}{3}$	$0.30
835.25	4) $3.43875	0.751	0.716	80.
$854.04	0.85969	$2.253	$2.15	$80.30
	$2.57906	180.25	200.	
	157.50	$182.50	$202.15	
	$160.08			

$160.08	$854.04
182.50	625.03
202.15	$229.01 *Ans.*
80.30	
$625.03	

4. A note of $1247.50, dated Mar. 10, 1897, payable on demand, with interest at 5%, has the following endorsements: $350.40, Apr. 14, 1897; $212.85, June 16, 1897; $316.45, Aug. 25, 1898. What is due Oct. 18, 1897?

yr.	mo.	dy.	yr.	mo.	dy.	yr.	mo.	dy.	yr.	mo.	dy.
1897	10	18	1897	10	18	1897	10	18	1897	10	18
1897	3	10	1897	4	14	1897	6	16	1897	8	25
	7	8		6	4		4	2		1	23

7 mo. 8 dy.	6 mo. 4 dy.	4 mo. 2 dy.	1 mo. 23 dy.
$0.035 $0.001\frac{1}{3}$	$0.03 $0.000\frac{2}{3}$	$0.02 $0.000\frac{1}{3}$	$0.005 $0.003\frac{5}{6}$
$0.001\frac{1}{3}$	$0.000\frac{2}{3}$	$0.000\frac{1}{3}$	$0.003\frac{5}{6}$
$\$0.036\frac{1}{3}$	$\$0.030\frac{2}{3}$	$\$0.020\frac{1}{3}$	$\$0.008\frac{5}{6}$

[redacted]

$1247.50	$350.40	$212.85	$316.45
0.036⅓	0.030⅔	0.020⅓	0.008⅚
41583⅓	23360	7095	26370⅚
748500	1051200	425700	253160
374250	6 | $10.74560	6 | $4.32795	6 | $2.79530⅚
6 | $45.32583⅓	1.7909	0.72132	0.4659
7.5543	$8.9547	$3.60663	$2.3294
$37.7715	350.40	212.85	316.45
1247.50	$359.35	$216.46	$318.78
$1285.27			

$359.35	$1285.27
216.46	894.59
318.78	$390.68 *Ans.*
$894.59	

5. A note of $1648.25, dated Jan. 22, 1897, payable on demand, with interest at 5 %, has the following endorsements: $212.60, Mar. 1, 1897; $168.40, May 26, 1897; $244.40, Aug. 4, 1897; $744.80, Oct. 1, 1897. What is due Jan. 22, 1898?

yr. mo. dy.	yr. mo. dy.	yr. mo. dy.	yr. mo. dy.	yr. mo. dy.
1898 1 22	1898 1 22	1898 1 22	1898 1 22	1898 1 22
1897 1 22	1897 3 1	1897 5 26	1897 8 4	1897 10 1
1 0 0	10 21	7 26	5 18	3 21
1 yr.	10 mo. 21 dy.	7 mo. 26 dy.	5 mo. 18 dy.	3 mo. 21 dy.
$0.06	$0.05 0.0035	$0.035 0.004⅓	$0.025 0.003	$0.015 0.0035
	0.0035	0.004⅓	0.003	0.0035
	$0.0535	$0.039⅓	$0.028	$0.0185

$1648.25	$212.60	$168.40	$244.40	$744.80
0.06	0.0535	0.039⅓	0.028	0.0185
6 | $98.8950	106300	5613⅓	195520	372400
16.4825	63780	151560	48880	595840
$82.4125	106300	50520	6 | $6.84320	74480
1648.25	6 | $11.374100	6 | $6.62373⅓	1.1405	6 | $13.778800
$1730.66	1.8957	1.10395	$5.7027	2.2965
	$9.4784	$5.51978	244.40	$11.4823
	212.60	168.40	$250.10	744.80
	$222.08	$173.92		$756.28

$222.08	$1730.66
173.92	1402.38
250.10	$328.28 *Ans.*
756.28	
$1402.38	

[redacted]

Exercise 130. Page 284.

1. A note of $2000, dated Jan. 22, 1896, and drawing interest at 6%, had the following endorsements: May 20, 1896, $100; July 20, 1896, $325; Nov. 2, 1896, $20; Dec. 23, 1896, $125. Find the balance due Mar. 1, 1897.

yr.	mo.	dy.
1896	5	20
1896	1	22
	3	28

0.019⅔

$0.019⅔
2000
1333⅓
38000
$39.333⅓
2000.
$2039.33
100.
$1939.33

yr.	mo.	dy.
1896	7	20
1896	5	20
	2	0

0.01

$1939.33
0.01
$19.3933
1939.33
$1958.72
325.
$1633.72

yr.	mo.	dy.
1896	11	2
1896	7	20
	3	12

0.017

$1633.72
0.017
1143604
163372
$27.77324

Payment less than interest.

yr.	mo.	dy.
1896	12	23
1896	11	2
	1	21

0.0085

$1633.72
0.0085
816860
1306976
$13.886620
27.77
1633.72
$1675.38
$20 + $125 = 145.
$1530.38

yr.	mo.	dy.
1897	3	1
1896	12	23
	2	8

0.011⅓

$1530.38
0.011⅓
51012⅔
153038
153038
$17.34430⅔
1530.38
$1547.72 Ans.

[redacted]

2. A note of \$1662.50, dated Jan. 15, 1896, and drawing interest at $5\frac{1}{2}\%$, had the following endorsements: Apr. 30, 1896, \$25; June 24, 1896, \$25; Sept. 2, 1896, \$625; Jan. 30, 1897, \$700. Find the balance due May 12, 1897.

yr.	mo.	dy.
1896	4	30
1896	1	15
	3	15

0.0175

$$\begin{array}{r} \$1662.50 \\ 0.0175 \\ \hline 831250 \\ 1163750 \\ 166250 \\ \hline 12\,|\,\$29.093750 \\ 2.42448 \\ \hline \$26.66927 \end{array}$$

Payment less than interest.

yr.	mo.	dy.
1896	6	24
1896	4	30
	1	24

0.009

$$\begin{array}{r} \$1662.50 \\ 0.009 \\ \hline 12\,|\,\$14.96250 \\ 1.2469 \\ \hline \$13.7156 \\ 26.6693 \\ 1662.50 \\ \hline \$1702.88 \\ \$25 + \$25 = \quad 50. \\ \hline \$1652.88 \\ 0.011\frac{1}{3} \\ \hline 55096 \\ 165288 \\ 165288 \\ \hline 12\,|\,\$18.73264 \\ 1.56105 \\ \hline \$17.17159 \\ 1652.88 \\ \hline \$1670.05 \\ 625. \\ \hline \$1045.05 \\ 0.024\frac{2}{3} \\ \hline 69670 \\ 418020 \\ 209010 \\ \hline 12\,|\,\$25.77790 \\ 2.1481 \\ \hline \$23.6298 \\ 1045.05 \\ \hline \$1068.68 \end{array}$$

yr.	mo.	dy.
1896	9	2
1896	6	24
	2	8

$0.011\frac{1}{3}$

yr.	mo.	dy.
1897	1	30
1896	9	2
	4	28

$0.024\frac{2}{3}$

yr.	mo.	dy.
1897	5	12
1897	1	30
	3	12

0.017

$$\begin{array}{r} \$1068.68 \\ 700. \\ \hline \$368.68 \\ 0.017 \\ \hline 258076 \\ 36868 \\ \hline 12\,|\,\$6.26756 \\ 0.52229 \\ \hline \$5.74527 \\ 368.68 \\ \hline \$374.43 \end{array}$$ *Ans.*

3. A note of $4560, dated Jan. 22, 1896, and drawing interest at 5%, had the following endorsements: Jan. 11, 1897, $2000; Aug. 31, 1897, $500; Jan. 15, 1898, $1200; Mar. 4, 1898, $860. Find the balance due June 15, 1898.

yr.	mo.	dy.
1897	1	11
1896	1	22
	11	19

0.058⅙

yr.	mo.	dy.
1897	8	31
1897	1	11
	7	20

0.038⅓

yr.	mo.	dy.
1898	1	15
1897	8	31
	4	14

0.022⅓

yr.	mo.	dy.
1898	3	4
1898	1	15
	1	19

0.008⅙

yr.	mo.	dy.
1898	6	15
1898	3	4
	3	11

0.016⅚

$4560.
0.058⅙
760
36480
22800
6 | $265.240
44.207
$221.033
4560.
$4781.03
2000.
$2781.03
0.038⅓
92701
2224824
834309
6 | $106.60615
17.7677
$88.8384
2781.03
$2869.87
500.
$2369.87
0.022⅓
78995⅔
473974
473974
6 | $52.92709⅔
8.82118
$44.10591
2369.87
$2413.98
1200.
$1213.98
0.008⅙
20233
.971184
6 | $9.91417
1.65236
$8.26181

$8.26
1213.98
$1222.24
860.
$362.24
0.016⅚
30186⅔
217344
36224
6 | $6.09770⅔
1.0163
$5.0814
362.24
$367.32 Ans.

[redacted]

4. A note of \$785.50, dated Jan. 30, 1896, and drawing interest at 5%, had the following endorsements: July 17, 1896, \$100; Jan. 29, 1897, \$100; Dec. 31, 1897, \$20; Mar. 16, 1898, \$300; June 18, 1898, \$50. Find the balance due July 23, 1898.

yr.	mo.	dy.
1896	7	17
1896	1	30
	5	17

$0.027\frac{5}{6}$

$$\begin{array}{r} \$785.50 \\ \underline{0.027\frac{5}{6}} \\ 65458\frac{1}{3} \\ 549850 \\ 157100 \\ 6\,|\,\$21.86308\frac{1}{3} \\ \underline{3.64385} \\ \$18.21923 \\ \underline{785.50} \\ \$803.72 \\ \underline{100.} \\ \$703.72 \\ \underline{0.032} \\ 140744 \\ 211116 \\ 6\,|\,\$22.51904 \\ \underline{3.753} \\ \$18.766 \end{array}$$

yr.	mo.	dy.
1897	1	29
1896	7	17
	6	12

0.032

yr.	mo.	dy.
1897	12	31
1897	1	29
	11	2

$0.055\frac{1}{3}$

$$\begin{array}{r} \$18.77 \\ \underline{703.72} \\ \$722.49 \\ \underline{100.} \\ \$622.49 \\ \underline{0.055\frac{1}{3}} \\ 20749\frac{2}{3} \\ 311245 \\ 311245 \\ 6\,|\,\$34.44444\frac{2}{3} \\ \underline{5.74074} \\ \$28.7037 \end{array}$$

Payment less than interest.

yr.	mo.	dy.
1898	3	16
1897	12	31
	2	15

0.0125

$$\begin{array}{r} \$622.49 \\ \underline{0.0125} \\ 311245 \\ 124498 \\ 62249 \\ 6\,|\,\$7.781125 \\ \underline{1.296854} \\ \$6.484271 \\ 28.7037 \\ \underline{622.49} \\ \$657.68 \\ \$20+\$300=\underline{320.} \\ \$337.68 \\ \underline{0.015\frac{1}{3}} \\ 11256 \\ 168840 \\ 33768 \\ 6\,|\,\$5.17776 \\ \underline{0.86296} \\ \$4.3148 \end{array}$$

yr.	mo.	dy.
1898	6	18
1898	3	16
	3	2

$0.015\frac{1}{3}$

yr.	mo.	dy.
1898	7	23
1898	6	18
	1	5

$0.005\frac{5}{6}$

$$\begin{array}{r} \$4.31 \\ \underline{337.68} \\ \$341.99 \\ \underline{50.} \\ \$291.99 \\ \underline{0.005\frac{5}{6}} \\ 24332\frac{1}{2} \\ 145995 \\ 6\,|\,\$1.70327\frac{1}{2} \\ \underline{0.28388} \\ \$1.41939 \\ \underline{291.99} \\ \$293.41 \end{array}$$

Ans.

5. A note of $300.25, dated Aug. 4, 1896, and drawing interest at $4\frac{1}{2}$%, had the following endorsements: Oct. 14, 1896, $100; July 21, 1897, $100; Oct. 11, 1897, $50; Jan. 19, 1898, $50. Find the amount due July 22, 1898.

yr.	mo.	dy.
1896	10	14
1896	8	4
	2	10

$0.011\frac{2}{3}$

$$\begin{array}{r} \$300.25 \\ 0.011\frac{2}{3} \\ \hline 20016\frac{2}{3} \\ 30025 \\ 30025 \\ 4\,\underline{)\,\$3.50291} \\ 0.87573 \\ \hline \$2.62718 \\ 300.25 \\ \hline \$302.88 \\ 100. \\ \hline \$202.88 \\ 0.046\frac{1}{6} \\ \hline 3381\frac{1}{3} \\ 121728 \\ 81152 \\ 4\,\underline{)\,\$9.36629\frac{1}{3}} \\ 2.34157 \\ \hline \$7.02472 \\ 202.88 \\ \hline \$209.90 \\ 100. \\ \hline \$109.90 \\ 0.013\frac{1}{3} \\ \hline 3663\frac{1}{3} \\ 32970 \\ 10990 \\ 4\,\underline{)\,\$1.46533\frac{1}{3}} \\ 0.36633 \\ \hline \$1.09900 \\ 109.90 \\ \hline \$111. \\ 50. \\ \hline \$61. \\ 0.016\frac{1}{3} \\ \hline 20\frac{1}{3} \\ 366 \\ 61 \\ 4\,\underline{)\,\$0.996\frac{1}{3}} \\ 0.249 \\ \hline \$0.747 \end{array}$$

yr.	mo.	dy.
1897	7	21
1896	10	14
	9	7

$0.046\frac{1}{6}$

yr.	mo.	dy.
1897	10	11
1897	7	21
	2	20

$0.013\frac{1}{3}$

yr.	mo.	dy.
1898	1	19
1897	10	11
	3	8

$0.016\frac{1}{3}$

yr.	mo.	dy.
1898	7	22
1898	1	19
	6	3

0.0305

$$\begin{array}{r} \$0.75 \\ 61. \\ \hline \$61.75 \\ 50. \\ \hline \$11.75 \\ 0.0305 \\ \hline 5875 \\ 3525 \\ 4\,\underline{)\,\$0.358375} \\ 0.089594 \\ \hline \$0.268781 \\ 11.75 \\ \hline \end{array}$$

$12.02 Ans.

[illegible]	[illegible]	12
	5	10

0.026⅔

yr.	mo.	dy.
1896	12	26
1896	7	22
	5	4

0.025⅔

yr.	mo.	dy.
1897	8	24
1896	12	26
	7	28

0.039⅔

yr.	mo.	dy.
1897	10	6
1897	8	24
	1	12

0.007

yr.	mo.	dy.
1898	4	14

9836
29508
6 | $ 39.344
6.557
$ 32.787
1475.40
$ 1508.19
370.
$ 1138.19
0.025⅔
758791⅓
569095
227638
6 | $ 29.213541⅓
4.86892
$ 24.34462
1138.19
$ 1162.53
426.50
$ 736.03
0.039⅔
49068⅔
662427
220809
6 | $ 29.19585⅔
4.86597
$ 24.32988
736.03

[redacted]

7. A note of \$5762.45, dated Jan. 2, 1896, and drawing interest at 5 %, had the following endorsements: May 17, 1896, \$500; Oct. 12, 1896, \$750; Feb. 4, 1897, \$1000; Aug. 25, 1897, \$1250; Mar. 1, 1898, \$1500; June 15, 1898, \$1050. Find the balance due Oct. 2, 1898.

yr.	mo.	dy.
1896	5	17
1896	1	2
	4	15

0.0225

yr.	mo.	dy.
1896	10	12
1896	5	17
	4	25

0.024$\frac{1}{6}$

yr.	mo.	dy.
1897	2	4
1896	10	12
	3	22

0.018$\frac{2}{3}$

yr.	mo.	dy.
1897	8	25
1897	2	4
	6	21

0.0335

yr.	mo.	dy.
1898	3	1
1897	8	25
	6	6

0.031

yr.	mo.	dy.
1898	6	15
1898	3	1
	3	14

0.017$\frac{1}{3}$

yr.	mo.	dy.
1898	10	2
1898	6	15
	3	17

0.017$\frac{5}{6}$

\$5762.45
0.0225
2881225
1152490
1152490
6 | \$129.655125
21.6092
\$108.0459
5762.45
\$5870.50
500.
\$5370.50
0.024$\frac{1}{6}$
89508$\frac{1}{3}$
2148200
1074100
6 | \$129.78708$\frac{1}{3}$
21.63118
\$108.15590
5370.50
\$5478.66
750.
\$4728.66
0.018$\frac{2}{3}$
315244
3782928
472866
6 | \$88.26832
14.71139
\$73.55693
4728.66
\$4802.22
1000.
\$3802.22
0.0335
1901110
1140666
1140666
6 | \$127.374370
21.22906
\$106.14531

\$106.15
3802.22
\$3908.37
1250.
\$2658.37
0.031
265837
797511
6 | \$82.40947
13.73491
\$68.67456
2658.37
\$2727.04
1500.
\$1227.04
0.017$\frac{1}{3}$
40901$\frac{1}{3}$
858928
122704
6 | \$21.26869$\frac{1}{3}$
3.54478
\$17.72391
1227.04
\$1244.76
1050.
\$194.76
0.017$\frac{5}{6}$
16230
136332
19476
6 | \$3.47322
0.57887
\$2.89435
194.76
\$197.65 *Ans.*

[redacted]

Exercise 131. Page 287.

1. Find, by the New Hampshire Rule, and also by the Vermont Rule, the amount due Sept. 22, 1896, on a note for $1750, dated June 6, 1892, with interest annually at 6%, which has the following endorsements: Aug. 12, 1893, $300; Dec. 23, 1893, $200; Jan. 15, 1895, $50; Apr. 23, 1896, $800.

(By the New Hampshire Rule.)

Principal,		$1750.00		
1st annual interest,			$105.00	
Int. on 1st annual interest for 1 yr.,				$6.30
2d annual interest,			105.00	
		$1750.00	$210.00	$6.30
Payment Aug. 12, 1893,	$300.00			
Int. on payment June 6, 1894,	14.70			
Payment Dec. 23, 1893,	200.00			
Int. on payment June 6, 1894,	5.43			
Amt. of payments June 6, 1894,	$520.13=	$303.83+	$210.00+	$6.30
Principal June 6, 1894,		$1446.17		
3d annual interest,			$86.77	
		$1532.94		
As payment Jan. 15, 1895, does not exceed the annual interest, deduct payment without interest,		50.00		
Principal June 6, 1895,		$1482.94		
4th annual interest,			$88.98	
Payment Apr. 23, 1896,	$800.00			
Int. on payment June 6, 1896,	5.73			
Amt. of payment June 6, 1896,	$805.73=	$716.75+	$88.98	
Principal June 6, 1896,		$766.19		
5th annual interest,		13.54		
Amt. due Sept. 22, 1896,		$779.73	*Ans.*	

[redacted]

(By the Vermont Rule.)

Principal June 6, 1894,		$1446.17	
3d annual interest,			$86.77
Payment Jan. 15, 1895,	$50.00		
Int. on payment June 6, 1895,	1.18		
Amt. of payment June 6, 1895,	$51.18=		$51.18
		$1446.17+	$35.59
Principal June 6, 1895,		$1481.76	
4th annual interest,			$88.91
Payment Apr. 23, 1896,	$800.00		
Int. on payment June 6, 1896,	5.73		
Amt. of payment June 6, 1896,	$805.73=	$716.82+	$88.91
Principal June 6, 1896,		$764.94	
5th annual interest,		13.51	
Amt. due Sept. 22, 1896,		$778.45	*Ans.*

2. Find by the Connecticut Rule the amount due Sept. 22, 1896, on a note for $1500, dated Aug. 9, 1892, with interest annually at 6%, which has the following endorsements: Mar. 17, 1893, $250; Apr. 19, 1894, $50; Sept. 21, 1895, $500; June 26, 1896, $600.

Principal,		$1500.00
Int. on principal to Aug. 9, 1893,		90.00
Amt. of principal Aug. 9, 1893,		$1590.00
Payment Mar. 17, 1893,	$250.00	
Int. on payment to Aug. 9, 1893,	5.92	
Amt. of payment Aug. 9, 1893,		$255.92
New principal Aug. 9, 1893,		$1334.08
Int. on principal to Aug. 9, 1894,		80.04
Amt. of principal Aug. 9, 1894,		$1414.12
Payment Apr. 19, 1894 (less than interest),		50.00
New principal Aug. 9, 1894,		$1364.12
Int. on principal Sept. 21, 1895,		91.40
Amt. of principal Sept. 21, 1895,		$1455.52
Payment Sept. 21, 1895,		500.00
New principal Sept. 21, 1895,		$955.52
Int. on principal to June 26, 1896,		43.79
Amt. of principal June 26, 1896,		$999.31
Payment June 26, 1896,		600.00
New principal June 26, 1896,		$399.31
Int. on principal to Sept. 22, 1896,		5.72
Amt. due Sept. 22, 1896,		$405.03 Ans.

[redacted]

Exercise 132. Page 290.

1. What is the cost of 25 shares of Boston and Maine R.R. stock at 167, brokerage $\frac{1}{4}$?

$\$167 + \$0.25 = \$167.25.$

$$\begin{array}{r} \$167.25 \\ 25 \\ \hline 83625 \\ 33450 \\ \hline \$4181.25 \end{array}$$ *Ans.*

2. How many shares of Illinois Central R.R. stock at $101\frac{7}{8}$ can be bought for \$20,400, brokerage $\frac{1}{8}$?

$\$101\frac{7}{8} + \$\frac{1}{8} = \$102.$

$$\begin{array}{r} 200 \\ 102\overline{)20400} \\ 204 \\ \hline 00 \end{array}$$

200 *Ans.*

3. What is the annual income from 150 shares of Lake Shore and Michigan Southern Ry. stock that pays an annual dividend of 6%?

Each share pays \$6 dividend.

$150 \times \$6 = \$900.$ *Ans.*

4. How much must be invested in 6% stock at 107 to yield an annual income of \$240, brokerage $\frac{1}{4}$?

\$6 is the dividend from 1 share.

\$240 dividend requires $\frac{240}{6} = 40$ shares.

$\$107 + \$0.25 = \$107.25.$

$$\begin{array}{r} \$107.25 \\ 40 \\ \hline \$4290.00 \end{array}$$ *Ans.*

5. What per cent does the investment yield, if Lake Shore and Michigan Southern Ry. stock is bought at 170? The stock pays 6% dividend; no brokerage reckoned.

Each \$170 invested pays \$6 dividend.

$$\begin{array}{r} 0.0353 \\ 170\overline{)6.00} \\ 510 \\ \hline 900 \\ 850 \\ \hline 500 \end{array}$$

3.53%. *Ans.*

6. Find the cost of 350 shares of Chicago, Milwaukee and St. Paul Ry. stock at $93\frac{3}{8}$, brokerage $\frac{1}{8}$.

$\$93\frac{3}{8} + \$\frac{1}{8} = \$93\frac{1}{2} = \$93.50.$

$$\begin{array}{r} \$93.50 \\ 350 \\ \hline 467500 \\ 2805 \\ \hline \$32725.00 \end{array}$$ *Ans.*

7. Find the cost of 165 shares of Michigan Central R.R. stock at $105\frac{3}{4}$, brokerage $\frac{1}{8}$.

$\$105\frac{3}{4} + \$\frac{1}{8} = \$105\frac{7}{8} = \$105.875.$

$$\begin{array}{r} \$105.875 \\ 165 \\ \hline 529375 \\ 635250 \\ 105875 \\ \hline \$17469.375 \end{array}$$

\$17,469.38. *Ans.*

8. Find the cost of 35 shares of Reading R.R. stock at $23\frac{1}{2}$, brokerage $\frac{1}{8}$.

$\$23\frac{1}{2} + \$\frac{1}{8} = \$23\frac{5}{8} = \$23.625.$

$$\begin{array}{r} \$23.625 \\ 35 \\ \hline 118125 \\ 70875 \\ \hline \$826.875 \end{array}$$

$826.88. *Ans.*

9. What is the cost of 25 U. S. 4% registered 1925 bonds of $1000 each, at $127\frac{1}{8}$, brokerage $\frac{1}{8}$?

$\$127\frac{1}{8} + \$\frac{1}{8} = \$127\frac{1}{4} = \$127.25.$

$10 \times \$127.25 = \$1272.50.$

$25 \times \$1272.50 = \$31,812.50.$ *Ans.*

10. What is the cost of 40 Northern Pacific R.R. 1st mortgage 6% registered bonds of $1000 each, at $119\frac{7}{8}$, brokerage $\frac{1}{8}$?

$\$119\frac{7}{8} + \$\frac{1}{8} = \$120.$

$10 \times \$120 = \$1200.$

$40 \times \$1200 = \$48,000.$ *Ans.*

11. What per cent income does the investment of Example 10 yield?

Each $120 invested yields an income of $6.

$$\begin{array}{r} 0.05 \\ 120\overline{)6.00} \\ 600 \\ \hline \end{array}$$

5%. *Ans.*

12. What is the annual income received from the investment of Example 10?

Each bond yields

6% of $1000 = $60.

$40 \times \$60 = \$2400.$ *Ans.*

13. What is the annual income from 200 shares of Chicago and Northwestern Ry. stock that pays an annual dividend of 5%?

Each share pays $5 dividend.

$200 \times \$5 = \$1000.$ *Ans.*

14. What is the cost of the investment of Example 13 at $122\frac{7}{8}$, brokerage $\frac{1}{8}$?

$\$122\frac{7}{8} + \$\frac{1}{8} = \$123.$

$200 \times \$123 = \$24,600.$ *Ans.*

15. What per cent income does the investment of Example 13 yield?

Each $123 invested yields $5 income.

$$\begin{array}{r} 0.0406 \\ 123\overline{)5.00} \\ 492 \\ \hline 800 \\ 738 \\ \hline 62 \end{array}$$

4.07%. *Ans.*

16. How many shares of New York Central stock can be bought for $4757.50 at $107\frac{7}{8}$, brokerage $\frac{1}{4}$?

$\$107\frac{7}{8} + \$\frac{1}{4} = \$108\frac{1}{8} = \$108.125.$

$$\begin{array}{r} 44 \\ 108125\overline{)4757500} \\ 432500 \\ \hline 432500 \\ 432500 \\ \hline \end{array}$$

44 *Ans.*

57375

114750
114750

18. What is the annual income from the investme

Each bond yields 7 % of \$500 = \$35.

$12 \times \$35 = \$420$. *Ans.*

19. What sum of money must be invested in Nor 1st mortgage 6's at 119½ to produce an annual income age ⅛?

Each bond of \$1000 yields \$60 income.

\$2400 income requires $\frac{2400}{60} = 40$ bonds.

$\$119\frac{1}{2} + \$\frac{1}{8} = \$119\frac{5}{8} = \$119.625$.

Each bond costs $10 \times \$119.625 = \$1196.25$.

\$1196.25
40

\$47850.00 *Ans.*

20. What sum of money must be invested in Wal 5 % bonds at 107¼ to produce an annual

21. What sum of money must be invested in Louisville and Nashville R.R. unified gold 4% bonds at $84\frac{1}{4}$ to produce an annual income of \$320, brokerage $\frac{1}{4}$?

Each bond of \$1000 yields \$40 income.

\$320 income requires $\frac{320}{40} = 8$ bonds.

$$\$84\tfrac{1}{4} + \$\tfrac{1}{4} = \$84\tfrac{1}{2} = \$84.50.$$

Each bond costs $10 \times \$84.50 = \$845$.

$$\begin{array}{r} \$845 \\ 8 \\ \hline \$6760 \end{array} \quad \textit{Ans.}$$

22. What sum of money must be invested in St. Louis and San Francisco Ry. general mortgage 5% bonds at $100\frac{1}{2}$ to produce an annual income of \$600, brokerage $\frac{1}{8}$?

Each bond of \$1000 yields \$50 income.

\$600 income requires $\frac{600}{50} = 12$ bonds.

$$\$100\tfrac{1}{2} + \$\tfrac{1}{8} = \$100\tfrac{5}{8} = \$100.625.$$

Each bond costs $10 \times \$100.625 = \$1006.25$.

$$\begin{array}{r} \$1006.25 \\ 12 \\ \hline 201250 \\ 100625 \\ \hline \$12075.00 \end{array} \quad \textit{Ans.}$$

23. How many shares of Chicago and Northwestern Ry. stock can be bought for \$14,670 at $122\frac{1}{8}$, brokerage $\frac{1}{8}$? What is the brokerage? If 5% dividends are paid, what per cent on his investment does the purchaser receive?

$$\$122\tfrac{1}{8} + \$\tfrac{1}{8} = \$122\tfrac{1}{4} = \$122.25.$$

$$\begin{array}{r} 120 \\ 12225\overline{)1467000} \\ 12225 \\ \hline 24450 \\ 24450 \\ \hline 0 \end{array}$$

120 shares. *Ans.*

Brokerage = $\frac{1}{8}$% of \$12,000 = \$15. *Ans.*

Each \$122.25 invested yields \$5 dividend.

$$\begin{array}{r} 0.0408 \\ 12225\overline{)500.00} \\ 48900 \\ \hline 110000 \\ 97800 \\ \hline 12200 \end{array}$$

4.08% Ans.

[redacted]

24. How many shares of Michigan Central R.R. stock can be bought for $16,940 at $105\frac{3}{4}$, brokerage $\frac{1}{8}$? What is the brokerage? If 4% dividends are paid, what per cent on his investment does the purchaser receive?

$$\$105\tfrac{3}{4} + \$\tfrac{1}{8} = \$105\tfrac{7}{8} = \$105.875.$$

```
              160
105875)16940000
         105875
          635250
          635250
               0
```

160 shares. *Ans.*

Brokerage = $\frac{1}{8}$% of $16,000 = $20. *Ans.*

Each $105.875 invested yields $4 dividend.

```
             0.0377
105875)4000.00
        317625
         823750
         741125
          826250
          741125
           85125
```

3.78%. *Ans.*

25. What is the cost of 40 shares of Central R.R. of New Jersey stock at $92\frac{7}{8}$, brokerage $\frac{1}{4}$? What is the brokerage? If 6% dividends are paid, what per cent on his investment does the purchaser receive?

$$\$92\tfrac{7}{8} + \$\tfrac{1}{4} = \$93\tfrac{1}{8} = \$93.125.$$

```
 $93.125
      40
$3725.000  Ans.
```

Brokerage = $\frac{1}{4}$% of $4000 = $10. *Ans.*

Each $93.125 invested yields $6 dividend.

```
           0.0644
93125)6000.00
       558750
        412500
        372500
         400000
         372500
          27500
```

6.44%. *Ans.*

26. What is the cost of 250 shares of Pullman Palace Car Co. stock at $171\frac{1}{4}$, brokerage $\frac{1}{8}$? What is the brokerage? If 8% dividends are paid, what per cent on his investment does the purchaser receive?

$$\$171\tfrac{1}{4} + \$\tfrac{1}{8} = \$171\tfrac{3}{8} = \$171.375.$$

$$\begin{array}{r} \$171.375 \\ 250 \\ \hline 8568750 \\ 342750 \\ \hline \$42843.75 \end{array}\ \textit{Ans.}$$

Brokerage = $\frac{1}{8}$% of \$25,000 = \$31.25. *Ans.*

Each \$171.375 invested yields \$8 dividend.

$$\begin{array}{r} 0.0466 \\ 171375)\overline{8000.00} \\ 685500 \\ \hline 1145000 \\ 1028250 \\ \hline 1167500 \\ 1028250 \\ \hline 139250 \end{array}$$

4.67%. *Ans.*

27. What per cent on his investment does a purchaser receive who buys New York, New Haven and Hartford R.R. stock at $180\frac{1}{2}$, if annual dividends of 8% are declared?

Each \$180.50 invested yields \$8 dividend.

$$\begin{array}{r} 0.0443 \\ 18050)\overline{800.00} \\ 72200 \\ \hline 78000 \\ 72200 \\ \hline 58000 \\ 54150 \\ \hline 3850 \end{array}$$

4.43%. Ans.

$$\begin{array}{r} \$1072.50 \\ 20 \\ \hline \$21450.00 \end{array}\ \textit{Ans.}$$

Each \$107.25 invested yields \$4.50 income.

$$\begin{array}{r} 0.0419 \\ 10725)\overline{450.00} \\ \underline{42900} \\ 21000 \\ \underline{10725} \\ 102750 \\ \underline{96525} \\ 6225 \end{array}$$

29. When Mexican Central Ry. 1st mortgage 4 % b
at $62\frac{1}{2}$, how much must be invested to produce an an
\$200, brokerage $\frac{1}{4}$? What per cent on his investme
chaser receive?

Each bond of \$1000 yields \$40 income.

\$200 income requires $\frac{200}{40} = 5$ bonds.

$$\$62\tfrac{1}{2} + \$\tfrac{1}{4} = \$62\tfrac{3}{4} = \$62.75.$$

Each bond costs $10 \times \$62.75 = \$627.50$.

$$\begin{array}{r} \$627.50 \\ 5 \\ \hline \$3137.50 \end{array}\ \textit{Ans.}$$

Each \$62.75 invested yields \$4 income.

$$0.0637$$

30. When West Shore R.R. 1st mortgage 4 % bonds are selling at $108\frac{3}{4}$, how much must be invested to produce an annual income of \$800, brokerage $\frac{1}{8}$?

Each bond of \$1000 yields \$40 income.

\$800 income requires $\frac{800}{40} = 20$ bonds.

$$\$108\tfrac{3}{4} + \$\tfrac{1}{8} = \$108\tfrac{7}{8} = \$108.875.$$

Each bond costs $10 \times \$108.875 = \$1088.75$.

$$\begin{array}{r} \$1088.75 \\ 20 \\ \hline \$21775.00 \end{array} \quad \textit{Ans.}$$

31. When New England Tel. and Tel. Co. 6 % bonds are selling at $101\frac{1}{8}$, how much must be invested to produce an annual income of \$900, brokerage $\frac{1}{8}$?

Each bond of \$1000 yields \$60 income.

\$900 income requires $\frac{900}{60} = 15$ bonds.

$$\$101\tfrac{1}{8} + \$\tfrac{1}{8} = \$101\tfrac{1}{4} = \$101.25.$$

Each bond costs $10 \times \$101.25 = \$1012.50$.

$$\begin{array}{r} \$1012.50 \\ 15 \\ \hline 506250 \\ 101250 \\ \hline \$15187.50 \end{array} \quad \textit{Ans.}$$

32. If a man buys a 6 % bond at 120, what rate of interest does he receive on the money invested?

Each \$120 invested yields \$6 interest.

$$\begin{array}{r} 0.05 \\ 120\overline{)6.00} \\ \underline{600} \end{array}$$

5 %. *Ans.*

33. If 3 % bonds are at $88\frac{1}{2}$, what rate per cent interest will a purchaser receive on his money?

Each \$88.50 invested yields \$3 interest.

$$\begin{array}{r} 0.0338 \\ 885\overline{)30.00} \\ \underline{2655} \\ 3450 \\ \underline{2655} \\ 7950 \\ \underline{7080} \\ 870 \end{array}$$

3.38 %. Ans.

[redacted]

34. If an 8 % stock is at 150, what rate per cent interest will a purchaser receive on his money ?

Each $ 150 invested yields $ 8 interest.

```
       0.0533
150)8.00
     750
      500
      450
       500
       450
        50
```

5.33 %. *Ans.*

35. If a 10 % stock is at 175, what rate per cent interest will an investor receive on his money ?

Each $ 175 invested yields $ 10 interest.

```
      0.0571
175)10.00
     875
     1250
     1225
       250
       175
        75
```

5.71 %. *Ans.*

36. If a $4\frac{1}{2}$ % stock is at 85, what rate per cent interest will a purchaser receive on his money ?

Each $ 85 invested yields $ 4.50 interest.

```
     0.0529
85)4.50
    425
     250
     170
      800
      765
       35
```

5.29 %. *Ans.*

37. If 7% bonds are at 114, what rate per cent interest will a purchaser receive on his money?

Each \$114 invested yields \$7 interest.

```
       0.0614
114)7.00
     684
      160
      114
       460
       456
         4
```

6.14%. *Ans.*

38. If 6% bonds are at 130, what rate per cent interest will a purchaser receive on his money?

Each \$130 invested yields \$6 interest.

```
       0.0461
130)6.00
     520
      800
      780
       200
       130
        70
```

4.62%. *Ans.*

39. If \$8000 5% stocks are sold at 90 and the proceeds invested in $3\frac{1}{2}$% stocks at 60, find the increase or decrease in income.

Income from 5% stock = 5% of \$8000 = \$400.

Proceeds from 5% stock = 80 × \$90 = \$7200.

\$0.60 is paid for \$1 of $3\frac{1}{2}$% stock.

Therefore, \$7200 is paid for $\frac{\$7200}{0.60}$ = \$12,000 of $3\frac{1}{2}$% stock.

Income from $3\frac{1}{2}$% stock = $3\frac{1}{2}$% of \$12,000 = \$420.

\$420 − \$400 = \$20, increase in income. *Ans.*

40. If \$10,000 $3\frac{1}{2}$% bonds are sold at 65, and the proceeds invested in 8% bonds at 130, find the increase or decrease in income.

Income from $3\frac{1}{2}$% bonds = $3\frac{1}{2}$% of \$10,000 = \$350.

Proceeds from $3\frac{1}{2}$% bonds = 100 × \$65 = \$6500.

\$1.30 is paid for \$1 of 8% bonds.

Therefore, \$6500 is paid for $\frac{\$6500}{1.30}$ = \$5000 of 8% bonds.

Income from 8% bonds = 8% of \$5000 = \$400.

\$400 − \$350 = \$50, increase in income. *Ans.*

[redacted]

41. If $8000 $4\frac{1}{2}\%$ stocks are sold at 70 and the proceeds invested in 10% stocks at 160, find the increase or decrease in income.

Income from $4\frac{1}{2}\%$ stock = $4\frac{1}{2}\%$ of $8000 = $360.
Proceeds from $4\frac{1}{2}\%$ stock = 80 × $70 = $5600.
$1.60 is paid for $1 of 10% stock.
Therefore, $5600 is paid for $\frac{\$5600}{1.60} = \$3500$ of 10% stock.
Income from 10% stock = 10% of $3500 = $350.
$360 − $350 = $10, decrease in income. *Ans.*

42. If $6000 6% bonds are sold at 90, and the proceeds invested in 10% bonds at 135, find the increase or decrease in income.

Income from 6% bonds = 6% of $6000 = $360.
Proceeds from 6% bonds = 60 × $90 = $5400.
$1.35 is paid for $1 of 10% bonds.
Therefore, $5400 is paid for $\frac{\$5400}{1.35} = \$4000$ of 10% bonds.
Income from 10% bonds = 10% of $4000 = $400.
$400 − $360 = $40, increase in income. *Ans.*

43. Find the rate of interest obtained by investing in a 5% bond at 124.

Each $124 invested yields $5 interest.

$$\begin{array}{r} 0.0403 \\ 124\overline{)5.00} \\ \underline{496} \\ 400 \\ \underline{372} \\ 28 \end{array}$$

4.03%. *Ans.*

44. What is the price of stock if $7000 stock can be bought for $5880?

$7000 stock = 70 shares.

$$\begin{array}{r} 7\cancel{0})588\cancel{0} \\ \hline 84 \end{array}$$

84. *Ans.*

45. Find the amount received for 100 mining shares issued at $15 a share and sold at $2\frac{1}{4}\%$ discount.

$$\begin{array}{r} 0.0225 \\ \underline{15} \\ 1125 \\ \underline{225} \\ 0.3375 \end{array} \qquad \begin{array}{r} \$15. \\ \underline{0.3375} \\ \$14.6625 \\ \underline{100} \\ \$1466.25 \end{array}$$

Ans.

46. How much $3\frac{1}{2}\%$ stock must be sold at $75\frac{1}{8}$ to buy \$5000 4% stock at $94\frac{3}{8}$, brokerage $\frac{1}{8}$ on each transaction?

1 share of 4% stock costs $\$94\frac{3}{8} + \$\frac{1}{8} = \$94\frac{1}{2}$.
1 share of $3\frac{1}{2}\%$ stock sells for $\$75\frac{1}{8} - \$\frac{1}{8} = \$75$.
Therefore, the amount of stock required

$$= \frac{94\frac{1}{2} \times \$5000}{75} = \frac{\overset{63}{\cancel{189}}}{\cancel{2}} \times \$\frac{\overset{100}{\cancel{\overset{200}{\cancel{5000}}}}}{\underset{\cancel{3}}{\cancel{75}}} = \$6300. \textit{ Ans.}$$

47. How much stock must be sold at $76\frac{1}{8}$ to raise a sum sufficient to discount a note for \$1075, due in 53 days, with grace, and discounted at $5\frac{1}{2}\%$?

The time of the note to run is 56 days.
The discount is the interest on \$1075 for 56 days at $5\frac{1}{2}\%$; or

$$\frac{11}{12} \times 9\frac{1}{3} \times \$1.075 = \$9.20.$$

The proceeds is $\$1075 - \$9.20 = \$1065.80$.
1 share of the stock sells for $\$76\frac{1}{8}$.
Therefore, the number of shares of stock is $\frac{1065.80}{76.125} = 14$ shares; and the amount of the stock is $14 \times \$100 = \$1400$. *Ans.*

```
           14
76125)1065800
       76125
      304550
      304500
          50
```

48. A broker bought five \$1000 bonds at $88\frac{1}{2}$. At what price must he sell them to gain \$100, brokerage $\frac{1}{8}$ on each transaction?

If the broker is to gain \$100 on the transaction, he must gain $\frac{1}{5}$ of $\$100 = \$20$ on each bond; that is, \$2 on each \$100 of the face value of the bonds. He must also pay $\frac{1}{8} + \frac{1}{8}$ for brokerage.

Therefore, the selling price must be $88\frac{1}{2} + 2 + \frac{1}{8} + \frac{1}{8} = 90\frac{3}{4}$. *Ans.*

49. If a broker buys bonds at $87\frac{7}{8}$, at what price must he sell them to make $12\frac{1}{2}\%$ profit, brokerage $\frac{1}{8}$ on each transaction?

The price of the bonds is $87\frac{7}{8} + \frac{1}{8} = 88$.

$$88 + 12\tfrac{1}{2}\% \text{ of } 88 = 88 + 11 = 99.$$

The selling price of the bonds is, therefore, $99 + \frac{1}{8} = 99\frac{1}{8}$. *Ans.*

50. Which is the more profitable stock for investment, a 4 % at 85 or a 3 % at 63 ? a $3\frac{1}{2}$ % at $67\frac{1}{4}$ or a 4 % at $81\frac{1}{2}$?

Each \$85 invested in the 4 % stock yields \$4 interest; each \$63 in the 3 % stock yields \$3 interest.

$$\begin{array}{r} 0.0470 \\ 85\overline{)4.00} \\ 340 \\ \hline 600 \\ 595 \\ \hline 50 \end{array} \qquad \begin{array}{r} 0.0476 \\ 63\overline{)3.00} \\ 252 \\ \hline 480 \\ 441 \\ \hline 390 \\ 378 \\ \hline 12 \end{array}$$

Therefore, the 4 % stock yields 4.71 % interest; and the 3 %, 4.76 %.
Therefore, the 3 % is the more profitable investment.

Each \$67.25 invested in the $3\frac{1}{2}$ % stock yields \$3.50 interest; each \$81.50 in the 4 % stock yields \$4 interest.

$$\begin{array}{r} 0.0520 \\ 6725\overline{)350.00} \\ 33625 \\ \hline 13750 \\ 13450 \\ \hline 3000 \end{array} \qquad \begin{array}{r} 0.0490 \\ 815\overline{)40.00} \\ 3260 \\ \hline 7400 \\ 7335 \\ \hline 650 \end{array}$$

Therefore, the $3\frac{1}{2}$ % stock yields 5.20 % interest; and the 4 %, 4.91 %.
Therefore, the $3\frac{1}{2}$ % stock is the more profitable investment.

51. Find the price of a $4\frac{1}{2}$ % bond to be as profitable an investment as a $3\frac{1}{2}$ % bond at $88\frac{1}{2}$.

A $3\frac{1}{2}$ % bond at $88\frac{1}{2}$ yields $\frac{3\frac{1}{2}}{88\frac{1}{2}}$ of 100 % interest.

The price of a $4\frac{1}{2}$ % bond to yield $\frac{3\frac{1}{2}}{88\frac{1}{2}}$ of 100 % interest must be

$$4\tfrac{1}{2} \div \frac{3\frac{1}{2}}{88\frac{1}{2}} = \frac{9}{\cancel{2}} \times \frac{\cancel{2}}{7} \times \frac{177}{2} = \frac{1593}{14} = 113\tfrac{11}{14}. \text{ Ans.}$$

52. Find the price of a 5 % bond to be as profitable an investment as a 3 % bond at $89\frac{1}{2}$.

A 3 % bond at $89\frac{1}{2}$ yields $\frac{3}{89\frac{1}{2}}$ of 100 % interest.

The price of a 5 % bond to yield $\frac{3}{89\frac{1}{2}}$ of 100 % interest must be

$$5 \div \frac{3}{89\frac{1}{2}} = 5 \times \frac{1}{3} \times \frac{179}{2} = \frac{895}{6} = 149\tfrac{1}{6}. \text{ Ans.}$$

[redacted]

53. Find the price of a $3\frac{1}{2}\%$ bond to be as profitable an investment as a 6% bond at par.

A 6% bond at par yields 6% interest.

The price of a $3\frac{1}{2}\%$ bond to yield 6% interest must be

$$3\tfrac{1}{2} \div \frac{6}{100} = \frac{7}{\cancel{2}} \times \frac{\overset{25}{\cancel{\overset{50}{\cancel{100}}}}}{\underset{3}{\cancel{6}}} = \frac{175}{3} = 58\tfrac{1}{3}. \quad \textit{Ans.}$$

54. Find the loss in buying \$80,000 worth of bonds at $91\frac{5}{8}$ and selling at 90, brokerage $\frac{1}{8}$ on each transaction.

The cost for every \$100 of the bonds is $\$91\frac{5}{8} + \$\frac{1}{8} = \$91\frac{3}{4}$.

The price to the seller for every \$100 of the bonds is

$$\$90 - \$\tfrac{1}{8} = \$89\tfrac{7}{8}.$$

The loss on every \$100 of the bonds is $\$91\frac{3}{4} - \$89\frac{7}{8} = \$1\frac{7}{8} = \$1.875$.

$$\begin{array}{r} \$1.875 \\ 800 \\ \hline \$1500.000 \end{array} \quad \textit{Ans.}$$

55. Which is the better investment, a 5% stock at $137\frac{1}{4}$ or a $3\frac{1}{2}\%$ stock at $91\frac{1}{2}$? What rate of interest will be received from each investment?

Each \$137.25 invested in the 5% stock yields \$5 interest; each \$91.50 in the $3\frac{1}{2}\%$ stock yields \$3.50 interest.

$$\begin{array}{r} 0.0364 \\ 13725)\overline{500.00} \\ 41175 \\ \hline 88250 \\ 82350 \\ \hline 59000 \\ 54900 \\ \hline 4100 \end{array} \qquad \begin{array}{r} 0.0382 \\ 915)\overline{35.00} \\ 2745 \\ \hline 7550 \\ 7320 \\ \hline 2300 \\ 1830 \\ \hline 470 \end{array}$$

Therefore, the 5% stock yields 3.64% interest; the $3\frac{1}{2}\%$ stock yields 3.83%; and the $3\frac{1}{2}\%$ stock is the better investment. *Ans.*

56. A person invests \$7370 in the purchase of a stock at 92. What will be his loss if he sells at 90, brokerage $\frac{1}{8}$ on each transaction?

The buying price is $92 + \frac{1}{8} = 92\frac{1}{8}$.

The number of shares is $\dfrac{7370}{92\frac{1}{8}} = 80$.

The selling price is $90 - \frac{1}{8} = 89\frac{7}{8}$.

Therefore, the loss on each share is $\$92\frac{1}{8} - \$89\frac{7}{8} = \$2\frac{1}{4}$.

Therefore, the loss on 80 shares is $80 \times \$2\frac{1}{4} = \$180$. Ans.

[redacted]

57. How much stock must be sold at $90\frac{5}{8}$ so that when the seller invests the proceeds in a mortgage at 6% he will receive \$543.75 annual income?

The face of the 6% mortgage is \$543.75 ÷ 0.06 = \$9062.50.

The number of shares of stock at $90\frac{5}{8}$ that must be sold to amount to \$9062.50 is $\frac{9062.50}{90.625} = 100$ shares.

100 shares amount to \$10,000. *Ans.*

58. A person invests $\frac{2}{7}$ of his money at 6%, $\frac{2}{5}$ at $4\frac{1}{2}$%, and the rest at $3\frac{1}{2}$%. What per cent does he receive on the whole amount?

$$\frac{2}{7}+\frac{2}{5}=\frac{10+14}{35}=\frac{24}{35}. \qquad \frac{35}{35}-\frac{24}{35}=\frac{11}{35}.$$

On the whole amount he receives

$$\tfrac{2}{7} \text{ of } 6\% + \tfrac{2}{5} \text{ of } 4\tfrac{1}{2}\% + \tfrac{11}{35} \text{ of } 3\tfrac{1}{2}\% = 1\tfrac{5}{7}\% + 1\tfrac{4}{5}\% + 1\tfrac{1}{10}\%$$
$$= 3\tfrac{50+56+7}{70}\% = 4\tfrac{43}{70}\% = 4.61\%. \quad \textit{Ans.}$$

59. How many shares of stock must a man sell at $107\frac{1}{4}$, that when he invests the proceeds in 3% stock at $71\frac{1}{2}$ he may receive an annual income of \$900?

Each share of the 3% yields \$3 income.

\$900 income requires $\frac{900}{3}$ or 300 shares.

The cost of 300 shares at $71\frac{1}{2}$ is $300 \times \$71\frac{1}{2}$.

Therefore, the number of shares he must sell at $107\frac{1}{4}$ is

$$\frac{300 \times 71\frac{1}{2}}{107\frac{1}{4}} = \overset{100}{\cancel{300}} \times \frac{\cancel{143}}{\cancel{2}} \times \frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{429}}} = 200. \quad \textit{Ans.}$$

Exercise 133. Page 296.

1. Find the cost of a sight draft on New York of \$1100, exchange $\frac{1}{4}$% premium.

Exchange = $\frac{1}{4}$% of \$1100 = \$2.75.

Cost of draft = \$1100 + \$2.75 = \$1102.75. *Ans.*

2. Find the cost of a sight draft on New Orleans of \$1350, exchange $\frac{1}{4}$% discount.

Exchange = $\frac{1}{4}$% of \$1400 = \$3.50.

Cost of draft = \$1350 − \$3.50 = \$1346.50. *Ans.*

3. Find the cost of a draft on Boston of \$1600, payable 30 days after sight with grace, interest 6%, exchange $\frac{1}{4}$% premium.

Discount on \$1600 for 33 dy. at $6\% = 5\frac{1}{2} \times \$1.60 = \$8.80$.
Proceeds $= \$1600 - \$8.80 = \$1591.20$.
Exchange $= \frac{1}{4}\%$ of $\$1600 = \$4$.
Cost of draft $= \$1591.20 + \$4 = \$1595.20$. *Ans.*

4. Find the cost of a draft of \$500, payable 60 days after sight with grace, interest 7%, exchange $\frac{1}{2}$% discount.

Discount on \$500 for 63 dy. at $6\% = 10.5 \times \$0.50 = \$5.25$.

$$\begin{array}{r|r} 6 & \$5.25 \\ & 0.875 \\ \hline & \$6.125 \end{array}$$

Exchange $= \frac{1}{2}\%$ of $\$500 = \$2.50$.
Total discount $= \$6.13 + \$2.50 = \$8.63$.
Cost of draft $= \$500 - \$8.63 = \$491.37$. *Ans.*

5. Find the cost of a draft of \$1200, payable 90 days after sight with grace, interest 7%, exchange $\frac{1}{2}$% premium.

Discount on \$1200 for 93 dy. at $6\% = 15.5 \times \$1.20$; at $7\% = 15.5 \times \$1.40$.

$$\begin{array}{r} \$15.5 \\ 1.40 \\ \hline 6200 \\ 155 \\ \hline \$21.700 \end{array}$$

Proceeds $= \$1200 - \$21.70 = \$1178.30$.
Exchange $= \frac{1}{2}\%$ of $\$1200 = \$6.00$.
Cost of draft $= \$1178.30 + \$6 = \$1184.30$. *Ans.*

6. Find the cost of a draft of \$950, payable in 30 days with grace, interest $4\frac{1}{2}$%, exchange at par.

Discount on \$950 for 33 dy. at $6\% = 5.5 \times \$0.95$.

$$\begin{array}{r|r} & \$0.95 \\ & 5.5 \\ \hline & 475 \\ & 475 \\ 4 & \$5.225 \\ & 1.306 \\ \hline & \$3.919 \end{array}$$

$$\begin{array}{r} \$950. \\ 3.92 \\ \hline \$946.08 \end{array}$$ Ans.

[redacted]

7. Find the cost of a draft of \$725, payable in 60 days with grace, interest 5%, exchange ¼% discount.

Discount on \$725 for 63 dy. at 6% = 10.5 × \$0.725.

```
    $0.725
      10.5
    ------
      3625
     725
  |--------
 6| $7.6125
  |  1.26875
  ---------
   $6.34375
```

Exchange = ¼% of \$800 = \$2.

Total discount = \$6.34 + \$2 = \$8.34.

Cost of draft = \$725 − \$8.34 = \$716.66. *Ans.*

8. Find the cost of a draft of \$810, payable in 90 days with grace, interest 5½%, exchange ¼% premium.

Discount on \$810 for 93 dy. at 6% = 15.5 × \$0.81.

```
     $15.5
      0.81
     -----
       155
     1240                     $810.
   |-------                     11.51
12 | $12.555                  -------
   |   1.046                  $798.49
   --------
    $11.509
```

Exchange = ¼% of \$900 = \$2.25.

Cost of draft = \$798.49 + \$2.25 = \$800.74. *Ans.*

9. Find the face of a draft, payable 30 days after sight with grace, that can be bought for \$274, interest 6%, exchange at par.

Discount on \$1 for 33 dy. at 6% = \$0.0055; and the proceeds of \$1 = \$1 − \$0.0055 = \$0.9945.

Face of draft = $\$\frac{274}{0.9945}$ = \$275.52. *Ans.*

```
            275.51
      -------------
 9945)2740000.
      19890
      -----
       75100
       69615
       -----
        54850
        49725
        -----
         51250
         49725
         -----
          15250
           9945
          -----
           5305
```

10. Find the face of a draft, payable 60 days after sight with grace, that can be bought for \$ 1250, interest 7 %, exchange $\frac{1}{4}$% premium.

Discount on \$ 1 for 63 dy. at 6 % = \$ 0.0105; at 7 % = \$ 0.01225; and proceeds of \$ 1 = \$ 1 − \$ 0.01225 = \$ 0.98775.

Exchange on \$ 1 = \$ 0.0025; and cost of \$ 1 = \$ 0.98775 + \$ 0.0025 = \$ 0.99025.

Face of draft = \$ $\frac{1250}{0.99025}$ = \$ 1262.31. *Ans.*

```
            1262.30
99025)125000000.
      99025
      259750
      198050
       617000
       594150
        228500
        198050
         304500
         297075
           74250
```

11. Find the face of a draft, payable 60 days after date with grace, that can be bought for \$ 1125, interest $5\frac{1}{2}$ %, exchange $\frac{1}{4}$% discount.

Discount on \$ 1 for 63 dy. at 6 % = 0.0105; at $5\frac{1}{2}$ % = \$ 0.009625; and proceeds of \$ 1 = \$ 1 − \$ 0.009625 = \$ 0.990375.

Exchange on \$ 1 = \$ 0.0025; and cost of \$ 1 = \$ 0.990375 − \$ 0.0025 = \$ 0.987875.

Face of draft = \$ $\frac{1125}{0.987875}$ = \$ 1138.81. *Ans.*

```
              1138.80
987875)1125000000.
       987875
       1371250
        987875
        3833750
        2963625
         8701250
         7903000
          7982500
          7903000
            795000
```

[redacted]

12. Find the face of a draft, payable 30 days after date with grace, that can be bought for \$520, interest 4%, exchange $\frac{1}{2}$% premium.

Discount on \$1 for 33 dy. at 6% = \$0.0055; at 4% = \$0.0036$\frac{2}{3}$; and proceeds of \$1 = \$1 − \$0.0036$\frac{2}{3}$ = \$0.9963$\frac{1}{3}$.

Exchange on \$1 = \$0.005; and cost of \$1 = \$0.9963$\frac{1}{3}$ + \$0.005 = \$1.0013$\frac{1}{3}$.

Face of draft $= \$\frac{520}{1.001\frac{1}{3}} = \$\frac{1560}{3.004} = \$519.31$. *Ans.*

```
           519.30
    3004)1560000.
         15020
          5800
          3004
          27960
          27036
            9240
            9012
             2280
```

13. Find the face of a draft, payable 90 days after date with grace, that can be bought for \$10,000, interest 4$\frac{1}{2}$%, exchange at par.

Discount on \$1 for 93 dy. at 6% = \$0.0155; at 4$\frac{1}{2}$% = \$0.011625; and proceeds of \$1 = \$1 − \$0.011625 = \$0.988375.

Face of draft $= \$\frac{10000}{0.988375} = \$10,117.62$. *Ans.*

```
                  10117.61
    988375)10000000000.
            988375
             1162500
              988375
              1741250
               988375
              7528750
              6918625
               6101250
               5930250
                1710000
                 988375
                 721625
```

Exercise 134. Page 298.

1. Find the cost of a sight draft on London for £320 10 s. 6 d.

£320 10 s. 6 d. = £320.525.

320.525 × \$4.865 = \$1559.35. *Ans.*

```
    320.525
      4.865
    1602625
   1923150
  2564200
 1282100
1559.354125
```

2. Find the cost of a sight draft on Paris for 8000 francs.

$$8000 \text{ fr.} = 8000 \times \$\frac{1}{5.18\frac{1}{8}}$$

$$= 8000 \times \$\frac{8}{41.45} = \$\frac{64000}{41.45}$$

= \$1544.03. *Ans.*

```
          1544.02
4145)6400000.
     4145
     22550
     20725
      18250
      16580
       16700
       16580
         12000
          8290
          3710
```

3. Find the cost of a sight draft on Hamburg for 2876 reichsmarks.

4 reichsmarks = \$0.955.

∴ 1 reichsmark = \$0.23875.

```
  $0.23875
      2876
    143250
   167125
  191000
  47750
$686.64500   $686.65. Ans.
```

4. Find the cost of a sight draft on Amsterdam for 6486 guilders.

```
 $0.40375
     6486
   242250
  323000
 161500
 242250
$2618.72250
```

\$2618.72. *Ans.*

5. Find the cost of a sight draft on Glasgow for £5876 10 s.

£5876 10 s. = £5876.5.

5876.5 × \$4.865 = \$28,589.17. *Ans.*

```
   5876.5
    4.865
   293825
  352590
 470120
235060
28589.1725
```

6. Find the cost of a sight draft on Paris for 12,842 francs.

$$12{,}842 \text{ fr.} = \$\frac{12842}{5.18\frac{1}{8}}$$

= \$2478.55. *Ans.*

```
               2478.55
518125)1284200000.
       1036250
        2479500
        2072500
         4070000
         3626875
          4431250
          4145000
           2862500
           2590625
            2718750
            2590625
             128125
```

7. Find the cost of a sight draft on Berlin for 4885 reichsmarks.

1 reichsmark = $0.23875.

$0.23875
4885
119375
191000
191000
95500
$1166.29375

$1166.29. *Ans.*

8. Find the cost of a sight draft on Rotterdam for 8282 guilders.

$0.40375
8282
80750
323000
80750
323000
$3343.85750

$3343.86. *Ans.*

9. Find the cost of a sight draft on Liverpool for £ 1242 12 s. 6 d.

£ 1242 12 s. 6 d. = £ 1242.625.

1242.625 × $4.865
= $6045.37. *Ans.*

1242.625
4.865
6213125
7455750
9941000
4970500
6045.370625

10. Find the cost of a sight draft on Paris for 2685 francs.

$$2685 \text{ fr.} = \$\frac{2685}{5.18\frac{1}{8}} = \$518.21. \textit{ Ans.}$$

518.21
518125)268500000.
2590625
943750
518125
4256250
4145000
1112500
1036250
762500
518125
244375

11. Find the face of a sight draft on Glasgow that can be bought for $2000.

$$\$2000 = £\frac{2000}{4.865} = £411.0997$$
$$= £411 \ 2\text{ s. } \textit{ Ans.}$$

411.0996
4865)2000000.
19460
5400
4865
5350
4865
48500
43785
47150
43785
33650
29190
4460

411.0997
20
1.9940
12
11.928

12. Find the face of a sight draft on London that can be bought for $4000.

Twice as large a draft can be bought for $4000 as for $2000.

$2000 will buy a draft (from Ex. 11) of £411 1 s. 11.928 d.

$$\begin{array}{lll} \pounds 411 & 1\text{ s.} & 11.928\text{ d.} \\ & & \quad\quad\quad 2 \\ \hline \pounds 822 & 3\text{ s.} & 11.856\text{ d.} \end{array}$$

£822 3 s. $11\frac{7}{8}$ d. *Ans.*

13. Find the cost of a sixty-day draft on London for £150, when sixty-day bills are quoted at $4.81\frac{1}{4}$, and the broker's commission is $\frac{1}{8}$% of the cost of the draft.

$150 \times \$4.81\frac{1}{4} = \$721.88.$

$\frac{1}{8}\%$ of $\$721.88 = \$0.90.$

$$\begin{array}{r} \$4.8125 \\ 150 \\ \hline 2406250 \\ 48125 \\ \hline \$721.8750 \end{array} \qquad \begin{array}{r} \$721.88 \\ 0.90 \\ \hline \$722.78 \end{array} \textit{ Ans.}$$

14. How large a sight draft on Paris can be bought for $2840?

$2840 \times 5.18\frac{1}{8}$ fr. = 14,714.75 fr. *Ans.*

$$\begin{array}{r} 5.18125 \\ 2840 \\ \hline 20725000 \\ 4145000 \\ 1036250 \\ \hline 14714.75000 \end{array}$$

15. How large a sixty-day draft on Paris can be bought for $1500, when sixty-day bills are quoted at $5.17\frac{3}{8}$?

$1500 \times 5.17\frac{3}{8}$ fr. = 7760.625 fr. *Ans.*

$$\begin{array}{r} 5.17375 \\ 1500 \\ \hline 258687500 \\ 517375 \\ \hline 7760.62500 \end{array}$$

16. How large a sight draft on Berlin can be bought for $8000?

4 reichsmarks = $0.955.

∴ 1 reichsmark = $0.23875.

$$\$8000 = \frac{8000}{0.23875} \text{ reichsmarks}$$

$$= \frac{8000}{0.23\frac{7}{8}} \text{ reichsmarks}$$

$$= \frac{64000}{1.91} \text{ reichsmarks}$$

= 33,507.85 reichsmarks. *Ans.*

$$\begin{array}{r} 33507.85 \\ 191\overline{)6400000.} \\ 573 \\ \hline 670 \\ 573 \\ \hline 970 \\ 955 \\ \hline 1500 \\ 1337 \\ \hline 1630 \\ 1528 \\ \hline 1020 \\ 955 \\ \hline 65 \end{array}$$

17. How large a sixty-day draft on Hamburg can be bought for \$2500, when German sixty-day drafts are quoted at 0.95?

$$4 \text{ reichsmarks} = \$0.95.$$

$$\therefore\ 1 \text{ reichsmark} = \$0.23\tfrac{3}{4}.$$

$$\$2500 = \frac{2500}{0.23\frac{3}{4}} \text{ reichsmarks}$$

$$= \frac{10000}{0.95} \text{ reichsmarks}$$

$$= 10{,}526.32 \text{ reichsmarks.}$$

Ans.

```
      10526.31
95)1000000.
    95
     500
     475
      250
      190
       600
       570
        300
        285
         150
          95
          55
```

18. How large a sight draft on Amsterdam can be bought for \$2200?

$$\$2200 = \frac{2200}{0.40\frac{3}{8}} \text{ guilders}$$

$$= \frac{17600}{3.23} \text{ guilders}$$

$$= 5448.92 \text{ guilders.}$$ *Ans.*

```
        5448.91
323)1760000.
    1615
     1450
     1292
      1580
      1292
       2880
       2584
        2960
        2907
          530
          323
          207
```

19. How large a sixty-day draft on Rotterdam can be bought for \$1200, when a sixty-day draft on Holland is quoted at $0.40\frac{1}{8}$?

$$\$1200 = \frac{1200}{0.40\frac{1}{8}} \text{ guilders}$$

$$= \frac{9600}{3.21} \text{ guilders}$$

$$= 2990.65 \text{ guilders.}$$ *Ans.*

```
       2990.65
321)960000.
    642
    3180
    2889
     2910
     2889
      2100
      1926
       1740
       1605
        135
```

Exercise 135. Page 300.

1. Find the equated time for the payment of \$250 due in 3 mo., \$400 due in 6 mo., \$700 due in 8 mo.

$$\begin{array}{rl}
\$250 \times 0 = & \\
\$400 \times 3 = & \$1200 \\
\underline{\$700} \times 5 = & \underline{3500} \\
\$1350 & \$4700 \\
 & 3\tfrac{13}{27}
\end{array}$$

$3\tfrac{13}{27}$ mo. = 3 mo. 14 dy.

Hence, the equated time is 3 mo. 14 dy. after 3 mo.; that is, 6 mo. 14 dy. *Ans.*

2. Find the equated time for the payment of \$300 due in 30 days, \$500 due in 60 days, and \$200 due in 90 days.

$$\begin{array}{rl}
\$300 \times 00 = & \\
\$500 \times 30 = & \$15000 \\
\underline{\$200} \times 60 = & \underline{12000} \\
\$1000 & \$27000 \\
 & 27
\end{array}$$

Hence, the equated time is 27 dy. after 30 dy.; that is, 57 dy. *Ans.*

3. Find the equated time for the payment of \$325 due now, \$200 due in 30 days, \$460 due in 60 days, and \$150 due in 90 days.

$$\begin{array}{rl}
\$325 \times 00 = & \\
\$200 \times 30 = & \$6000 \\
\$460 \times 60 = & 27600 \\
\underline{\$150} \times 90 = & \underline{13500} \\
\$1135 & \$47100 \\
 & 41.49
\end{array}$$

Hence, the equated time is 41 dy. *Ans.*

4. Find the equated time for the payment of \$240 due May 10, \$420 due July 2, \$310 due Sept. 14, and \$600 due Oct. 1.

$$\begin{array}{rl}
\$240 \times 00 = & \\
\$420 \times 53 = & \$22260 \\
\$310 \times 127 = & 39370 \\
\underline{\$600} \times 144 = & \underline{86400} \\
\$1570 & \$148030 \\
 & 94.3
\end{array}$$

Hence, the equated time is 94 dy. after May 10; that is, Aug. 12. *Ans.*

[redacted]

5. Find the equated time for the payment of $275 due June 21, $175 due July 16, $200 due Aug. 6, and $150 due Sept. 3.

$$\begin{array}{rr}
\$275 \times 00 = & \\
\$175 \times 25 = & \$4375 \\
\$200 \times 46 = & 9200 \\
\$150 \times 74 = & \underline{11100} \\
\$800 & \overline{)\$24675} \\
 & 30.8
\end{array}$$

Hence, the equated time is 31 dy. after June 21; that is, July 22. *Ans.*

6. Find the equated time for the payment of $112.30 due July 6, $115.25 due July 30, $232.15 due Sept. 4, and $102.36 due Oct. 1.

$$\begin{array}{rr}
\$112.30 \times 00 = & \\
\$115.25 \times 24 = & \$2766.00 \\
\$232.15 \times 60 = & 13929.00 \\
\underline{\$102.36 \times 87 =} & \underline{8905.32} \\
\$562.06 & \overline{)\$25600.32} \\
 & 45.55
\end{array}$$

Hence, the equated time is 46 dy. after July 6; that is, Aug. 21. *Ans.*

7. A owes B $200 due in 10 mo. If he pays $120 in 4 mo., when should he pay the balance?

By paying $120 in 4 mo. A loses the use of $120 for 6 mo., which is equal to the use of $720 for 1 mo. Therefore, he is entitled to keep the balance ($80) $\frac{720}{80}$ mo. = 9 mo. after its maturity.

Hence, he should pay the balance in 19 mo. *Ans.*

8. A owed B $2000 payable in 4 mo., but at the end of 1 mo. he paid him $500, at the end of 2 mo. $500, and at the end of 3 mo. $500. In how many months is the balance due?

$$\begin{array}{rr}
\$500 \times 3 = & \$1500 \\
\$500 \times 2 = & 1000 \\
\$500 \times 1 = & 500 \\
\hline
\$1500 & \$3000
\end{array}$$

Therefore, he is entitled to keep the balance ($500) $\frac{3000}{500}$ mo. = 6 mo. after its maturity.

Hence, the balance is due in 10 mo. *Ans.*

9. A man, Feb. 11, 1898, gave a note for $1700 payable in 4 mo.; but he paid Mar. 22, $400, Apr. 20, $220, May 10, $300. When was the balance due?

Note was due June 11, 1898.

$$\begin{array}{rcr} \$400 \times 81 &=& \$32400 \\ \$220 \times 52 &=& 11440 \\ \underline{\$300} \times 32 &=& \underline{9600} \\ \$920 & & \$53440 \end{array}$$

Therefore, he is entitled to keep the balance ($780) $\frac{53440}{780}$ dy. = 69 dy. after its maturity.

Hence, the balance was due 69 dy. after June 11, 1898; that is, Aug. 19, 1898. *Ans.*

10. A man, Jan. 4, 1898, gave a note for $2500 payable in 6 mo.; but he paid Feb. 4, $200, Mar. 4, $400, Apr. 4, $600, May 4, $500, and June 4, $300. When was the balance due?

Note was due July 4, 1898.

$$\begin{array}{rcr} \$200 \times 5 &=& \$1000 \\ \$400 \times 4 &=& 1600 \\ \$600 \times 3 &=& 1800 \\ \$500 \times 2 &=& 1000 \\ \underline{\$300} \times 1 &=& \underline{300} \\ \$2000 & & \$5700 \end{array}$$

Therefore, he is entitled to keep the balance ($500) $\frac{5700}{500}$ mo. = 11.4 mo. after its maturity.

Hence, the balance was due 11.4 mo. = 11 mo. 12 dy. after July 4, 1898; that is, June 16, 1899. *Ans.*

Exercise 136. Page 302.

1. Find the time for the payment of the balance of an account if the debit and credit sides, when equated, stand

	Dr.		Cr.
May 17, 1897	$950	Apr. 12, 1897 . . .	$1000

The difference between the equated times is 35 dy.

The balance of account is $1000 − $950 = $50.

If the account were settled at the later date, May 17, 1897, the $1000 would have been on interest 35 dy., which is equivalent to having the balance, $50, on interest $\frac{1000}{50}$ of 35 dy. = 700 dy.

Hence, the balance should begin to draw interest 700 dy. before May 17, 1897; that is, June 17, 1895. *Ans.*

[redacted]

2. Find the time for the payment of the balance of an account if the debit and credit sides, when equated, stand

	Dr.		Cr.
Apr. 12, 1897	\$950	May 17, 1897	\$1000

The difference between the equated times is 35 dy.

The balance of account is \$1000 – \$950 = \$50.

If the account were settled at the later date, May 17, 1897, the \$950 would have been on interest 35 dy., which is equivalent to having the balance, \$50, on interest $\frac{950}{50}$ of 35 dy. = 665 dy.

Hence, the balance should remain unpaid 665 dy. after May 17, 1897; that is, until Mar. 13, 1899. *Ans.*

3. Find the time for the payment of the balance of an account if the debit and credit sides, when equated, stand

	Dr.		Cr.
May 30, 1898	\$1000	June 23, 1898	\$920

The difference between the equated times is 24 dy.

The balance of account is \$1000 – \$920 = \$80.

If the account were settled at the later date, June 23, 1898, the \$1000 would have been on interest 24 dy., which is equivalent to having the balance, \$80, on interest $\frac{1000}{80}$ of 24 dy. = 300 dy.

Hence, the balance should begin to draw interest 300 dy. before June 23, 1898; that is, Aug. 27, 1897. *Ans.*

4. Find the time for the payment of the balance of an account if the debit and credit sides, when equated, stand

	Dr.		Cr.
July 6, 1897	\$500	Apr. 14, 1897	\$480

The difference between the equated times is 83 dy.

The balance of account is \$500 – \$480 = \$20.

If the account were settled at the later date, July 6, 1897, the \$480 would have been on interest 83 dy., which is equivalent to having the balance, \$20, on interest $\frac{480}{20}$ of 83 dy. = 1992 dy.

Hence, the balance should remain unpaid 1992 dy. after July 6, 1897; that is, until Dec. 20, 1902. *Ans.*

5. Find the time for the payment of the balance of an account if the debit and credit sides, when equated, stand

	Dr.		Cr.
Aug. 13, 1897	$875	Sept. 13, 1897	$600

The difference between the equated times is 31 dy.

The balance of account is $875 – $600 = $275.

If the account were settled at the later date, Sept. 13, 1897, the $875 would have been on interest 31 dy., which is equivalent to having the balance, $275, on interest $\frac{875}{275}$ of 31 dy. = 99 dy.

Hence, the balance should begin to draw interest 99 dy. before Sept. 13, 1897; that is, June 6, 1897. *Ans.*

6. Find the time for the payment of the balance of an account if the debit and credit sides, when equated, stand

	Dr.		Cr.
May 28, 1898	$500	June 4, 1898	$550

The difference between the equated times is 7 dy.

The balance of account is $550 – $500 = $50.

If the account were settled at the later date, June 4, 1898, the $500 would have been on interest 7 dy., which is equivalent to having the balance, $50, on interest $\frac{500}{50}$ of 7 dy. = 70 dy.

Hence, the balance should remain unpaid 70 dy. after June 4, 1898; that is, until Aug. 13, 1898. *Ans.*

7. Find the time for the payment of the balance of an account if the debit and credit sides, when equated, stand

	Dr.		Cr.
Apr. 4, 1898	$400	June 6, 1898	$300

The difference between the equated times is 63 dy.

The balance of account is $400 – $300 = $100.

If the account were settled at the later date, June 6, 1898, the $400 would have been on interest 63 dy., which is equivalent to having the balance, $100, on interest $\frac{400}{100}$ of 63 dy. = 252 dy.

Hence, the balance should begin to draw interest 252 dy. before June 6, 1898; that is, Sept. 27, 1897. *Ans.*

8. Find the time for the payment of the balance of an account if the debit and credit sides, when equated, stand

	Dr.		Cr.
Mar. 12, 1898	$750	Feb. 4, 1898	$500

The difference between the equated times is 36 dy.

The balance of account is $750 − $500 = $250.

If the account were settled at the later date, Mar. 12, 1898, the $500 would have been on interest 36 dy., which is equivalent to having the balance, $250, on interest $\frac{500}{250}$ of 36 dy. = 72 dy.

Hence, the balance should remain unpaid 72 dy. after Mar. 12, 1898; that is, until May 23, 1898. *Ans.*

9. Find the time for the payment of the balance of an account if the debit and credit sides, when equated, stand

	Dr.		Cr.
Feb. 4, 1898	$750	Mar. 12, 1898	$500

The difference between the equated times is 36 dy.

The balance of account is $750 − $500 = $250.

If the account were settled at the later date, Mar. 12, 1898, the $750 would have been on interest 36 dy., which is equivalent to having the balance, $250, on interest $\frac{750}{250}$ of 36 dy. = 108 dy.

Hence, the balance should begin to draw interest 108 dy. before Mar. 12, 1898; that is, Nov. 24, 1897. *Ans.*

Exercise 137. Page 303.

1. Find the cash balance of the following account, reckoning interest at 6%:

1897.		Dr.	Int.	1897.		Cr.	Int.
Apr. 5.	To mdse.,	$250.00	$8.18	Apr. 20.	By cash,	$200.00	$2.00
Apr. 27.	To mdse.,	610.00	5.89	Apr. 30.	By cash,	500.00	4.17
June 1.	To mdse.,	200.00	0.60	June 4.	By cash,	400.00	1.00
June 19.	To bal. acct.,	40.00		June 19.	By bal. int.,		1.95
		$1100.00	$9.12			$1100.00	$9.12

Hence, the cash balance is $40.00 − $1.95 = $38.05. *Ans.*

2. Find the cash balance of the following account, reckoning interest at 6%:

1897.		Dr.	Int.	1897.		Cr.	Int.
Jan. 15.	To mdse. 3 mo.,	$ 250.00	$ 7.46	Apr. 26.	By cash,	$ 150.00	$ 4.20
Feb. 25.	To mdse. 3 mo.,	98.50	2.28	May 17.	By cash,	150.00	3.68
Mar. 8.	To mdse. 3 mo.,	300.00	6.25	July 7.	By cash,	200.00	3.20
				Oct. 11.	By bal. acct.,	148.50	
				Oct. 11.	By bal. int.,		4.91
		$ 648.50	$ 15.99			$ 648.50	$ 15.99

Hence, the cash balance is $ 148.50 + $ 4.91 = $ 153.41. *Ans.*

3. Find the cash balance of the following account, reckoning interest at 6%:

1897.		Dr.	Int.	1897.		Cr.	Int.
Jan. 2.	To mdse. 60 dy.,	$ 100.00	$ 2.58	Feb. 25.	By cash,	$ 100.00	$ 2.68
Mar. 8.	To mdse. 60 dy.,	200.00	2.90	Mar. 22.	By cash,	150.00	3.38
May 10.	To mdse. 30 dy.,	150.00	1.35	June 21.	By cash,	200.00	1.40
June 2.	To mdse.,	95.00	0.97	Aug. 2.	By bal. acct.,	95.00	
				Aug. 2.	By bal. int.,		0.39
		$ 545.00	$ 7.75			$ 545.00	$ 7.75

Hence, the cash balance is $ 95.00 + $ 0.39 = $ 95.39. *Ans.*

Exercise 138. Page 305.

Find the balance on deposit Jan. 1, 1898, on the following account:

1. Interest being 4%, computed quarterly. Deposited Jan. 1, 1897, $ 125; Mar. 22, 1897, $ 40; June 8, 1897, $ 35; July 30, 1897, $ 85; Sept. 24, 1897, $ 65. Withdrawn Apr. 2, 1897, $ 110; June 30, 1897, $ 40; Oct. 22, 1897, $ 10; Dec. 17, 1897, $ 25.

Date.	Deposited.		Withdrawn.		Interest.		Balance.	
1897.								
Jan. 1,	$ 125	00					$ 125	00
Mar. 22,	40	00					165	00
Apr. 1,					$ 1	25	166	25
Apr. 2,			$ 110	00			56	25
June 8,	35	00					91	25
June 30,			40	00			51	25
July 1,					0	51	51	76
July 30,	85	00					136	76
Sept. 24,	65	00					201	76
Oct. 1,					0	52	202	28
Oct. 22,			10	00			192	28
Dec. 17,			25	00			167	28
1898.								
Jan. 1,					1	67	168	95

[redacted]

2. Interest being 3%, computed quarterly. Deposited Jan. 1, 1897, $200; Feb. 14, 1897, $125; Mar. 10, 1897, $75; May 31, 1897, $50; Aug. 2, 1897, $100. Withdrawn May 7, 1897, $25; June 22, 1897, $40; Oct. 2, 1897, $50; Nov. 4, 1897, $65; Dec. 14, 1897, $75.

DATE.	DEPOSITED.	WITHDRAWN.	INTEREST.	BALANCE.
1897.				
Jan. 1,	$200 00			$200 00
Feb. 14,	125 00			325 00
Mar. 10,	75 00			400 00
Apr. 1,			$1 50	401 50
May 7,		$25 00		376 50
May 31,	50 00			426 50
June 22,		40 00		386 50
July 1,			2 82	389 32
Aug. 2,	100 00			489 32
Oct. 1,			2 92	492 24
Oct. 2,		50 00		442 24
Nov. 4,		65 00		377 24
Dec. 14,		75 00		302 24
1898.				
Jan. 1,			2 27	304 51

3. Interest being 3%, computed semi-annually. Deposited Jan. 1, 1897, $425; May 10, 1897, $15; Sept. 24, 1897, $200; Oct. 5, 1897, $25; Nov. 15, 1897, $65. Withdrawn Feb. 1, 1897, $25; Mar. 20, 1897, $45; Aug. 2, 1897, $50; Aug. 28, 1897, $125; Dec. 10, 1897, $100.

DATE.	DEPOSITED.	WITHDRAWN.	INTEREST.	BALANCE.
1897.				
Jan. 1,	$425 00			$425 00
Feb. 1,		$25 00		400 00
Mar. 20,		45 00		355 00
May 10,	15 00			370 00
July 1,			$5 32	375 32
Aug. 2,		50 00		325 32
Aug. 28,		125 00		200 32
Sept. 24,	200 00			400 32
Oct. 5,	25 00			425 32
Nov. 15,	65 00			490 32
Dec. 10,		100 00		390 32
1898.				
Jan. 1,			3 00	393 32

4. Interest being 3 %, computed annually. Deposited Jan. 1, 1897, $266.50; May 3, 1897, $122.50; Aug. 2, 1897, $57; Aug. 9, 1897, $108; Sept. 4, 1897, $64.50. Withdrawn June 15, 1897, $40; Oct. 8, 1897, $75; Nov. 1, 1897, $60; Dec. 4, 1897, $85; Dec. 20, 1897, $142.

Date.	Deposited.		Withdrawn.		Interest.		Balance.	
1897.								
Jan. 1,	$266	50					$266	50
May 3,	122	50					389	00
June 15,			$40	00			349	00
Aug. 2,	57	00					406	00
Aug. 9,	108	00					514	00
Sept. 4,	64	50					578	50
Oct. 8,			75	00			503	50
Nov. 1,			60	00			443	50
Dec. 4,			85	00			358	50
Dec. 20,			142	00			216	50
1898.								
Jan. 1,					$6	49	222	99

Exercise 139. Page 309.

1. Find the square root of 2916.

29 16(54
25
104)416
416

2. Find the square root of 7921.

79 21(89
64
169)1521
1521

3. Find the square root of 494,209.

49 42 09(703
49
1403)4209
4209

4. Find the square root of 20,164.

2 01 64(142
1
24)101
96
282)564
564

[redacted]

5. Find the square root of 3,345,241.

```
     3 34 52 41(1829
     1
   28)234
      224
  362)1052
       724
 3649)32841
      32841
```

6. Find the square root of 125,457.64.

```
     12 54 57.64(354.2
     9
   65)354
      325
  704)2957
      2816
 7082)14164
      14164
```

7. Find the square root of 47,320,641.

```
      47 32 06 41(6879
      36
   128)1132
       1024
  1367)10806
        9569
 13749)123741
       123741
```

8. Find the square root of 21,609.

```
    2 16 09(147
    1
  24)116
      96
 287)2009
     2009
```

9. Find the square root of 53.7289.

```
     53.72 89(7.33
     49
  143)472
      429
 1463)4389
      4389
```

10. Find the square root of 883.2784.

```
     8 83.27 84(29.72
     4
   49)483
      441
  587)4227
      4109
 5942)11884
      11884
```

11. Find the square root of 1.97262025.

```
      1.97 26 20 25(1.4045
      1
    24)97
       96
  2804)12620
       11216
 28085)140425
       140425
```

12. Find the square root of 0.0002090916.

```
0.00 02 09 09 16(0.01446
     1
 24)109
     96
284)1309
    1136
2886)17316
     17316
```

13. Find the square root of 2.

```
2.00 00 00(1.414213
 1
 24)100
     96
 281)400
     281
 2824)11900
      11296
  2828)6040
       5656
        3840
        2828
        10120
         8484
```

14. Find the square root of 5.

```
5.00 00 00(2.236067
 4
 42)100
     84
 443)1600
     1329
 4466)27100
      26796
  4472)30400
       26832
        35680
        31304
```

15. Find the square root of 0.3.

```
0.30 00 00 00(0.547722
  25
104)500
    416
1087)8400
     7609
10947)79100
      76629
 10954)24710
       21908
        28020
        21908
```

16. Find the square root of $3\frac{1}{4}$.

```
3.25 00 00(1.802775
1
28)225
   224
3602)10000
     7204
 3604)27960
      25228
       27320
       25228
        20920
        18020
```

17. Find the square root of $8\frac{5}{6}$.

```
8.83 33 33(2.972092
4
49)483
   441
587)4233
    4109
5942)12433
     11884
 5944)54933
      53496
       14373
       11888
```

[redacted]

18. Find the square root of 0.9.

0.90 00 00 00(0.948683
81
184)900
736
1888)16400
15104
18966)129600
113796
18972)158040
151776
62640
56916

19. Find the square root of $\frac{4}{9}$.

$\sqrt{\frac{4}{9}} = \frac{2}{3} = 0.666667.$

20. Find the square root of $\frac{5}{9}$.

0.55 55 55 55(0.745355
49
144)655
576
1485)7955
7425
14903)53055
44709
14906)83465
74530
89355
74530

21. Find the square root of $\frac{1}{2}$.

0.50 00 00 00(0.707106
49
1407)10000
9849
14141)15100
14141
14142)95900
84852

22. Find the square root of $\frac{3}{5}$.

0.60 00 00 00(0.774596
49
147)1100
1029
1544)7100
6176
15485)92400
77425
15490)149750
139410
103400
92940

23. Find the square root of $\frac{3}{4}$.

0.75 00 00 00(0.866025
64
166)1100
996
1726)10400
10356
17320)44000
34640
93600
86600

24. Find the square root of $\frac{2}{3}$.

0.66 66 66 66(0.816496
64
161)266
161
1626)10566
9756
16324)81066
65296
16328)157706
146952
107546
97968

Exercise 140. Page 315.

1. Find the cube root of 1331.

$$\begin{array}{r|l}
 & 1\,331(11 \\
 & 1 \\
\hline
3 \times 10^2 = 300 & 331 \\
3 \times (10 \times 1) = 30 & \\
1^2 = 1 & \\
\hline
331 & 331
\end{array}$$

2. Find the cube root of 1728.

$$\begin{array}{r|l}
 & 1\,728(12 \\
 & 1 \\
\hline
3 \times 10^2 = 300 & 728 \\
3 \times (10 \times 2) = 60 & \\
2^2 = 4 & \\
\hline
364 & 728
\end{array}$$

3. Find the cube root of 12.167.

$$\begin{array}{r|l}
 & 12.167(2.3 \\
 & 8 \\
\hline
3 \times 20^2 = 1200 & 4167 \\
3 \times (20 \times 3) = 180 & \\
3^2 = 9 & \\
\hline
1389 & 4167
\end{array}$$

4. Find the cube root of 300.763.

$$\begin{array}{r|l}
 & 300.763(6.7 \\
 & 216 \\
\hline
3 \times 60^2 = 10800 & 84763 \\
3 \times (60 \times 7) = 1260 & \\
7^2 = 49 & \\
\hline
12109 & 84763
\end{array}$$

5. Find the cube root of 148,877.

$$\begin{array}{r|l}
 & 148\,877(53 \\
 & 125 \\
\hline
3 \times 50^2 = 7500 & 23877 \\
3 \times (50 \times 3) = 450 & \\
3^2 = 9 & \\
\hline
7959 & 23877
\end{array}$$

6. Find the cube root of 2,048,383.

$$\begin{array}{r|l}
 & 2\,048\,383(127 \\
 & 1 \\
\hline
3 \times 10^2 = 300 & 1048 \\
3 \times (10 \times 2) = 60 & \\
2^2 = 4 & \\
\hline
364 & 728 \\
64 & \\
\hline
3 \times 120^2 = 43200 & 320383 \\
3 \times (120 \times 7) = 2520 & \\
7^2 = 49 & \\
\hline
45769 & 320383
\end{array}$$

[redacted]

7. Find the cube root of 59.776471.

		59. 776 471(3.91
		27
$3 \times 30^2 =$	2700	32776
$3 \times (30 \times 9) =$	810	
$9^2 =$	81	
	3591	32319
	891	457471
$3 \times 390^2 =$	456300	
$3 \times (390 \times 1) =$	1170	
$1^2 =$	1	
	457471	457471

8. Find the cube root of 304,957.115891.

		304 957.115 891(67.31
		216
$3 \times 60^2 =$	10800	88957
$3 \times (60 \times 7) =$	1260	
$7^2 =$	49	
	12109	84763
	1309	4194115
$3 \times 670^2 =$	1346700	
$3 \times (670 \times 3) =$	6030	
$3^2 =$	9	
	1352739	4058217
	6039	135898891
$3 \times 6730^2 =$	135878700	
$3 \times (6730 \times 1) =$	20190	
$1^2 =$	1	
	135898891	135898891

9. Find the cube root of 0.007821346625.

	0.007 821 346 625(0.1985
	1
$3 \times 10^2 = 300$	6821
$3 \times (10 \times 9) = 270$	
$9^2 = 81$	
651	5859
351	962346
$3 \times 190^2 = 108300$	
$3 \times (190 \times 8) = 4560$	
$8^2 = 64$	
112924	903392
4624	58954625
$3 \times 1980^2 = 11761200$	
$3 \times (1980 \times 5) = 29700$	
$5^2 = 25$	
11790925	58954625

10. Find the cube root of 104.600290750613.

	104.600 290 750 613(4.7117
	64
$3 \times 40^2 = 4800$	40600
$3 \times (40 \times 7) = 840$	
$7^2 = 49$	
5689	39823
889	777290
$3 \times 470^2 = 662700$	
$3 \times (470 \times 1) = 1410$	
$1^2 = 1$	
664111	664111
1411	113179750
$3 \times 4710^2 = 66552300$	
$3 \times (4710 \times 1) = 14130$	
$1^2 = 1$	
66566431	66566431
14131	46613319613
$3 \times 47110^2 = 6658056300$	
$3 \times (47110 \times 7) = 989310$	
$7^2 = 49$	
6659045659	46613319613

11. Find the cube root of 17,183,498,535,125.

		17 183 498 535 125(25805
		8
$3 \times 20^2 =$	1200	9183
$3 \times (20 \times 5) =$	300	
$5^2 =$	25	
	1525	7625
	325	1558498
$3 \times 250^2 =$	187500	
$3 \times (250 \times 8) =$	6000	
$8^2 =$	64	
	193564	1548512
	6064	9986535125
$3 \times 25800^2 =$	1996920000	
$3 \times (25800 \times 5) =$	387000	
$5^2 =$	25	
	1997307025	9986535125

12. Find the cube root of 122,615.327232.

		122 615.327 232(49.68
		64
$3 \times 40^2 =$	4800	58615
$3 \times (40 \times 9) =$	1080	
$9^2 =$	81	
	5961	53649
	1161	4966327
$3 \times 490^2 =$	720300	
$3 \times (490 \times 6) =$	8820	
$6^2 =$	36	
	729156	4374936
	8856	591391232
$3 \times 4960^2 =$	73804800	
$3 \times (4960 \times 8) =$	119040	
$8^2 =$	64	
	73923904	591391232

13. Find the cube root of 116,400.

	116 400(48.8259
	64
$3 \times 40^2 = 4800$	52400
$3 \times (40 \times 8) = 960$	
$8^2 = 64$	
5824	46592
1024	5808000
$3 \times 480^2 = 691200$	
$3 \times (480 \times 8) = 11520$	
$8^2 = 64$	
702784	5622272
11584	185728000
$3 \times 4880^2 = 71443200$	
$3 \times (4880 \times 2) = 29280$	
$2^2 = 4$	
71472484	142944968
29284	427830320
$3 \times 4882^2 = 71501772$	357508860
	703214600
	643515948

[redacted]

14. Find the cube root of 22,406,807.

		22 406 807(281.9205
		8
$3 \times 20^2 =$	1200	14406
$3 \times (20 \times 8) =$	480	
$8^2 =$	64	
	1744	13952
	544	454807
$3 \times 280^2 =$	235200	
$3 \times (280 \times 1) =$	840	
$1^2 =$	1	
	236041	236041
	841	218766000
$3 \times 2810^2 =$	23688300	
$3 \times (2810 \times 9) =$	75870	
$9^2 =$	81	
	23764251	213878259
	75951	4887741000
$3 \times 28190^2 =$	2384028300	
$3 \times (28190 \times 2) =$	169140	
$2^2 =$	4	
	2384197444	4768394888
	169144	11934611200
$3 \times 28192^2 =$	2384366592	11921832960

15. Find the cube root of 10.

$$
\begin{array}{rr|l}
 & & 10.000(2.1544 \\
 & & 8 \\
3 \times 20^2 = & 1200 & 2000 \\
3 \times (20 \times 1) = & 60 & \\
1^2 = & 1 & \\
 & 1261 & 1261 \\
 & 61 & 739000 \\
3 \times 210^2 = & 132300 & \\
3 \times (210 \times 5) = & 3150 & \\
5^2 = & 25 & \\
 & 135475 & 677375 \\
 & 3175 & 61625000 \\
3 \times 2150^2 = & 13867500 & \\
3 \times (2150 \times 4) = & 25800 & \\
4^2 = & 16 & \\
 & 13893316 & 55573264 \\
 & 25816 & 60517360 \\
3 \times 2154^2 = & 13919148 & 55676592
\end{array}
$$

16. Find the cube root of $3\frac{5}{8}$.

$$\sqrt[3]{3\tfrac{5}{8}} = \sqrt[3]{\frac{29}{8}} = \frac{\sqrt[3]{29}}{2} = \frac{3.0723}{2} = 1.5362. \quad \textit{Ans.}$$

$$
\begin{array}{rr|l}
 & & 29.000(3.0723 \\
 & & 27 \\
3 \times 300^2 = & 270000 & 2000000 \\
3 \times (300 \times 7) = & 6300 & \\
7^2 = & 49 & \\
 & 276349 & 1934443 \\
 & 6349 & 65557000 \\
3 \times 3070^2 = & 28274700 & \\
3 \times (3070 \times 2) = & 18420 & \\
2^2 = & 4 & \\
 & 28293124 & 56586248 \\
 & 18424 & 89707520 \\
3 \times 3072^2 = & 28311552 & 84934656
\end{array}
$$

[redacted]

17. Find the cube root of $8\frac{1}{3}$.

		8.333 333(2.0274
		8
$3 \times 200^2 =$	120000	333333
$3 \times (200 \times 2) =$	1200	
$2^2 =$	4	
	121204	242408
	1204	90925333
$3 \times 2020^2 =$	12241200	
$3 \times (2020 \times 7) =$	42420	
$7^2 =$	49	
	12283669	85985683
	42469	49396503
$3 \times 2027^2 =$	12326187	49304748

18. Find the cube root of 5.

		5.000(1.7099
		1
$3 \times 10^2 =$	300	4000
$3 \times (10 \times 7) =$	210	
$7^2 =$	49	
	559	3913
	259	87000000
$3 \times 1700^2 =$	8670000	
$3 \times (1700 \times 9) =$	45900	
$9^2 =$	81	
	8715981	78443829
	45981	85561710
$3 \times 1709^2 =$	8762043	78858387

19. Find the cube root of $\frac{5}{9}$.

		0.555 555(0.8221
		512
$3 \times 80^2 =$	19200	43555
$3 \times (80 \times 2) =$	480	
$2^2 =$	4	
	19684	39368
	484	41875
$3 \times 82^2 =$	20172	40344
		15315

20. Find the cube root of $7\frac{3}{5}$.

		7.600(1.966
		1
$3 \times 10^2 =$	300	6600
$3 \times (10 \times 9) =$	270	
$9^2 =$	81	
	651	5859
	351	741000
$3 \times 190^2 =$	108300	
$3 \times (190 \times 6) =$	3420	
$6^2 =$	36	
	111756	670536
	3456	704640
$3 \times 196^2 =$	115248	691488

21. Find the cube root of $\frac{3}{4}$.

		0.750 000 000(0.9085
		729
$3 \times 900^2 =$	2430000	21000000
$3 \times (900 \times 8) =$	21600	
$8^2 =$	64	
	2451664	19613312
	21664	13866880
$3 \times 908^2 =$	2473392	12366960

Exercise 141. Page 321.

1. Find the area of a parallelogram, base 18 in., altitude 11 in.

Area = (18 × 11) sq. in. = 198 sq. in. *Ans.*

2. Find the area of a triangle, base 16 in., altitude 12 in.

Area = $\frac{1}{2}$(16 × 12) sq. in. = 96 sq. in. *Ans.*

3. Find the area of a rectangle, base 24 in., altitude 18 in.

Area = (24 × 18) sq. in. = 432 sq. in. *Ans.*

4. Find the area of a square, side 18 in.

Area = (18 × 18) sq. in. = 324 sq. in. *Ans.*

5. Find the area of a rhombus, diagonals 8 in. and 10 in.

Area = $\frac{1}{2}$(8 × 10) sq. in. = 40 sq. in. *Ans.*

[redacted]

6. Find the area of a triangle, sides 12 in., 11 in., and 10 in., respectively.

The half sum of the sides is $\frac{1}{2}(12 + 11 + 10)$ in. $= 16.5$ in.

$$\text{Area} = \sqrt{16.5 \times 4.5 \times 5.5 \times 6.5} \text{ sq. in.}$$
$$= \sqrt{2654.4375} \text{ sq. in.} = 51.52 \text{ sq. in. } \textit{Ans.}$$

7. Find the area of a regular hexagon, side 4 in.

Apothem $= 0.8660 \times 4$ in. $= 3.464$ in.
Perimeter $= 6 \times 4$ in. $= 24$ in.
Area $= \frac{1}{2}(24 \times 3.464)$ sq. in. $= 41.568$ sq. in. *Ans.*

8. Find the area of a regular octagon, side 2 in.

Apothem $= 1.2071 \times 2$ in. $= 2.4142$ in.
Perimeter $= 8 \times 2$ in. $= 16$ in.
Area $= \frac{1}{2}(16 \times 2.4142)$ sq. in. $= 19.3136$ sq. in. *Ans.*

9. Find the area of a triangle, base 185 yd., altitude 154 yd.

Area $= \frac{1}{2}(185 \times 154)$ sq. yd. $= 14{,}245$ sq. yd. *Ans.*

$$2\,\underline{|154}\quad\;\; \begin{array}{r} 185 \\ \underline{77} \\ 1295 \\ \underline{1295} \\ 14245 \end{array}$$
$$77$$

10. Find the area of a square, side 212 yd.

Area $= (212 \times 212)$ sq. yd. $= 44{,}944$ sq. yd. *Ans.*

$$\begin{array}{r} 212 \\ \underline{212} \\ 424 \\ 212 \\ \underline{424} \\ 44944 \end{array}$$

11. Find the area of a rectangle, base 106 yd., altitude 66 yd.

Area $= (106 \times 66)$ sq. yd. $= 6996$ sq. yd. *Ans.*

$$\begin{array}{r} 106 \\ \underline{66} \\ 636 \\ \underline{636} \\ 6996 \end{array}$$

12. Find the area of a parallelogram, base 24 ft., altitude 18 ft.

Area = (24×18) sq. ft. = 432 sq. ft. *Ans.*

$$\begin{array}{r} 24 \\ \underline{18} \\ 192 \\ \underline{24} \\ 432 \end{array}$$

13. Find the area of an equilateral triangle, side 132 yd.

Apothem = 0.2887×132 yd. = 38.1084 yd.
Perimeter = 3×132 yd. = 396 yd.
Area = $\frac{1}{2}(396 \times 38.1084)$ sq. yd. = 7545.4632 sq. yd. *Ans.*

$$\begin{array}{r} 0.2887 \\ \underline{132} \\ 5774 \\ 8661 \\ \underline{2887} \\ 38.1084 \end{array} \qquad \begin{array}{r|l} 2 & 396 \\ \hline & 198 \end{array} \qquad \begin{array}{r} 38.1084 \\ \underline{198} \\ 3048672 \\ 3429756 \\ \underline{381084} \\ 7545.4632 \end{array}$$

14. Find the area of a right triangle, base 164 ft., perpendicular 150 ft.

Area = $\frac{1}{2}(164 \times 150)$ sq. ft. = 12,300 sq. ft. *Ans.*

$$\begin{array}{r} 164 \\ \underline{150} \\ 8200 \\ 164 \\ 2\,\underline{)24600} \\ 12300 \end{array}$$

15. Find the area of a regular pentagon, side $5\frac{1}{2}$ in.

Apothem = $0.6882 \times 5\frac{1}{2}$ in. = 3.7851 in.
Perimeter = 5×5.5 in. = 27.5 in.
Area = $\frac{1}{2}(27.5 \times 3.7851)$ sq. in. = 52.0451 sq. in. *Ans.*

$$\begin{array}{r} 0.6882 \\ \underline{5.5} \\ 34410 \\ \underline{34410} \\ 3.78510 \end{array} \qquad \begin{array}{r} 3.7851 \\ \underline{27.5} \\ 189255 \\ 264957 \\ 75702 \\ 2\,\underline{)104.09025} \\ 52.0451 \end{array}$$

16. Find the area of a parallelogram, base 122 yd., altitude 76 yd.

Area = (122 × 76) sq. yd. = 9272 sq. yd. *Ans.*

$$\begin{array}{r} 122 \\ \underline{76} \\ 732 \\ \underline{854} \\ 9272 \end{array}$$

17. Find the area of a regular decagon, side $2\frac{1}{2}$ in.

Apothem = 1.5388 × 2.5 in. = 3.847 in.
Perimeter = 10 × 2.5 in. = 25 in.
Area = $\frac{1}{2}$ (25 × 3.847) sq. in. = 48.0875 sq. in. *Ans.*

$$\begin{array}{r} 1.5388 \\ \underline{2.5} \\ 76940 \\ \underline{30776} \\ 3.84700 \end{array} \qquad \begin{array}{r} 3.847 \\ \underline{25} \\ 19235 \\ 7694 \\ 2\,\underline{|96.175} \\ 48.0875 \end{array}$$

18. Find the area of a triangle, base 82^{cm}, altitude 51^{cm}.

Area = $\frac{1}{2}(82 \times 51)^{qcm} = 2091^{qcm}$. *Ans.*

$$\begin{array}{r} 2\,\underline{|82} \\ 41 \end{array} \qquad \begin{array}{r} 51 \\ \underline{41} \\ 51 \\ \underline{204} \\ 2091 \end{array}$$

19. Find the area of a rhombus, diagonals 16 ft. and 12 ft.

Area = $\frac{1}{2}$ (16 × 12) sq. ft. = 96 sq. ft. *Ans.*

20. Find the area of a circle, diameter 72 ft.

Area = (3.1416 × 36 × 36) sq. ft. = 4071.5136 sq. ft. *Ans.*

$$\begin{array}{r} 36 \\ \underline{36} \\ 216 \\ \underline{108} \\ 1296 \end{array} \qquad \begin{array}{r} 3.1416 \\ \underline{1296} \\ 188496 \\ 282744 \\ 62832 \\ \underline{31416} \\ 4071.5136 \end{array}$$

21. Find the area of a trapezoid, parallel sides 106 ft. and 56 ft., respectively, altitude 48 ft.

Sum of bases = 106 ft. + 56 ft. = 162 ft.

Area = ½ (48 × 162) sq. ft. = 3888 sq. ft. *Ans.*

$$2\,\underline{|162}\qquad\qquad \begin{array}{r} 48 \\ \underline{81} \\ 48 \\ \underline{384} \\ 3888 \end{array}$$
$$81$$

22. Find the number of hektars in a triangular field, one side of which is 82.1^{m}, and the distance to this side from the opposite corner 47.3^{m}.

Area = ½ $(82.1 \times 47.3)^{qm} = 1941.665^{qm} = 0.1942^{ha}$. *Ans.*

$$\begin{array}{r} 47.3 \\ \underline{82.1} \\ 473 \\ 946 \\ \underline{3784} \\ 2\,|\underline{3883.33} \\ 1941.665 \end{array}$$

23. Find the number of acres in a triangular field, one side of which is 343.6 ft., and the distance to this side from the opposite corner 163.2 ft.

Area = ½ (343.6 × 163.2) sq. ft. = 28,037.76 sq. ft. = 0.644 A. *Ans.*

$$2\,\underline{|163.2}\qquad \begin{array}{r} 343.6 \\ \underline{81.6} \\ 20616 \\ 3436 \\ \underline{27488} \\ 28037.76 \end{array} \qquad \begin{array}{r} 0.643 \\ 4356\cancel{0})\overline{2803.776} \\ \underline{26136} \\ 19017 \\ \underline{17424} \\ 15936 \\ \underline{13068} \\ 2868 \end{array}$$
$$81.6$$

24. Find the area of a circle that has a radius of 10 in.; of a circle that has a diameter of 10 ft.; of a circle that has a circumference of 30 in.

Area = (3.1416 × 10 × 10) sq. in. = 314.16 sq. in. *Ans.*

Area = (0.7854 × 10 × 10) sq. ft. = 78.54 sq. ft. *Ans.*

$$\text{Area} = \left(0.7854 \times \frac{30}{3.1416} \times \frac{30}{3.1416}\right) \text{ sq. in.} = 71.620 \text{ sq. in. } \textit{Ans.}$$

$$\cancel{0.7854} \times \frac{\overset{15}{\cancel{30}}}{\underset{\underset{\cancel{2}}{\cancel{4}}}{\cancel{3.1416}}} \times \frac{\overset{15}{\cancel{30}}}{3.1416} = \frac{225}{3.1416} = 225 \times 0.31831.$$

$$\begin{array}{r} 0.31831 \\ 225 \\ \hline 159155 \\ 63662 \\ 63662 \\ \hline 71.61975 \end{array}$$

25. A horse is tied by a rope 27.8^{m} long; over what part of a hektar can he graze?

$$\text{Area} = (3.1416 \times 27.8 \times 27.8)^{qm} = 2427.95^{qm} = 0.2428^{ha}. \; \textit{Ans.}$$

$$\begin{array}{r} 27.8 \\ 27.8 \\ \hline 2224 \\ 1946 \\ 556 \\ \hline 772.84 \end{array} \qquad\qquad \begin{array}{r} 772.84 \\ 3.1416 \\ \hline 463704 \\ 77284 \\ 309136 \\ 77284 \\ 231852 \\ \hline 2427.954144 \end{array}$$

26. How many square feet in a circle that has a diameter of $17\frac{2}{3}$ yd.?

$$17\tfrac{2}{3} \text{ yd.} = 53 \text{ ft.}$$

$$\text{Area} = (0.7854 \times 53 \times 53) \text{ sq. ft.} = 2206.1886 \text{ sq. ft. } \textit{Ans.}$$

$$\begin{array}{r} 53 \\ 53 \\ \hline 159 \\ 265 \\ \hline 2809 \end{array} \qquad\qquad \begin{array}{r} 0.7854 \\ 2809 \\ \hline 70686 \\ 62832 \\ 15708 \\ \hline 2206.1886 \end{array}$$

27. How many square feet in a circle that has a circumference of 117 yd.?

$$117 \text{ yd.} = 351 \text{ ft.}$$

$$\text{Diameter} = \frac{351}{3.1416} \text{ ft.}$$

$$\text{Area} = \left(0.7854 \times \frac{351}{3.1416} \times \frac{351}{3.1416}\right) \text{ sq. ft.} = 9804.0276 \text{ sq. ft. } \textit{Ans.}$$

$$\cancel{0.7854} \times \frac{351}{\underset{4}{\cancel{3.1416}}} \times \frac{351}{3.1416} = \frac{123201}{4} \times \frac{1}{3.1416} = 30800.25 \times 0.31831.$$

$$\begin{array}{r} 0.31831 \\ \underline{30800.25} \\ 159155 \\ 63662 \\ 254648 \\ \underline{95493} \\ 9804.0275775 \end{array}$$

28. Find the area of a triangle whose sides are 73 ft., 57 ft., and 48 ft.

The half sum of the sides $= \frac{1}{2}(73 + 57 + 48)$ ft. $= 89$ ft.

$$\begin{aligned} \text{Area} &= \sqrt{89 \times 16 \times 32 \times 41} \text{ sq. ft.} = \sqrt{1868288} \text{ sq. ft.} \\ &= 1366.853 \text{ sq. ft. } \textit{Ans.} \end{aligned}$$

$$\begin{array}{r} 89 \\ \underline{16} \\ 534 \\ \underline{89} \\ 1424 \end{array} \qquad \begin{array}{r} 1424 \\ \underline{32} \\ 2848 \\ \underline{4272} \\ 45568 \end{array} \qquad \begin{array}{r} 45568 \\ \underline{41} \\ 45568 \\ \underline{182272} \\ 1868288 \end{array}$$

```
      1 86 82 88(1366.853
      1
   23)86
      69
  266)1782
      1596
 2726)18688
      16356
  2732)23320
       21856
        14640
        13660
          9800
          8196
```

[redacted]

29. Find the number of hektars in a triangular field whose sides are 37.5^{m}, 91.7^{m}, and 78.9^{m}.

The half sum of the sides $= \frac{1}{2}(37.5 + 91.7 + 78.9)^{m} = 104.05^{m}$.

Area $= \sqrt{104.05 \times 66.55 \times 12.35 \times 25.15}^{qm} = \sqrt{2150775.55281875}^{qm}$
$= 1466.5^{qm} = 0.1467^{ha}$. *Ans.*

```
     104.05
      66.55
      52025
     52025
    62430
   62430
   6924.5275
      12.35
   346226375
  207735825
 138490550
 69245275
 85517.914625
```

```
   85517.914625
          25.15
   427589573125
  85517914625
 427589573125
171035829250
2150775.55281875
```

```
       2 15 07 75.55 28 18 75(1466.5
       1
   24)115
       96
  286)1907
      1716
 2926)19175
      17556
29325)161955
      146625
```

30. Find the number of hektars in a triangular field whose sides are 67.5^{m}, 81.2^{m}, and 102.7^{m}.

The half sum of the sides $= \frac{1}{2}(67.5 + 81.2 + 102.7)^{m} = 125.7^{m}$.

Area $= \sqrt{125.7 \times 58.2 \times 44.5 \times 23}^{qm} = \sqrt{7487659.89}^{qm}$
$= 2736.3^{qm} = 0.2736^{ha}$. *Ans.*

```
    125.7
     58.2
     2514
   10056
   6285
   7315.74
     44.5
   3657870
  2926296
 2926296
 325550.43
        23
  97665129
 65110086
7487659.89
```

```
      7 48 76 59.89(2736.3
      4
   47)348
      329
  543)1976
      1629
 5466)34759
      32796
54723)196389
      164169
```

31. Find the number of acres in a triangular field whose sides are 227 ft., 342 ft., and 416 ft.

The half sum of the sides $= \frac{1}{2}(227 + 342 + 416)$ ft. $= 492.5$ ft.

$$\text{Area} = \sqrt{492.5 \times 265.5 \times 150.5 \times 76.5}\text{ sq. ft.} = \sqrt{1505458178.4375}\text{ sq. ft.}$$

$$= 38{,}800.23\text{ sq. ft.} = \frac{38800.23}{43560}\text{ A.} = 0.8907\text{ A. } Ans.$$

```
        492.5
        265.5
        -----
        24625
       24625
      29550
      9850
    ---------
    130758.75
        150.5
    ---------
     65379375
   65379375
  13075875
 ------------
 19679191.875
         76.5
 ------------
  98395959375
 118075151250
137754343125
---------------
1505458178.4375
```

```
      15 05 45 81 78.43 75(38800.23
       9
68)  605
     544
768)6145
    6144
776002)1817843
       1552004
7760043)26583975
        23280129
```

```
            0.8907
43560)3880.023
      34848
       39522
       39204
         31830
         30492
```

32. Find the number of acres in a triangular field whose sides are 79.08 ch., 57.03 ch., and 102.19 ch.

The half sum of the sides $= \frac{1}{2}(79.08 + 57.03 + 102.19)$ ch. $= 119.15$ ch.

$$\text{Area} = \sqrt{119.15 \times 40.07 \times 62.12 \times 16.96}\text{ sq. ch.}$$

$$= \sqrt{5030031.2603456}\text{ sq. ch.} = 2242.77\text{ sq. ch.} = 224.277\text{ A. } Ans.$$

```
      119.15
       40.07
      ------
       83405
     47660
    --------
    4774.3405
       62.12
    ---------
     95486810
    47743405
   95486810
  286460430
 --------------
 296582.031860
```

```
    296582.03186
           16.96
    ------------
    177949219116
   266923828674
  177949219116
  29658203186
 ---------------
 5030031.2603456
```

```
       5 03 00 31.26 03 45 60(2242.77
       4
42)103
    84
444)1900
    1776
4482)12431
      8964
44847)346726
      313929
448547)3279703
       3139829
```

[redacted]

33. Find the number of square rods in a triangle whose sides ar
7 rd. 2 yd.; 6 rd. 5 yd.; and 9 rd. $4\frac{1}{2}$ ft.

7 rd. 2 yd. = 40.5 yd.; 6 rd. 5 yd. = 38 yd.; 9 rd. $4\frac{1}{2}$ ft. = 51 yd.

The half sum of the sides $= \frac{1}{2}(40.5 + 38 + 51)$ yd. = 64.75 yd.

$$\text{Area} = \sqrt{64.75 \times 24.25 \times 26.75 \times 13.75}\ \text{sq. yd.}$$
$$= \sqrt{577534.58984375}\ \text{sq. yd.} = 759.9569\ \text{sq. yd.}$$
$$= \frac{759.9569}{30.25}\ \text{sq. rd.} = 25.12\ \text{sq. rd.}\quad \textit{Ans.}$$

```
      64.75
      24.25
      -----
      32375
     12950
    25900
   12950
 ----------
  1570.1875
      26.75
 ----------
   78509375
  109913125
 94211250
31403750
 -------------
 42002.515625
        13.75
 -------------
    210012578125
   294017609375
  126007546875
 42002515625
 ----------------
 577534.58984375
```

```
        57 75 34.58 98 43 75(759.9569
        49
145)875
     725
1509)15034
     13581
15189)145358
      136701
151985)865798
       759925
1519906)10587343
         9119436
15199129)146790775
         136792161
```

```
         25.12
3025)75995.69
     6050
     15495
     15125
      3706
      3025
       6819
       6050
        769
```

34. One diagonal of a trapezium is 10 rd., and the perpendiculars upon it from the opposite corners are 6 rd. and 8 rd. Find the area.

Area of 1st triangle $= \frac{1}{2}(10 \times 6)$ sq. rd. $= 30$ sq. rd.

Area of 2d triangle $= \frac{1}{2}(10 \times 8)$ sq. rd. $= 40$ sq. rd.

Area of trapezium $= 30$ sq. rd. $+ 40$ sq. rd. $= 70$ sq. rd. *Ans.*

35. Find the area of a lot of land in the shape of a trapezium, if one diagonal is 108 ft., and the perpendiculars upon it from the opposite corners are 55 ft. and 60 ft.

Area of 1st triangle $= \frac{1}{2}(108 \times 55)$ sq. ft. $= 2970$ sq. ft.

Area of 2d triangle $= \frac{1}{2}(108 \times 60)$ sq. ft. $= 3240$ sq. ft.

Area of trapezium $= 2970$ sq. ft. $+ 3240$ sq. ft. $= 6210$ sq. ft. *Ans.*

36. What is the area of the ground covered by a tent, the base of which is a regular heptagon 25 ft. on a side?

Apothem $= 1.0382 \times 25$ ft. $= 25.955$ ft.

Perimeter $= 7 \times 25$ ft. $= 175$ ft.

Area $= \frac{1}{2}(175 \times 25.955)$ sq. ft. $= 2271.0625$ sq. ft. *Ans.*

$$2\underline{|175}$$
$$87.5$$

$$\begin{array}{r} 25.955 \\ \underline{87.5} \\ 129775 \\ 181685 \\ \underline{207640} \\ 2271.0625 \end{array}$$

37. How many paving stones will be required to pave a rectangular court 60 ft. long and 40 ft. wide, if each stone is in the shape of a regular hexagon 5 in. on a side?

Area of court $= (60 \times 40)$ sq. ft. $= 2400$ sq. ft.

Apothem $= 0.8660 \times 5$ in. $= 4.33$ in.

Perimeter $= 6 \times 5$ in. $= 30$ in.

Area of stone $= \frac{1}{2}(30 \times 4.33)$ sq. in. $= 64.95$ sq. in.

Number of stones $= \dfrac{2400 \times 144}{64.95} = 5322.$ *Ans.*

$$\begin{array}{r} 144 \\ \underline{2400} \\ 57600 \\ \underline{288} \\ 345600 \end{array}$$

$$\begin{array}{r} 5321. \\ 6495\overline{)34560000.} \\ \underline{32475} \\ 20850 \\ \underline{19485} \\ 13650 \\ \underline{12990} \\ 6600 \\ \underline{6495} \end{array}$$

[redacted]

38. At \$225 an acre, what is the value of a field in the shape of a regular pentagon 250 yd. on a side?

Apothem $= 0.6882 \times 250$ yd. $= 172.05$ yd.
Perimeter $= 5 \times 250$ yd. $= 1250$ yd.
Area $= \frac{1}{2}(1250 \times 172.05)$ sq. yd. $= 107{,}531.25$ sq. yd. $= 22.217$ A.

```
2|1250              172.05
   625                 625
                     86025
                    34410
                   103230
                   107531.25
```

1 A. $= 160 \times 30\frac{1}{4}$ sq. yd. $= 4840$ sq. yd.

```
         22.217                  22.217
484∅)10753.125                      225
     968                         111085
     1073                        44434
      968                       44434
      1051                      4998.825
       968
        832                     $4998.83. Ans.
        484
        3485
        3388
          97
```

39. A rectangular field 100 yd. wide contains $3\frac{1}{8}$ A. What is its length?

$$3\tfrac{1}{8}\text{ A.} = 3\tfrac{1}{8} \times 4840 \text{ sq. yd.}$$

$$\frac{3\frac{1}{8}\times 4840}{100} = \frac{\cancel{25}\times \overset{605}{\cancel{4840}}}{\cancel{8}\times \underset{4}{\cancel{100}}} = \frac{605}{4} = 151\tfrac{1}{4}. \qquad 151\tfrac{1}{4}\text{ yd. } \textit{Ans.}$$

40. The dimensions of a rectangle are 45 yd. and 28 yd. What is the length of its diagonal?

$$\sqrt{45^2 + 28^2} = \sqrt{2025 + 784} = \sqrt{2809} = 53.$$

```
      28 09(53
      25
103)309
    309           53 yd. Ans.
```

41. A field has the shape of a right triangle, and the two legs are 75 yd. and 60 yd., respectively. What decimal of an acre does the field contain?

$$\text{Area} = \tfrac{1}{2}(75 \times 60) \text{ sq. yd.} = \frac{\frac{1}{2} \times 75 \times 60}{4840} \text{ A.}$$

$$\frac{1}{2} \times \overset{15}{\cancel{75}} \times \overset{15}{\cancel{60}} \times \frac{1}{\underset{\underset{242}{\cancel{968}}}{\cancel{4840}}} = \frac{225}{484} = 0.46488. \qquad 0.46488 \text{ A. } \textit{Ans.}$$

42. Compare the areas of a square and an equilateral triangle, if the perimeter of each is 60 ft.

Side of square $= \frac{1}{4}$ of 60 ft. $= 15$ ft.

Area of square $= (15 \times 15)$ sq. ft.

Side of triangle $= 20$ ft.

Apothem $= 0.2887 \times 20$ ft.

Area of triangle $= \frac{1}{2}(60 \times 0.2887 \times 20)$ sq. ft.

$\therefore$ area square : area triangle

$$= 15 \times 15 : \tfrac{1}{2}(60 \times 0.2887 \times 20) = 3 : 2.3096. \textit{ Ans.}$$

$$\frac{\overset{3}{\cancel{15} \times \cancel{15}}}{\underset{2}{\cancel{30}} \times 0.2887 \times \underset{4}{\cancel{20}}} = \frac{3}{2.3096}.$$

43. Find the area of a field in the shape of a trapezoid, if the altitude is 240 yd., and the parallel sides are 510 yd. and 725 yd., respectively.

Sum of bases $= 510$ yd. $+ 725$ yd. $= 1235$ yd.

Area $= \frac{1}{2}(1235 \times 240)$ sq. yd. $= 148{,}200$ sq. yd. *Ans.*

$$2\,\underline{|240}$$
$$120$$

$$\begin{array}{r} 1235 \\ \underline{120} \\ 24700 \\ \underline{1235} \\ 148200 \end{array}$$

[redacted]

44. The legs of a right triangle are each equal to 12 ft. Find the hypotenuse.

Hypotenuse $= \sqrt{12^2 + 12^2}$ ft. $= \sqrt{144 + 144}$ ft. $= \sqrt{288}$ ft.
$= 16.97056$ ft. *Ans.*

2 88(16.97056
1
26)188
156
329)3200
2961
3387)23900
23709
19100

33940)191000
169700
213000
203640

45. A city lot in the shape of a right triangle has for its base 119 ft., and for its perpendicular 120 ft. Find the area and the hypotenuse of the lot.

Area $= \frac{1}{2}(119 \times 120)$ sq. ft. $= 7140$ sq. ft. *Ans.*

Hypotenuse $= \sqrt{119^2 + 120^2}$ ft. $= \sqrt{14161 + 14400}$ ft. $= \sqrt{28561}$ ft.
$= 169$ ft. *Ans.*

2 85 61(169
1
26)185
156
329)2961
2961

46. Find the base and the area of a right triangle, hypotenuse 130 yd., and perpendicular 112 yd.

Base $= \sqrt{130^2 - 112^2}$ yd. $= \sqrt{16900 - 12544}$ yd. $= \sqrt{4356}$ yd.
$= 66$ yd. *Ans.*

43 56(66
36
126)756
756

Area $= \frac{1}{2}(112 \times 66)$ sq. yd. $= 3696$ sq. yd. *Ans.*

$$2\underline{|112} \qquad 56$$

$$\begin{array}{r} 56 \\ \underline{66} \\ 336 \\ \underline{336} \\ 3696 \end{array}$$

47. Find the perpendicular and the area of a right triangle, hypotenuse 164 ft., and base 160 ft.

Perpendicular $= \sqrt{164^2 - 160^2}$ ft. $= \sqrt{26896 - 25600}$ ft. $= \sqrt{1296}$ ft. $= 36$ ft. *Ans.*

$$\begin{array}{r} 12\ 96(36 \\ 9 \\ 66)\overline{3\ 96} \\ \underline{3\ 96} \end{array}$$

Area $= \frac{1}{2}(160 \times 36)$ sq. ft. $= 2880$ sq. ft. *Ans.*

48. Find the hypotenuse and the area of a right triangle, base 100 yd., and perpendicular 105 yd.

Hypotenuse $= \sqrt{100^2 + 105^2}$ yd. $= \sqrt{10000 + 11025}$ yd. $= \sqrt{21025}$ yd. $= 145$ yd. *Ans.*

$$\begin{array}{r} 2\ 10\ 25(145 \\ 1 \\ 24)\overline{1\ 10} \\ 96 \\ 285)\overline{14\ 25} \\ \underline{14\ 25} \end{array}$$

Area $= \frac{1}{2}(100 \times 105)$ sq. yd. $= 5250$ sq. yd. *Ans.*

49. Find the hypotenuse and the area of a right triangle, base 96 ft., and perpendicular 110 ft.

Hypotenuse $= \sqrt{96^2 + 110^2}$ ft.
$= \sqrt{9216 + 12100}$ ft.
$= \sqrt{21316}$ ft. $= 146$ ft. *Ans.*

$$\begin{array}{r} 2\ 13\ 16(146 \\ 1 \\ 24)\overline{1\ 13} \\ 96 \\ 286)\overline{17\ 16} \\ \underline{17\ 16} \end{array}$$

Area $= \frac{1}{2}(96 \times 110)$ sq. ft.
$= 5280$ sq. ft. *Ans.*

$$2\underline{|110} \qquad 55$$

$$\begin{array}{r} 96 \\ \underline{55} \\ 480 \\ \underline{480} \\ 5280 \end{array}$$

$$45\overline{)270}$$
$$225$$
$$509\overline{)4581}$$
$$4581$$

Area = $\frac{1}{2}$ (259 × 660) sq. yd. = 85,470 sq. yd.

2 | 660	259	
330	330	484
	7770	
	777	
	85470	

51. A rectangular field is 345 yd. long an
is the length of its diagonal?

$$\text{Diagonal} = \sqrt{345^2 + 152^2}\text{ yd.} = \sqrt{119025 +}$$
$$= \sqrt{142129}\text{ y}$$

14 21 29(377

52. The legs of a right triangle are 44 ft. 4 in. and 13 ft. 9 in., respectively. Find the length of its hypotenuse.

44 ft. 4 in. = 532 in.; 13 ft. 9 in. = 165 in.

$$\text{Hypotenuse} = \sqrt{532^2 + 165^2}\text{ in.} = \sqrt{283024 + 27225}\text{ in.}$$
$$= \sqrt{310249}\text{ in.} = 557\text{ in.} = 46\text{ ft. }5\text{ in. } \textit{Ans.}$$

```
      31 02 49(557
      25
     105)602
         525
    1107)7749
         7749
```

53. The hypotenuse of a right triangle is 7 ft. 1 in., and one leg is 6 ft. 5 in. Find the other leg and the area.

7 ft. 1 in. = 85 in. ; 6 ft. 5 in. = 77 in.

$$\text{Leg} = \sqrt{85^2 - 77^2}\text{ in.} = \sqrt{7225 - 5929}\text{ in.}$$
$$= \sqrt{1296}\text{ in.} = 36\text{ in.} = 3\text{ ft. } \textit{Ans.}$$

```
     12 96(36
      9
    66)396
       396
```

$$\text{Area} = \tfrac{1}{2}\left(3 \times 6\tfrac{5}{12}\right)\text{ sq. ft.} = 9\tfrac{5}{8}\text{ sq. ft.} = 9\text{ sq. ft. }90\text{ sq. in. } \textit{Ans.}$$

54. The hypotenuse of a right triangle is 3 ft. 1 in., and one leg is 2 ft. 11 in. Find the other leg and the area.

3 ft. 1 in. = 37 in. ; 2 ft. 11 in. = 35 in.

$$\text{Leg} = \sqrt{37^2 - 35^2}\text{ in.} = \sqrt{1396 - 1225}\text{ in.} = \sqrt{144}\text{ in.} = 12\text{ in.} = 1\text{ ft. } \textit{Ans.}$$

$$\text{Area} = \tfrac{1}{2}\left(2\tfrac{11}{12} \times 1\right)\text{ sq. ft.} = 1\tfrac{11}{24}\text{ sq. ft.} = 1\text{ sq. ft. }66\text{ sq. in. } \textit{Ans.}$$

55. The area of a lot in the shape of a right triangle is 1560 sq. yd., and the base is 80 yd. Find the perpendicular and the hypotenuse.

$$\text{Perpendicular} = \frac{\cancel{2} \times \overset{39}{\cancel{1560}}}{\underset{\cancel{2}}{\cancel{80}}}\text{ yd.} = 39\text{ yd. } \textit{Ans.}$$

$$\text{Hypotenuse} = \sqrt{80^2 + 39^2}\text{ yd.} = \sqrt{6400 + 1521}\text{ yd.}$$
$$= \sqrt{7921}\text{ yd.} = 89\text{ yd. } \textit{Ans.}$$

$$\begin{array}{r@{}l} & 79\,21(89 \\ & \underline{64} \\ 169) & 1521 \\ & \underline{1521} \end{array}$$

56. The area of a right triangle is 60 sq. in., and one leg is 8 in. Find the hypotenuse and the other leg.

$$\text{Leg} = \frac{\cancel{2} \times \overset{15}{\cancel{60}}}{\underset{\cancel{2}}{\cancel{8}}}\text{ in.} = 15\text{ in. } \textit{Ans.}$$

$$\text{Hypotenuse} = \sqrt{15^2 + 8^2}\text{ in.} = \sqrt{225 + 64}\text{ in.} = \sqrt{289}\text{ in.} = 17\text{ in. } \textit{Ans.}$$

$$\begin{array}{r@{}l} & 2\,89(17 \\ & \underline{1} \\ 27) & 189 \\ & \underline{189} \end{array}$$

57. The length and diagonal of a rectangular field are 60 rd. and 65 rd., respectively. What is its area?

$$\text{Breadth} = \sqrt{65^2 - 60^2}\text{ rd.} = \sqrt{4225 - 3600}\text{ rd.} = \sqrt{625}\text{ rd.} = 25\text{ rd.}$$

$$\text{Area} = (60 \times 25)\text{ sq. rd.} = 1500\text{ sq. rd.} = 9\tfrac{3}{8}\text{ A. } \textit{Ans.}$$

58. What is the length of a side of a square that contains 390,625 sq. ft.?

$$\text{Side} = \sqrt{390625}\text{ ft.} = 625\text{ ft. } \textit{Ans.}$$

$$\begin{array}{r@{}l} & 39\,06\,25(625 \\ & \underline{36} \\ 122) & 306 \\ & \underline{244} \\ 1245) & 6225 \\ & \underline{6225} \end{array}$$

59. Express to six places of decimals the length of the diagonal of a square in terms of a side.

$$\text{Diagonal} = \sqrt{1^2+1^2} = \sqrt{1+1} = \sqrt{2} = 1.414213. \textit{ Ans.}$$

```
        2.00(1.414213
        1
       ----
   24)100               2828)6040
       96                    5656
      ----                   ----
   281)400                   3840
       281                   2828
      -----                  -----
  2824)11900                10120
       11296                 8484
       -----                 -----
         6040                1636
```

60. The hypotenuse of a right triangle is 95 ft., and the two legs are as 3 to 4. Find the legs and the area.

$$(\text{Base})^2 : (\text{Perpendicular})^2 = 3^2 : 4^2 = 9 : 16.$$

$$(\text{Base})^2 + (\text{Perpendicular})^2 = 95^2 = 9025.$$

$$(\text{Base})^2 = \frac{9}{\cancel{25}} \text{ of } \overset{361}{\cancel{9025}} = 3249. \quad \text{Base} = \sqrt{3249} \text{ ft.} = 57 \text{ ft. } \textit{Ans.}$$

$$(\text{Perpendicular})^2 = \frac{16}{\cancel{25}} \times \overset{361}{\cancel{9025}} = 5776.$$

$$\text{Perpendicular} = \sqrt{5776} \text{ ft.} = 76 \text{ ft. } \textit{Ans.}$$

$$\text{Area} = \tfrac{1}{2}(76 \times 57) \text{ sq. ft.} = 2166 \text{ sq. ft. } \textit{Ans.}$$

```
2|76                    57
  38                    38
                       ---
                       456
                      171
                      ----
                      2166
```

61. St. Mark's Square in Venice has the shape of a trapezoid. The parallel sides are 61 yd. and 90 yd., respectively, and the altitude is 192 yd. What is its area?

Sum of bases = 61 yd. + 90 yd. = 151 yd.

Area = $\tfrac{1}{2}$ (192 × 151) sq. yd. = 14,496 sq. yd. = 2.995 A. Ans.

[redacted]

2)192	151	2.405
96	96	[illegible]
	906	[illegible]
	1359	[illegible]
	14496	[illegible]
		[illegible]
		[illegible]
		2400
		2420
		20

62. The perimeter of a regular hexagon is 45 in. Find its area.

Side = ⅙ of 45 in. = 7.5 in. Apothem = 0.866 × 7.5 in. = 6.495 in.
Area = ½ (45 × 6.495) sq. in. = 146.1375 sq. in. *Ans.*

0.866	6.495
7.5	45
4330	32475
6062	25980
6.4950	2)292.275
	146.1375

63. A circular pond contains 12 acres. Express its diameter in feet.

Area = 12 × 43,560 sq. ft.

Radius = $\sqrt{0.31831 \times 12 \times 43560}$ ft.

= $\sqrt{166387.0032}$ ft. = 407.905 ft.

Diameter = 2 × 407.905 ft. = 815.81 ft. *Ans.*

43560	
12	
87120	16 63 87.00 32(407.905
43560	16
522720	807)6387
0.31831	5649
522720	8149)73800
636620	73341
222817	815805)4593200
63662	4079025
63662	514175
159155	
166387.00320	

64. What is the diameter of a circle whose radius is 1262 sq. ft.?

Radius $= \sqrt{0.31831 \times 1262}$ ft. $= \sqrt{401.70722}$ ft. $= 20.0426$ ft.

Diameter $= 2 \times 20.0426$ ft. $= 40.085$ ft. *Ans.*

```
 0.31831
    1262
   63662
 190986
 63662
31831
401.70722
```

```
    4 01.70 72 20(20.0426
    4
4004)17072
     16016
40082)105620
       80164
400846)2545600
       2405076
```

65. What is the diameter of a circle whose area is 2206 sq. ft.?

Radius $= \sqrt{0.31831 \times 2206}$ ft. $= \sqrt{702.19186}$ ft. $= 26.4989$ ft.

Diameter $= 2 \times 26.4989$ ft. $= 52.998$ ft. *Ans.*

```
 0.31831
    2206
  190986
 63662
63662
702.19186
```

```
   7 02.19 18 60(26.4989
   4
46)302
   276
524)2619
    2096
5289)52318
     47601
52988)471760
      423904
529969)4785600
       4769721
         15879
```

Exercise 142. Page 327.

1. Find the volume of a triangular prism, height 11 in., and sides of the ends 2 in., 3 in., and 4 in., respectively.

Half sum of sides of base $= \frac{1}{2}(2 + 3 + 4)$ in. $= 4.5$ in.

Area of base $= \sqrt{4.5 \times 2.5 \times 1.5 \times 0.5}$ sq. in $= \sqrt{8.4375}$ sq. in.
$= 2.9047$ sq. in.

Volume $= (11 \times 2.9047)$ cu. in. $= 31.9517$ cu. in. Ans.

[redacted]


```
   4.5              8.43 75(2.9047
   2.5              4
  ----         49)443
   225              441
   90          5804)27500
  ----               23216
  11.25        58087)428400
   1.5                406609
  ----                 21791
  5625
 1125
 ------
 16.875
   0.5
 ------
 8.4375
```

2. Find the capacity in bushels of a bin 6 ft. long, the end of which is a square 3 ft. 3 in. on a side.

$$3 \text{ ft. } 3 \text{ in.} = 3\tfrac{1}{4} \text{ ft.}$$

$$\text{Volume} = (6 \times 3\tfrac{1}{4} \times 3\tfrac{1}{4}) \text{ cu. ft.} = \left(\overset{3}{\cancel{6}} \times \frac{13}{\underset{2}{\cancel{4}}} \times \frac{13}{4}\right) \text{ cu. ft.}$$

$$= \frac{507}{8} \text{ cu. ft.} = 63.375 \text{ cu. ft.}$$

$$\begin{aligned} \tfrac{4}{5} \text{ of } 63.375 &= 50.7 \\ \tfrac{1}{2} \text{ of } 0.01 \text{ of } 50.7 &= \underline{\ 0.2535} \\ & \ 50.9535 \end{aligned}$$

50.9535 bu. *Ans.*

3. Find the lateral surface and the volume of a regular pyramid, base a regular hexagon 9 in. on a side, altitude 40 in., and slant height 40.75 in.

Perimeter of base = 6 × 9 in. = 54 in.

Lateral surface = ½ (54 × 40.75) sq. in. = 1100.25 sq. in. *Ans.*

```
2|54          40.75
  27             27
              -----
              28525
              8150
             -------
             1100.25
```

Apothem of base = 0.8660 × 9 in. = 7.794 in.

Area of base = ½ (54 × 7.794) sq. in. = 210.438 sq. in.

$$\begin{array}{r|l} 2 & 54 \\ \hline & 27 \end{array} \qquad \begin{array}{r} 7.794 \\ 27 \\ \hline 54558 \\ 15588 \\ \hline 210.438 \end{array}$$

Volume = $\frac{1}{3}(210.438 \times 40)$ cu. in. = 2805.84 cu. in. *Ans.*

$$\begin{array}{r|l} 3 & 210.438 \\ \hline & 70.146 \\ & 40 \\ \hline & 2805.840 \end{array}$$

4. Find the number of cubic yards in a prism, base a square 200 ft. on a side, height 40 ft.

Volume = $(200 \times 200 \times 40)$ cu. ft. = 1,600,000 cu. ft.

$= \frac{1600000}{27}$ cu. yd. = $59{,}259\frac{7}{27}$ cu. yd. *Ans.*

$$\begin{array}{r} 59259 \\ 27)\overline{1600000} \\ \underline{135} \\ 250 \\ \underline{243} \\ 70 \\ \underline{54} \\ 160 \\ \underline{135} \\ 250 \\ \underline{243} \\ 7 \end{array}$$

5. How many square yards of canvas are required for a conical tent 9 ft. 11 in. high, diameter of base 20 ft.?

9 ft. 11 in. = 119 in.; 20 ft. = 240 in. Radius = $\frac{1}{2}$ of 240 in. = 120 in.

Slant height = $\sqrt{119^2 + 120^2}$ in. = $\sqrt{14161 + 14400}$ in.

= $\sqrt{28561}$ in. = 169 in.

$$\begin{array}{rl} & 2\ 85\ 61(169 \\ & \underline{1} \\ 26) & 1\ 85 \\ & \underline{1\ 56} \\ 329) & 29\ 61 \\ & \underline{29\ 61} \end{array}$$

[redacted]

Lateral surface $= \frac{1}{2}(3.1416 \times 240 \times 169)$ sq. in. $= 63{,}711.648$ sq. in.

$= \dfrac{63711.648}{9 \times 144}$ sq. yd. $= 49.16$ sq. yd. *Ans.*

2 | 240	3.1416	144	49.160
120	120	9	1296)63711.648
	628320	1296	5184
	31416		11871
	376.992		11664
	169		2076
	3392928		1296
	2261952		7804
	376992		7776
	63711.648		288

6. Find the volume and the lateral surface of a frustum of a regular pyramid, bases squares 24 in. and 12 in. on a side, respectively, altitude $17\frac{1}{2}$ in., slant height $18\frac{1}{2}$ in.

Area of lower base $= (2 \times 2)$ sq. ft. $= 4$ sq. ft.

Area of upper base $= (1 \times 1)$ sq. ft. $= 1$ sq. ft.

$$\sqrt{4 \times 1} = \sqrt{4} = 2.$$

Volume $= \frac{1}{3} \times \frac{17\frac{1}{2}}{12}(4 + 1 + 2)$ cu. ft. $= \left(\frac{1}{3} \times \frac{17\frac{1}{2}}{12} \times 7\right)$ cu. ft.

$= 3\frac{29}{72}$ cu. ft. $= 3$ cu. ft. 696 cu. in. *Ans.*

$$\tfrac{1}{3} \times \tfrac{35}{2} \times \tfrac{1}{12} \times 7 = \tfrac{245}{72} = 3\tfrac{29}{72}.$$

Perimeter of lower base $= 4 \times 2$ ft. $= 8$ ft.

Perimeter of upper base $= 4 \times 1$ ft. $= 4$ ft.

Half sum of perimeters of bases $= \frac{1}{2}(8 + 4)$ ft. $= 6$ ft.

Lateral surface $= \frac{1}{2}\left(6 \times \frac{18\frac{1}{2}}{12}\right)$ sq. ft. $= 4\frac{5}{8}$ sq. ft. $= 4$ sq. ft. 90 sq. in. *Ans.*

$$\frac{1}{2} \times 6 \times \frac{18\frac{1}{2}}{12} = \frac{1}{2} \times \cancel{6} \times \frac{37}{2} \times \frac{1}{\underset{2}{\cancel{12}}} = \frac{37}{8} = 4\tfrac{5}{8}.$$

7. Find the volume and the lateral surface of a frustum of a right cone, radii of bases 50^{cm} and 30^{cm}, respectively, altitude 48^{cm}, and slant height 52^{cm}.

Area of lower base $= (3.1416 \times 50^2)^{qcm}$.
Area of upper base $= (3.1416 \times 30^2)^{qcm}$.
Square root of product of areas of bases
$= \sqrt{3.1416 \times 50^2 \times 3.1416 \times 30^2} = 3.1416 \times 50 \times 30 = 3.1416 \times 1500.$

Volume $= \frac{1}{3} \times 48 \times (3.1416 \times 2500 + 3.1416 \times 900 + 3.1416 \times 1500)^{ccm}$
$= [\frac{1}{3} \times 48 \times 3.1416 \times (2500 + 900 + 1500)]^{ccm}$
$= (\frac{1}{3} \times 48 \times 3.1416 \times 4900)^{ccm} = 24{,}630.144^{ccm}$. *Ans.*

3 | 48	4900	3.1416
16	16	78400
	29400	12566400
	49	251328
	78400	219912
		24630.144

Perimeter of lower base $= 3.1416 \times 100^{cm} = 314.16^{cm}$.
Perimeter of upper base $= 3.1416 \times 60^{cm} = 188.496^{cm}$.
Half sum of perimeters of bases $= \frac{1}{2}(314.16 + 188.496)^{cm} = 251.328^{cm}$.
Lateral surface $= \frac{1}{2}(251.328 \times 52)^{qcm} = 6534.528^{qcm}$. *Ans.*

2 | 52	251.328
26	26
	1507968
	502656
	6534.528

8. Find the volume and the surface of a sphere whose diameter is 17.2^{cm}.

Surface $= (3.1416 \times 17.2 \times 17.2)^{qcm} = 929.411^{qcm}$. *Ans.*
Volume $= (\frac{1}{6} \times 3.1416 \times 17.2 \times 17.2 \times 17.2)^{ccm} = 26{,}643.114^{ccm}$. *Ans.*

17.2	295.84
17.2	3.1416
344	177504
1204	29584
172	118336
295.84	29584
	88752
	929.410944
	$28\frac{2}{3}$
6 | 172	619607296
$28\frac{2}{3}$	7435287552
	1858821888
	26643.118728

[redacted]

9. A right cylinder is 3 ft. 2 in. in diameter and 4 ft. 6 in. high Find its volume and its lateral surface.

3 ft. 2 in. $= 3\frac{1}{6}$ ft.; 4 ft. 6 in. $= 4\frac{1}{2}$ ft.

Radius $= \frac{1}{2}$ of $3\frac{1}{6}$ ft. $= 1\frac{7}{12}$ ft.

Volume $= (4\frac{1}{2} \times 3.1416 \times 1\frac{7}{12} \times 1\frac{7}{12})$ cu. ft.

$$= \left(\frac{\overset{3}{\cancel{9}}}{\cancel{2}} \times \overset{\overset{0.1309}{\cancel{0.2618}}}{\cancel{3.1416}} \times \frac{19}{\cancel{12}} \times \frac{19}{\underset{4}{\cancel{12}}}\right) \text{ cu. ft.} = 35.4412 \text{ cu. ft. } \textit{Ans.}$$

Lateral surface $= (4\frac{1}{2} \times 3.1416 \times 3\frac{1}{6})$ sq. ft.

$$= \left(\frac{9}{\cancel{2}} \times \overset{\overset{0.2618}{\cancel{0.5236}}}{\cancel{3.1416}} \times \frac{19}{\cancel{6}}\right) \text{ sq. ft.} = 44.7678 \text{ sq. ft. } \textit{Ans.}$$

$$\begin{array}{r}
19 \\
\underline{19} \\
171 \\
\underline{19} \\
361 \\
\underline{3} \\
1083 \\
\underline{0.1309} \\
9747 \\
3249 \\
1083 \\
4\ \boxed{141.7647} \\
35.4412
\end{array}
\qquad
\begin{array}{r}
19 \\
\underline{9} \\
171
\end{array}
\qquad
\begin{array}{r}
0.2618 \\
\underline{171} \\
2618 \\
18326 \\
\underline{2618} \\
44.7678
\end{array}$$

10. Find the length of an edge of a cubical vessel that will hold a ton of water.

1 cu. ft. of water weighs $62\frac{1}{2}$ lb.

Therefore, 1 lb. of water occupies $\frac{1}{62\frac{1}{2}}$ cu. ft. and 2000 lb. occupy $\left(2000 \times \frac{1}{62\frac{1}{2}}\right)$ cu. ft. $= \left(\overset{16}{\cancel{2000}} \times \frac{2}{\cancel{125}}\right)$ cu. ft. $= 32$ cu. ft.

An edge of the vessel therefore $= \sqrt[3]{32}$ ft. $= 3.17480$ ft. *Ans.*

$$
\begin{array}{rr|l}
 & & 32.000(3.17480 \\
 & & 27 \\
3 \times 30^2 = & 2700 & 5000 \\
3 \times (30 \times 1) = & 90 & \\
1^2 = & 1 & \\
 & 2791 & 2791 \\
 & 91 & 2209000 \\
3 \times 310^2 = & 288300 & \\
3 \times (310 \times 7) = & 6510 & \\
7^2 = & 49 & \\
 & 294859 & 2064013 \\
 & 6559 & 144987000 \\
3 \times 3170^2 = & 30146700 & \\
3 \times (3170 \times 4) = & 38040 & \\
4^2 = & 16 & \\
 & 30184756 & 120739024 \\
 & 38056 & 242479760 \\
2 \times 3174^2 = & 30222828 & 241782624 \\
 & & 6971360
\end{array}
$$

11. A rectangular tank 6 ft. long and $4\frac{1}{2}$ ft. wide holds 108 cu. ft. of water. What is the height of the tank?

$$\text{Height} = \left(\frac{108}{6 \times 4\frac{1}{2}}\right) \text{ ft.} = \frac{\overset{\overset{2}{\cancel{12}}}{\cancel{108}} \times 2}{\cancel{6} \times \cancel{9}} \text{ ft.} = 4 \text{ ft. } Ans.$$

12. Find the total surface of a regular pyramid, base a square 5 ft. on a side, and slant height 20 ft.

Perimeter of base $= 4 \times 5$ ft. $= 20$ ft.
Lateral surface $= \frac{1}{2}(20 \times 20)$ sq. ft. $= 200$ sq. ft.
Area of base $= (5 \times 5)$ sq. ft. $= 25$ sq. ft.
Total surface $= 200$ sq. ft. $+ 25$ sq. ft. $= 225$ sq. ft. *Ans.*

13. The circumference of the base of a right cone is 12 ft., and the height of the cone is 12 ft. Find the volume.

$$\text{Radius of base} = \frac{12}{2 \times 3.1416}.$$

$$\text{Area of base} = \left(3.1416 \times \frac{12}{2 \times 3.1416} \times \frac{12}{2 \times 3.1416}\right) \text{ sq. ft.}$$

[redacted] [redacted]

$$\text{Volume} = \left(\frac{1}{3} \times 12 \times 3.1416 \times \frac{12}{2 \times 3.1416} \times \frac{12}{2 \times 3.1416}\right) \text{ cu. ft.}$$

$= 45.83664$ cu. ft. *Ans.*

$$\frac{1}{\cancel{3}} \times \overset{\overset{\cancel{2}}{\cancel{4}}}{\cancel{12}} \times \cancel{3.1416} \times \frac{12}{\cancel{2} \times \cancel{3.1416}} \times \frac{12}{\cancel{2} \times 3.1416} = \frac{144}{3.1416}$$

$$= 144 \times 0.31831 = 45.83664.$$

$$\begin{array}{r} 0.31831 \\ 144 \\ \hline 127324 \\ 127324 \\ 31831 \\ \hline 45.83664 \end{array}$$

14. Find the surface of a megaphone in the shape of a frustum of a right cone, diameters of the upper and lower bases 24 in. and 3 in., respectively, slant height 30 in.

Perimeter of upper base = 3.1416×3 in.
Perimeter of lower base = 3.1416×24 in.
Sum of perimeters of bases = 3.1416×27 in.
Lateral surface = $\frac{1}{2}(3.1416 \times 27 \times 30)$ sq. in.
= 1272.348 sq. in. = 8 sq. ft. 120.348 sq. in. *Ans.*

$$\begin{array}{r} 27 \\ 30 \\ \hline 2\,\big)\,810 \\ \hline 405 \end{array} \qquad\qquad \begin{array}{r} 3.1416 \\ 405 \\ \hline 157080 \\ 125664 \\ \hline 1272.3480 \end{array}$$

15. Find the difference between the volume of a frustum of a regular pyramid, bases squares 8 ft. and 6 ft., respectively, on a side, and altitude 9 ft., and the volume of a right prism, base a square 7 ft. on a side, altitude 9 ft.

Area of upper base = (6×6) sq. ft. = 36 sq. ft.
Area of lower base = (8×8) sq. ft. = 64 sq. ft.
Square root of product of areas of bases = $\sqrt{36 \times 64} = 6 \times 8 = 48.$
Volume of frustum of pyramid

$$= \frac{1}{3} \times 9 \times (36 + 64 + 48) \text{ cu. ft.} = \left(\frac{1}{\cancel{3}} \times \overset{3}{\cancel{9}} \times 148\right) \text{ cu. ft.} = 444 \text{ cu. ft.}$$

Volume of prism = $(9 \times 7 \times 7)$ cu. ft. = 441 cu. ft.
Therefore, the frustum of the pyramid is the larger by
444 cu. ft. − 441 cu. ft. = 3 cu. ft. *Ans.*

16. Find the surface and the volume of a sphere whose diameter is 28 in.

Surface $= (3.1416 \times 28 \times 28)$ sq. in. $= 2463.0144$ sq. in. *Ans.*
Volume $= (\frac{1}{6} \times 3.1416 \times 28 \times 28 \times 28)$ cu. in. $= 11{,}494.0672$ cu. in. *Ans.*

$$\begin{array}{r} 28 \\ \underline{28} \\ 224 \\ \underline{56} \\ 784 \end{array} \qquad \begin{array}{r} 3.1416 \\ \underline{784} \\ 125664 \\ 251328 \\ \underline{219912} \\ 2463.0144 \\ \underline{4\frac{2}{3}} \\ 16420096 \\ \underline{98520576} \\ 11494.0672 \end{array}$$

$$6\,\underline{|28}\qquad 4\tfrac{2}{3}$$

17. Find the ratio of the volume of a cube of wood 15 in. on an edge to the volume of the largest sphere that can be turned from it. Find the ratio of their surfaces.

$$\frac{\text{Volume of cube}}{\text{Volume of sphere}} = \frac{15^3}{0.5236 \times 15^3} = \frac{1}{0.5236}.\quad \textit{Ans.}$$

$$\frac{\text{Surface of cube}}{\text{Surface of sphere}} = \frac{6 \times 15^2}{3.1416 \times 15^2} = \frac{6}{3.1416} = \frac{1}{0.5236}.\quad \textit{Ans.}$$

18. Find the ratio of the volume of a cube of wood to the volume of the largest right cylinder that can be turned from it. Find the ratio of their surfaces.

$$\frac{\text{Volume of cube}}{\text{Volume of cylinder}} = \frac{1^3}{1 \times 3.1416 \times \frac{1}{2} \times \frac{1}{2}} = \frac{1}{0.7854}.\quad \textit{Ans.}$$

$$\frac{\text{Surface of cube}}{\text{Surface of cylinder}} = \frac{6 \times 1^2}{2 \times 3.1416 \times (\frac{1}{2})^2 + 3.1416 \times 1}$$

$$= \frac{6}{\frac{3}{2} \times 3.1416} = \frac{1}{\frac{1}{4} \times 3.1416} = \frac{1}{0.7854}.\quad \textit{Ans.}$$

19. Find the ratio of the volume of a right cylinder of wood to the volume of the largest right cone that can be turned from it. Find the ratio of their lateral surfaces.

$$\frac{\text{Volume of cylinder}}{\text{Volume of cone}} = \frac{1 \times 3.1416 \times 1^2}{\frac{1}{3} \times 1 \times 3.1416 \times 1^2} = \frac{1}{3}.\quad \textit{Ans.}$$

$$\frac{\text{Lateral surface of cylinder}}{\text{Lateral surface of cone}} = \frac{1 \times 3.1416 \times 1}{\frac{1}{2} \times 1 \times 3.1416 \times 1} = \frac{1}{2}.\quad \textit{Ans.}$$

[redacted]

20. Find the length of an edge of a cube that contains 100 cu. in.

		100.000(4.64158
		64
$3 \times 40^2 =$	4800	36000
$3 \times (40 \times 6) =$	720	
$6^2 =$	36	
	5556	33336
	756	2664000
$3 \times 460^2 =$	634800	
$3 \times (460 \times 4) =$	5520	
$4^2 =$	16	
	640336	2561344
	5536	102656000
$3 \times 4640^2 =$	64588800	
$3 \times (4640 \times 1) =$	13920	
$1^2 =$	1	
	64602721	64602721
	13921	380532790
$3 \times 4641^2 =$	64616643	323083215
		574495750
		516933144
		57562606

4.64159 in. *Ans.*

21. The Great Pyramid of Egypt was originally made in the form of a regular pyramid, altitude $480\frac{3}{4}$ ft., and base a square 764 ft. on a side. Find in acres the area of the ground covered by the pyramid. Find in cubic yards the volume, and in square yards the lateral surface of the pyramid.

$$\text{Area of base} = (764 \times 764) \text{ sq. ft.} = \frac{\overset{191}{\cancel{764}} \times \overset{382}{\cancel{764}}}{\underset{5445}{\cancel{\cancel{43560}}}} \text{ A.} = \frac{72962}{5445} \text{ A.}$$

$$= 13\tfrac{2177}{5445} \text{ A.} = 13.4 \text{ A. } \textit{Ans.}$$

$$\text{Volume} = (\tfrac{1}{3} \times 764 \times 764 \times 480\tfrac{3}{4}) \text{ cu. ft.} = \frac{\tfrac{1}{3} \times 764 \times 764 \times 480\tfrac{3}{4}}{27} \text{ cu. yd.}$$

$$= \frac{764 \times \overset{191}{\cancel{764}} \times \overset{641}{\cancel{1923}}}{\cancel{3} \times 27 \times \cancel{4}} \text{ cu. yd.} = \frac{93537284}{27} \text{ cu. yd.}$$

$$= 3{,}464{,}343\tfrac{23}{27} \text{ cu. yd. } \textit{Ans.}$$

Slant height $= \sqrt{480.75^2 + 382^2}$ ft. $= \sqrt{231120.5625 + 145924}$ ft.

$= \sqrt{377044.5625}$ ft. $= 614.04$ ft.

Lateral surface $= (\frac{1}{2} \times 4 \times 764 \times 614.04)$ sq. ft. $= \dfrac{2 \times 764 \times \overset{204.68}{\cancel{614.04}}}{\underset{3}{\cancel{9}}}$ sq. yd.

$= \dfrac{312751.04}{3}$ sq. yd. $= 104{,}250.35$ sq. yd. *Ans.*

22. The mast of a ship is 80 ft. high, and the diameters of its ends are 4 ft. 6 in. and 2 ft., respectively. Find its value at 75 cents a cubic foot.

Area of lower base $= (0.7854 \times 4.5^2)$ sq. ft. $= (0.7854 \times 20.25)$ sq. ft.
Area of upper base $= (0.7854 \times 2^2)$ sq. ft. $= (0.7854 \times 4)$ sq. ft.
Square root of product of areas of bases

$= \sqrt{0.7854 \times 20.25 \times 0.7854 \times 4}$ sq. ft. $= (0.7854 \times 4.5 \times 2)$ sq. ft.
$= (0.7854 \times 9)$ sq. ft.

Sum of areas of bases plus square root of their product
$= (0.7854 \times 20.25 + 0.7854 \times 4 + 0.7854 \times 9)$ sq. ft.
$= 0.7854 \times (20.25 + 4 + 9)$ sq. ft. $= (0.7854 \times 33.25)$ sq. ft.

Volume $= (\frac{1}{3} \times 80 \times 0.7854 \times 33.25)$ cu. ft. $= 696.388$ cu. ft.

$$\begin{array}{r} 33.25 \\ 80 \\ \hline 2660.00 \end{array} \qquad \begin{array}{r} 3\,\underline{|0.7854} \\ 0.2618 \\ 2660 \\ \hline 157080 \\ 15708 \\ 5236 \\ \hline 696.388 \end{array}$$

Value $= 696.388 \times \$0.75 = \$522.29$. *Ans.*

$$\begin{array}{r} 696.388 \\ 0.75 \\ \hline 3481940 \\ 4874716 \\ \hline 522.29100 \end{array}$$

23. A spherical shot 6 in. in diameter is melted and cast into a cylinder 3 in. in diameter. What is the height of this cylinder?

[redacted]

Volume of shot $= (0.5236 \times 6^3)$ cu. in.

Volume of cylinder $=$ (height $\times\ 0.7854 \times 3^2$) cu. in.

Height of cylinder $= \dfrac{0.5236 \times 6^3}{0.7854 \times 3^2}$ in. $= 16$ in. *Ans.*

$$\frac{\overset{2}{\cancel{0.5236}} \times \overset{2}{\cancel{6}} \times \overset{2}{\cancel{6}} \times \overset{2}{\cancel{6}}}{\underset{3}{\cancel{0.7854}} \times \cancel{3} \times \cancel{3}} = 16.$$

24. A cylindrical pail 14 in. high holds 2 cu. ft. of water. What is the diameter of its base?

Volume $= 2$ cu. ft. $= (2 \times 1728)$ cu. in.

Volume $= [14 \times 0.7854 \times (\text{diameter})^2]$ cu. in.

Diameter $= \sqrt{\dfrac{2 \times 1728}{14 \times 0.7854}}$ in. $= \sqrt{314.3075}$ in. $= 17.73$ in. *Ans.*

$$\frac{2 \times 1728}{14 \times 0.7854} = \frac{\cancel{2} \times \overset{288}{\cancel{1728}} \times 10000}{\underset{7}{\cancel{14}} \times \underset{1309}{\cancel{7854}}} = \frac{2880000}{9163} = 314.3075.$$

```
        3 14.30 75(17.728
        1
     27)214
        189
    347)2530
        2429
   3542)10175
         7084
  35448)309100
        283584
         25516
```

25. A regular pyramid 14 in. high has for its base an equilateral triangle 6 in. on a side. What is its volume?

Half sum of sides of base $= \frac{1}{2}(6 + 6 + 6)$ in. $= 9$ in.

Area of base $= \sqrt{9 \times 3 \times 3 \times 3}$ sq. in. $= \sqrt{243}$ sq. in. $= 15.588$ sq. in.

Volume $= (\frac{1}{3} \times 14 \times 15.588)$ cu. in. $= 72.744$ cu. in. *Ans.*

```
       2 43(15.588
       1
       ---
     25)143
        125
        ----
    305)1800
        1525
        -----
   3108)27500
        24864
        ------
  31168)263600
        249344
        ------
         14256
```

```
3|15.588
    5.196
       14
    -----
    20784
     5196
   ------
   72.744
```

26. A right prism 8 in. high has for its base a trapezoid whose altitude is 4 in., and whose parallel sides are 5 in. and 3 in., respectively. What is the volume of the prism in cubic inches?

Sum of bases of trapezoid = 5 in. + 3 in. = 8 in.
Area of base = $\frac{1}{2}(8 \times 4)$ sq. in. = 16 sq. in.
Volume = (8×16) cu. in. = 128 cu. in. *Ans.*

27. A rectangular room is 18 ft. long, 16 ft. wide, and 12 ft. high. What is the distance from the upper right-hand corner to the opposite lower left-hand corner?

Diagonal of floor = $\sqrt{18^2 + 16^2}$ ft.

Diagonal of room = $\sqrt{(\sqrt{18^2 + 16^2})^2 + 12^2}$ ft. = $\sqrt{18^2 + 16^2 + 12^2}$ ft.
= $\sqrt{324 + 256 + 144}$ ft. = $\sqrt{724}$ ft. = 26.907 ft. *Ans.*

```
        7 24(26.907
        4
        ---
     46)324
        276
        ----
    529)4800
        4761
        ------
  53807)390000
        376649
        ------
         13351
```

[redacted]

28. A conical spire 40 ft. high has a base 15 ft. in diameter. Find the cost at 5 cents a square inch of gilding the spire.

Slant height $= \sqrt{40^2 + 7.5^2}$ ft. $= \sqrt{1600 + 56.25}$ ft. $= \sqrt{1656.25}$ ft.
$= 40.7$ ft.

$$\begin{array}{l} 16\ 56.25(40.7 \\ 16 \\ \hline 807)5625 \end{array}$$

Circumference of base $= 3.1416 \times 15$ ft.

Lateral surface $= (\frac{1}{2} \times 40.7 \times 3.1416 \times 15)$ sq. ft. $= 958.9734$ sq. ft.

$$\begin{array}{r} 40.7 \\ 15 \\ \hline 2035 \\ 407 \\ \hline 610.5 \end{array} \qquad \begin{array}{r} 2\,\underline{|3.1416} \\ 1.5708 \\ 610.5 \\ \hline 78540 \\ 15708 \\ 94248 \\ \hline 958.97340 \end{array}$$

\$0.05 per sq. in. $= 144 \times$ \$0.05 per sq. ft. $=$ \$7.20 per sq. ft.

$$\begin{array}{r} 958.9734 \\ 7.20 \\ \hline 191794680 \\ 67128138 \\ \hline 6904.608480 \end{array}$$

\$6904.61. *Ans.*

Exercise 143. Page 330.

1. If the diameter of the moon is reckoned at 2000 mi., and that of the earth at 8000 mi., find the ratio of their surfaces and the ratio of their volumes.

$2000^2 : 8000^2 = 1^2 : 4^2 = 1 : 16$. *Ans.*
$2000^3 : 8000^3 = 1^3 : 4^3 = 1 : 64$. *Ans.*

2. If the diameters of two circles are 20 in. and 40 in., find the ratio of their circumferences and of their areas.

$20 : 40 = 1 : 2$. *Ans.* $\qquad 20^2 : 40^2 = 1^2 : 2^2 = 1 : 4$. *Ans.*

3. If the areas of two circles are 8000 sq. in. and 36,000 sq. in., respectively, find the ratio of their diameters.

$$\sqrt{8000} : \sqrt{36000} = \sqrt{4} : \sqrt{18} = 2 : 4.242 = 1 : 2.121. \textit{ Ans.}$$

$$\begin{array}{r l}
 & 18.00(4.242 \\
 & 16 \\
82) & 200 \\
 & 164 \\
844) & 3600 \\
 & 3376 \\
8482) & 22400 \\
 & 16964 \\
 & 5436
\end{array}$$

4. If the volumes of two spheres are 100 cu. in. and 1000 cu. in., respectively, find the ratio of their diameters.

$$\sqrt[3]{100} : \sqrt[3]{1000} = \sqrt[3]{1} : \sqrt[3]{10} = 1 : 2.154. \textit{ Ans.}$$

$$\begin{array}{r r | l}
 & & 10.000(2.154 \\
 & & 8 \\
3 \times 20^2 = & 1200 & 2000 \\
3 \times (20 \times 1) = & 60 & \\
1^2 = & 1 & \\
 & 1261 & 1261 \\
 & 61 & 739000 \\
3 \times 210^2 = & 132300 & \\
3 \times (210 \times 5) = & 3150 & \\
5^2 = & 25 & \\
 & 135475 & 677375 \\
 & 3175 & 616250 \\
3 \times 215^2 = & 138675 & 554700 \\
 & & 61550
\end{array}$$

5. If an ox 7 ft. in girth weighs 1500 lb., what will be the girth of a similar ox that weighs 2500 lb.?

$$\sqrt[3]{1500} : \sqrt[3]{2500} = 7 \text{ ft.} : ?.$$

$$\sqrt[3]{1} : \sqrt[3]{\tfrac{2500}{1500}} = 7 \text{ ft.} : ?.$$

$$\sqrt[3]{1} : \sqrt[3]{1\tfrac{2}{3}} = 7 \text{ ft.} : ?.$$

$$1 : 1.185 = 7 \text{ ft.} : ?.$$

$$1.185 \times 7 \text{ ft.} = 8.295 \text{ ft.}$$

8.3 ft. Ans.

[redacted]

	1.666 666(1.185
	1
$3 \times 10^2 = 300$	666
$3 \times (10 \times 1) = 30$	
$1^2 = 1$	
331	331
31	335666
$3 \times 110^2 = 36300$	
$3 \times (110 \times 8) = 2640$	
$8^2 = 64$	
39004	312032
2704	236346
$3 \times 118^2 = 41772$	208860
	27486

6. The surface of a pyramid is 560 sq. in. What is the surface of a similar pyramid whose volume is 27 times as great ?

$\sqrt[3]{1} : \sqrt[3]{27} = 1 : 3.$

$1^2 : 3^2 = 560$ sq. in. : ?.

$1 : 9 = 560$ sq. in. : ?. 9×560 sq. in. $= 5040$ sq. in. *Ans.*

7. The volume of a pyramid is 1331 cu. in. What is the volume of a similar pyramid whose surface is 4 times as great ?

$\sqrt{1} : \sqrt{4} = 1 : 2.$

$1^3 : 2^3 = 1331$ cu. in. : ?.

$1 : 8 = 1331$ cu. in. : ?. 8×1331 cu. in. $= 10{,}648$ cu. in. *Ans.*

8. If a well-proportioned man 5 ft. 10 in. high weighs 160 lb., what should a man 6 ft. high weigh, to the nearest tenth of a pound ? What should be the height, to the nearest tenth of an inch, of a man who weighs 210 lb. ?

5 ft. 10 in. = 70 in.; 6 ft. = 72 in.

$70^3 : 72^3 = 160$ lb. : ?.

$343000 : 373248 = 160$ lb. : ?.

$$\frac{373248 \times \overset{4}{\cancel{160}}\text{ lb.}}{\underset{8575}{\cancel{343000}}} = \frac{1492992}{8575}\text{ lb.} = 174.1\text{ lb. } \textit{Ans.}$$

$$\sqrt[3]{160} : \sqrt[3]{210} = 70 \text{ in.} : ?.$$

$$\sqrt[3]{1} : \sqrt[3]{1.3125} = 70 \text{ in.} : ?.$$

$$1 : 1.095 = 70 \text{ in.} : ?.$$

$$\frac{1.095 \times 70 \text{ in.}}{1} = 76.65 \text{ in.}$$

76.6 in. = 6 ft. 4.6 in. *Ans.*

$$\begin{array}{rr|l}
 & & 1.312\,500(1.095 \\
 & & 1 \\
3 \times 100^2 = & 30000 & 312500 \\
3 \times (100 \times 9) = & 2700 & \\
9^2 = & 81 & \\
 & 32781 & 295029 \\
 & 2781 & 174710 \\
3 \times 109^2 = & 35643 &
\end{array}$$

9. A three-gallon jug and a one-gallon jug are similar. Find to three decimals the ratio of their diameters.

$$\sqrt[3]{3} : \sqrt[3]{1} = \sqrt[3]{1} : \sqrt[3]{\tfrac{1}{3}} = 1 : 0.693. \textit{ Ans.}$$

$$\begin{array}{rr|l}
 & & 0.333\,333\,333(0.693 \\
 & & 216 \\
3 \times 60^2 = & 10800 & 117333 \\
3 \times (60 \times 9) = & 1620 & \\
9^2 = & 81 & \\
 & 12501 & 112509 \\
 & 1701 & 4824333 \\
3 \times 690^2 = & 1428300 & \\
3 \times (690 \times 3) = & 6210 & \\
3^2 = & 9 & \\
 & 1434519 & 4303557 \\
 & & 520776
\end{array}$$

10. Two hills have exactly the same shape; one is 900 ft. high, the other 1200 ft. Find the ratio of their surfaces, and also the ratio of their volumes.

$$900^2 : 1200^2 = 3^2 : 4^2 = 9 : 16. \textit{ Ans.}$$

$$900^3 : 1200^3 = 3^3 : 4^3 = 27 : 64. \textit{ Ans.}$$

11. A ball 3 in. in diameter weighs 4 lb.; another ball of the same metal weighs 9 lb. Find the diameter of the second ball to the nearest thousandth of an inch.

$$\sqrt[3]{4} : \sqrt[3]{9} = 3 \text{ in.} : ?.$$

$$\sqrt[3]{1} : \sqrt[3]{2.25} = 3 \text{ in.} : ?.$$

$$1 : 1.3103 = 3 \text{ in.} : ?.$$

$$1.3103 \times 3 \text{ in.} = 3.9309 \text{ in.} \qquad 3.931 \text{ in. } \textit{Ans.}$$

$$\begin{array}{rr|l}
 & & 2.250(1.3103 \\
 & & 1 \\
3 \times 10^2 = & 300 & 1250 \\
3 \times (10 \times 3) = & 90 & \\
3^2 = & 9 & \\
 & 399 & 1197 \\
 & 99 & 53000 \\
3 \times 130^2 = & 50700 & \\
3 \times (130 \times 1) = & 390 & \\
1^2 = & 1 & \\
 & 51091 & 51091 \\
 & 391 & 1909000000 \\
3 \times 13100^2 = & 514830000 & \\
3 \times (13100 \times 3) = & 117900 & \\
3^2 = & 9 & \\
 & 514947909 & 1544843727 \\
 & & 364156273
\end{array}$$

12. If Apollo's altar were a perfect cube 10 ft. on an edge, what would be the edge of a new cubical altar containing twice as much stone?

$$\sqrt[3]{1} : \sqrt[3]{2} = 10 \text{ ft.} : ?.$$

$$1 : 1.2599 = 10 \text{ ft.} : ?.$$

$$1.2599 \times 10 \text{ ft.} = 12.599 \text{ ft.} = 12 \text{ ft. } 7.188 \text{ in. } \textit{Ans.}$$

[redacted]

$$
\begin{array}{rr|l}
 & & 2.000(1.2599 \\
 & & 1 \\
3 \times 10^2 = & 300 & 1000 \\
3 \times (10 \times 2) = & 60 & \\
2^2 = & 4 & \\
 & 364 & 728 \\
 & 64 & 272000 \\
3 \times 120^2 = & 43200 & \\
3 \times (120 \times 5) = & 1800 & \\
5^2 = & 25 & \\
 & 45025 & 225125 \\
 & 1825 & 46875000 \\
3 \times 1250^2 = & 4687500 & \\
3 \times (1250 \times 9) = & 33750 & \\
9^2 = & 81 & \\
 & 4721331 & 42491979 \\
 & 33831 & 43830210 \\
3 \times 1259^2 = & 4755243 & 42797187 \\
 & & 1033023
\end{array}
$$

13. A man standing 40 ft. from a building 24 ft. wide observed that, when he closed one eye, the width of the building hid from view 90 rd. of fence which was parallel to the width of the building. Find the distance from the eye of the observer to the fence.

$24 : 40 = 90 \text{ rd.} : ?$ $\qquad \dfrac{\overset{5}{\cancel{40}} \times \overset{30}{\cancel{90}} \text{ rd.}}{\underset{\cancel{3}}{\cancel{24}}} = 150 \text{ rd.}$ *Ans.*

14. A bushel measure and a peck measure are of the same shape. Find the ratio of their heights.

$1 \text{ bu.} = 4 \text{ pk.} \qquad \sqrt[3]{4} : \sqrt[3]{1} = \sqrt[3]{1} : \sqrt[3]{0.25} = 1 : 0.63.$ *Ans.*

$$
\begin{array}{rr|l}
 & & 0.250\,000(0.63 \\
 & & 216 \\
3 \times 60^2 = & 10800 & 34000 \\
3 \times (60 \times 3) = & 540 & \\
3^2 = & 9 & \\
 & 11349 & 34047
\end{array}
$$

[redacted]

15. If the height and the diameter of a cylinder are both doubled, in what ratio is the volume altered?

Volume = height × 0.7854 × diameter × diameter.

Of the larger cylinder the volume = twice the height × 0.7854 × twice the diameter × twice the diameter.

Therefore, the larger cylinder is 8 times the smaller.

Exercise 144. Page 334.

1. Change $\frac{3}{11}$, $\frac{13}{75}$, $\frac{29}{127}$, $\frac{135}{64}$ to continued fractions.

3)11(3
9
2)3(1
2
1)2(2
2

$$\therefore \frac{3}{11} = \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{2}}} \quad \textit{Ans.}$$

13)75(5
65
10)13(1
10
3)10(3
9
1)3(3
3

$$\therefore \frac{13}{75} = \cfrac{1}{5 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{3}}}} \quad \textit{Ans.}$$

29)127(4
116
11)29(2
22
7)11(1
7
4)7(1
4
3)4(1
3
1)3(3
3

$$\therefore \frac{29}{127} = \cfrac{1}{4 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3}}}}}} \quad \textit{Ans.}$$

$$\frac{135}{64} = 2\frac{7}{64}.$$

7)64(9
63
1)7(7
7

$$\therefore \frac{135}{64} = 2 + \cfrac{1}{9 + \cfrac{1}{7}} \quad \textit{Ans.}$$

2. Find the approximate values of $\frac{20}{27}$; $\frac{53}{47}$; $\frac{734}{851}$.

20)27(1
20
7)20(2
14
6)7(1
6
1)6(6
6

$$\therefore \frac{20}{27} = \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{6}}}}$$

$$\frac{1}{1} = 1. \qquad \cfrac{1}{1 + \cfrac{1}{2}} = \frac{2}{3}. \qquad \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1}}} = \frac{3}{4}.$$

$1, \frac{2}{3}, \frac{3}{4}$. *Ans.*

$$\frac{53}{47} = 1\tfrac{6}{47}.$$

6)47(7
42
5)6(1
5
1)5(5
5

$$\therefore 1\tfrac{6}{47} = 1 + \cfrac{1}{7 + \cfrac{1}{1 + \cfrac{1}{5}}}$$

$$1 = 1. \qquad 1 + \frac{1}{7} = \frac{8}{7}. \qquad 1 + \cfrac{1}{7 + \cfrac{1}{1}} = \frac{9}{8}.$$

$1, \frac{8}{7}, \frac{9}{8}$. *Ans.*

734)851(1
734
117)734(6
702
32)117(3
96
21)32(1
21
11)21(1
11
10)11(1
10
1)10(10
10

$$\therefore \frac{734}{851} = \cfrac{1}{1 + \cfrac{1}{6 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{10}}}}}}}$$

$$\frac{1}{1} = 1.$$

$$\cfrac{1}{1 + \cfrac{1}{6}} = \frac{6}{7}.$$

$$\cfrac{1}{1 + \cfrac{1}{6 + \cfrac{1}{3}}} = \frac{19}{22}.$$

$$\cfrac{1}{1 + \cfrac{1}{6 + \cfrac{1}{3 + \cfrac{1}{1}}}} = \frac{25}{29}.$$

$$\cfrac{1}{1 + \cfrac{1}{6 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{1}}}}} = \frac{44}{51}.$$

$$\cfrac{1}{1 + \cfrac{1}{6 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1}}}}}} = \frac{69}{80}.$$

$1, \frac{6}{7}, \frac{19}{22}, \frac{25}{29}, \frac{44}{51}, \frac{69}{80}$. *Ans.*

[redacted]

3. Find a series of fractions approximating to 0.236 ; 0.2361 ; 1.609.

$$0.236 = \tfrac{236}{1000} = \tfrac{59}{250}.$$

```
59)250(4
   236
    14)59(4
       56
        3)14(4
          12
           2)3(1
             2
             1)2(2
               2
```

$$\therefore \tfrac{59}{250} = \cfrac{1}{4 + \cfrac{1}{4 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{2}}}}}$$

$$\frac{1}{4} = \frac{1}{4}.$$

$$\cfrac{1}{4 + \cfrac{1}{4}} = \frac{4}{17}.$$

$$\cfrac{1}{4 + \cfrac{1}{4 + \cfrac{1}{4}}} = \frac{17}{72}.$$

$$\cfrac{1}{4 + \cfrac{1}{4 + \cfrac{1}{4 + \cfrac{1}{1}}}} = \frac{21}{89}$$

$\frac{1}{4}$, $\frac{4}{17}$, $\frac{17}{72}$, $\frac{21}{89}$. *Ans.*

$$0.2361 = \tfrac{2361}{10000}.$$

```
2361)10000(4
      9444
       556)2361(4
           2224
            137)556(4
                548
                  8)137(17
                    136
                      1)8(8
                        8
```

$$\therefore \tfrac{2361}{10000} = \cfrac{1}{4 + \cfrac{1}{4 + \cfrac{1}{4 + \cfrac{1}{17 + \cfrac{1}{8}}}}}$$

$$\frac{1}{4} = \frac{1}{4}.$$

$$\cfrac{1}{4 + \cfrac{1}{4}} = \frac{4}{17}.$$

$$\cfrac{1}{4 + \cfrac{1}{4 + \cfrac{1}{4}}} = \frac{17}{72}.$$

$$\cfrac{1}{4 + \cfrac{1}{4 + \cfrac{1}{4 + \cfrac{1}{17}}}} = \frac{293}{1241}.$$

$\frac{1}{4}$, $\frac{4}{17}$, $\frac{17}{72}$, $\frac{293}{1241}$. *Ans.*

$$1.609 = 1 + \tfrac{609}{1000}.$$

609)1000(1
609
391)609(1
391
218)391(1
218
173)218(1
173
45)173(3
135
38)45(1
38
7)38(5
35
3)7(2
6
1)3(3
3

$$\therefore\ 1 + \tfrac{609}{1000} = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{2 + \cfrac{1}{3}}}}}}}}}$$

$$1 + \frac{1}{1} = 2.$$

$$1 + \cfrac{1}{1 + \cfrac{1}{1}} = \frac{3}{2}.$$

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1}}} = \frac{5}{3}.$$

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1}}}} = \frac{8}{5}.$$

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3}}}}} = \frac{29}{18}.$$

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1}}}}}} = \frac{37}{23}.$$

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{5}}}}}}} = \frac{214}{133}.$$

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{2}}}}}}}} = \frac{465}{289}.$$

$2, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{29}{18}, \frac{37}{23}, \frac{214}{133}, \frac{465}{289}$. Ans.

4. Find a series of fractions approximating to 0.382 ; 1.732 ; 0.6253.

$$0.382 = \tfrac{382}{1000} = \tfrac{191}{500}.$$

```
191)500(2
    382
    118)191(1
        118
         73)118(1
             73
             45)73(1
                45
                28)45(1
                   28
                   17)28(1
                      17
                      11)17(1
                         11
                          6)11(1
                             6
                             5)6(1
                               5
                               1)5(5
                                 5
```

$$\therefore \tfrac{191}{500} = \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \frac{1}{5}}}}}}}}}}$$

$$\frac{1}{2} = \frac{1}{2}.$$

$$\cfrac{1}{2 + \frac{1}{1}} = \frac{1}{3}.$$

$$\cfrac{1}{2 + \cfrac{1}{1 + \frac{1}{1}}} = \frac{2}{5}.$$

$$\cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \frac{1}{1}}}} = \frac{3}{8}.$$

$$\cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \frac{1}{1}}}}} = \frac{5}{13}.$$

$$\cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \frac{1}{1}}}}}} = \frac{8}{21}.$$

$$\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{1}}}}}}} = \frac{13}{34}.$$

$$\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{1}}}}}}}} = \frac{21}{55}.$$

$$\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{1}}}}}}}}} = \frac{34}{89}.$$

$\frac{1}{2}, \frac{1}{3}, \frac{2}{5}, \frac{3}{8}, \frac{5}{13}, \frac{8}{21}, \frac{13}{34}, \frac{21}{55}, \frac{34}{89}$. *Ans.*

$$1.732 = 1\tfrac{183}{250}.$$

```
183)250(1
    183
     67)183(2
        134
         49)67(1
            49
            18)49(2
               36
               13)18(1
                  13
                   5)13(2
                     10
                      3)5(1
                        3
                        2)3(1
                          2
                          1)2(2
                            2
```

$$\therefore\ 1\tfrac{183}{250} = 1 + \cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{2}}}}}}}}}$$

[redacted]

$$1+\frac{1}{1}=2.$$

$$1+\cfrac{1}{1+\cfrac{1}{2}}=\frac{5}{3}.$$

$$1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1}}}=\frac{7}{4}.$$

$$1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2}}}}=\frac{19}{11}.$$

$$1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1}}}}}=\frac{26}{15}.$$

$$1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2}}}}}}=\frac{71}{41}.$$

$$1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1}}}}}}}=\frac{97}{56}.$$

$$1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1}}}}}}}}=\frac{168}{97}.$$

$2,\ \frac{5}{3},\ \frac{7}{4},\ \frac{19}{11},\ \frac{26}{15},\ \frac{71}{41},\ \frac{97}{56},\ \frac{168}{97}$. *Ans.*

$$0.6253=\frac{6253}{10000}.$$

```
6253)10000(1
     6253
     3747)6253(1
          3747
          2506)3747(1
               2506
               1241)2506(2
                    2482
                      24)1241(51
                         1224
                           17)24(1
                              17
                               7)17(2
                                 14
                                  3)7(2
                                    6
                                    1)3(3
                                      3
```

$$\therefore\ \frac{6253}{10000}=\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{51+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{3}}}}}}}}}$$

$$\frac{1}{1} = 1.$$

$$\cfrac{1}{1+\cfrac{1}{1}} = \frac{1}{2}.$$

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1}}} = \frac{2}{3}.$$

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2}}}} = \frac{5}{8}.$$

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{51}}}}} = \frac{257}{411}.$$

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{51+\cfrac{1}{1}}}}}} = \frac{262}{419}.$$

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{51+\cfrac{1}{1+\cfrac{1}{2}}}}}}} = \frac{781}{1249}.$$

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{51+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{2}}}}}}}} = \frac{1824}{2917}.$$

$1, \frac{1}{2}, \frac{2}{3}, \frac{5}{8}, \frac{257}{411}, \frac{262}{419}, \frac{781}{1249}, \frac{1824}{2917}$. *Ans.*

5. Find the approximate values of $\frac{171}{457}$; $\frac{313}{757}$; $\frac{711}{873}$; $\frac{937}{113}$.

```
171)457(2
    342
    115)171(1
        115
         56)115(2
            112
              3)56(18
                54
                 2)3(1
                   2
                   1)2(2
                     2
```

$$\therefore \frac{171}{457} = \cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{18+\cfrac{1}{1+\cfrac{1}{2}}}}}}$$

$$\frac{1}{2} = \frac{1}{2}. \qquad \cfrac{1}{2+\cfrac{1}{1}} = \frac{1}{3}.$$

[redacted]

$$\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2}}}=\frac{3}{8}.$$

$$\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{18}}}}=\frac{55}{147}.$$

$$\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{18+\cfrac{1}{1}}}}}=\frac{58}{155}.$$

$\frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{8}$, $\frac{55}{147}$, $\frac{58}{155}$. *Ans.*

```
613)757(1
    613
    144)613(4
        576
         37)144(3
            111
             33)37(1
                33
                 4)33(8
                   32
                    1)4(4
                      4
```

$$\therefore \frac{613}{757}=\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{8+\cfrac{1}{4}}}}}}$$

$$\frac{1}{1}=1. \qquad \cfrac{1}{1+\cfrac{1}{4}}=\frac{4}{5}.$$

$$\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{3}}}=\frac{13}{16}.$$

$$\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{1}}}}=\frac{17}{21}.$$

$$\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{8}}}}}=\frac{149}{184}.$$

1, $\frac{4}{5}$, $\frac{13}{16}$, $\frac{17}{21}$, $\frac{149}{184}$. *Ans.*

$$\frac{711}{813}=\frac{237}{271}.$$

```
237)271(1
    237
     34)237(6
        204
         33)34(1
            33
             1)33(33
               33
```

$$\therefore \frac{237}{271}=\cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{1+\cfrac{1}{33}}}}$$

$$\frac{1}{1}=1. \qquad \cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{1}}}=\frac{7}{8}.$$

$$\cfrac{1}{1+\cfrac{1}{6}}=\frac{6}{7}.$$

1, $\frac{6}{7}$, $\frac{7}{8}$. *Ans.*

$\frac{937}{113} = 8\frac{33}{113}$.

33)113(3
99
14)33(2
28
5)14(2
10
4)5(1
4
1)4(4
4

$$\therefore 8\tfrac{33}{113} = 8 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{4}}}}}.$$

$8 = 8.$

$8 + \frac{1}{3} = 8\frac{1}{3} = \frac{25}{3}.$

$$8 + \cfrac{1}{3 + \cfrac{1}{2}} = 8\tfrac{2}{7} = \frac{58}{7}.$$

$$8 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{2}}} = 8\tfrac{5}{17} = \frac{141}{17}.$$

$$8 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{1}}}} = 8\tfrac{7}{24} = \frac{199}{24}.$$

$8, \frac{25}{3}, \frac{58}{7}, \frac{141}{17}, \frac{199}{24}.$ *Ans.*

6. Find the proper fraction that, when changed to a continued fraction, will have 2, 3, 5, 6, 7 as quotients.

$$\cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{6 + \cfrac{1}{7}}}}} = \frac{709}{1640}. \quad \textit{Ans.}$$

$$\frac{1}{6\frac{1}{7}} = \frac{7}{43}; \quad \frac{1}{5\frac{7}{43}} = \frac{43}{222};$$

$$\frac{1}{3\frac{43}{222}} = \frac{222}{709}; \quad \frac{1}{2\frac{222}{709}} = \frac{709}{1640}.$$

7. Find a series of fractions approximating to the ratio of the pound troy (5760 gr.) to the pound avoirdupois (7000 gr.).

$\frac{5760}{7000} = \frac{144}{175}$.

144)175(1
144
31)144(4
124
20)31(1
20
11)20(1
11
9)11(1
9
2)9(4
8
1)2(2
2

$$\therefore \frac{144}{175} = \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{2}}}}}}}$$

[redacted]

$$\frac{1}{1} = 1. \qquad \cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1}}} = \frac{5}{6}. \qquad \cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1}}}} = \frac{9}{11}.$$

$$\cfrac{1}{1+\cfrac{1}{4}} = \frac{4}{5}.$$

$$\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1}}}}} = \frac{14}{17}. \qquad \cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{4}}}}}} = \frac{65}{79}.$$

$1, \frac{4}{5}, \frac{5}{6}, \frac{9}{11}, \frac{14}{17}, \frac{65}{79}$. *Ans.*

8. Find a series of fractions approximating to the ratio of the side of a square to its diagonal; that ratio being 1 : 1.414214 nearly.

$$\frac{1}{1.414214} = \frac{1000000}{1414214} = \frac{7071}{10000}.$$

```
7071)10000(1
     7071
     2929)7071(2
          5858
          1213)2929(2
               2426
                503)1213(2
                    1006
                     207)503(2
                         414
                          89)207(2
                             178
                              29)89(3
                                 87
                                  2)29(14
                                    28
                                     1)2(2
                                       2
```

$$\therefore \frac{7071}{10000} = \cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{14+\cfrac{1}{2}}}}}}}}$$

$$\frac{1}{1} = 1.$$

$$\cfrac{1}{1+\cfrac{1}{2}} = \frac{2}{3}.$$

$$\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{2}}} = \frac{5}{7}.$$

$$\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2}}}} = \frac{12}{17}$$

$$\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{3}}}}}}} = \frac{239}{338}.$$

$$\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2}}}}} = \frac{29}{41}.$$

$$\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2}}}}}} = \frac{70}{99}.$$

$$\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{14}}}}}}}} = \frac{3416}{4831}.$$

1, $\frac{2}{3}$, $\frac{5}{7}$, $\frac{12}{17}$, $\frac{29}{41}$, $\frac{70}{99}$, $\frac{239}{338}$, $\frac{3416}{4831}$. *Ans.*

9. Find a series of fractions approximating to the ratio of the ar to the square chain, from the equality 1 ar = 0.2471 sq. ch.

$$0.2471 = \tfrac{2471}{10000}.$$

```
2471)10000(4
      9884
       116)2471(21
           2436
             35)116(3
                105
                 11)35(3
                    33
                     2)11(5
                       10
                        1)2(2
                          2
```

$$\therefore \tfrac{2471}{10000} = \cfrac{1}{4+\cfrac{1}{21+\cfrac{1}{3+\cfrac{1}{3+\cfrac{1}{5+\cfrac{1}{2}}}}}}$$

[redacted]

$$\frac{1}{4}=\frac{1}{4}.\qquad \cfrac{1}{4+\cfrac{1}{21}}=\frac{21}{85}.\qquad \cfrac{1}{4+\cfrac{1}{21+\cfrac{1}{3}}}=\frac{64}{259}.$$

$$\cfrac{1}{4+\cfrac{1}{21+\cfrac{1}{3+\cfrac{1}{3}}}}=\frac{213}{862}.\qquad \cfrac{1}{4+\cfrac{1}{21+\cfrac{1}{3+\cfrac{1}{3+\cfrac{1}{5}}}}}=\frac{1129}{4569}.$$

$\frac{1}{4}, \frac{21}{85}, \frac{64}{259}, \frac{213}{862}, \frac{1129}{4569}$. *Ans.*

10. Find a series of fractions approximating to the ratio of the weight of the 48-pound shot to the weight of the French shot of 24^{kg}.

$$48 \text{ lb.} = 48 \times 0.45359^{kg} = 21.77232^{kg}.$$

```
907)1000(1
    907
     93)907(9
        837
         70)93(1
            70
            23)70(3
               69
                1)23(23
                  23
```

$$\frac{21.77232}{24}=\frac{907}{1000}.$$

$$\therefore \frac{907}{1000}=\cfrac{1}{1+\cfrac{1}{9+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{23}}}}}$$

$$\frac{1}{1}=1.\qquad \cfrac{1}{1+\cfrac{1}{9}}=\frac{9}{10}.\qquad \cfrac{1}{1+\cfrac{1}{9+\cfrac{1}{1}}}=\frac{10}{11}.\qquad \cfrac{1}{1+\cfrac{1}{9+\cfrac{1}{1+\cfrac{1}{3}}}}=\frac{39}{43}.$$

$1, \frac{9}{10}, \frac{10}{11}, \frac{39}{43}$. *Ans.*

11. If the mean diameter of the Earth is reckoned at 7912 mi., and that of Mars 4189 mi., find a series of fractions approximating to the ratio of the mean diameters of these two planets.

```
4189)7912(1
     4189
     3723)4189(1
          3723
           466)3723(7
               3262
                461)466(1
                    461
                      5)461(92
                        460
                          1)5(5
                            5
```

$$\therefore \frac{4189}{7912}=\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{7+\cfrac{1}{1+\cfrac{1}{92+\cfrac{1}{5}}}}}}$$

$$\frac{1}{1}=1.$$

$$\cfrac{1}{1+\cfrac{1}{1}}=\frac{1}{2}.$$

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{7}}}=\frac{8}{15}.$$

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{7+\cfrac{1}{1}}}}=\frac{9}{17}.$$

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{7+\cfrac{1}{1+\cfrac{1}{92}}}}}=\frac{836}{1579}.$$

$1, \frac{1}{2}, \frac{8}{15}, \frac{9}{17}, \frac{836}{1579}$. *Ans.*

12. Find a series of fractions approximating to the ratio of a cubic yard to a cubic meter from the equality

$$1 \text{ cu. yd.} = 0.76453^{\text{cbm}}.$$

$$0.76453 = \tfrac{76453}{100000}.$$

```
76453)100000(1
      76453
      23547)76453(3
            70641
             5812)23547(4
                  23248
                    299)5812(19
                        5681
                         131)299(2
                             262
                              37)131(3
                                 111
                                  20)37(1
                                     20
                                     17)20(1
                                        17
                                         3)17(5
                                           15
                                            2)3(1
                                              2
                                              1)2(2
                                                2
```

$$\therefore \frac{76453}{100000}=\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{19+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{5+\cfrac{1}{1+\cfrac{1}{2}}}}}}}}}}}$$

[redacted]

$$\frac{1}{1} = 1.$$

$$\cfrac{1}{1 + \cfrac{1}{3}} = \frac{3}{4}.$$

$$\cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{4}}} = \frac{13}{17}.$$

$$\cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{19}}}} = \frac{250}{327}.$$

$$\cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{19 + \cfrac{1}{2}}}}} = \frac{513}{671}.$$

$$\cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{19 + \cfrac{1}{2 + \cfrac{1}{3}}}}}} = \frac{1789}{2340}.$$

$$\cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{19 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{1}}}}}}} = \frac{2302}{3011}.$$

$$\cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{19 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{1}}}}}}}} = \frac{4091}{5351}.$$

$$\cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{19 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{5}}}}}}}}} = \frac{22757}{29766}.$$

$$\cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{19 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{1}}}}}}}}}} = \frac{26848}{35117}.$$

$1, \frac{3}{4}, \frac{13}{17}, \frac{250}{327}, \frac{513}{671}, \frac{1789}{2340}, \frac{2302}{3011}, \frac{4091}{5351}, \frac{22757}{29766}, \frac{26848}{35117}$. *Ans.*

13. Find a series of fractions approximating to the ratio of the kilometer to the mile, from the equality $1^{m} = 1.09362$ yd.

$1^{m} = 1.09362$ yd. $\quad 1^{km} = 1093.62$ yd. $\quad \therefore\ 1^{km} = 0.621$ mi.

$$0.621 = \tfrac{621}{1000}.$$

621)1000(1
621
379)621(1
379
242)379(1
242
137)242(1
137
105)137(1
105
32)105(3
96
9)32(3
27
5)9(1
5
4)5(1
4
1)4(4
4

$$\therefore\ \tfrac{621}{1000} = \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{4}}}}}}}}}$$

$$\frac{1}{1} = 1.$$

$$\cfrac{1}{1+\cfrac{1}{1}} = \frac{1}{2}.$$

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1}}} = \frac{2}{3}.$$

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1}}}} = \frac{3}{5}.$$

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1}}}}} = \frac{5}{8}.$$

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3}}}}}} = \frac{18}{29}.$$

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{3}}}}}}} = \frac{59}{95}.$$

[redacted]

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{3+\frac{1}{1}}}}}}}}=\frac{77}{124}.\qquad \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{3+\cfrac{1}{1+\frac{1}{1}}}}}}}}}=\frac{136}{219}$$

$1,\ \frac{1}{2},\ \frac{2}{3},\ \frac{3}{5},\ \frac{5}{8},\ \frac{18}{29},\ \frac{59}{95},\ \frac{77}{124},\ \frac{136}{219}.$ *Ans.*

14. Find the proper fraction that, if changed to a continued fraction, will have as quotients 1, 7, 5, 2.

$$\cfrac{1}{1+\cfrac{1}{7+\cfrac{1}{5+\frac{1}{2}}}}=\frac{79}{90}.\ \textit{Ans.}$$

$$\frac{1}{5\frac{1}{2}}=\frac{2}{11};\qquad \frac{1}{7\frac{2}{11}}=\frac{11}{79};\qquad \frac{1}{1\frac{11}{79}}=\frac{79}{90}.$$

15. Find a series of fractions approximating to 0.5236; approximating to 0.7854.

$$0.5236=\frac{5236}{10000}=\frac{1309}{2500}.$$

```
1309)2500(1
     1309
     1191)1309(1
          1191
           118)1191(10
               1180
                 11)118(10
                    110
                      8)11(1
                         8
                         3)8(2
                           6
                           2)3(1
                             2
                             1)2(2
                               2
```

$$\therefore \frac{1309}{2500}=\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{10+\cfrac{1}{10+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\frac{1}{2}}}}}}}}$$

$\frac{1}{1} = 1.$ $\qquad \dfrac{1}{1+\frac{1}{1}} = \frac{1}{2}.$ $\qquad \dfrac{1}{1+\dfrac{1}{1+\frac{1}{10}}} = \frac{11}{21}.$

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{10+\cfrac{1}{10}}}} = \frac{111}{212}. \qquad \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{10+\cfrac{1}{10+\cfrac{1}{1}}}}} = \frac{122}{233}.$$

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{10+\cfrac{1}{10+\cfrac{1}{1+\cfrac{1}{2}}}}}} = \frac{355}{678}. \qquad \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{10+\cfrac{1}{10+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1}}}}}}} = \frac{477}{911}.$$

$1, \frac{1}{2}, \frac{11}{21}, \frac{111}{212}, \frac{122}{233}, \frac{355}{678}, \frac{477}{911}.$ *Ans.*

$$0.7854 = \tfrac{7854}{10000} = \tfrac{3927}{5000}.$$

```
3927)5000(1
     3927
     1073)3927(3
          3219
           708)1073(1
                708
                365)708(1
                    365
                    343)365(1
                        343
                         22)343(15
                            330
                             13)22(1
                                13
                                 9)13(1
                                   9
                                   4)9(2
                                     8
                                     1)4(4
                                       4
```

$$\therefore \tfrac{3927}{5000} = \cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{15+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{4}}}}}}}}}}$$

[redacted]

$$\frac{1}{1} = 1.$$

$$\cfrac{1}{1+\cfrac{1}{3}} = \frac{3}{4}.$$

$$\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1}}} = \frac{4}{5}.$$

$$\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1}}}} = \frac{7}{9}.$$

$$\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1}}}}} = \frac{11}{14}.$$

$$\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{15}}}}}} = \frac{172}{219}.$$

$$\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{15+\cfrac{1}{1}}}}}}} = \frac{183}{233}.$$

$$\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{15+\cfrac{1}{1+\cfrac{1}{1}}}}}}}} = \frac{355}{452}.$$

$$\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{15+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2}}}}}}}}} = \frac{893}{1137}.$$

$1, \frac{3}{4}, \frac{4}{5}, \frac{7}{9}, \frac{11}{14}, \frac{172}{219}, \frac{183}{233}, \frac{355}{452}, \frac{893}{1137}$. *Ans.*

16. Find a series of fractions approximating to the continued fraction that has as quotients 7, 2, 1, 2, 6, 4; that has as quotients 1, 2, 3, 4, 5, 6.

$$\frac{1}{7}=\frac{1}{7}. \qquad \cfrac{1}{7+\cfrac{1}{2}}=\frac{2}{15}. \qquad \cfrac{1}{7+\cfrac{1}{2+\cfrac{1}{1}}}=\frac{3}{22}. \qquad \cfrac{1}{7+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2}}}}=\frac{8}{59}.$$

$$\cfrac{1}{7+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{6}}}}}=\frac{51}{376}. \qquad \cfrac{1}{7+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{6+\cfrac{1}{4}}}}}}=\frac{212}{1563}.$$

$\frac{1}{7}, \frac{2}{15}, \frac{3}{22}, \frac{8}{59}, \frac{51}{376}, \frac{212}{1563}$. *Ans.*

$$\frac{1}{1}=1. \qquad \cfrac{1}{1+\cfrac{1}{2}}=\frac{2}{3}. \qquad \cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3}}}=\frac{7}{10}. \qquad \cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4}}}}=\frac{30}{43}.$$

$$\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{5}}}}}=\frac{157}{225}. \qquad \cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{6}}}}}}=\frac{972}{1393}.$$

$1, \frac{2}{3}, \frac{7}{10}, \frac{30}{43}, \frac{157}{225}, \frac{972}{1393}$. *Ans.*

Exercise 145. Page 337.

1. Change 4852 of the common scale to the scale of 7.

7	4852	
7	693	remainder 1.
7	99	remainder 0.
7	14	remainder 1.
	2	remainder 0.

20,101. *Ans.*

2. Change 4852 of the common scale to the scale of 2.

2	4852	
2	2426	remainder 0.
2	1213	remainder 0.
2	606	remainder 1.
2	303	remainder 0.
2	151	remainder 1.
2	75	remainder 1.
2	37	remainder 1.
2	18	remainder 1.
2	9	remainder 0.
2	4	remainder 1.
2	2	remainder 0.
	1	remainder 0.

1,001,011,110,100. *Ans.*

3. Change 4852 of the common scale to the scale of 9.

9	4852	
9	539	remainder 1.
9	59	remainder 8.
	6	remainder 5.

6581. *Ans.*

4. Change 4852 of the common scale to the scale of 3.

3	4852	
3	1617	remainder 1.
3	539	remainder 0.
3	179	remainder 2.
3	59	remainder 2.
3	19	remainder 2.
3	6	remainder 1.
	2	remainder 0.

20,122,201. *Ans.*

5. Change 4852 of the common scale to the scale of 6.

6	4852	
6	808	remainder 4.
6	134	remainder 4.
6	22	remainder 2.
	3	remainder 4.

34,244. *Ans.*

6. Change 4852 of the common scale to the scale of 5.

5	4852	
5	970	remainder 2.
5	194	remainder 0.
5	38	remainder 4.
5	7	remainder 3.
	1	remainder 2.

123,402. *Ans.*

7. Change 4852 of the common scale to the scale of 8.

8	4852	
8	606	remainder 4.
8	75	remainder 6.
8	9	remainder 3.
	1	remainder 1.

11,364. *Ans.*

[redacted]

8. Change 4852 of the common scale to the scale of 4.

4 | 4852
4 | 1213 remainder 0.
4 | 303 remainder 1.
4 | 75 remainder 3.
4 | 18 remainder 3.
4 | 4 remainder 2.
1 remainder 0.

1,023,310. *Ans.*

9. Change 54,231 of the scale of 6 to the common scale.

10 | 54231
10 | 3235 remainder 5.
10 | 202 remainder 3.
10 | 11 remainder 4.
0 remainder 7.

7435. *Ans.*

10. Change 54,231 of the scale of 7 to the common scale.

10 | 54231
10 | 3635 remainder 7.
10 | 251 remainder 9.
10 | 16 remainder 4.
1 remainder 3.

13,497. *Ans.*

11. Change 54,231 of the scale of 8 to the common scale.

10 | 54231
10 | 4334 remainder 1.
10 | 342 remainder 8.
10 | 26 remainder 6.
2 remainder 2.

22,681. *Ans.*

12. Change 54,231 of the scale of 9 to the common scale.

10 | 54231
10 | 4830 remainder 1.
10 | 438 remainder 1.
10 | 38 remainder 9.
3 remainder 5.

35,911. *Ans.*

Perform the following arithmetical processes:

13. Add 67,814; 76,406; 88,718 (scale of 9).

67814
76406
88718

255140 *Ans.*

14. Add 44,231; 13,432; 12,304 (scale of 5).

44231
13432
12304

131022 *Ans.*

15. Subtract 77,614 from 114,672 (scale of 8).

114672
77614

15056 *Ans.*

16. Subtract 52,515 from 112,252 (scale of 6).

112252
52515

15333 *Ans.*

17. Multiply 14,612 by 6502 (scale of 7).

[redacted]


```
     14612
      6502
    ------
     32524
   113263
  131205
  ---------
  142600124  Ans.
```

18. Multiply 72,645 by 46,723 (scale of 8).

```
       72645
       46723
     -------
      260357
     165512
    633603
   540736
  353224
  ----------
  4360713777  Ans.
```

19. Divide 162,542 by 6522 (scale of 7).

```
           16
  6522)162542
       6522
       -----
       64322
       55365
       -----
        5624
```

$16\frac{5624}{6522}$. *Ans.*

20. Divide 468,722 by 5432 (scale of 9).

```
           77
  5432)468722
       42345
       -----
       45262
       42345
       -----
        2816
```

$77\frac{2816}{5432}$. *Ans.*

Exercise 146. Page 339.

1. Find the seventh term of the series 3, 5, 7, etc.

$3 + (6 \times 2) = 3 + 12 = 15$. *Ans.*

2. Find the fifteenth term of the series 2, 7, 12, etc.

$2 + (14 \times 5) = 2 + 70 = 72$. *Ans.*

3. Find the sixth term of the series 2, $2\frac{5}{7}$, $3\frac{3}{7}$, etc.

$2 + (5 \times \frac{5}{7}) = 2 + 3\frac{4}{7} = 5\frac{4}{7}$. *Ans.*

4. Find the twentieth term of the series 2, $3\frac{1}{4}$, $4\frac{1}{2}$, etc.

$2 + (19 \times 1\frac{1}{4}) = 2 + 23\frac{3}{4} = 25\frac{3}{4}$. *Ans.*

5. Find the seventh term of the series 21, 19, 17, etc.

$21 - (6 \times 2) = 21 - 12 = 9$. *Ans.*

6. Find the twelfth term of the series 18, $17\frac{1}{3}$, $16\frac{2}{3}$, etc.

$18 - (11 \times \frac{2}{3}) = 18 - 7\frac{1}{3} = 10\frac{2}{3}$. *Ans.*

7. If the first term of a series is 5, and the common difference $2\frac{1}{4}$, find the thirteenth and the eighteenth terms.

$$13\text{th term} = 5 + (12 \times 2\tfrac{1}{4}) = 5 + 27 = 32.$$

$$18\text{th term} = 5 + (17 \times 2\tfrac{1}{4}) = 5 + 38\tfrac{1}{4} = 43\tfrac{1}{4}.$$

8. If the fourth term of a series is 18, and the common difference 3, find the seventh and eleventh terms.

The seventh term is the fourth term of the series whose first term is 18; the eleventh term is the eighth term of this series.

$$\begin{aligned}\text{7th term} &= 18 + (3 \times 3)\\ &= 18 + 9 = 27.\\ \text{11th term} &= 18 + (7 \times 3)\\ &= 18 + 21 = 39.\end{aligned}$$

9. If the fifth term of a decreasing series is 52, and the common difference $3\frac{1}{2}$, find the twelfth and eighteenth terms.

The twelfth term is the eighth term of the series whose first term is 52; the eighteenth term is the fourteenth term of this series.

$$\begin{aligned}\text{12th term} &= 52 - (7 \times 3\tfrac{1}{2})\\ &= 52 - 24\tfrac{1}{2} = 27\tfrac{1}{2}.\\ \text{18th term} &= 52 - (13 \times 3\tfrac{1}{2})\\ &= 52 - 45\tfrac{1}{2} = 6\tfrac{1}{2}.\end{aligned}$$

10. If the fourth term of a series is 14, and the twelfth term 38, what is the common difference?

$$\frac{38 - 14}{8} = 3. \quad \textit{Ans.}$$

11. Find the common difference in a series if the fourth term is 12 and the seventh term 27.

$$\frac{27 - 12}{3} = 5. \quad \textit{Ans.}$$

12. Find the common difference in a series if the first term is 20 and the fourth term 40.

$$\frac{40 - 20}{3} = 6\tfrac{2}{3}. \quad \textit{Ans.}$$

13. Find the common difference in a series if the first term is 2 and the eleventh term 20.

$$\frac{20 - 2}{10} = 1\tfrac{4}{5}. \quad \textit{Ans.}$$

14. Find the common difference in a series if the third term is 7 and the eighth term $12\frac{1}{2}$.

$$\frac{12\frac{1}{2} - 7}{5} = 1\tfrac{1}{10}. \quad \textit{Ans.}$$

15. Find the common difference in a series if the first term is 1 and the fourth term 19.

$$\frac{19 - 1}{3} = 6. \quad \textit{Ans.}$$

Exercise 147. Page 340.

1. Find the sum of 1, 5, 9, etc., to twenty terms.

20th term $= 1 + (19 \times 4) = 1 + 76 = 77.$

Sum $= 20 \times \frac{1}{2}(1 + 77) = 20 \times 39 = 780.$ *Ans.*

2. Find the sum of 4, $5\frac{1}{2}$, 7, etc., to eight terms.

8th term $= 4 + (7 \times 1\frac{1}{2}) = 4 + 10\frac{1}{2} = 14\frac{1}{2}.$

Sum $= 8 \times \frac{1}{2}(4 + 14\frac{1}{2}) = 8 \times 9\frac{1}{4} = 74.$ *Ans.*

3. Find the sum of 8, $7\frac{2}{3}$, $7\frac{1}{3}$, etc., to sixteen terms.

16th term $= 8 - (15 \times \frac{1}{3}) = 8 - 5 = 3.$

Sum $= 16 \times \frac{1}{2}(8 + 3) = 16 \times 5\frac{1}{2} = 88.$ *Ans.*

[redacted]

4. Find the sum of 20, $18\frac{1}{4}$, $16\frac{1}{2}$, etc., to seven terms.

7th term $= 20 - (6 \times 1\frac{3}{4}) = 20 - 10\frac{1}{2} = 9\frac{1}{2}$.
Sum $= 7 \times \frac{1}{2}(20 + 9\frac{1}{2}) = 7 \times 14\frac{3}{4} = 103\frac{1}{4}$. *Ans.*

5. Find the sum of the first twenty natural numbers.

Sum $= 20 \times \frac{1}{2}(1 + 20) = 20 \times 10\frac{1}{2} = 210$. *Ans.*

6. Find the sum of the natural numbers from 37 to 53 both inclusive.

Sum $= 17 \times \frac{1}{2}(37 + 53) = 17 \times 45 = 765$. *Ans.*

7. Find the sum of a series of thirty terms, if the first term is 21 and the last 59.

Sum $= 30 \times \frac{1}{2}(21 + 59) = 30 \times 40 = 1200$. *Ans.*

8. Find the sum of the series whose first two terms are 3 and 9 and the last term 75.

Number of terms $= 1 + \dfrac{75 - 3}{6} = 1 + 12 = 13$.

Sum $= 13 \times \frac{1}{2}(3 + 75) = 13 \times 39 = 507$. *Ans.*

9. Find the sum of a series of twenty terms whose third and fifth terms are 10 and 15, respectively.

Common difference $= \dfrac{15 - 10}{2} = 2\frac{1}{2}$.

1st term $= 10 - (2 \times 2\frac{1}{2}) = 10 - 5 = 5$.
20th term $= 5 + (19 \times 2\frac{1}{2}) = 5 + 47\frac{1}{2} = 52\frac{1}{2}$.
Sum $= 20 \times \frac{1}{2}(5 + 52\frac{1}{2}) = 20 \times 28\frac{3}{4} = 575$. *Ans.*

10. A body falls through a space of $16\frac{1}{12}$ ft. in the first second of its fall, and in each succeeding second $32\frac{1}{6}$ ft. more than in the second just before. How far will a stone fall in the seventh second? How far in seven seconds?

7th term $= 16\frac{1}{12}$ ft. $+ (6 \times 32\frac{1}{6}$ ft.$) = 16\frac{1}{12}$ ft. $+ 193$ ft. $= 209\frac{1}{12}$ ft. *Ans.*
Sum $= 7 \times \frac{1}{2}(16\frac{1}{12} + 209\frac{1}{12})$ ft. $= 7 \times 112\frac{7}{12}$ ft. $= 788\frac{1}{12}$ ft. *Ans.*

11. A travels 8 miles the first day, 11 miles the second, 14 miles the third, and so on, and overtakes in 17 days B who started at the same time, and traveled at a uniform rate. What is B's rate per day?

17th term $= 8$ mi. $+ (16 \times 3$ mi.$) = 8$ mi. $+ 48$ mi. $= 56$ mi.

Sum $= 17 \times \frac{1}{2}(8 + 56)$ mi. $= 17 \times 32$ mi.

$$\frac{17 \times 32}{17} \text{ mi.} = 32 \text{ mi. } \textit{Ans.}$$

[redacted]

12. In a potato race 100 potatoes are placed in a straight line 3 ft. distant from each other. A boy, starting from a basket 3 ft. from the first potato, is required to pick them up one by one and carry them to the basket. To finish the race how far must the boy run?

First term is 6 ft., common difference 6 ft., and number of terms 100.
100th term $= 6$ ft. $+ (99 \times 6$ ft.$) = 100 \times 6$ ft. $= 600$ ft.
Sum $= 100 \times \frac{1}{2}(6 + 600)$ ft. $= 100 \times 303$ ft. $= 30{,}300$ ft. *Ans.*

13. How many times a day does a clock strike that strikes the hours only?

For half a day, sum $= 12 \times \frac{1}{2}(1 + 12) = 12 \times 6\frac{1}{2} = 78$.
For whole day, $2 \times 78 = 156$. *Ans.*

14. A body falls through a space of 4.9^m in the first second of its fall, and in each succeeding second 9.8^m more than in the second just before. A stone dropped from a balloon was 35 seconds in reaching the ground. How high was the balloon?

35th term $= 4.9^m + (34 \times 9.8)^m = 4.9^m + 333.2^m = 338.1^m$.
Sum $= 35 \times \frac{1}{2}(4.9^m + 338.1^m) = 35 \times 171.5^m = 6002.5^m$. *Ans.*

Exercise 148. Page 342.

1. Find the eighth term of the series 2, 6, 18, etc.

$$2 \times 3^7 = 2 \times 2187 = 4374. \quad \textit{Ans.}$$

2. Find the fifth term of the series 8, 4, 2, etc.

$$8 \times (\tfrac{1}{2})^4 = 8 \times \tfrac{1}{16} = \tfrac{1}{2}. \quad \textit{Ans.}$$

3. Find the seventh term of the series 2, 3, $4\frac{1}{2}$, etc.

$$2 \times (\tfrac{3}{2})^6 = 2 \times \tfrac{729}{64} = \tfrac{729}{32} = 22\tfrac{25}{32}. \quad \textit{Ans.}$$

4. Find the sixth term of the series 4, $2\frac{2}{3}$, $1\frac{7}{9}$, etc.

$$4 \times (\tfrac{2}{3})^5 = 4 \times \tfrac{32}{243} = \tfrac{128}{243}. \quad \textit{Ans.}$$

5. Find the eighth term of the series 4, 10, 25, etc.

$$4 \times (\tfrac{5}{2})^7 = 4 \times \tfrac{78125}{128} = \tfrac{78125}{32} = 2441\tfrac{13}{32}. \quad \textit{Ans.}$$

6. Find the fifth term of the series $\frac{1}{4}$, $\frac{1}{16}$, $\frac{1}{64}$, etc.

$$\tfrac{1}{4} \times (\tfrac{1}{4})^4 = \tfrac{1}{4} \times \tfrac{1}{256} = \tfrac{1}{1024}. \quad \textit{Ans.}$$

7. Find the ninth term of the series 4, 2, 1, etc.

$$4 \times \left(\frac{1}{2}\right)^8 = 2^2 \times \frac{1}{2^8} = \frac{1}{2^6} = \frac{1}{64}. \quad \textit{Ans.}$$

8. Find the sixth term of the series 6, 9, $13\frac{1}{2}$, etc.

$$6 \times (\tfrac{3}{2})^5 = 6 \times \tfrac{243}{32} = \tfrac{729}{16} = 45\tfrac{9}{16}. \quad \textit{Ans.}$$

9. Write the first six terms of the geometrical series whose fi and sixth terms are 112 and 224, respectively.

$$\text{Ratio} = 2. \qquad \text{1st term} = \frac{112}{2^4} = 7.$$

Therefore, the series is 7, 14, 28, 56, 112, 224. *Ans.*

10. The seventh and ninth terms of a geometrical series are 1 and 144, respectively. Find the twelfth term.

$$\text{9th term} = \text{7th term} \times (\text{ratio})^2.$$

$$\therefore (\text{ratio})^2 = \tfrac{144}{100}. \qquad \therefore \text{ratio} = \tfrac{12}{10} = \tfrac{6}{5}.$$

$$\text{12th term} = 144 \times (\tfrac{6}{5})^3 = 144 \times \tfrac{216}{125} = 248.832. \quad \textit{Ans.}$$

11. A capital of \$1000 is increased by $\frac{1}{10}$ of itself each year. Wh will it be at the beginning of the fifth year?

$$1000 \times \$(\tfrac{11}{10})^4 = 1000 \times \$\tfrac{14641}{10000} = \$1464.10. \quad \textit{Ans.}$$

12. A capital of \$1000 is increased by $\frac{6}{100}$ of itself each year. Wh will it be at the beginning of the sixth year?

$$1000 \times \$(\tfrac{106}{100})^5 = 1000 \times \$\tfrac{13382255776}{10000000000} = \$1338.23. \quad \textit{Ans.}$$

Exercise 149. Page 343.

1. Find the sum of 2, 6, 18, etc., to six terms.

$$\text{6th term} = 2 \times 3^5 = 2 \times 243 = 486.$$

$$\text{Sum} = \frac{3 \times 486 - 2}{3 - 1} = \frac{1458 - 2}{2} = \frac{1456}{2} = 728. \quad \textit{Ans.}$$

2. Find the sum of 1, 2, 4, etc., to nine terms.

$$\text{9th term} = 1 \times 2^8 = 1 \times 256 = 256.$$

$$\text{Sum} = \frac{2 \times 256 - 1}{2 - 1} = \frac{512 - 1}{1} = 511. \quad \textit{Ans.}$$

3. Find the sum of 3, 9, 27, etc., to five terms.

$$\text{5th term} = 3 \times 3^4 = 3 \times 81 = 243.$$

$$\text{Sum} = \frac{3 \times 243 - 3}{3 - 1} = \frac{729 - 3}{2} = \frac{726}{2} = 363. \quad \textit{Ans.}$$

4. Find the sum of 2, 3, $4\frac{1}{2}$, etc., to eight terms.

$$\text{8th term} = 2 \times (\tfrac{3}{2})^7 = 2 \times \tfrac{2187}{128} = \tfrac{2187}{64} = 34\tfrac{11}{64}.$$

$$\text{Sum} = \frac{\frac{3}{2} \times 34\frac{11}{64} - 2}{\frac{3}{2} - 1} = \frac{51\frac{33}{128} - 2}{\frac{1}{2}} = 2 \times 49\tfrac{33}{128} = 98\tfrac{33}{64}. \quad \textit{Ans.}$$

[redacted]

5. Find the sum of 1, $\frac{1}{3}$, $\frac{1}{9}$, etc., to eight terms.

8th term $= 1 \times (\frac{1}{3})^7 = \frac{1}{2187}$.

$$\text{Sum} = \frac{1 - \frac{1}{3} \times \frac{1}{2187}}{1 - \frac{1}{3}} = \frac{1 - \frac{1}{6561}}{\frac{2}{3}} = \frac{\overset{3280}{\cancel{6560}}}{\underset{2187}{\cancel{6561}}} \times \frac{\cancel{3}}{\cancel{2}} = \frac{3280}{2187} = 1\tfrac{1093}{2187}. \quad \textit{Ans.}$$

6. Find the sum of 1, $\frac{1}{2}$, $\frac{1}{4}$, etc., to ten terms.

10th term $= 1 \times (\frac{1}{2})^9 = \frac{1}{512}$.

$$\text{Sum} = \frac{1 - \frac{1}{2} \times \frac{1}{512}}{1 - \frac{1}{2}} = \frac{1 - \frac{1}{1024}}{\frac{1}{2}} = 2 \times \frac{1023}{1024} = \frac{1023}{512} = 1\tfrac{511}{512}. \quad \textit{Ans.}$$

7. Find the sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{9}$, etc., to eight terms.

7th term $= \frac{1}{2} \times (\frac{2}{3})^7 = \frac{1}{2} \times \frac{128}{2187} = \frac{64}{2187}$.

$$\text{Sum} = \frac{\frac{1}{2} - \frac{2}{3} \times \frac{64}{2187}}{1 - \frac{2}{3}} = \frac{\frac{1}{2} - \frac{128}{6561}}{\frac{1}{3}} = \cancel{3} \times \frac{6305}{\underset{4374}{\cancel{13122}}} = \frac{6305}{4374} = 1\tfrac{1931}{4374}. \quad \textit{Ans.}$$

8. Find the sum of the first six terms of the series whose first term is 3 and ratio 5.

6th term $= 3 \times 5^5 = 3 \times 3125 = 9375$.

$$\text{Sum} = \frac{5 \times 9375 - 3}{5 - 1} = \frac{46875 - 3}{4} = \frac{46872}{4} = 11{,}718. \quad \textit{Ans.}$$

9. Find the sum of the first eight terms of the series whose first term is 3 and ratio $\frac{1}{3}$.

8th term $= 3 \times (\frac{1}{3})^7 = 3 \times \frac{1}{2187} = \frac{1}{729}$.

$$\text{Sum} = \frac{3 - \frac{1}{3} \times \frac{1}{729}}{1 - \frac{1}{3}} = \frac{3 - \frac{1}{2187}}{\frac{2}{3}} = \frac{3}{2} \times 2\tfrac{2186}{2187} = \frac{\cancel{3}}{\cancel{2}} \times \frac{\overset{3280}{\cancel{6560}}}{\underset{729}{\cancel{2187}}} = \frac{3280}{729} = 4\tfrac{364}{729}. \quad \textit{Ans.}$$

10. A man saved in one year \$64, and in each succeeding year, for 9 years more, $1\frac{1}{2}$ times as much as in the preceding year. Find the whole amount he saved.

Number of terms is 10.

10th term $= 64 \times (\frac{3}{2})^9 = 2^6 \times \frac{3^9}{2^9} = \frac{3^9}{2^3} = \frac{19683}{8} = 2460\frac{3}{8}$.

$$\text{Sum} = \frac{\frac{3}{2} \times 2460\frac{3}{8} - 64}{\frac{3}{2} - 1} = \frac{3690\frac{9}{16} - 64}{\frac{1}{2}} = 2 \times 3626\tfrac{9}{16} = 7253\tfrac{1}{8}.$$

\$$7253\frac{1}{8}$ = \$7253.13. *Ans.*

$$s = \frac{\frac{2}{3}}{1 - \frac{2}{3}} = \frac{\frac{2}{3}}{\frac{1}{3}} = 2.$$

3. **Find the sum of the infinite series $\frac{1}{4}$**

$$s = \frac{\frac{1}{4}}{1 - \frac{1}{4}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}.$$

4. **Find the sum of the infinite series $\frac{1}{5}$,**

$$s = \frac{\frac{1}{5}}{1 - \frac{1}{5}} = \frac{\frac{1}{5}}{\frac{4}{5}} = \frac{1}{4}. \ A$$

5. **Find the sum of the infinite series 0.**

$$s = \frac{0.17}{1 - 0.01} = \frac{0.17}{0.99} =$$

6. **Find the sum of the infinite series 0.**

$$s = \frac{0.21}{1 - 0.01} = \frac{0.21}{0.99} =$$

7. **Find the sum of the infinite series 0.**

$$s = \frac{0.9}{1 - 0.1} = \frac{0.9}{0.9} = 1$$

8. **Find the sum of the infinite series 0.**

$$s = \frac{0.23}{1 - 0.01} = \frac{0.23}{0.99} =$$

Exercise 151. Page 349.

1. $\log 70 = 1.8451$. *Ans.*
2. $\log 101 = 2.0043$. *Ans.*
3. $\log 333 = 2.5224$. *Ans.*
4. $\log 3491 = 3.5428 + (\frac{1}{10} \text{ of } 13) = 3.5429$. *Ans.*
5. $\log 1866 = 3.2695 + (\frac{6}{10} \text{ of } 23) = 3.2709$. *Ans.*
6. $\log 6897 = 3.8382 + (\frac{7}{10} \text{ of } 6) = 3.8386$. *Ans.*
7. $\log 9901 = 3.9956 + (\frac{1}{10} \text{ of } 5) = 3.9957$. *Ans.*
8. $\log 4389 = 3.6415 + (\frac{9}{10} \text{ of } 10) = 3.6424$. *Ans.*
9. $\log 1111 = 3.0453 + (\frac{1}{10} \text{ of } 39) = 3.0457$. *Ans.*
10. $\log 58{,}343 = 4.7657 + (\frac{48}{100} \text{ of } 7) = 4.7660$. *Ans.*
11. $\log 77{,}860 = 4.8910 + (\frac{60}{100} \text{ of } 5) = 4.8913$. *Ans.*
12. $\log 30{,}127 = 4.4786 + (\frac{27}{100} \text{ of } 14) = 4.4790$. *Ans.*
13. $\log 730.84 = 2.8633 + (\frac{84}{100} \text{ of } 6) = 2.8638$. *Ans.*
14. $\log 0.008765 = 7.9425 + (\frac{5}{10} \text{ of } 5) - 10 = 7.9428 - 10$. *Ans.*
15. $\log 8.0808 = 0.9074 + (\frac{8}{100} \text{ of } 5) = 0.9074$. *Ans.*
16. $\log 5.0009 = 0.6990 + (\frac{9}{100} \text{ of } 8) = 0.6991$. *Ans.*
17. $\log 0.3769 = 9.5752 + (\frac{9}{10} \text{ of } 11) - 10 = 9.5762 - 10$. *Ans.*
18. $\log 0.070707 = 8.8494 + (\frac{7}{100} \text{ of } 6) - 10 = 8.8494 - 10$. *Ans.*
19. $\log 0.03723 = 8.5705 + (\frac{3}{10} \text{ of } 12) - 10 = 8.5709 - 10$. *Ans.*
20. $\log 98.871 = 1.9948 + (\frac{71}{100} \text{ of } 4) = 1.9951$. *Ans.*

Exercise 152. Page 353.

1. Find antilog 3.9017.

   The number corresponding to the mantissa 9015 is 7970.
   The number corresponding to the mantissa 9020 is 7980.
   The difference between these numbers is 10,
   and $7970 + \frac{2}{5} \text{ of } 10 = 7974$. *Ans.*

2. Find antilog 1.2076.

   The number corresponding to the mantissa 2068 is 1610.
   The number corresponding to the mantissa 2095 is 1620.
   The difference between these numbers is 10,
   and $1610 + \frac{8}{27} \text{ of } 10 = 1613$.
   Therefore, the number required is 16.13. *Ans.*

[redacted]

3. Find antilog 0.4442.

The number corresponding to the mantissa 4440 is 2780.
The number corresponding to the mantissa 4456 is 2790.
The difference between these numbers is 10,

and $2780 + \frac{2}{16}$ of $10 = 2781$.

Therefore, the number required is 2.781. *Ans.*

4. Find antilog 1.0090.

The number corresponding to the mantissa 0086 is 1020.
The number corresponding to the mantissa 0128 is 1030.
The difference between these numbers is 10,

and $1020 + \frac{4}{42}$ of $10 = 1021$.

Therefore, the number required is 10.21. *Ans.*

5. Find antilog 4.8697.

The number corresponding to the mantissa 8692 is 7400.
The number corresponding to the mantissa 8698 is 7410.
The difference between these numbers is 10,

and $7400 + \frac{5}{6}$ of $10 = 7408$.

Therefore, the number required is 74,080. *Ans.*

6. Find antilog 1.9214.

The number corresponding to the mantissa 9212 is 8340.
The number corresponding to the mantissa 9217 is 8350.
The difference between these numbers is 10,

and $8340 + \frac{2}{5}$ of $10 = 8344$.

Therefore, the number required is 83.44. *Ans.*

7. Find antilog 2.9850.

The number corresponding to the mantissa 9850 is 9660.
Therefore, the number required is 966. *Ans.*

8. Find antilog 4.5388.

The number corresponding to the mantissa 5378 is 3450.
The number corresponding to the mantissa 5391 is 3460.
The difference between these numbers is 10,

and $3450 + \frac{10}{13}$ of $10 = 3458$.

Therefore, the number required is 34,580. *Ans.*

9. Find antilog 0.8550.

The number corresponding to the mantissa 8549 is 7160.
The number corresponding to the mantissa 8555 is 7170.
The difference between these numbers is 10,

and $7160 + \frac{1}{6}$ of $10 = 7162$.

Therefore, the number required is 7.162. *Ans.*

10. Find antilog 9.9992 – 10.

The number corresponding to the mantissa 9991 is 9980.
The number corresponding to the mantissa 9996 is 9990.
The difference between these numbers is 10,

and $9980 + \frac{1}{5}$ of $10 = 9982$.

Therefore, the number required is 0.9982. *Ans.*

11. Find antilog 7.0016 – 10.

The number corresponding to the mantissa 0000 is 1000.
The number corresponding to the mantissa 0043 is 1010.
The difference between these numbers is 10,

and $1000 + \frac{16}{43}$ of $10 = 1004$.

Therefore, the number required is 0.001004. *Ans.*

12. Find antilog 9.2618 – 10.

The number corresponding to the mantissa 2601 is 1820.
The number corresponding to the mantissa 2625 is 1830.
The difference between these numbers is 10,

and $1820 + \frac{17}{24}$ of $10 = 1827$.

Therefore, the number required is 0.1827. *Ans.*

13. Find antilog 8.7324 – 10.

The number corresponding to the mantissa 7324 is 5400.
Therefore, the number required is 0.054. *Ans.*

14. Find antilog 9.5555 – 10.

The number corresponding to the mantissa 5551 is 3590.
The number corresponding to the mantissa 5563 is 3600.
The difference between these numbers is 10,

and $3590 + \frac{4}{12}$ of $10 = 3593$.

Therefore, the number required is 0.3593. *Ans.*

15. Find antilog 6.0216 – 10.

The number corresponding to the mantissa 0212 is 1050.
The number corresponding to the mantissa 0253 is 1060.
The difference between these numbers is 10,

and $1050 + \frac{4}{41}$ of $10 = 1051$.

Therefore, the number required is 0.0001051. *Ans.*

16. Find antilog 7.0080 – 10.

The number corresponding to the mantissa 0043 is 1010.
The number corresponding to the mantissa 0086 is 1020.
The difference between these numbers is 10,

and $1010 + \frac{37}{43}$ of $10 = 1019$.

Therefore, the number required is 0.001019. *Ans.*

17. Find antilog 8.2361 − 10.

The number corresponding to the mantissa 2355 is 1720.
The number corresponding to the mantissa 2380 is 1730.
The difference between these numbers is 10,

and $1720 + \frac{6}{25}$ of $10 = 1722$.

Therefore, the number required is 0.01722. *Ans.*

18. Find antilog 9.4513 − 10.

The number corresponding to the mantissa 4502 is 2820.
The number corresponding to the mantissa 4518 is 2830.
The difference between these numbers is 10,

and $2820 + \frac{11}{16}$ of $10 = 2827$.

Therefore, the number required is 0.2827. *Ans.*

Exercise 153. Page 353.

1. Find by logarithms the value of 948.22 × 0.4387.

$$\begin{aligned} \log\ 948.22 &= 2.9769 \\ \log\ 0.4387 &= \underline{9.6422 - 10} \\ & 2.6191 \qquad = \log 416.\ \textit{Ans.} \end{aligned}$$

2. Find by logarithms the value of 1.9704 × 0.0786.

$$\begin{aligned} \log\ 1.9704 &= 0.2946 \\ \log\ 0.0786 &= \underline{8.8954 - 10} \\ & 9.1900 - 10 = \log 0.1549.\ \textit{Ans.} \end{aligned}$$

3. Find by logarithms the value of 380.25 × 0.00673.

$$\begin{aligned} \log\ 380.25 &= 2.5801 \\ \log 0.00673 &= \underline{7.8280 - 10} \\ & 0.4081 \qquad = \log 2.559.\ \textit{Ans.} \end{aligned}$$

4. Find by logarithms the value of 270.05 × 0.0087.

$$\begin{aligned} \log\ 270.05 &= 2.4315 \\ \log\ 0.0087 &= \underline{7.9395 - 10} \\ & 0.3710 \qquad = \log 2.349.\ \textit{Ans.} \end{aligned}$$

5. Find by logarithms the value of 11.163 × 0.3333.

$$\begin{aligned} \log\ 11.163 &= 1.0478 \\ \log\ 0.3333 &= \underline{9.5228 - 10} \\ & 0.5706 \qquad = \log 3.721.\ \textit{Ans.} \end{aligned}$$

6. Find by logarithms the value of 777.78 × 0.0787.

$$\begin{aligned} \log\ 777.78 &= 2.8909 \\ \log\ 0.0787 &= \underline{8.8960 - 10} \\ & 1.7869 \qquad = \log 61.21.\ \textit{Ans.} \end{aligned}$$

7. Find by logarithms the value of 216.21×0.76312.

$$\begin{array}{rl} \log 216.21 = & 2.3349 \\ \log 0.76312 = & \underline{9.8826 - 10} \\ & 2.2175 \qquad = \log 165. \ \textit{Ans.} \end{array}$$

8. Find by logarithms the value of 0.56127×1.2312.

$$\begin{array}{rl} \log 0.56127 = & 9.7492 - 10 \\ \log 1.2312 = & \underline{0.0903 \qquad\quad} \\ & 9.8395 - 10 = \log 0.691. \ \textit{Ans.} \end{array}$$

9. Find by logarithms the value of 0.86311×56.371.

$$\begin{array}{rl} \log 0.86311 = & 9.9361 - 10 \\ \log 56.371 = & \underline{1.7511 \qquad\quad} \\ & 1.6872 \qquad = \log 48.67. \ \textit{Ans.} \end{array}$$

10. Find by logarithms the value of 59.795×0.7955.

$$\begin{array}{rl} \log 59.795 = & 1.7767 \\ \log 0.7955 = & \underline{9.9007 - 10} \\ & 1.6774 \qquad = \log 47.58. \ \textit{Ans.} \end{array}$$

11. Find by logarithms the value of 2.6537×0.2313.

$$\begin{array}{rl} \log 2.6537 = & 0.4238 \\ \log 0.2313 = & \underline{9.3642 - 10} \\ & 9.7880 - 10 = \log 0.6137. \ \textit{Ans.} \end{array}$$

12. Find by logarithms the value of 37.587×12.371.

$$\begin{array}{rl} \log 37.587 = & 1.5750 \\ \log 12.371 = & \underline{1.0924} \\ & 2.6674 \qquad = \log 464.9. \ \textit{Ans.} \end{array}$$

13. Find by logarithms the value of 89.313×2.3781.

$$\begin{array}{rl} \log 89.313 = & 1.9510 \\ \log 2.3781 = & \underline{0.3762} \\ & 2.3272 \qquad = \log 212.4. \ \textit{Ans.} \end{array}$$

14. Find by logarithms the value of 9.1765×0.089.

$$\begin{array}{rl} \log 9.1765 = & 0.9627 \\ \log 0.089 = & \underline{8.9494 - 10} \\ & 9.9121 - 10 = \log 0.8168. \ \textit{Ans.} \end{array}$$

[redacted]

15. Find by logarithms the value of $4786 \times 5.4187 \times 0.00218 \times 0.8652$.

$$\begin{aligned}
\log 4786 &= 3.6799 \\
\log 5.4187 &= 0.7339 \\
\log 0.00218 &= 7.3385 - 10 \\
\log 0.8652 &= \underline{9.9371 - 10} \\
&\ 1.6894 \quad = \log 48.91. \quad \textit{Ans.}
\end{aligned}$$

16. Find by logarithms the value of $3.1416 \times 7.77 \times 184 \times 0.01865$.

$$\begin{aligned}
\log 3.1416 &= 0.4971 \\
\log 7.77 &= 0.8904 \\
\log 184 &= 2.2648 \\
\log 0.01865 &= \underline{8.2707 - 10} \\
&\ 1.9230 \quad = \log 83.76. \quad \textit{Ans.}
\end{aligned}$$

17. Find by logarithms the value of $0.7854 \times 129.6 \times 63.45 \times 0.0021$.

$$\begin{aligned}
\log 0.7854 &= 9.8951 - 10 \\
\log 129.6 &= 2.1126 \\
\log 63.45 &= 1.8025 \\
\log 0.0021 &= \underline{7.3222 - 10} \\
&\ 1.1324 \quad = \log 13.57. \quad \textit{Ans.}
\end{aligned}$$

18. Find by logarithms the value of $1842.65 \times 9.876 \times 0.843 \times 0.0265$.

$$\begin{aligned}
\log 1842.65 &= 3.2654 \\
\log 9.876 &= 0.9946 \\
\log 0.843 &= 9.9258 - 10 \\
\log 0.0265 &= \underline{8.4232 - 10} \\
&\ 2.6090 \quad = \log 406.5. \quad \textit{Ans.}
\end{aligned}$$

19. Find by logarithms the value of $12.48 \times 44.63 \times 32.78 \times 0.004587$.

$$\begin{aligned}
\log 12.48 &= 1.0962 \\
\log 44.63 &= 1.6496 \\
\log 32.78 &= 1.5156 \\
\log 0.004587 &= \underline{7.6615 - 10} \\
&\ 1.9229 \quad = \log 83.74. \quad \textit{Ans.}
\end{aligned}$$

20. Find by logarithms the value of $0.9876 \times 0.8765 \times 0.7654 \times 0.6543$.

$$\begin{aligned}
\log 0.9876 &= 9.9946 - 10 \\
\log 0.8765 &= 9.9428 - 10 \\
\log 0.7654 &= 9.8839 - 10 \\
\log 0.6543 &= \underline{9.8158 - 10} \\
&\ 9.6371 - 10 \quad = \log 0.4336. \quad \textit{Ans.}
\end{aligned}$$

Exercise 154. Page 354.

1. Find by logarithms the value of 5.06^3.

$$\begin{array}{rl} \log 5.06 = & 0.7042 \\ & \underline{3} \\ & 2.1126 \quad = \log 129.6. \ \textit{Ans.} \end{array}$$

2. Find by logarithms the value of 2.501^5.

$$\begin{array}{rl} \log 2.501 = & 0.3981 \\ & \underline{5} \\ & 1.9905 \quad = \log 97.84. \ \textit{Ans.} \end{array}$$

3. Find by logarithms the value of 1.716^7.

$$\begin{array}{rl} \log 1.716 = & 0.2345 \\ & \underline{7} \\ & 1.6415 \quad = \log 43.8. \ \textit{Ans.} \end{array}$$

4. Find by logarithms the value of 1.178^{10}.

$$\begin{array}{rl} \log 1.178 = & 0.0712 \\ & \underline{10} \\ & 0.7120 \quad = \log 5.153. \ \textit{Ans.} \end{array}$$

5. Find by logarithms the value of 7.6821^6.

$$\begin{array}{rl} \log 7.6821 = & 0.8855 \\ & \underline{6} \\ & 5.3130 \quad = \log 205{,}600. \ \textit{Ans.} \end{array}$$

6. Find by logarithms the value of 0.7685^6.

$$\begin{array}{rl} \log 0.7685 = & 9.8857 - 10 \\ & \underline{6} \\ & 9.3142 - 10 = \log 0.2061. \ \textit{Ans.} \end{array}$$

7. Find by logarithms the value of 0.9611^8.

$$\begin{array}{rl} \log 0.9611 = & 9.9828 - 10 \\ & \underline{8} \\ & 9.8624 - 10 = \log 0.7285. \ \textit{Ans.} \end{array}$$

8. Find by logarithms the value of 0.0231^2.

$$\begin{array}{rl} \log 0.0231 = & 8.3636 - 10 \\ & \underline{2} \\ & 6.7272 - 10 = \log 0.0005336. \ \textit{Ans.} \end{array}$$

[redacted]

9. Find by logarithms the value of 0.8567^3.

$$\begin{array}{r}\log 0.8567 = 9.9329 - 10\\ \underline{3}\\ 9.7987 - 10 = \log 0.629.\end{array}$$ *Ans.*

10. Find by logarithms the value of 0.5438^5.

$$\begin{array}{r}\log 0.5438 = 9.7354 - 10\\ \underline{5}\\ 8.6770 - 10 = \log 0.04753.\end{array}$$ *Ans.*

11. Find by logarithms the value of 2.861415^4.

$$\begin{array}{r}\log 2.861415 = 0.4566\\ \underline{4}\\ 1.8264 \quad = \log 67.05.\end{array}$$ *Ans.*

12. Find by logarithms the value of 3.79125^6.

$$\begin{array}{r}\log 3.79125 = 0.5788\\ \underline{6}\\ 3.4728 \quad = \log 2970.\end{array}$$ *Ans.*

13. Find by logarithms the value of 0.021875^5.

$$\begin{array}{r}\log 0.021875 = 8.3399 - 10\\ \underline{5}\\ 1.6995 - 10 = \log 0.000000005006.\end{array}$$ *Ans.*

14. Find by logarithms the value of 0.87152^7.

$$\begin{array}{r}\log 0.87152 = 9.9403 - 10\\ \underline{7}\\ 9.5821 - 10 = \log 0.382.\end{array}$$ *Ans.*

15. Find by logarithms the value of 0.95956^8.

$$\begin{array}{r}\log 0.95956 = 9.9821 - 10\\ \underline{8}\\ 9.8568 - 10 = \log 0.7192.\end{array}$$ *Ans.*

Exercise 155. Page 355.

1. Find by logarithms the value of $13^{\frac{1}{3}}$.

$$\begin{array}{r}\log 13 = 1.1139\\ 3\,\underline{|1.1139}\\ 0.3713\\ = \log 2.351.\end{array}$$ *Ans.*

2. Find by logarithms the value of $29^{\frac{1}{5}}$.

$$\begin{array}{r}\log 29 = 1.4624\\ 5\,\underline{|1.4624}\\ 0.2925\\ = \log 1.961.\end{array}$$ *Ans.*

3. Find by logarithms the value of $471^{\frac{1}{4}}$.

$$\begin{array}{r} \log 471 = 2.6730 \\ 4\,\underline{|2.6730} \\ 0.6683 \\ = \log 4.659.\ \textit{Ans.} \end{array}$$

4. Find by logarithms the value of $288^{\frac{1}{6}}$.

$$\begin{array}{r} \log 288 = 2.4594 \\ 6\,\underline{|2.4594} \\ 0.4099 \\ = \log 2.57.\ \textit{Ans.} \end{array}$$

5. Find by logarithms the value of $1019^{\frac{1}{7}}$.

$$\begin{array}{r} \log 1019 = 3.0082 \\ 7\,\underline{|3.0082} \\ 0.4297 \\ = \log 2.689.\ \textit{Ans.} \end{array}$$

6. Find by logarithms the value of $1281^{\frac{1}{8}}$.

$$\begin{array}{r} \log 1281 = 3.1075 \\ 8\,\underline{|3.1075} \\ 0.3884 \\ = \log 2.446.\ \textit{Ans.} \end{array}$$

7. Find by logarithms the value of $1862^{\frac{1}{9}}$.

$$\begin{array}{r} \log 1862 = 3.2700 \\ 9\,\underline{|3.2700} \\ 0.3633 \\ = \log 2.308.\ \textit{Ans.} \end{array}$$

8. Find by logarithms the value of $879^{\frac{1}{10}}$.

$$\begin{array}{r} \log 879 = 2.9440 \\ 10\,\underline{|2.9440} \\ 0.2944 \\ = \log 1.97.\ \textit{Ans.} \end{array}$$

9. Find by logarithms the value of $0.609^{\frac{1}{4}}$.

$$\begin{array}{rr} \log 0.609 = & 9.7846 - 10 \\ & 30. \quad\ - 30 \\ 4 & \underline{|39.7846 - 40} \\ & 9.9462 - 10 \\ & = \log 0.8834.\ \textit{Ans.} \end{array}$$

10. Find by logarithms the value of $0.8716^{\frac{1}{5}}$.

$$\begin{array}{rr} \log 0.8716 = & 9.9403 - 10 \\ & 40. \quad\ - 40 \\ 5 & \underline{|49.9403 - 50} \\ & 9.9881 - 10 \\ & = \log 0.973.\ \textit{Ans.} \end{array}$$

11. Find by logarithms the value of $0.021641^{\frac{1}{6}}$.

$$\begin{array}{rr} \log 0.021641 = & 8.3353 - 10 \\ & 50. \quad\ - 50 \\ 6 & \underline{|58.3353 - 60} \\ & 9.7226 - 10 \\ & = \log 0.528.\ \textit{Ans.} \end{array}$$

12. Find by logarithms the value of $0.9825^{\frac{1}{7}}$.

$$\begin{array}{rr} \log 0.9825 = & 9.9924 - 10 \\ & 60. \quad\ - 60 \\ 7 & \underline{|69.9924 - 70} \\ & 9.9989 - 10 \\ & = \log 0.9975.\ \textit{Ans.} \end{array}$$

13. Find by logarithms the value of $0.42184^{\frac{1}{8}}$.

$$\begin{array}{rr} \log 0.42184 = & 9.6251 - 10 \\ & 70. \quad\ - 70 \\ 8 & \underline{|79.6251 - 80} \\ & 9.9531 - 10 \\ & = \log 0.8976.\ \textit{Ans.} \end{array}$$

[redacted]

14. Find by logarithms the value of $0.02187^{\frac{1}{9}}$.

$$\begin{array}{rr} \log 0.02187 = & 8.3398 - 10 \\ & 80. \quad - 80 \\ 9 \,| & \underline{88.3398 - 90} \\ & 9.8156 - 10 \\ & = \log 0.6539. \; \textit{Ans.} \end{array}$$

15. Find by logarithms the value of $93.73^{\frac{1}{2}}$.

$$\begin{array}{rr} \log 93.73 = & 1.9719 \\ 2 \,| & \underline{1.9719} \\ & 0.9860 \\ & = \log 9.683. \; \textit{Ans.} \end{array}$$

16. Find by logarithms the value of $21.97^{\frac{5}{6}}$.

$$\begin{array}{rr} \log 21.97 = & 1.3418 \\ & \underline{5} \\ 6 \,| & \underline{6.7090} \\ & 1.1182 \\ & = \log 13.13. \; \textit{Ans.} \end{array}$$

17. Find by logarithms the value of $7.935^{\frac{5}{7}}$.

$$\begin{array}{rr} \log 7.935 = & 0.8996 \\ & \underline{5} \\ 7 \,| & \underline{4.4980} \\ & 0.6426 \\ & = \log 4.391. \; \textit{Ans.} \end{array}$$

18. Find by logarithms the value of $0.815^{\frac{3}{4}}$.

$$\begin{array}{rr} \log 0.815 = & 9.9112 - 10 \\ & \underline{3} \\ & 9.7336 - 10 \\ & 30. \quad - 30 \\ 4 \,| & \underline{39.7336 - 40} \\ & 9.9334 - 10 \\ & = \log 0.8578. \; \textit{Ans.} \end{array}$$

19. Find by logarithms the value of $2.8145^{\frac{2}{3}}$.

$$\begin{array}{rr} \log 2.8145 = & 0.4494 \\ & \underline{2} \\ 3 \,| & \underline{0.8988} \\ & 0.2996 \\ & = \log 1.993. \; \textit{Ans.} \end{array}$$

20. Find by logarithms the value of $0.04165^{\frac{9}{14}}$.

$$\begin{array}{rr} \log 0.04165 = & 8.6196 - 10 \\ & \underline{9} \\ & 7.5764 - 10 \\ & 130. \quad - 130 \\ 14 \,| & \underline{137.5764 - 140} \\ & 9.8269 - 10 \\ & = \log 0.6713. \; \textit{Ans.} \end{array}$$

21. Find by logarithms the value of $4{,}516{,}298^{\frac{1}{15}}$.

$$\begin{array}{rr} \log 4{,}516{,}298 = & 6.6548 \\ 15 \,| & \underline{6.6548} \\ & 0.4437 \\ & = \log 2.778. \; \textit{Ans.} \end{array}$$

Exercise 156. Page 357.

1. Find by logarithms the value of $\frac{56.407}{13.045}$.

$$\begin{aligned} \log 56.407 &= 1.7513 \\ \text{colog } 13.045 &= \underline{8.8846 - 10} \\ &\quad 0.6359 \\ &= \log 4.324. \quad \textit{Ans.} \end{aligned}$$

2. Find by logarithms the value of $\frac{857.06}{3079.8}$.

$$\begin{aligned} \log 857.06 &= 2.9330 \\ \text{colog } 3079.8 &= \underline{6.5114 - 10} \\ &\quad 9.4444 - 10 \\ &= \log 0.2783. \quad \textit{Ans.} \end{aligned}$$

3. Find by logarithms the value of $\frac{0.9387}{598.6}$.

$$\begin{aligned} \log 0.9387 &= 9.9726 - 10 \\ \text{colog } 598.6 &= \underline{7.2229 - 10} \\ &\quad 7.1955 - 10 \\ &= \log 0.001569. \quad \textit{Ans.} \end{aligned}$$

4. Find by logarithms the value of $\frac{3069}{0.7891}$.

$$\begin{aligned} \log 3069 &= 3.4870 \\ \text{colog } 0.7891 &= \underline{0.1028} \\ &\quad 3.5898 \\ &= \log 3889. \quad \textit{Ans.} \end{aligned}$$

5. Find by logarithms the value of $\frac{75.46 \times 0.0765}{93.08 \times 98.071}$.

$$\begin{aligned} \log 75.46 &= 1.8777 \\ \log 0.0765 &= 8.8837 - 10 \\ \text{colog } 93.08 &= 8.0312 - 10 \\ \text{colog } 98.071 &= \underline{8.0084 - 10} \\ &\quad 6.8010 - 10 \\ &= \log 0.0006324. \quad \textit{Ans.} \end{aligned}$$

6. Find by logarithms the value of $\frac{98 \times 537 \times 0.0079}{67309 \times 0.0947}$.

$$\begin{aligned} \log 98 &= 1.9912 \\ \log 537 &= 2.7300 \\ \log 0.0079 &= 7.8976 - 10 \\ \text{colog } 67309 &= 5.1719 - 10 \\ \text{colog } 0.0947 &= \underline{1.0237} \\ &\quad 8.8144 - 10 \\ &= \log 0.06523. \quad \textit{Ans.} \end{aligned}$$

7. Find by logarithms the value of $\frac{314 \times 7.18 \times 8132}{519 \times 827 \times 3.215}$.

$$\begin{aligned} \log 314 &= 2.4969 \\ \log 7.18 &= 0.8561 \\ \log 8132 &= 3.9102 \\ \text{colog } 519 &= 7.2848 - 10 \\ \text{colog } 827 &= 7.0825 - 10 \\ \text{colog } 3.215 &= \underline{9.4928 - 10} \\ &\quad 1.1233 \\ &= \log 13.28. \quad \textit{Ans.} \end{aligned}$$

8. Find by logarithms the value of $\frac{212 \times 2.16 \times 8002}{536 \times 351 \times 7.256}$.

$$\begin{aligned} \log 212 &= 2.3263 \\ \log 2.16 &= 0.3345 \\ \log 8002 &= 3.9032 \\ \text{colog } 536 &= 7.2708 - 10 \\ \text{colog } 351 &= 7.4547 - 10 \\ \text{colog } 7.256 &= \underline{9.1393 - 10} \\ &\quad 0.4288 \\ &= \log 2.684. \quad \textit{Ans.} \end{aligned}$$

[redacted]

9. Find by logarithms the value of $\left(\frac{61}{73}\right)^4$.

$$\begin{array}{rl} \log 61 = & 1.7853 \\ \text{colog}\, 73 = & 8.1367 - 10 \\ \hline & 9.9220 - 10 \\ & \quad 4 \\ \hline & 9.6880 - 10 \end{array}$$

$= \log 0.4876$. *Ans.*

10. Find by logarithms the value of $\left(\frac{13}{71}\right)^3$.

$$\begin{array}{rl} \log 13 = & 1.1139 \\ \text{colog}\, 71 = & 8.1487 - 10 \\ \hline & 9.2626 - 10 \\ & \quad 3 \\ \hline & 7.7878 - 10 \end{array}$$

$= \log 0.006134$. *Ans.*

11. Find by logarithms the value of $\left(5\frac{5}{11}\right)^2$.

$5\frac{5}{11} = \frac{60}{11}$.

$$\begin{array}{rl} \log 60 = & 1.7782 \\ \text{colog}\, 11 = & 8.9586 - 10 \\ \hline & 0.7368 \\ & \quad 2 \\ \hline & 1.4736 \end{array}$$

$= \log 29.76$. *Ans.*

12. Find by logarithms the value of $\left(4\frac{4}{31}\right)^3$.

$4\frac{4}{31} = \frac{128}{31}$.

$$\begin{array}{rl} \log 128 = & 2.1072 \\ \text{colog}\ \ 31 = & 8.5086 - 10 \\ \hline & 0.6158 \\ & \quad 3 \\ \hline & 1.8474 \end{array}$$

$= \log 70.37$. *Ans.*

13. Find by logarithms the value of $\left(\frac{412}{617}\right)^5$.

$$\begin{array}{rl} \log 412 = & 2.6149 \\ \text{colog}\, 617 = & 7.2097 - 10 \\ \hline & 9.8246 - 10 \\ & \quad 5 \\ \hline & 9.1230 - 10 \end{array}$$

$= \log 0.1327$. *Ans.*

14. Find by logarithms the value of $\left(\frac{83}{97}\right)^8$.

$$\begin{array}{rl} \log 83 = & 1.9191 \\ \text{colog}\, 97 = & 8.0132 - 10 \\ \hline & 9.9323 - 10 \\ & \quad 8 \\ \hline & 9.4584 - 10 \end{array}$$

$= \log 0.2873$. *Ans.*

15. Find by logarithms the value of $\left(\frac{507}{622}\right)^3$.

$$\begin{array}{rl} \log 507 = & 2.7050 \\ \text{colog}\, 622 = & 7.2062 - 10 \\ \hline & 9.9112 - 10 \\ & \quad 3 \\ \hline & 9.7336 - 10 \end{array}$$

$= \log 0.5415$. *Ans.*

16. Find by logarithms the value of $\left(\frac{1741}{1816}\right)^8$.

$$\begin{array}{rl} \log 1741 = & 3.2408 \\ \text{colog}\, 1816 = & 6.7409 - 10 \\ \hline & 9.9817 - 10 \\ & \quad 8 \\ \hline & 9.8536 - 10 \end{array}$$

$= \log 0.7138$. *Ans.*

[redacted]

17. Find by logarithms the value of

$$\frac{19.258 \times 3.1416 \times 812.72}{716.4 \times 8.002 \times 21.465}.$$

$$\begin{aligned}
\log 19.258 &= 1.2846 \\
\log 3.1416 &= 0.4971 \\
\log 812.72 &= 2.9100 \\
\text{colog } 716.4 &= 7.1449 - 10 \\
\text{colog } 8.002 &= 9.0968 - 10 \\
\text{colog } 21.465 &= \underline{8.6683 - 10} \\
& 9.6017 - 10 \\
&= \log 0.3996. \; \textit{Ans.}
\end{aligned}$$

18. Find by logarithms the value of

$$\frac{2018 \times 0.00261 \times 1728}{1412 \times 0.0965 \times 0.08621}.$$

$$\begin{aligned}
\log 2018 &= 3.3050 \\
\log 0.00261 &= 7.4166 - 10 \\
\log 1728 &= 3.2375 \\
\text{colog } 1412 &= 6.8502 - 10 \\
\text{colog } 0.0965 &= 1.0155 \\
\text{colog } 0.08621 &= \underline{1.0644} \\
& 2.8892 \\
&= \log 774.8. \; \textit{Ans.}
\end{aligned}$$

19. Find by logarithms the value of

$$\frac{44816 \times 17.265 \times 181}{28754 \times 1.2871 \times 206.45}.$$

$$\begin{aligned}
\log 44816 &= 4.6514 \\
\log 17.265 &= 1.2371 \\
\log 181 &= 2.2577 \\
\text{colog } 28754 &= 5.5413 - 10 \\
\text{colog } 1.2871 &= 9.8904 - 10 \\
\text{colog } 206.45 &= \underline{7.6852 - 10} \\
& 1.2631 \\
&= \log 18.33. \; \textit{Ans.}
\end{aligned}$$

20. Find by logarithms the value of

$$\frac{216.1 \times 5280 \times 144.2}{187.42 \times 4622.6 \times 156.8}.$$

$$\begin{aligned}
\log 216.1 &= 2.3347 \\
\log 5280 &= 3.7226 \\
\log 144.2 &= 2.1590 \\
\text{colog } 187.42 &= 7.7271 - 10 \\
\text{colog } 4622.6 &= 6.3351 - 10 \\
\text{colog } 156.8 &= \underline{7.8047} - 10 \\
& 0.0832 \\
&= \log 1.211. \; \textit{Ans.}
\end{aligned}$$

21. Find by logarithms the value of

$$\frac{5982.55 \times 0.02987 \times 0.9852}{42.875 \times 34.62 \times 28.47}.$$

$$\begin{aligned}
\log 5982.55 &= 3.7769 \\
\log 0.02987 &= 8.4753 - 10 \\
\log 0.9852 &= 9.9935 - 10 \\
\text{colog } 42.875 &= 8.3678 - 10 \\
\text{colog } 34.62 &= 8.4607 - 10 \\
\text{colog } 28.47 &= \underline{8.5456 - 10} \\
& 7.6198 - 10 \\
&= \log 0.004167. \; \textit{Ans.}
\end{aligned}$$

22. Find by logarithms the value of

$$\frac{14.718 \times 48.67 \times 96.542}{2746.2 \times 0.0467 \times 2.1876}.$$

$$\begin{aligned}
\log 14.718 &= 1.1678 \\
\log 48.67 &= 1.6872 \\
\log 96.542 &= 1.9847 \\
\text{colog } 2746.2 &= 6.5613 - 10 \\
\text{colog } 0.0467 &= 1.3307 \\
\text{colog } 2.1876 &= \underline{9.6601} - 10 \\
& 2.3918 \\
&= \log 246.5. \; \textit{Ans.}
\end{aligned}$$

[redacted] [redacted]

23. Find by logarithms the value of

$$\sqrt{\frac{83.25 \times 4267 \times 0.008576}{0.0327 \times 687.5 \times 0.005003}}.$$

$$\begin{array}{lrl}
\log & 83.25 & = 1.9204 \\
\log & 4267 & = 3.6301 \\
\log & 0.008576 & = 7.9333 - 10 \\
\text{colog} & 0.0327 & = 1.4855 \\
\text{colog} & 687.5 & = 7.1627 - 10 \\
\text{colog} & 0.005003 & = 2.3008 \\
 & 2 & \underline{|4.4328} \\
 & & \ 2.2164 \\
 & & = \log 164.6.\ \textit{Ans.}
\end{array}$$

24. Find by logarithms the value of

$$\sqrt[3]{\frac{4.163^2 \times 17.74^4 \times 0.7183^{\frac{1}{2}}}{3.013^2 \times 34.34 \times 0.08137^{\frac{1}{2}}}}.$$

$$\begin{array}{lrl}
\log & 4.163^2 & = 1.2388 \\
\log & 17.74^4 & = 4.9960 \\
\log & 0.7183^{\frac{1}{2}} & = 9.9282 - 10 \\
\text{colog} & 3.013^2 & = 9.0420 - 10 \\
\text{colog} & 34.34 & = 8.4642 - 10 \\
\text{colog} & 0.08137^{\frac{1}{2}} & = 0.5447 \\
 & 3 & \underline{|4.2139} \\
 & & \ 1.4046 \\
 & & = \log 25.39.\ \textit{Ans.}
\end{array}$$

25. Find by logarithms the value of

$$\sqrt[4]{\frac{0.7132 \times 9.245 \times 0.5477^2}{76.93 \times 0.000173^{\frac{1}{3}} \times 0.01}}.$$

$$\begin{array}{lrl}
\log & 0.7132 & = 9.8532 - 10 \\
\log & 9.245 & = 0.9659 \\
\log & 0.5477^2 & = 9.4772 - 10 \\
\text{colog} & 76.93 & = 8.1139 - 10 \\
\text{colog} & 0.000173^{\frac{1}{3}} & = 1.2540 \\
\text{colog} & 0.01 & = 2.0000 \\
 & 4 & \underline{|1.6642} \\
 & & \ 0.4161 \\
 & & = \log 2.607.\ \textit{Ans.}
\end{array}$$

26. Find by logarithms the value of

$$\sqrt[5]{\frac{65.02^2 \times 0.002753 \times 97.98^{\frac{1}{2}}}{7.298 \times 0.04754 \times 8.156^2}}.$$

$$\begin{array}{lrl}
\log & 65.02^2 & = 3.6260 \\
\log & 0.002753 & = 7.4398 - 10 \\
\log & 97.98^{\frac{1}{2}} & = 0.9956 \\
\text{colog} & 7.298 & = 9.1368 - 10 \\
\text{colog} & 0.04754 & = 1.3229 \\
\text{colog} & 8.156^2 & = 8.1770 - 10 \\
 & 5 & \underline{|0.6981} \\
 & & \ 0.1396 \\
 & & = \log 1.379.\ \textit{Ans.}
\end{array}$$

27. Find by logarithms the value of

$$\sqrt[6]{\frac{23.79^2 \times 0.00756 \times 0.4648^{\frac{1}{3}}}{4723^{\frac{1}{2}} \times 0.6571 \times 0.8246^{1}}}.$$

$$\begin{array}{lrl}
\log & 23.79^2 & = 2.7528 \\
\log & 0.00756 & = 7.8785 - 10 \\
\log & 0.4648^{\frac{1}{3}} & = 9.8891 - 10 \\
\text{colog} & 4723^{\frac{1}{2}} & = 8.1629 - 10 \\
\text{colog} & 0.6571 & = 0.1823 \\
\text{colog} & 0.8246^{\frac{1}{2}} & = \underline{0.0209} \\
 & & \ 8.8865 - 10 \\
 & & \ 50. \quad\ - 50 \\
 & 6 & \underline{|58.8865 - 60} \\
 & & \ 9.8144 - 10 \\
 & & = \log 0.6523.\ \textit{Ans.}
\end{array}$$

[redacted]

28. Find by logarithms the value of

$$\sqrt[7]{\frac{0.6012 \times 0.6012^{\frac{1}{2}} \times 0.6012^{\frac{1}{3}}}{0.5926 \times 0.5926^{\frac{1}{2}} \times 0.5926^{\frac{1}{3}}}}$$

$$\begin{aligned}
\log\ 0.6012 &= 9.7790-10\\
\log\ 0.6012^{\frac{1}{2}} &= 9.8895-10\\
\log\ 0.6012^{\frac{1}{3}} &= 9.9263-10\\
\operatorname{colog}\ 0.5926 &= 0.2272\\
\operatorname{colog}\ 0.5926^{\frac{1}{2}} &= 0.1136\\
\operatorname{colog}\ 0.5926^{\frac{1}{3}} &= \underline{0.0757}\\
7\,&\underline{\big|0.0113}\\
&\ 0.0016\\
&= \log 1.004.\ \textit{Ans.}
\end{aligned}$$

29. Find by logarithms the value of

$$\left(\frac{0.03214 \times 3.718^{3} \times 0.07824^{\frac{1}{2}}}{0.05142 \times 0.4728^{\frac{1}{2}} \times 1.239^{3}}\right)^{\frac{3}{4}}.$$

$$\begin{aligned}
\log\ 0.03214 &= 8.5071-10\\
\log\ 3.718^{3} &= 1.7109\\
\log\ 0.07824^{\frac{1}{2}} &= 9.4467-10\\
\operatorname{colog}\ 0.05142 &= 1.2888\\
\operatorname{colog}\ 0.4728^{\frac{1}{2}} &= 0.1626\\
\operatorname{colog}\ 1.239^{3} &= \underline{9.7207-10}\\
&\ 0.8368\\
&\ \underline{3}\\
4\,&\underline{\big|2.5104}\\
&\ 0.6276\\
&= \log 4.242.\ \textit{Ans.}
\end{aligned}$$

30. Find by logarithms the value of

$$\left(\frac{0.07986 \times 0.7555^{\frac{1}{3}} \times 0.5557^{\frac{1}{3}}}{0.06897 \times 0.5777^{\frac{1}{3}} \times 0.05698^{2}}\right)^{\frac{3}{7}}.$$

$$\begin{aligned}
\log\ 0.07986 &= 8.9023-10\\
\log\ 0.7555^{\frac{1}{3}} &= 9.9594-10\\
\log\ 0.5557^{\frac{1}{3}} &= 9.9150-10\\
\operatorname{colog}\ 0.06897 &= 1.1614\\
\operatorname{colog}\ 0.5777^{\frac{1}{3}} &= 0.0794\\
\operatorname{colog}\ 0.05698^{2} &= \underline{2.4886}\\
&\ 2.5061\\
&\ \underline{3}\\
7\,&\underline{\big|7.5183}\\
&\ 1.0740\\
&= \log 11.86.\ \textit{Ans.}
\end{aligned}$$

31. Find by logarithms the value of

$$\left(\frac{0.07543 \times 0.7689^{\frac{1}{2}} \times 0.8965^{2}}{0.06987 \times 0.07986^{\frac{1}{2}} \times 0.9867^{\frac{2}{3}}}\right)^{\frac{5}{6}}.$$

$$\begin{aligned}
\log 0.07543 &= 8.8776-10\\
\log 0.7689^{\frac{1}{2}} &= 9.9429-10\\
\log 0.8965^{2} &= 9.9052-10\\
\operatorname{colog} 0.06987 &= 1.1557\\
\operatorname{colog} 0.07986^{\frac{1}{2}} &= 0.5488\\
\operatorname{colog} 0.9867^{\frac{2}{3}} &= \underline{0.0039}\\
&\ 0.4341\\
&\ \underline{5}\\
6\,&\underline{\big|2.1705}\\
&\ 0.3618\\
&= \log 2.301.\ \textit{Ans.}
\end{aligned}$$

[redacted]

Exercise 157. Page 360.

1. Find the compound interest on \$1280 for 7 years at $4\frac{1}{2}\%$.

$$A = 1280 \times 1.045^7.$$
$$\log\ 1280 = 3.1072$$
$$\log 1.045^7 = \underline{0.1337}$$
$$3.2409$$
$$= \log 1742.$$

\$1742 − \$1280 = \$462. *Ans.*

2. Find the compound interest on \$2645 for 5 years at $3\frac{1}{2}\%$.

$$A = 2645 \times 1.035^5.$$
$$\log\ 2645 = 3.4224$$
$$\log 1.035^5 = \underline{0.0745}$$
$$3.4969$$
$$= \log 3140.$$

\$3140 − \$2645 = \$495. *Ans.*

3. Find the amount of \$848 for 6 years at 5% compound interest.

$$A = 848 \times 1.05^6.$$
$$\log\ 848 = 2.9284$$
$$\log 1.05^6 = \underline{0.1272}$$
$$3.0556$$
$$= \log 1137.$$

\$1137. *Ans.*

4. Find the amount of \$3600 for 5 years at $5\frac{1}{2}\%$ compound interest.

$$A = 3600 \times 1.055^5.$$
$$\log\ 3600 = 3.5563$$
$$\log 1.055^5 = \underline{0.1165}$$
$$3.6728$$
$$= \log 4708.$$

\$4708. *Ans.*

5. What principal will amount to \$720 in 4 years at 6% compound interest?

$$720 = P \times 1.04^6.$$
$$\therefore\ P = \frac{720}{1.04^6}.$$
$$\log\ 720 = 2.8573$$
$$\text{colog } 1.04^6 = \underline{9.8980} - 10$$
$$2.7553$$
$$= \log 569.3.$$

\$569.30. *Ans.*

6. What principal will amount to \$1640 in 6 years at 3% compound interest?

$$1640 = P \times 1.03^6.$$
$$\therefore\ P = \frac{1640}{1.03^6}.$$
$$\log\ 1640 = 3.2148$$
$$\text{colog } 1.03^6 = \underline{9.9232} - 10$$
$$3.1380$$
$$= \log 1374.$$

\$1374. *Ans.*

7. At what rate of interest will \$648 amount to \$788.20 in 5 years at compound interest?

$$788.20 = 648 \times (1 + r)^5.$$
$$\therefore (1 + r)^5 = \frac{788.20}{648},$$
and
$$1 + r = \sqrt[5]{\frac{788.20}{648}}.$$
$$\log 788.20 = 2.8966$$
$$\text{colog}\ \ 648 = 7.1884 - 10$$
$$5 \,\underline{|0.0850}$$
$$0.0170$$
$$= \log 1.04.$$

Therefore, the required rate of interest is 4%. *Ans.*

8. At what rate of interest will \$2415 amount to \$3237 in 6 years at compound interest?

$$3237 = 2415 \times (1 + r)^6.$$

$$\therefore (1 + r)^6 = \frac{3237}{2415},$$

and $$1 + r = \sqrt[6]{\frac{3237}{2415}}.$$

$$\begin{aligned} \log 3237 &= 3.5101 \\ \text{colog } 2415 &= 6.6171 - 10 \\ 6 &\,\underline{|\,0.1272} \\ &\ 0.0212 \\ &= \log 1.05. \end{aligned}$$

Therefore, the required rate of interest is 5%. *Ans.*

9. In what time at $4\frac{1}{2}$% compound interest will \$1265 amount to \$1576?

$\log 1576 = \log 1265 + n \times \log 1.045,$
$n \times \log 1.045 = \log 1576 + \text{colog } 1265.$

$$n = \frac{\log 1576 + \text{colog } 1265}{\log 1.045}$$

$$= \frac{3.1976 + 6.8979 - 10}{0.0191}$$

$$= \frac{0.0955}{0.0191} = 5.$$

The required time is 5 years. *Ans.*

10. In what time at 5% compound interest will \$1845 amount to \$2413?

$\log 2413 = \log 1845 + n \times \log 1.05,$
$n \times \log 1.05 = \log 2413 + \text{colog } 1845.$

$$n = \frac{\log 2413 + \text{colog } 1845}{\log 1.05}$$

$$= \frac{3.3825 + 6.7340 - 10}{0.0212}$$

$$= \frac{0.1165}{0.0212} = 5.4953.$$

5.4953 yr. = 5 yr. 5 mo. 28 dy. *Ans.*

Exercise 158. Page 363.

1. A man deposits \$60 in a savings bank, and draws out his whole account at the end of 8 years, with 4% compound interest. What amount does he receive?

The amount of \$1 for 8 yr. at 4% is \$1.36857.

$$\begin{array}{r} \$1.36857 \\ 60 \\ \hline \$82.11420 \end{array}$$

\$82.11. *Ans.*

2. What will \$100 amount to in 7 years with interest at 8% per annum, compounded semi-annually?

The amount of \$1 for 14 yr. at 4% is \$1.73168.

$$\begin{array}{r} \$1.73168 \\ 100 \\ \hline \$173.16800 \end{array}$$

\$173.17. *Ans.*

[redacted]

3. In how many years will a sum of money double itself at 6%, compounded annually?

By the table, in a little less than 12 yr. *Ans.*

4. In how many years will a sum of money treble itself at 6%, compounded annually?

By the table, in a little less than 19 yr. *Ans.*

5. In how many years will \$87 amount to \$99 at 3%, compounded annually?

Since \$87 amounts to \$99, \$1 amounts to $\$\frac{99}{87} = \$1.13793$. By the table, \$1 will in 4 yr. at 3% amount to \$1.12551.

Hence, the required time is a little more than 4 yr. *Ans.*

6. In how many years will \$100 amount to \$175 at 4%, compounded annually?

Since \$100 amounts to \$175, \$1 amounts to $\$\frac{175}{100} = \$1.75000$. By the table, \$1 will in 14 yr. at 4% amount to \$1.73168.

Hence, the required time is a little more than 14 yr. *Ans.*

7. At what rate per cent will a sum of money double itself in 12 years, compound interest?

\$1 will in 12 yr. amount to \$2. By the table, \$1 will in 12 yr. at 6% amount to \$2.01220. Hence, the required rate is 6%, nearly. *Ans.*

8. At what rate will a sum of money treble itself in 19 years, compound interest?

\$1 will in 19 yr. amount to \$3. By the table, \$1 will in 19 yr. at 6% amount to \$3.02560. Hence, the required rate is 6%, nearly. *Ans.*

9. At what rate will \$80 at compound interest amount to \$110 in 8 years?

Since \$80 amounts to \$110, \$1 amounts to $\$\frac{110}{80} = \$1.37500$. By the table, \$1 will in 8 yr. at 4% amount to \$1.36857. Hence, the required rate is 4%, nearly. *Ans.*

10. What sum must be invested at 5%, compound interest, to amount to \$1200 in 7 years?

The amount of \$1 for 7 yr. at 5% is \$1.40710. Since \$1.40710 is the amount of \$1, \$1200 is the amount of $\$\frac{1200}{1.40710} = \$852.83$. *Ans.*

11. What sum must be invested at 4%, compound interest, to amount to \$2000 in 10 years? To amount to \$5000 in 8 years?

The amount of \$1 for 10 yr. at 4% is \$1.48024. Since \$1.48024 is the amount of \$1, \$2000 is the amount of $\$\frac{2000}{1.48024} = \$1351.13$. *Ans.*

The amount of \$1 for 8 yr. at 4% is \$1.36857. Since \$1.36857 is the amount of \$1, \$5000 is the amount of $\$\frac{5000}{1.36857} = \$3653.45$. *Ans.*

12. At what rate compound interest will \$462.50 yield \$277.98 interest in 12 years?

The amount of \$462.50 for 12 yr. is \$462.50 + \$277.98 = \$740.48. Since the amount of \$462.50 is \$740.48, the amount of \$1 is $\$\frac{740.48}{462.50} = \$1.60103$. By the table, \$1 will in 12 yr. at 4% amount to \$1.60103. Hence, the required rate is 4%. *Ans.*

13. What principal will in 10 years at 6% amount to \$3612.22, interest being compounded semi-annually?

The amount of \$1 for 20 yr. at 3% is \$1.80611. Since \$1.80611 is the amount of \$1, \$3612.22 is the amount of $\$\frac{3612.22}{1.80611} = \$2000$. *Ans.*

14. In what time at 5% will \$1250 amount to \$2000, interest being compounded semi-annually?

Since \$1250 amounts to \$2000, \$1 amounts to $\$\frac{2000}{1250} = \$1.60000$. By the table, \$1 at 2½% will in 19 yr. amount to \$1.59865. Hence, the required time is nearly 19 half years, or 9½ years, nearly. *Ans.*

15. At what rate per annum will \$500 amount to \$779.83 in 9 years, interest being compounded semi-annually?

Since \$500 amounts to \$779.83, \$1 amounts to $\$\frac{779.83}{500} = \$1.55966$.

By the table, \$1 will in 18 yr. at 2½% amount to \$1.55966. Hence, the required rate is 2½% semi-annually, or 6% annually. 6%. *Ans.*

[redacted]

Exercise 159. Page 366.

1. Find the present value of an annuity of \$300 for 6 years, if money is worth 5%.

$$P = \frac{300}{0.05} \times \frac{1.05^6 - 1}{1.05^6}.$$

$$\begin{aligned} \log 1.05 &= 0.0212 \\ &\underline{\quad\;\; 6} \\ 0.1272 &= \log 1.34. \end{aligned}$$

$$\therefore P = \frac{300}{0.05} \times \frac{0.34}{1.34}.$$

$$\begin{aligned} \log 300 &= 2.4771 \\ \log 0.34 &= 9.5315 - 10 \\ \text{colog } 0.05 &= 1.3010 \\ \text{colog } 1.34 &= \underline{9.8728} - 10 \\ 3.1824 &= \log 1522. \end{aligned}$$

\$1522. *Ans.*

2. Find the present value of an annuity of \$600 for 4 years, if money is worth $5\frac{1}{2}$%.

$$P = \frac{600}{0.055} \times \frac{1.055^4 - 1}{1.055^4}.$$

$$\begin{aligned} \log 1.055 &= 0.0233 \\ &\underline{\quad\;\; 4} \\ 0.0932 &= \log 1.239. \end{aligned}$$

$$\therefore P = \frac{600}{0.055} \times \frac{0.239}{1.239}.$$

$$\begin{aligned} \log 600 &= 2.7782 \\ \log 0.239 &= 9.3784 - 10 \\ \text{colog } 0.055 &= 1.2596 \\ \text{colog } 1.239 &= \underline{9.9068} - 10 \\ 3.3230 &= \log 2104. \end{aligned}$$

\$2104. *Ans.*

3. Find the present value of an annuity of \$800 for 5 years, if money is worth 6%.

$$P = \frac{800}{0.06} \times \frac{1.06^5 - 1}{1.06^5}.$$

$$\begin{aligned} \log 1.06 &= 0.0253 \\ &\underline{\quad\;\; 5} \\ 0.1265 &= \log 1.338. \end{aligned}$$

$$\therefore P = \frac{800}{0.06} \times \frac{0.338}{1.338}.$$

$$\begin{aligned} \log 800 &= 2.9031 \\ \log 0.338 &= 9.5289 - 10 \\ \text{colog } 0.06 &= 1.2218 \\ \text{colog } 1.338 &= \underline{9.8735} - 10 \\ 3.5273 &= \log 3368. \end{aligned}$$

\$3368. *Ans.*

4. Find the present value of a perpetual scholarship of \$900, if money is worth $3\frac{1}{2}$%.

$$P = \frac{900}{0.035}.$$

$$\begin{aligned} \log 900 &= 2.9542 \\ \text{colog } 0.035 &= \underline{1.4559} \\ 4.4101 &= \log 25{,}710. \end{aligned}$$

\$25,710. *Ans.*

5. Find the present value of a perpetual fellowship of \$3200, if money is worth $4\frac{1}{4}$%.

$$P = \frac{3200}{0.0425}.$$

$$\begin{aligned} \log 3200 &= 3.5051 \\ \text{colog } 0.0425 &= \underline{1.3716} \\ 4.8767 &= \log 75{,}280. \end{aligned}$$

\$75,280. *Ans.*

6. What is the value of a sinking fund, if \$25,000 is set apart yearly for 7 years at $4\frac{1}{2}\%$ compound interest?

$$A = \frac{25000 \times (1.045^7 - 1)}{0.045}.$$

$$\begin{aligned} \log 1.045 &= 0.0191 \\ &\underline{\quad\quad 7} \\ &0.1337 = \log 1.361. \end{aligned}$$

$$\therefore\ A = \frac{25000 \times 0.361}{0.045}.$$

$$\begin{aligned} \log 25000 &= 4.3979 \\ \log 0.361 &= 9.5575 - 10 \\ \text{colog}\, 0.045 &= \underline{1.3468} \\ &\ 5.3022 \\ &= \log 200{,}500. \end{aligned}$$

\$200,500. *Ans.*

7. What is the value of a sinking fund, if \$18,000 is set apart yearly for 5 years at $3\frac{1}{2}\%$ compound interest?

$$A = \frac{18000 \times (1.035^5 - 1)}{0.035}.$$

$$\begin{aligned} \log 1.035 &= 0.0149 \\ &\underline{\quad\quad 5} \\ &0.0745 = \log 1.187. \end{aligned}$$

$$\therefore\ A = \frac{18000 \times 0.187}{0.035}.$$

$$\begin{aligned} \log 18000 &= 4.2553 \\ \log 0.187 &= 9.2718 - 10 \\ \text{colog}\, 0.035 &= \underline{1.4559} \\ &\ 4.9830 \\ &= \log 96{,}160. \end{aligned}$$

\$96,160. *Ans.*

Exercise 160. Page 369.

1. Find the present value of an annuity of \$900 for 15 years at 4%.

$$\begin{array}{r} \$11.11839 \\ \underline{900} \\ \$10006.55100 \end{array}$$

\$10,006.55. *Ans.*

2. Find the present value of an annuity of \$1500 for 12 years at 4%.

$$\begin{array}{r} \$9.38507 \\ \underline{1500} \\ 469253500 \\ \underline{938507} \\ \$14077.60500 \end{array}$$

\$14,077.61. *Ans.*

3. Find the present value of an annual pension of \$144 for 10 years at $3\frac{1}{2}\%$.

$$\begin{array}{r} \$8.31661 \\ \underline{144} \\ 3326644 \\ 3326644 \\ \underline{831661} \\ \$1197.59184 \end{array}$$

\$1197.59. *Ans.*

4. Find the present value of a scholarship of \$200 for 25 years at $3\frac{1}{2}\%$.

$$\begin{array}{r} \$16.48152 \\ \underline{200} \\ \$3296.30400 \end{array}$$

\$3296.30. *Ans.*

[redacted] [redacted]

5. Find the present value of an annuity of \$2500 for 30 years at 4 %.

$$\begin{array}{r} \$17.29203 \\ 2500 \\ \hline 864601500 \\ 3458406 \\ \hline \$43230.07500 \end{array}$$

\$43,230.08. *Ans.*

6. Find the present value of an annuity of \$250 for 12 years at $3\frac{1}{2}$ %.

$$\begin{array}{r} \$9.66333 \\ 250 \\ \hline 48316650 \\ 1932666 \\ \hline \$2415.83250 \end{array}$$

\$2415.83. *Ans.*

7. A person 22 years old has a life annuity of \$750. Find its present value at 4 %.

The expectancy of life for a person 22 yr. old is about 40 yr.

$$\begin{array}{r} \$19.79277 \\ 750 \\ \hline 98963850 \\ 13854939 \\ \hline \$14844.57750 \end{array}$$

\$14,844.58. *Ans.*

8. A person 35 years old has a life annuity of \$1800. Find its present value at 4 %.

The expectancy of life for a person 35 yr. old is about 31 yr.

$$\begin{array}{r} \$17.58849 \\ 1800 \\ \hline 1407079200 \\ 1758849 \\ \hline \$31659.28200 \end{array}$$

\$31,659.28. *Ans.*

9. A person 53 years old has a life annuity of \$500. Find its present value at 4 %.

The expectancy of life for a person 53 yr. old is about 19 yr.

$$\begin{array}{r} \$13.13394 \\ 500 \\ \hline \$6566.97000 \end{array}$$

\$6566.97. *Ans.*

10. A person 75 years old has a life annuity of \$2400. Find its present value at $3\frac{1}{2}$ %.

The expectancy of life for a person 75 yr. old is about 7 yr.

$$\begin{array}{r} \$6.11454 \\ 2400 \\ \hline 244581600 \\ 1222908 \\ \hline \$14674.89600 \end{array}$$

\$14,674.90. *Ans.*

11. A boy 15 years old has a life annuity of \$3250. Find its present value at 4 %.

The expectancy of life for a person 15 yr. old is about 45 yr.

$$P = \frac{3250}{0.04} \times \frac{1.04^{45} - 1}{1.04^{45}}.$$

$$\begin{array}{r} \log 1.04 = 0.0170 \\ 45 \\ \hline 850 \\ 680 \\ \hline 0.7650 = \log 5.821. \end{array}$$

$$\therefore\ P = \frac{3250}{0.04} \times \frac{4.821}{5.821}.$$

$$\begin{aligned}
\log 3250 &= 3.5119\\
\log 4.821 &= 0.6831\\
\text{colog}\, 0.04 &= 1.3979\\
\text{colog}\, 5.821 &= \underline{9.2350 - 10}\\
&\ 4.8279 = \log 67{,}280.
\end{aligned}$$

$67,280. *Ans.*

12. A person 22 years old pays $4948.19 for a life annuity. If interest is 4 %, find the amount of the annuity.

The expectancy of life for a person 22 yr. old is about 40 yr. The present value of an annuity of $1 per annum at 4 % for 40 yr. is $ 19.79277.

Therefore, $4948.19 is the present value of an annuity of

$$\$\frac{4948.19}{19.79277} = \$250. \ \textit{Ans.}$$

```
              250
1979277)494819000
        3958554
         9896360
         9896385
              00
```

13. A person 29 years old pays $7465.84 for a life annuity. If interest is 4 %, find the amount of the annuity.

The expectancy of life for a person 29 yr. old is about 35 yr.

The present value of an annuity of $1 per annum at 4 % for 35 yr. is $18.66461.

Therefore, $7465.84 is the present value of an annuity of

$$\$\frac{7465.84}{18.66461} = \$400. \ \textit{Ans.}$$

```
              400
1866461)746584000
        7465844
             00
```

14. A person 35 years old pays $9368.14 for a life annuity. If interest is $3\frac{1}{2}$ %, find the amount of the annuity.

The expectancy of life for a person 35 yr. old is about 31 yr.

The present value of an annuity of $1 per annum at $3\frac{1}{2}$ % for 31 yr. is $18.73628.

Therefore, $9368.14 is the present value of an annuity of

$$\$\frac{9368.14}{18.73628} = \$500. \ \textit{Ans.}$$

```
              500
1873628)936814000
        9368140
             00
```

15. A person 44 years old pays $5933.35 for a life annuity. If interest is $3\frac{1}{2}$ %, find the amount of the annuity.

The expectancy of life for a person 44 yr. old is about 25 yr.

The present value of an annuity of $1 per annum at $3\frac{1}{2}$ % for 25 yr. is $16.48152.

Therefore, $5933.35 is the present value of an annuity of

$$\$\frac{5933.35}{16.48152} = \$360. \ \textit{Ans.}$$

```
              360
1648152)593335000
        4944456
         9888940
         9888912
             280
```

[redacted]

Exercise 161. Page 371.

1. Find the cost at compound interest of a coöperative bank share that matured in 10 years, when money was worth $4\frac{1}{2}\%$.

10 yr. = 120 mo.

The rate of interest was $4\frac{1}{2}\%$ yearly or 0.375 % monthly.

$$A = \frac{1 \times (1.00375^{120} - 1)}{1.00375 - 1}.$$

$$\begin{aligned} \log 1.00375 &= 0.0016 \\ &120 \\ \hline &320 \\ &16 \\ \hline &0.1920 \\ &= \log 1.556. \end{aligned}$$

$$\therefore\ A = \frac{0.556}{0.00375}.$$

$$\begin{aligned} \log\ 0.556 &= 9.7451 - 10 \\ \text{colog}\ 0.00375 &= 2.4260 \\ \hline &2.1711 \\ &= \log 148.3. \end{aligned}$$

\$ 148.30. *Ans.*

2. Find the cost at compound interest of a coöperative bank share that matured in $11\frac{1}{2}$ years, when money was worth 5 %.

$11\frac{1}{2}$ yr. = 138 mo.

The rate of interest was 5 % yearly, or $\frac{5}{12}\%$ monthly.

$$A = \frac{1 \times (1.00\tfrac{5}{12}{}^{138} - 1)}{1.00\tfrac{5}{12} - 1}.$$

$$\begin{aligned} \log 1.00\tfrac{5}{12} &= 0.0018 \\ &138 \\ \hline &144 \\ &54 \\ &18 \\ \hline &0.2484 = \log 1.772. \end{aligned}$$

$$\therefore\ A = \frac{0.772}{0.00\frac{5}{12}} = \frac{77.2}{\frac{5}{12}} = \frac{12 \times 77.2}{5}.$$

$$\begin{aligned} \log\ 12 &= 1.0792 \\ \log 77.2 &= 1.8876 \\ \text{colog}\ 5 &= 9.3010 - 10 \\ \hline &2.2678 = \log 185.3. \end{aligned}$$

\$ 185.30. *Ans.*

3. How much more does it cost to borrow \$ 2000 from a coöperative bank, monthly interest being \$ 12, and the shares maturing in 10 years, than to borrow \$ 2000 at compound interest for 10 years, if money is worth 5 % in both cases?

To borrow \$ 2000, the shareholder must own 10 shares; and he pays monthly \$ 10 + \$ 12, or \$ 22, for 10 yr., that is 120 mo.

The rate of interest is 5 % yearly or $\frac{5}{12}\%$ monthly.

$$A = \frac{22 \times (1.00\tfrac{5}{12}{}^{120} - 1)}{1.00\tfrac{5}{12} - 1}.$$

$$\begin{aligned} \log 1.00\tfrac{5}{12} &= 0.0018 \\ &120 \\ \hline &360 \\ &18 \\ \hline &0.2160 = \log 1.644. \end{aligned}$$

$$\therefore A = \frac{22 \times 0.644}{0.00\frac{5}{12}} = \frac{12 \times 22 \times 64.4}{5}.$$

$$\begin{aligned}
\log\ 12 &= 1.0792\\
\log\ 22 &= 1.3424\\
\log 64.4 &= 1.8089\\
\text{colog}\ 5 &= \underline{9.3010} - 10\\
&\ 3.5315 = \log 3400.
\end{aligned}$$

Hence, the cost of borrowing \$ 2000 from the coöperative bank is \$ 3400.

$$A = 2000 \times 1.05^{10}.$$

$$\begin{aligned}
\log\ 2000 &= 3.3010\\
\log 1.05^{10} &= \underline{0.2120}\\
&\ 3.5130 = \log 3258.
\end{aligned}$$

Hence, the cost of borrowing \$ 2000 at compound interest is \$ 3258.

\$ 3400 − \$ 3258 = \$ 142. *Ans.*

MISCELLANEOUS PROBLEMS.

1. Make six different numbers with the digits 1, 2, 3, and find their sum.

$$\begin{array}{r}
123\\
132\\
213\\
231\\
312\\
\underline{321}\\
1332
\end{array}\ \textit{Ans.}$$

2. Make six different numbers with the digits 2, 3, 5, and find, by logarithms, their continued product.

$$235 \times 253 \times 325$$
$$\times\ 352 \times 523 \times 532.$$

$$\begin{aligned}
\log 235 &= 2.3711\\
\log 253 &= 2.4031\\
\log 325 &= 2.5119\\
\log 352 &= 2.5465\\
\log 523 &= 2.7185\\
\log 532 &= \underline{2.7259}\\
&\ 15.2770
\end{aligned}$$

$$= \log 1{,}892{,}000{,}000{,}000{,}000.$$

3. Make six different numbers with the digits 8, 7, 3, and find, by logarithms, their continued product.

$$873 \times 837 \times 783$$
$$\times\ 738 \times 387 \times 378.$$

$$\begin{aligned}
\log 873 &= 2.9410\\
\log 837 &= 2.9227\\
\log 783 &= 2.8938\\
\log 738 &= 2.8681\\
\log 387 &= 2.5877\\
\log 378 &= \underline{2.5775}\\
&\ 16.7908
\end{aligned}$$

$$= \log 61{,}770{,}000{,}000{,}000{,}000.$$

4. Find, by logarithms, the missing term in each of the following proportions:

(i)

$$7.13 : 3.57 :: 4.18 : ?.$$

$$\frac{3.57 \times 4.18}{7.13} = 2.093.\ \textit{Ans.}$$

$$\begin{aligned}
\log 3.57 &= 0.5527\\
\log 4.18 &= 0.6212\\
\text{colog}\ 7.13 &= \underline{9.1469} - 10\\
&\ 0.3208
\end{aligned}$$

$$= \log 2.093.$$

[redacted]

(ii)

$5.89 : 76.3 :: ? : 38.7.$

$$\frac{5.89 \times 38.7}{76.3} = 2.987. \quad \textit{Ans.}$$

$$\begin{aligned} \log 5.89 &= 0.7701 \\ \log 38.7 &= 1.5877 \\ \text{colog } 76.3 &= \underline{8.1175} - 10 \\ &\ 0.4753 \\ &= \log 2.987. \end{aligned}$$

(iii)

$7.37 : ? :: 86.1 : 43.7.$

$$\frac{7.37 \times 43.7}{86.1} = 3.741. \quad \textit{Ans.}$$

$$\begin{aligned} \log 7.37 &= 0.8675 \\ \log 43.7 &= 1.6405 \\ \text{colog } 86.1 &= \underline{8.0650} - 10 \\ &\ 0.5730 \\ &= \log 3.741. \end{aligned}$$

(iv)

$? : 69.7 :: 3.79 : 29.4.$

$$\frac{69.7 \times 3.79}{29.4} = 8.984. \quad \textit{Ans.}$$

$$\begin{aligned} \log 69.7 &= 1.8432 \\ \log 3.79 &= 0.5786 \\ \text{colog } 29.4 &= \underline{8.5317} - 10 \\ &\ 0.9535 = \log 8.984. \end{aligned}$$

5. Find, by logarithms, the value of $0.08^{\frac{1}{3}}$; $2734^{\frac{1}{3}}$; $21.97^{\frac{1}{3}}$; $7^{3.6}$; $9.71^{\frac{7}{3}}$; $7.936^{\frac{5}{7}}$.

$\frac{1}{3} \times \log 0.08 = \frac{1}{3} \times (8.9031 - 10) = 9.6344 - 10 = \log 0.4309.$

$\frac{1}{3} \times \log 2734 = \frac{1}{3} \times 3.4368 = 1.1456 = \log 13.98.$

$\frac{1}{3} \times \log 21.97 = \frac{1}{3} \times 1.3418 = 0.4473 = \log 2.801.$

$3.6 \times \log 7 = 3.6 \times 0.8451 = 3.0424 = \log 1103.$

$\frac{7}{3} \times \log 9.71 = \frac{7}{3} \times 0.9872 = 2.3035 = \log 201.1.$

$\frac{5}{7} \times \log 7.936 = \frac{5}{7} \times 0.8996 = 0.6426 = \log 4.391.$

6. Find the value of

$$\sqrt[5]{\frac{4.79^2 \times 3.1416 \times 12.72}{0.5236 \times 14.28}}.$$

$$\begin{aligned} \log\ 4.79^2 &= 1.3606 \\ \log 3.1416 &= 0.4971 \\ \log\ 12.72 &= 1.1045 \\ \text{colog } 0.5236 &= 0.2810 \\ \text{colog } 14.28 &= \underline{8.8453} - 10 \\ 5\,&\underline{)\,2.0885} \\ &\ 0.4177 \\ &= \log 2.616. \quad \textit{Ans.} \end{aligned}$$

7. If the air-line distance between two points is 1534 ft., and the difference of level is 34 ft., what is the horizontal distance between the two points?

$$\begin{aligned} &\sqrt{1534^2 - 34^2} \text{ ft.} \\ &= \sqrt{2353156 - 1156} \text{ ft.} \\ &= \sqrt{2352000} \text{ ft.} \\ &= 1533.623 \text{ ft.} \quad \textit{Ans.} \end{aligned}$$

```
    2 35 20 00(1533.623
    1
25)135
   125
303)1020
    909
3063)11100
      9189
 3066)19110
       18396
         7140
         6132
        10080
         9198
```

8. If the road distance is 1 mi., and the rise 347 ft., find the horizontal distance.

$\sqrt{5280^2 - 347^2}$ ft.

$= \sqrt{27878400 - 120409}$ ft.

$= \sqrt{27757991}$ ft.

$= 5268.585$ ft. *Ans.*

```
    27 75 79 91(5268.585
    25
102)275
    204
1046)7179
     6276
10528)90391
      84224
 10536)61670
       52680
        89900
        84288
         56120
         52680
```

9. If the road distance is half a mile, and the horizontal distance 2513 ft., find the difference of level.

$\frac{1}{2}$ mi. = 2640 ft.

$\sqrt{2640^2 - 2513^2}$ ft.

$= \sqrt{6969600 - 6315169}$ ft.

$= \sqrt{654431}$ ft. $= 808.97$ ft.

Ans.

```
     65 44 31(808.97
     64
1608)14431
     12864
1616)15670
     14544
      11260
```

10. The diagonal of a rectangular floor is 34.6 ft., and the width is 17.8 ft. Find the length of the floor.

$\sqrt{34.6^2 - 17.8^2}$ ft.

$= \sqrt{1197.16 - 316.84}$ ft.

$= \sqrt{880.32}$ ft. $= 29.67$ ft.

Ans.

```
   8 80.32(29.67
   4
49)480
   441
586)3932
    3516
592)4160
    4144
```

[redacted]

11. The height of a tower on the bank of a river is 55 ft., and the length of a line from the top of the tower to the opposite bank is 78 ft. Find the breadth of the river.

$$\sqrt{78^2 - 55^2}\text{ ft.} = \sqrt{6084 - 3025}\text{ ft.}$$
$$= \sqrt{3059}\text{ ft.} = 55.31\text{ ft. } \textit{Ans.}$$

```
      30 59(55.31
      25
     ----
  105)559
      525
     -----
 1103)3400
      3309
      ----
 1106)910
```

12. The number of seamen at Portsmouth is 800, at Charlestown 404, and at Brooklyn 756. A ship is commissioned whose complement is 490 seamen. Determine the number to be drafted from each place to obtain a proportionate number from each.

$$800 + 404 + 756 = 1960. \qquad \tfrac{404}{1960} \times 490 = 101, \text{ C.}$$
$$\tfrac{800}{1960} \times 490 = 200, \text{ P.} \qquad \tfrac{756}{1960} \times 490 = 189, \text{ B.}$$

13. Show, without division, that 36,432 contains 8, 9, 11 as factors.

$$432 = 54 \times 8.$$
$$3 + 6 + 4 + 3 + 2 = 18.$$
$$3 + 4 + 2 = 6 + 3. \qquad (\S\ 181)$$

14. Find the smallest multiplier that will make 47,250 a perfect cube.

$$47{,}250 = 2 \times 3^3 \times 5^3 \times 7.$$
$$2^2 \times 7^2 = 4 \times 49 = 196. \textit{ Ans.}$$

15. Find the proper fraction that, when reduced to a continued fraction, has for quotients 1, 3, 5, 7, 2, 4.

$$\cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{7 + \cfrac{1}{2 + \cfrac{1}{4}}}}}} \qquad \frac{1}{2 + \frac{1}{4}} = \frac{4}{9}; \qquad \frac{1}{7 + \frac{4}{9}} = \frac{9}{67}; \qquad \frac{1}{5 + \frac{9}{67}} = \frac{67}{344};$$

$$\frac{1}{3 + \frac{67}{344}} = \frac{344}{1099}; \qquad \frac{1}{1 + \frac{344}{1099}} = \frac{1099}{1443}. \quad \textit{Ans.}$$

[redacted]

16. If the meter is equal to 1.09362 yd., find a series of four fractions that will express more and more nearly the true ratio of the meter to the yard.

$$1.09362 = 1\tfrac{9362}{100000} = 1\tfrac{4681}{50000}.$$

```
4681)50000(10
     46810
      3190)4681(1
           3190
           1491)3190(2
                2982
                 208)1491(7
                     1456
                       35
```

$$\therefore\ 1\tfrac{4681}{50000} = 1 + \cfrac{1}{10 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{7}}}}$$

$$1 + \frac{1}{10} = \frac{11}{10}.$$

$$1 + \cfrac{1}{10 + \cfrac{1}{1}} = \frac{12}{11}.$$

$$1 + \cfrac{1}{10 + \cfrac{1}{1 + \cfrac{1}{2}}} = \frac{35}{32}.$$

$$1 + \cfrac{1}{10 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{7}}}} = \frac{257}{235}.$$

$\frac{11}{10}$, $\frac{12}{11}$, $\frac{35}{32}$, $\frac{257}{235}$. *Ans.*

17. Find the square factors contained in 33,075.

$33{,}075 = 3^3 \times 5^2 \times 7^2.$

$3^2 = 9,$

$5^2 = 25,$

$7^2 = 49,$

$3^2 \times 5^2 = 225,$

$3^2 \times 7^2 = 441,$

$5^2 \times 7^2 = 1225,$

$3^2 \times 5^2 \times 7^2 = 11{,}025.$

9, 25, 49, 225, 441, 1225, 11,025. *Ans.*

18. The height of St. Peter's, Rome, is $\frac{9}{110}$ of a mile, and that of St. Paul's, London, is $\frac{17}{264}$ of a mile. How many feet higher is St. Peter's than St. Paul's?

$\frac{9}{\cancel{110}}$ of $\overset{48}{\cancel{5280}}$ ft. $= 432$ ft

$\frac{17}{\cancel{264}}$ of $\overset{20}{\cancel{5280}}$ ft. $= 340$ ft.

432 ft. − 340 ft. = 92 ft. *Ans.*

19. How many days elapsed between the annular eclipse of May 15, 1836, and that of March 15, 1858?

[redacted]

yr.	mo.	dy.
1858	3	15
1836	5	15
21	10	0

During the interval there were five leap years, and in the ten months from May 15 to March 15 there are 304 days.

21 × 365 days = 7665 days. (7665 + 304 + 5) days = 7974 days. *Ans.*

20. In a gale, a flagstaff 60 ft. high snaps 28.8 ft. from the bottom; and, not being wholly broken off, the top touches the ground. If the ground is level, how far is the top from the bottom?

$$60 \text{ ft.} - 28.8 \text{ ft.} = 31.2 \text{ ft.}$$

$$\sqrt{31.2^2 - 28.8^2} \text{ ft.} = \sqrt{973.44 - 829.44} \text{ ft.} = \sqrt{144} \text{ ft.} = 12 \text{ ft. } \textit{Ans.}$$

21. Seventeen trees are standing in a straight line, 20 yd. apart; a man walks from the first to the second and back, then to the third and back, and so on. How far does he walk?

The distance is the sum of the terms of an arithmetical progression in which the first term is 40 yd., the common difference 40 yd., and the number of terms 16.

The 16th term = 40 yd. + 15 × 40 yd. = 40 yd. + 600 yd. = 640 yd.
The sum = 16 × ½ (40 yd. + 640 yd.) = 5440 yd. *Ans.*

22. A canal is 14¾ mi. long and 48 ft. wide. At one end is a lock 80 ft. by 24 ft., with a fall of 8 ft. 6 in. How many barges can pass through the lock before the water in the canal is lowered 1 in.?

The amount of water that is drained off in lowering the level 1 in. is $(14\frac{3}{4} \times 5280 \times 48 \times \frac{1}{12})$ cu. ft.

The amount of water that is wasted each time a barge passes through the lock is $(80 \times 24 \times 8\frac{1}{2})$ cu. ft.

Hence, the number of barges is

$$\frac{14\frac{3}{4} \times 5280 \times 48 \times \frac{1}{12}}{80 \times 24 \times 8\frac{1}{2}} = \frac{59 \times \overset{\overset{11}{\cancel{66}}}{\cancel{5280}} \times \overset{2}{\cancel{48}} \times 2}{\underset{2}{\cancel{4}} \times \cancel{80} \times \cancel{24} \times 17 \times \underset{\cancel{2}}{\cancel{12}}} = \frac{649}{34} = 19\tfrac{3}{34}.$$

19 barges. *Ans.*

23. Find the capacity, in liters and in bushels, of a box 1.7^{m} long, 87^{cm} wide, and 31^{cm} deep.

$$(170 \times 87 \times 31)^{ccm} = 458{,}490^{ccm} = 458.49^{l}. \textit{ Ans.}$$

$$458.49^{l} = 458.49 \times 0.908 \text{ qt.} = \frac{458.49 \times \overset{0.227}{\cancel{0.908}}}{\underset{8}{\cancel{32}}} \text{ bu.} = 13.01 \text{ bu. } \textit{Ans.}$$

[redacted]

87	458.49
31	0.227
87	320943
261	91698
2697	91698
170	8 | 104.07723
188790	13.00965
2697	
458490	

24. Find the number of kilograms of olive oil, specific gravity 0.915, required to fill a rectangular vessel 2.3^{m} long, 1.8^{m} wide, and 74^{cm} deep.

$(2.3 \times 1.8 \times 0.74)^{cbm} = 3.0636^{cbm}$.

3.0636^{cbm} of water weighs 3063.6^{kg}.

$0.915 \times 3063.6^{kg} = 2803.194^{kg}$. *Ans.*

2.3	3063.6
1.8	0.915
184	153180
23	30636
4.14	275724
0.74	2803.1940
1656	
2898	
3.0636	

25. How many tons in a block of marble 4 ft. long, 34 in. wide, 17.3 in. thick, specific gravity 2.73?

$$\text{Volume} = (48 \times 34 \times 17.3)\text{ cu. in.} = \frac{48 \times 34 \times 17.3}{1728}\text{ cu. ft.}$$

$$\text{Weight} = \frac{48 \times 34 \times 17.3}{1728} \times 2.73 \times 62\tfrac{1}{2}\text{ lb.}$$

$$= \frac{\cancel{48} \times \overset{17}{\cancel{34}} \times 173}{\underset{\underset{12}{\cancel{36}}}{\cancel{1728}} \times \underset{2}{\cancel{10}}} \times \frac{\overset{91}{\cancel{273}}}{\underset{4}{\cancel{100}}} \times \frac{\overset{5}{\cancel{125}}}{\cancel{2}} \times \frac{1}{2000}\text{ t.}$$

$$= \frac{267631}{192000}\text{ t.} = 1.394\text{ t. } Ans.$$

26. Find the surface of a sphere 18.3 in. in diameter.

$3.1416 \times (18.3 \times 18.3)$ sq. in. $= 1052.09$ sq. in. *Ans.*

$$\begin{array}{r} 18.3 \\ 18.3 \\ \hline 549 \\ 1464 \\ 183 \\ \hline 334.89 \end{array} \qquad\qquad \begin{array}{r} 334.89 \\ 3.1416 \\ \hline 200934 \\ 33489 \\ 133956 \\ 33489 \\ 100467 \\ \hline 1052.090424 \end{array}$$

27. Find the number of acres in a circular field 213 yd. 2 ft. in diameter.

Diameter is 213 yd. 2 ft. = 641 ft.

Radius is $\frac{1}{2}$ of 641 ft. = 320.5 ft.

1 A. = 43,560 sq. ft.

$$\text{Area} = \frac{3.1416 \times 320.5^2}{43560} \text{ A.} = 7.407 \text{ A.} \quad \textit{Ans.}$$

$$\begin{aligned} \log 3.1416 &= 0.4971 \\ \log 320.5^2 &= 5.0116 \\ \text{colog } 43{,}560 &= \underline{5.3609} - 10 \\ & 0.8696 \qquad = \log 7.407. \end{aligned}$$

28. How many cubic inches in a 10-inch globe? in a 20-inch globe? What is the ratio of their volumes?

The ratio of their volumes is $10^3 : 20^3 = 1^3 : 2^3 = 1 : 8$. *Ans.*

(0.5236×10^3) cu. in. = 523.6 cu. in. *Ans.*

8×523.6 cu. in. = 4188.8 cu. in. *Ans.*

29. How many balls 3 in. in diameter can be cast from a pig of iron 7 ft. long, 6.7 in. wide, 3.8 in. thick, if the waste in melting and casting is reckoned at $3\frac{1}{4}\%$?

7 ft. = 84 in.

$$\text{The number of balls} = \frac{84 \times 6.7 \times 3.8 \times 0.9675}{3^3 \times 0.5236}.$$

[redacted]

$$\begin{aligned}
\log\ 84 &= 1.9243\\
\log\ 6.7 &= 0.8261\\
\log\ 3.8 &= 0.5798\\
\log 0.9675 &= 9.9857 - 10\\
\text{colog}\ 27 &= 8.5686 - 10\\
\text{colog}\ 0.5236 &= 0.2810\\
\hline
&\ \ \ 2.1655 \qquad = \log 146.4.
\end{aligned}$$

Hence, the number of balls is 146. *Ans.*

30. Find the difference in length, at 80° F., of a glass rod and a steel rod, each 3 ft. long at 0° C., if the expansion at 100° C. is 0.00085 for glass and 0.0012 for steel.

$$80^\circ \text{ F.} = \tfrac{5}{9}(80^\circ - 32^\circ)\text{C.} = 26\tfrac{2}{3}^\circ \text{ C.}$$

$$0.0012 - 0.00085 = 0.00035.$$

$$\text{Difference in length} = \frac{26\frac{2}{3}}{100} \times 0.00035 \times 36 \text{ in.} = 0.00336 \text{ in. } \textit{Ans.}$$

$$\begin{array}{r}
36 \text{ in.}\\
0.00035\\
\hline
180\\
108\\
\hline
0.01260 \text{ in.}
\end{array}
\qquad\qquad
\begin{array}{r}
0.0126 \text{ in.}\\
0.26\frac{2}{3}\\
\hline
84\\
756\\
252\\
\hline
0.003360 \text{ in.}
\end{array}$$

31. A grain of gold is beaten into leaf to cover 56 sq. in. What weight will be required to gild the faces of a cube whose edge is $3\frac{1}{2}$ ft.?

$$6 \times (3\tfrac{1}{2} \times 3\tfrac{1}{2}) \text{ sq. ft.} = 6 \times \tfrac{7}{2} \times \tfrac{7}{2} \times 144 \text{ sq. in.}$$

Number of grains of gold required is

$$\frac{6 \times \frac{7}{2} \times \frac{7}{2} \times 144}{56} = \overset{3}{\cancel{6}} \times \frac{\cancel{7}}{\cancel{2}} \times \frac{7}{\cancel{2}} \times \overset{\overset{9}{\cancel{18}}}{\cancel{144}} \times \frac{1}{\underset{\cancel{8}}{\cancel{56}}} = 189.$$

189 gr. = 7 dwt. 21 gr. *Ans.*

32. What premium must be paid, at the rate of $4\frac{7}{8}$%, for insuring a vessel worth $100,000, in order that in the event of loss the owner may receive both the value of the ship and the premium?

[redacted]

100% of policy = policy (vessel and premium).
$4\frac{7}{8}\%$ of policy = premium.

$95\frac{1}{8}\%$ of policy = vessel.

$$\$100{,}000 \div 0.95\tfrac{1}{8} = \$105{,}124.84.$$

$$\$105{,}124.84 - \$100{,}000 = \$5124.84. \quad \textit{Ans.}$$

```
          105124.83
95125)10000000000.
      95125
       487500
       475625
        118750
         95125
         236250
         190250
          460000
          380500
           795000
           761000
            340000
            285375
             54625
```

33. By selling goods at 60 cents a pound, 8% is lost. What advance must be made in the price to gain 15%?

Cost = 60 cents $\div \frac{92}{100} = \frac{100}{92}$ of 60 cents.

$$\frac{\cancel{115}^{5}}{\cancel{100}} \times \frac{\cancel{100}}{\cancel{92}_{4}} \times \cancel{60}^{15} \text{ cents} = 75 \text{ cents.} \quad 75 \text{ cents} - 60 \text{ cents} = 15 \text{ cents.} \quad \textit{Ans.}$$

34. The sharpest grade on Mt. Washington Ry. is 1980 ft. to the mile. What fraction of a foot is the rise for each foot? What is the per cent of grade?

$\frac{1980}{5280}$ ft. $= \frac{3}{8}$ ft. *Ans.* $\qquad \frac{3}{8} = 37\frac{1}{2}\%$. *Ans.*

35. Find the square root, to four decimal places, of the reciprocal of 0.0043.

[redacted]

$$
\begin{array}{r}
232.55813953 \\
43\overline{)10000.} \\
\underline{86} \\
140 \\
\underline{129} \\
110 \\
\underline{86} \\
240 \\
\underline{215} \\
250 \\
\underline{215} \\
350 \\
\underline{344} \\
60 \\
\underline{43} \\
170 \\
\underline{129} \\
410 \\
\underline{387} \\
230 \\
\underline{215} \\
150 \\
\underline{129} \\
21
\end{array}
\qquad
\begin{array}{l}
2\ 32.55\ 81\ 39\ 53(15.2498 \\
1 \\
25)132 \\
\quad 125 \\
302)755 \\
\quad 604 \\
3044)15181 \\
\quad 12176 \\
30489)300539 \\
\quad 274401 \\
304988)2613853 \\
\quad 2439904
\end{array}
$$

15.2498. *Ans.*

36. The population of a city in 1890 was 12,298, showing a decrease of $8\frac{1}{3}\%$ on its population in 1880; in 1880 there was an increase of $7\frac{1}{2}\%$ on the census of 1870. What was its population in 1870?

Population in 1880 $= 12298 \div \frac{91\frac{2}{3}}{100}$.

Population in 1870

$$= \left(12298 \div \frac{91\frac{2}{3}}{100}\right) \div \frac{107\frac{1}{2}}{100} = \overset{\overset{26}{\cancel{1118}}}{\cancel{12298}} \times \frac{\overset{12}{\cancel{300}}}{\underset{11}{\cancel{275}}} \times \frac{\overset{40}{\cancel{200}}}{\underset{43}{\cancel{215}}} = 12,480. \textit{ Ans.}$$

37. Find the increase of income obtained by transferring 25 shares of 3 % stock at $94\frac{5}{8}$ to 4 % stock at $104\frac{7}{8}$, brokerage $\frac{1}{8}$ on each transaction.

25 × \$3 = \$75, income from the 3 % stock.
25 × \$94.50 = \$2362.50, proceeds from the 3 % stock.
\$1.05 is paid for \$1 worth of 4 % stock.
Hence, \$2362.50 is paid for \$2362.50 ÷ 1.05 = \$2250 stock.
4 % of \$2250 = \$90, income from 4 % stock.
\$90 − \$75 = \$15, increase of income. *Ans.*

[redacted]

38. Each person in breathing spoils the air of a closed room at the rate of about 8 cu. ft. a minute. An audience of 400 persons enter a closed hall 70 ft. by 40 ft., and 20 ft. high. How long will it take them to spoil the air?

$$\frac{\overset{35}{\cancel{70}} \times \overset{2}{\cancel{40}} \times \cancel{20}}{\underset{\cancel{20}}{\cancel{400}} \times \underset{\underset{2}{\cancel{4}}}{\cancel{8}}} = \frac{35}{2} = 17\tfrac{1}{2}.$$

$17\frac{1}{2}$ min. *Ans.*

39. How long can the windows and doors of a schoolroom be safely kept closed when occupied by 50 children, if the room is 25 ft. by 20 ft., and 10 ft. high?

$$\frac{\overset{5}{\cancel{25}} \times \overset{5}{\cancel{20}} \times \cancel{10}}{\underset{\cancel{2}}{\cancel{50}} \times \underset{2}{\cancel{8}}} = \frac{25}{2} = 12\tfrac{1}{2}.$$

$12\frac{1}{2}$ min. *Ans.*

40. A pays B \$230 as the present value of \$300 due in 5 years. Which gains by the payment, and how much, if interest is reckoned at 5 % compound interest?

The present value of \$300 due in 5 yr. at 5 % is

$$\frac{1}{1.27628} \text{ of } \$300 = \$\frac{300}{1.27628} = \$235.06.$$

$$\begin{array}{r@{\,}l}
 & 235.05 \\
127628\big) & \overline{30000000.} \\
 & \underline{255256} \\
 & 447440 \\
 & \underline{382884} \\
 & 645560 \\
 & \underline{638140} \\
 & 742000 \\
 & \underline{638140} \\
 & 103860
\end{array}$$

Therefore, A gains \$235.06 − \$230 = \$5.06. *Ans.*

41. Find the quantity of coal required by a steamer for a voyage of 4043 mi., if her rate per hour is 14.04 knots, and her consumption of coal 87 long tons per day.

The rate of the ship per day $= 24 \times 14.04$ knots $= 24 \times 14.04 \times 6086$ ft.

$$4043 \text{ mi.} = 4043 \times 5280 \text{ ft.}$$

Therefore, the number of days is $\dfrac{4043 \times 5280}{24 \times 14.04 \times 6086}$.

The number of long tons of coal is $\dfrac{4043 \times 5280 \times 87}{24 \times 14.04 \times 6086}$.

$$\begin{array}{rrl}
\log & 4043 = & 3.6067 \\
\log & 5280 = & 3.7226 \\
\log & 87 = & 1.9395 \\
\text{colog} & 24 = & 8.6198 - 10 \\
\text{colog} & 14.04 = & 8.8527 - 10 \\
\text{colog} & 6086 = & \underline{6.2157} - 10 \\
 & & 2.9570 \quad = \log 905.8.
\end{array}$$

905.8 l. t. *Ans.*

42. Find the area of a circular ring whose inner and outer diameters are 7.36 in. and 10.64 in., respectively.

$$\begin{aligned}
\text{Area} &= 0.7854 \times (10.64^2 - 7.36^2) \text{ sq. in.} \\
&= 0.7854 \times (113.2096 - 54.1696) \text{ sq. in.} \\
&= 0.7854 \times 59.04 \text{ sq. in.} = 46.37 \text{ sq. in. } \textit{Ans.}
\end{aligned}$$

10.64	7.36	0.7854
10.64	7.36	59.04
4256	4416	31416
6384	2208	70686
1064	5152	39270
113.2096	54.1696	46.370016

43. A and B can do a piece of work in $13\frac{1}{3}$ days; A and C in $10\frac{2}{3}$ days; A, B, and C in $7\frac{1}{2}$ days. In how many days can A do the work alone?

If A and B can do the work in $13\frac{1}{3}$ days, in 1 day they can do $\dfrac{1}{13\frac{1}{3}} = \dfrac{3}{40}$ of it.

If A and C can do the work in $10\frac{2}{3}$ days, in 1 day they can do $\dfrac{1}{10\frac{2}{3}} = \dfrac{3}{32}$ of it.

If A, B, and C can do the work in $7\frac{1}{2}$ days, in 1 day they can do $\dfrac{1}{7\frac{1}{2}} = \dfrac{2}{15}$ of it.

Hence, in 1 day B can do $\frac{2}{15} - \frac{3}{32} = \frac{19}{480}$ of the work.
Hence, in 1 day A can do $\frac{3}{40} - \frac{19}{480} = \frac{17}{480}$ of the work.
Therefore, it will take A $\frac{480}{17}$ days $= 28\frac{4}{17}$ days. *Ans.*

[redacted] [redacted]

44. If 3 men working 11 hours a day can reap 20 A. in 11 days, how many men working 12 hours a day can reap a field 360 yd. long and 320 yd. broad in 4 days?

$$20\text{ A.} = 20 \times 160 \times 30\tfrac{1}{4}\text{ sq. yd.}$$

$$\begin{array}{r|l} 12 & 11 \\ 4 & 11 \\ 20 \times 160 \times 30\frac{1}{4} & 360 \times 320 \end{array} \quad :: 3\text{ men} : ?.$$

$$\frac{11 \times 11 \times 360 \times 320 \times 3\text{ men}}{12 \times 4 \times 20 \times 160 \times 30\frac{1}{4}} = \frac{\cancel{11} \times \cancel{11} \times \overset{\overset{3}{\cancel{30}}}{\cancel{360}} \times \overset{2}{\cancel{320}} \times \cancel{4} \times 3\text{ men}}{\cancel{12} \times \cancel{4} \times \underset{\cancel{10}}{\cancel{20}} \times \cancel{160} \times \underset{\cancel{11}}{\cancel{121}}}$$

$$= 9\text{ men. } \textit{Ans.}$$

45. Find the area of a triangle whose sides are 12 in., 5 in., and 13 in., respectively.

Since $13^2 = 12^2 + 5^2$, the triangle is a right triangle.

Hence, area $= \frac{1}{2} \times (12 \times 5)$ sq. in. $= 30$ sq. in. *Ans.*

46. The four sides of a field measured in succession are 237 ft., 253 ft., 244 ft., and 261 ft., and the diagonal measured from the end of the first side to the end of the third side is 351 ft. Find the area of the field.

$$\frac{237 + 261 + 351}{2} = 424.5.$$

$$\text{Area of 1st triangle} = \sqrt{424.5 \times 187.5 \times 163.5 \times 73.5}\text{ sq. ft.}$$

$$\frac{253 + 244 + 351}{2} = 424.$$

$$\text{Area of 2d triangle} = \sqrt{424 \times 171 \times 180 \times 73}\text{ sq. ft.}$$

$$\begin{array}{rl} \log 424.5 = & 2.6279 \\ \log 187.5 = & 2.2730 \\ \log 163.5 = & 2.2135 \\ \log 73.5 = & 1.8663 \\ 2 \,| & \underline{8.9807} \\ & 4.4904 \end{array} \qquad \begin{array}{rl} \log 424 = & 2.6274 \\ \log 171 = & 2.2330 \\ \log 180 = & 2.2553 \\ \log 73 = & 1.8633 \\ 2 \,| & \underline{8.9790} \\ & 4.4895 \end{array}$$

$$= \log 30{,}930. \qquad\qquad = \log 30{,}860.$$

30,930 sq. ft. + 30,860 sq. ft. = 61,790 sq. ft. *Ans.*

47. The four sides of a field measured in succession are 361 ft., 561 ft., 443 ft., and 357 ft., and the distance from the beginning of the first side to the end of the second side is 682 ft. Find the area of the field.

$$\frac{361 + 561 + 682}{2} = 802.$$

$$\text{Area of triangle} = \sqrt{802 \times 441 \times 241 \times 120} \text{ sq. ft.}$$

$$\begin{aligned}
\log 802 &= 2.9042\\
\log 441 &= 2.6444\\
\log 241 &= 2.3820\\
\log 120 &= 2.0792\\
2 \,&\underline{|10.0098}\\
&5.0049 = \log 101{,}100.
\end{aligned}$$

$$\frac{443 + 357 + 682}{2} = 741.$$

$$\text{Area of triangle} = \sqrt{741 \times 298 \times 384 \times 59} \text{ sq. ft.}$$

$$\begin{aligned}
\log 741 &= 2.8698\\
\log 298 &= 2.4742\\
\log 384 &= 2.5843\\
\log \;\; 59 &= 1.7709\\
2 \,&\underline{|9.6992}\\
&4.8496 = \log 70{,}730.
\end{aligned}$$

101,100 sq. ft. + 70,730 sq. ft. = 171,830 sq. ft. *Ans.*

48. Find the altitude of a triangle, if each side is 1000 ft.

$\sqrt{1000^2 - 500^2}$ ft. $= \sqrt{1000000 - 250000}$ ft. $= \sqrt{750000}$ ft. $= 866.025$ ft. *Ans.*

```
          75 00 00(866.025
          64
    166)1100
         996
   1726)10400
        10356
  173202)440000
         346404
 1732045)9359600
         8660225
```

[redacted] [redacted]

49. Find the three altitudes of a triangle, if its sides are 17.8^{mm}, 23.6^{mm}, and 31.5^{mm}, respectively.

$$\frac{17.8 + 23.6 + 31.5}{2} = 36.45.$$

$$\text{Area} = \sqrt{36.45 \times 18.65 \times 12.85 \times 4.95}^{\,qmm}.$$

$\log 36.45 = 1.5617$
$\log 18.65 = 1.2707$
$\log 12.85 = 1.1089$
$\log\ \ 4.95 = 0.6946$
$2 \,|\, \underline{4.6359}$
$\log \text{area} = 2.3179$
$\text{colog } 8.9 = \underline{9.0506} - 10$
1.3685
$= \log 23.36.$
23.26^{mm}. *Ans.*

$\log \text{area} = 2.3179$
$\text{colog } 11.8 = \underline{8.9281} - 10$
1.2460
$= \log 17.62.$
17.62^{mm}. *Ans.*
$\log \text{area} = 2.3179$
$\text{colog } 15.75 = \underline{8.8027} - 10$
1.1206
$= \log 13.2.$
13.2^{mm}. *Ans.*

50. How many square inches in the surface of a sphere that has a radius of 12.37 in.?

$$\text{Area} = 3.1416 \times 4 \times 12.37^2.$$

$\log 3.1416 = 0.4971$
$\log\ \ \ 4 = 0.6021$
$\log 12.37^2 = \underline{2.1848}$
$3.2840 = \log 1923.$

1923 sq. in. *Ans.*

51. Find the area of the surface of the largest globe that can be turned out from a joist 4 in. by 6 in.

$$\begin{array}{r} 3.1416 \\ 16 \\ \hline 188496 \\ 31416 \\ \hline 50.2656 \end{array}$$

50.2656 sq. in. *Ans.*

52. How many cubic inches in a globe that has a diameter of 10 in.?

Volume $= 0.5236 \times 10^3$ cu. in. $= 523.6$ cu. in. *Ans.*

53. If a tree is round, and its girth is 17 ft. 6 in., find its diameter. Find the area of a cross section, and also the number of cubic feet in the largest sphere that can be cut from it.

$$\text{Diameter} = \frac{17.5}{3.1416}\text{ ft.} = 5.57\text{ ft. } \textit{Ans.}$$

$$\text{Area} = 0.7854 \times 5.57^2\text{ sq. ft.} = 24.37\text{ sq. ft. } \textit{Ans.}$$

$$\text{Volume} = 0.5236 \times 5.57^3\text{ cu. ft.} = 90.52\text{ cu. ft. } \textit{Ans.}$$

$$\begin{aligned} \log\ 17.5 &= 1.2430 \\ \text{colog}\ 3.1416 &= \underline{9.5029 - 10} \\ &\ 0.7459 \qquad = \log 5.57. \end{aligned}$$

$$\begin{aligned} \log 0.7854 &= 9.8951 - 10 \\ \log\ 5.57^2 &= \underline{1.4918} \\ &\ 1.3869 \qquad = \log 24.37. \end{aligned}$$

$$\begin{aligned} \log 0.5236 &= 9.7190 - 10 \\ \log\ 5.57^3 &= \underline{2.2377} \\ &\ 1.9567 \qquad = \log 90.52. \end{aligned}$$

54. Find the weight in kilograms and in pounds of an iron ball 21.5^{cm} in diameter, specific gravity 7.47; of a tin ball 13^{cm} in diameter, specific gravity 7.29; of a lead ball 17.3^{cm} in diameter, specific gravity 11.35; of a silver ball 1.31^{cm} in diameter, specific gravity 10.47.

Iron.

$$\text{Weight} = 7.47 \times 0.5236 \times (2.15^3)^{kg} = 38.86^{kg}. \textit{ Ans.}$$

$$38.86^{kg} = 38.86 \times 2.205\text{ lb.} = 85.68\text{ lb. } \textit{Ans.}$$

$$\begin{aligned} \log\ 7.47 &= 0.8733 \\ \log 0.5236 &= 9.7190 - 10 \\ \log\ 2.15^3 &= \underline{0.9972} \\ &\ 1.5895 \qquad = \log 38.86. \end{aligned}$$

$$\begin{aligned} \log\ 38.86 &= 1.5895 \\ \log\ 2.205 &= \underline{0.3434} \\ &\ 1.9329 \qquad = \log 85.68. \end{aligned}$$

[illegible]

$$= [illegible]$$

$$[illegible] = [illegible]$$

[illegible]

$$[illegible] = [illegible] \times 2.296\ [illegible] = [illegible]$$

$$\log [illegible] = [illegible]$$

$$\log [illegible] = [illegible] - 10$$

$$\log [illegible] = [illegible] - 10$$

$$[illegible] - 10 = \log 0.0[illegible]$$

$$\log [illegible] = [illegible] - 10$$

$$\log 2.296 = [illegible]$$

$$[illegible] - 10 = \log 0.02718$$

[redacted]

55. A slab of cast iron 4 ft. 2½ in. long, 17 in. wide, and 8⅓ in. thick, specific gravity 7.31, is cast into 2-lb. balls. If there is a loss of 5% in melting, how many balls are obtained, and what is the diameter of each?

The slab will make $\dfrac{50.5 \times 17 \times 25 \times 0.95 \times 62.5 \times 7.31}{2 \times 3 \times 1728} = 898$ balls.

The diameter will be $\sqrt[3]{\dfrac{50.5 \times 17 \times 25 \times 0.95}{0.5236 \times 3 \times 898}}$ in. $= 2.436$ in. *Ans.*

$$\begin{aligned}
\log 50.5 &= 1.7033\\
\log 17 &= 1.2304\\
\log 25 &= 1.3979\\
\log 0.95 &= 9.9777 - 10\\
\log 62.5 &= 1.7959\\
\log 7.31 &= 0.8639\\
\text{colog } 2 &= 9.6990 - 10\\
\text{colog } 3 &= 9.5229 - 10\\
\text{colog } 1728 &= 6.7625 - 10\\
& \overline{2.9535} = \log 898.4.
\end{aligned}$$

898 balls. *Ans.*

$$\begin{aligned}
\log 50.5 &= 1.7033\\
\log 17 &= 1.2304\\
\log 25 &= 1.3979\\
\log 0.95 &= 9.9777 - 10\\
\text{colog } 0.5236 &= 0.2810\\
\text{colog } 3 &= 9.5229 - 10\\
\text{colog } 898 &= 7.0467 - 10\\
3)\, & \underline{1.1599}\\
& 0.3866\\
& = \log 2.436.
\end{aligned}$$

56. How many pounds will a ball of iron 30 in. in diameter weigh, if the specific gravity of the iron is 7.31?

$$\frac{0.5236 \times 30^3 \times 7.31 \times 62.5}{1728} \text{ lb.} = 3738 \text{ lb. } \textit{Ans.}$$

$$\begin{aligned}
\log 0.5236 &= 9.7190 - 10\\
\log 30^3 &= 4.4313\\
\log 7.31 &= 0.8639\\
\log 62.5 &= 1.7959\\
\text{colog } 1728 &= 6.7625 - 10\\
& \overline{3.5726} \quad = \log 3738.
\end{aligned}$$

57. If the specific gravity of ice is 0.930, find the weight and the surface of each of three spheres of ice whose diameters are 1^{cm}, 10^{cm}, and 1^{m}.

$0.5236 \times (1^3)^{ccm} = 0.5236^{ccm}$.

$0.930 \times 523.6^{mg} = 486.948^{mg}$. *Ans.*

$0.5236 \times (10^3)^{ccm} = 523.6^{ccm}$.

$0.930 \times 523.6^{g} = 486.948^{g}$. *Ans.*

[redacted]

$$0.5236 \times (1^3)^{\text{cbm}} = 0.5236^{\text{cbm}}.$$

$$0.930 \times 523.6^{\text{kg}} = 486.948^{\text{kg}}. \quad \textit{Ans.}$$

$$\begin{array}{r} 523.6 \\ 0.93 \\ \hline 15708 \\ 47124 \\ \hline 486.948 \end{array}$$

$$3.1416 \times (1^2)^{\text{qcm}} = 3.1416^{\text{qcm}}. \quad \textit{Ans.}$$

$$3.1416 \times (10^2)^{\text{qcm}} = 314.16^{\text{qcm}}. \quad \textit{Ans.}$$

$$3.1416 \times (100^2)^{\text{qcm}} = 31{,}416^{\text{qcm}}. \quad \textit{Ans.}$$

58. Find the capacity in gallons of a round cistern 13 ft. in diameter and 9 ft. deep.

$$V = \frac{9 \times 3.1416 \times 6.5^2 \times 1728}{231} \text{ gal.} = 8933 \text{ gal.} \quad \textit{Ans.}$$

$$\begin{array}{rrl} \log & 9 = & 0.9542 \\ \log & 3.1416 = & 0.4971 \\ \log & 6.5^2 = & 1.6258 \\ \log & 1728 = & 3.2375 \\ \text{colog} & 231 = & \underline{7.6364} - 10 \\ & & 3.9510 \quad = \log 8933. \end{array}$$

59. A cylinder is 10 in. in diameter and 12 in. long. Find the area of each end, the lateral surface, the total surface, and the contents in gallons.

Area of end $= 0.7854 \times (10^2)$ sq. in. $= 78.54$ sq. in. *Ans.*

Lateral surface $= 3.1416 \times (10 \times 12)$ sq. in. $= 3.1416 \times 120$ sq. in.
$= 376.99$ sq. in. *Ans.*

Total surface $= 376.99$ sq. in. $+ 2 \times 78.54$ sq. in. $= 534.07$ sq. in. *Ans.*

$$\text{Volume} = \frac{\overset{4}{\cancel{12}} \times \overset{1.02}{\cancel{78.54}}}{\underset{77}{\cancel{231}}} \text{ gal.} = 4.08 \text{ gal.} \quad \textit{Ans.}$$

60. What must be the diameter of a cylinder 10 in. deep that it may hold 1 gallon?

$$231 = 10 \times 0.7854 \times D^2.$$

$$\therefore\ D = \sqrt{\frac{231}{7.854}} \text{ in.} = 5.424 \text{ in. } \textit{Ans.}$$

$$\begin{array}{rl}
\log\ 231 = & 2.3636 \\
\text{colog}\ 7.854 = & \underline{9.1049 - 10} \\
2 \,| & \overline{1.4685} \\
 & \overline{0.7343} \quad = \log 5.424.
\end{array}$$

61. Find the volume of a cylinder 8 in. in diameter and 11 in. high.

Volume $= (11 \times 0.7854 \times 8^2)$ cu. in. $= 552.92$ cu. in. *Ans.*

$$\begin{array}{r}
8 \\
\underline{8} \\
64 \\
\underline{11} \\
64 \\
\underline{64} \\
704
\end{array}
\qquad\qquad\qquad
\begin{array}{r}
0.7854 \\
\underline{704} \\
31416 \\
\underline{54978} \\
552.9216
\end{array}$$

62. Find the dimensions of three cylinders that have the diameters equal to the heights, and hold 1 gallon, 1 quart, and 1 liter, respectively.

$$V = 0.7854 \times D^2 \times H = 0.7854 \times D^3.$$

$$\therefore\ D = \sqrt[3]{\frac{V}{0.7854}}.$$

$$D = \sqrt[3]{\frac{231}{0.7854}} \text{ in.} = 6.65 \text{ in. } \textit{Ans.}$$

$$\begin{array}{rl}
\log\ 231 = & 2.3636 \\
\text{colog}\ 0.7854 = & \underline{0.1049} \\
3 \,| & \underline{2.4685} \\
 & 0.8228 = \log 6.65.
\end{array}$$

$$D = \sqrt[3]{\frac{57.75}{0.7854}} \text{ in.} = 4.19 \text{ in. } \textit{Ans.}$$

$$\begin{array}{rl}
\log\ 57.75 = & 1.7616 \\
\text{colog}\ 0.7854 = & \underline{0.1049} \\
3 \,| & \underline{1.8665} \\
 & 0.6222 = \log 4.19.
\end{array}$$

[redacted] [redacted]

$$D = \sqrt[3]{\frac{1000}{.7854}} = 10.84^{m}. \ \textit{Ans.}$$

$$\begin{aligned} \log\ 1000 &= 3.0000 \\ \text{colog}\ 0.7854 &= 0.1049 \\ 3\,\underline{|3.1049} & \\ 1.0350 &= \log 10.84. \end{aligned}$$

63. How many cubic yards in a pyramid 123 ft. high, with a square base 210 ft. on a side?

123 ft. = 41 yd.; 210 ft. = 70 yd.

Volume $= \frac{1}{3} \times (70^2 \times 41)$ cu. yd. $= 66{,}966\frac{2}{3}$ cu. yd. *Ans.*

$$\begin{array}{r} 70 \\ \underline{70} \\ 4900 \\ \underline{41} \\ 4900 \\ \underline{19600} \\ 200900 \end{array} \qquad \begin{array}{r} 3\,\underline{)200900} \\ 66966\frac{2}{3} \end{array}$$

64. Find the capacity of a cup, whose mouth is 4 in. square, and whose sides are four equilateral triangles.

Diagonal of base $= \sqrt{4^2 + 4^2}$ in. $= \sqrt{16 + 16}$ in. $= \sqrt{32}$ in.

Altitude of pyramid $= \sqrt{4^2 - (\frac{1}{2}\sqrt{32})^2}$ in. $= \sqrt{16 - 8}$ in. $= \sqrt{8}$ in.

Volume $= \frac{1}{3} \times (4^2 \times \sqrt{8})$ cu. in. $= 15.09$ cu. in. *Ans.*

$$\begin{aligned} \log\ 16 &= 1.2041 \\ \log \sqrt{8} &= 0.4516 \\ \text{colog}\ \ 3 &= \underline{9.5229} - 10 \\ 1.1786 &\qquad = \log 15.09. \end{aligned}$$

65. The largest of the Egyptian pyramids is 147^{m} high, with a base 231^{m} square. Find its volume in cubic meters.

Volume $= \frac{1}{3} \times (147 \times 231^2)^{cbm} = 2{,}614{,}689^{cbm}$. *Ans.*

$$\begin{array}{r} 231 \\ \underline{231} \\ 231 \\ 693 \\ \underline{462} \\ 53361 \end{array} \qquad \begin{array}{r} 3\,\underline{|147} \\ 49 \end{array} \qquad \begin{array}{r} 53361 \\ \underline{49} \\ 480249 \\ \underline{213444} \\ 2614689 \end{array}$$

66. The slant depth of a conical cup is 93^{mm}, and the diameter at the top 8^{cm}. What is its capacity ?

$$\text{Height} = \sqrt{9.3^2 - 4^2}^{\,cm} = \sqrt{86.49 - 16}^{\,cm} = \sqrt{70.49}^{\,cm}.$$

$$\text{Volume} = \tfrac{1}{3} \times (\sqrt{70.49} \times 0.7854 \times 8^2)^{ccm} = 140.7^{ccm} = 0.1407^{l}. \; \textit{Ans.}$$

$$\begin{aligned} \log \sqrt{70.49} &= 0.9241 \\ \log\ 0.7854 &= 9.8951 - 10 \\ \log\ 64 &= 1.8062 \\ \text{colog}\ 3 &= \underline{9.5229 - 10} \\ &\ 2.1483 \qquad = \log 140.7. \end{aligned}$$

67. The volume of a cone is 1^{cbm}; its height is equal to the radius of its base. Find the dimensions of the cone.

$$V = \tfrac{1}{3} \times 3.1416 \times R^2 \times H = \tfrac{1}{3} \times 3.1416 \times R^3.$$

$$\therefore\ R = \sqrt[3]{\frac{V}{\tfrac{1}{3} \times 3.1416}} = \sqrt[3]{\frac{V}{1.0472}}.$$

$$R = \sqrt[3]{\frac{1000000}{1.0472}}^{\,cm} = 98.48^{cm}. \; \textit{Ans.}$$

$$\begin{aligned} \log 1000000 &= 6.0000 \\ \text{colog}\ 1.0472 &= 9.9800 - 10 \\ 3\,&\underline{|\,5.9800} \\ &\ 1.9933 \qquad = \log 98.48. \end{aligned}$$

68. Find the capacity in pints of a cylinder, diameter 1.9375 in., height 2.4375 in. ; of a cylinder, diameter $3\tfrac{1}{8}$ in., height $3\tfrac{3}{8}$ in. ; of a cylinder, diameter $3\tfrac{11}{16}$ in., height $5\tfrac{1}{16}$ in.

$$1 \text{ pt.} = \tfrac{1}{8} \text{ of } 231 \text{ cu. in.} = 28.875 \text{ cu. in.}$$

$$\text{Volume} = \frac{0.7854 \times 1.9375^2 \times 2.4375}{28.875} \text{ pt.} = 0.2489 \text{ pt. } \textit{Ans.}$$

$$\begin{aligned} \log\ 0.7854 &= 9.8951 - 10 \\ \log 1.9375^2 &= 0.5746 \\ \log\ 2.4375 &= 0.3869 \\ \text{colog}\ 28.875 &= \underline{8.5395 - 10} \\ &\ 9.3961 - 10 = \log 0.2489. \end{aligned}$$

[redacted]

$$\text{Volume} = \frac{0.7854 \times 3.125^2 \times 3.625}{28.875} \text{ pt.} = 0.9632 \text{ pt. } \textit{Ans.}$$

$$\begin{aligned}
\log 0.7854 &= 9.8951 - 10 \\
\log 3.125^2 &= 0.9898 \\
\log 3.625 &= 0.5593 \\
\operatorname{colog} 28.875 &= 8.5395 - 10 \\
\hline
& 9.9837 - 10 = \log 0.9632.
\end{aligned}$$

$$\text{Volume} = \frac{0.7854 \times 3.8125^2 \times 5.0625}{28.875} \text{ pt.} = 2.002 \text{ pt. } \textit{Ans.}$$

$$\begin{aligned}
\log 0.7854 &= 9.8951 - 10 \\
\log 3.8125^2 &= 1.1624 \\
\log 5.0625 &= 0.7044 \\
\operatorname{colog} 28.875 &= 8.5395 - 10 \\
\hline
& 0.3014 \quad = \log 2.002.
\end{aligned}$$

69. Find the capacity, in pecks, of a cylinder, diameter 15.865 in., height 12.5 in.; of a cylinder, diameter 9.25 in., height 4.25 in.; of a cylinder, diameter 18.5 in., height 8 in.

$$1 \text{ pk.} = \tfrac{1}{4} \text{ of } 2150.42 \text{ cu. in.} = 537.605 \text{ cu. in.}$$

$$\text{Volume} = \frac{0.7854 \times 15.865^2 \times 12.5}{537.605} \text{ pk.} = 4.597 \text{ pk. } \textit{Ans.}$$

$$\begin{aligned}
\log 0.7854 &= 9.8951 - 10 \\
\log 15.865^2 &= 2.4010 \\
\log 12.5 &= 1.0969 \\
\operatorname{colog} 537.605 &= 7.2695 - 10 \\
\hline
& 0.6625 \quad = \log 4.597.
\end{aligned}$$

$$\text{Volume} = \frac{0.7854 \times 9.25^2 \times 4.25}{537.605} \text{ pk.} = 0.5311 \text{ pk. } \textit{Ans.}$$

$$\begin{aligned}
\log 0.7854 &= 9.8951 - 10 \\
\log 9.25^2 &= 1.9322 \\
\log 4.25 &= 0.6284 \\
\operatorname{colog} 537.605 &= 7.2695 - 10 \\
\hline
& 9.7252 - 10 = \log 0.5311.
\end{aligned}$$

[redacted]

$$\text{Volume} = \frac{0.7854 \times 18.5^2 \times 8}{537.605} \text{ pk.} = 4 \text{ pk. } \textit{Ans.}$$

$$\begin{aligned}
\log\ 0.7854 &= 9.8951 - 10 \\
\log\ 18.5^2 &= 2.5344 \\
\log\ 8 &= 0.9031 \\
\text{colog}\ 537.605 &= \underline{7.2695} - 10 \\
&\ 0.6021 \qquad = \log 4.
\end{aligned}$$

70. What must be the diameter of a circle to contain 78.54 sq. ft.? to contain 314.16 sq. ft.?

Area $= 0.7854 \times D^2$.	Area $= 0.7854 \times D^2$.
$78.54 = 0.7854 \times D^2$.	$314.16 = 0.7854 \times D^2$.
$\therefore D^2 = 100$.	$\therefore D^2 = 400$.
$D = 10$.	$D = 20$.
10 ft. *Ans.*	20 ft. *Ans.*

71. What must be the diameter of a circle to contain 1 A.? to contain 9 A.?

1 A. = 43,560 sq. ft.

Area $= 0.7854 \times D^2$. $\quad \therefore D = \sqrt{\dfrac{43560}{0.7854}}$.

$$\begin{aligned}
\log 43{,}560 &= 4.6391 \\
\text{colog}\ 0.7854 &= 0.1049 \\
&\ 2\,\underline{)4.7440} \\
&\ \ \ 2.3720 = \log 235.5. \quad 235.5 \text{ ft. } \textit{Ans.}
\end{aligned}$$

$\sqrt{9} = 3.$ $\quad 3 \times 235.5 \text{ ft.} = 706.5 \text{ ft.}$ *Ans.*

72. What must be the diameter of a circle to contain 1^{ha}? to contain 25^{ha}?

$1^{ha} = 10{,}000^{qm}$. $\quad \therefore D = \sqrt{\dfrac{10000}{0.7854}}$.

$$\begin{aligned}
\log\ 10000 &= 4.0000 \\
\text{colog}\ 0.7854 &= 0.1049 \\
&\ 2\,\underline{)4.1049} \\
&\ \ \ 2.0525 = \log 112.8. \quad 112.8^{m}. \textit{ Ans.}
\end{aligned}$$

$\sqrt{25} = 5.$ $\quad 5 \times 112.8^{m} = 564^{m}$. *Ans.*

73. Divide $1270 into parts proportional to $4\frac{1}{3}$, $5\frac{1}{6}$, $6\frac{2}{7}$.

$$42 \times (4\tfrac{1}{3},\ 5\tfrac{1}{6},\ 6\tfrac{2}{7}) = 182,\ 217,\ 264.$$

$$182 + 217 + 264 = 663.$$

$$\frac{\overset{14}{\cancel{182}}}{\underset{51}{\cancel{663}}} \text{ of } \$1270 = \$\frac{17780}{51} = \$348.63.$$

$$\frac{217}{663} \text{ of } \$1270 = \$\frac{275590}{663} = \$415.67.$$

$$\frac{\overset{88}{\cancel{264}}}{\underset{221}{\cancel{663}}} \text{ of } \$1270 = \$\frac{111760}{221} = \$505.70.$$

74. How much water will a hemispherical bowl hold that is 10 in. in diameter?

$\frac{1}{2}$ of $0.5236 \times (10^3)$ cu. in. $= 0.5236 \times 500$ cu. in. $= 261.8$ cu. in. *Ans.*

75. At 50 cents a square foot, what will it cost to gild a hemispherical dome 10 ft. in diameter?

$$\frac{1}{2} \times 3.1416 \times 10^2 \times \$\frac{1}{2} = \frac{1}{\cancel{2}} \times \overset{\overset{0.7854}{\cancel{1.5708}}}{\cancel{3.1416}} \times 100 \times \$\frac{1}{\cancel{2}} = \$78.54. \quad \textit{Ans.}$$

76. If the moon is a sphere 2170 miles in diameter, how many million bushels would it hold if hollow?

$$\text{Volume} = \frac{0.5236 \times (2170 \times 5280 \times 12)^3}{2150.42} \text{ bu.}$$

$$= 633{,}000{,}000{,}000{,}000{,}000{,}000 \text{ bu. } \textit{Ans.}$$

$$\begin{aligned}
\log\ 0.5236 &= \ \ 9.7190 - 10 \\
\log\ 2170^3 &= 10.0095 \\
\log\ 5280^3 &= 11.1678 \\
\log\ 12^3 &= \ \ 3.2376 \\
\text{colog}\ 2150.42 &= \ \ \underline{6.6675 - 10} \\
&\quad\ 20.8014
\end{aligned}$$

$$= \log 633{,}000{,}000{,}000{,}000{,}000{,}000.$$

77. If the earth is 7920 miles in diameter, and the air is 40 miles deep, how many cubic miles of air are there?

$$7920 + 80 = 8000.$$

$$\begin{aligned} \log\ 8000^3 &= 11.7093 \\ \log 0.5236 &= \underline{\ 9.7190 - 10} \\ &\ 11.4283 \\ &= \log 268,100,000,000. \end{aligned} \qquad \begin{aligned} \log\ 7920^3 &= 11.6961 \\ \log 0.5236 &= \underline{\ 9.7190} - 10 \\ &\ 11.4151 \\ &= \log 260,100,000,000. \end{aligned}$$

$$268,100,000,000 - 260,100,000,000 = 8,000,000,000. \textit{ Ans.}$$

78. What is the difference between 2 feet square and 2 square feet? between a foot square and a square foot? between half a foot square and 6 in. square?

"2 feet square" means a square 2 ft. on a side; "2 square feet," any surface equivalent in area to two squares each 1 foot on a side. A "foot square" is a square 1 ft. on a side; while a square foot is an equivalent area in any shape. "Half a foot square" is ambiguous. Half "a foot square" is half a square foot, while "half a foot" square is 6 inches square; that is, one-fourth a square foot. "6 in. square" is a square 6 in. on a side.

79. Find the volume of a frustum of a right pyramid whose lower base is a square 3 ft. on a side, upper base a square 2 ft. on a side, and height 4 ft.

$$\tfrac{1}{3} \times 4 \times (3^2 + 2^2 + \sqrt{3^2 \times 2^2}) = \tfrac{1}{3} \times 4 \times (9 + 4 + 6) = \tfrac{1}{3} \times 4 \times 19 = 25\tfrac{1}{3}.$$

$25\tfrac{1}{3}$ cu. ft. *Ans.*

80. Find the capacity in liquid quarts of a tin pan 10 in. in diameter at the top, 8 in. in diameter at the bottom, and 4 in. deep.

$$\tfrac{1}{3} \times 4 \times (0.7854 \times 10^2 + 0.7854 \times 8^2 + \sqrt{0.7854 \times 10^2 \times 0.7854 \times 8^2})$$

$$= \tfrac{1}{3} \times 4 \times 0.7854 \times (100 + 64 + 80)$$

$$= \frac{1}{\cancel{3}} \times 4 \times \overset{0.2618}{\cancel{0.7854}} \times 244 = 255.5168.$$

$$255.5168 \text{ cu. in.} = \frac{255.5168}{57.75} \text{ qt.} = 4.42 \text{ qt. } \textit{Ans.}$$

$$
\begin{array}{r}
4.42 \\
5775\overline{)25551.68} \\
\underline{23100} \\
24516 \\
\underline{23100} \\
14168 \\
\underline{11550} \\
2618
\end{array}
$$

81. How many hektoliters will a circular vat hold 5^m in diameter at the top, 4.57^m in diameter at the bottom, and 1.17^m deep?

$$
\begin{aligned}
&\tfrac{1}{3} \times 1.17 \times (0.7854 \times 5^2 + 0.7854 \times 4.57^2 \\
&\qquad + \sqrt{0.7854 \times 5^2 \times 0.7854 \times 4.57^2}) \\
&\qquad = 0.39 \times 0.7854 \times (5^2 + 4.57^2 + 5 \times 4.57) \\
&\qquad = 0.39 \times 0.7854 \times (25 + 20.8849 + 22.85) \\
&\qquad = 0.39 \times 0.7854 \times 68.7349 = 21.0539.
\end{aligned}
$$

$$
\begin{array}{r}
4.57 \\
\underline{4.57} \\
3199 \\
2285 \\
\underline{1828} \\
20.8849
\end{array}
\qquad
\begin{array}{r}
0.7854 \\
\underline{0.39} \\
70686 \\
\underline{23562} \\
0.306306
\end{array}
\qquad
\begin{array}{r}
68.7349 \\
\underline{0.306306} \\
4124094 \\
2062047 \\
4124094 \\
\underline{2062047} \\
21.0539122794
\end{array}
$$

$21.0539^{cbm} = 210.539^{hl}$. *Ans.*

82. If 4 cu. in. of iron weigh 1 lb. avoirdupois, what is the weight in grains of 1 cu. in. of iron? What is the specific gravity of the iron?

1 cu. in. of iron weighs $\frac{1}{4}$ lb. $= \frac{1}{4}$ of 7000 gr. $=$ 1750 gr. *Ans.*
1 cu. ft. of iron weighs $1728 \times \frac{1}{4}$ lb. $=$ 432 lb.
$432 \div 62\frac{1}{2} = 432 \times \frac{2}{125} = \frac{864}{125} = 6.912$. *Ans.*

$$
\begin{array}{r}
6.912 \\
125\overline{)864.} \\
\underline{750} \\
1140 \\
\underline{1125} \\
150 \\
\underline{125} \\
250 \\
\underline{250}
\end{array}
$$

[redacted]

83. If 4 cu. in. of iron weigh 1 lb., what is the diameter of a 6-lb. ball? of a 32-lb. ball?

$V = (6 \times 4)$ cu. in. $= 24$ cu. in.

$V = 0.5236\, D^3.$

$24 = 0.5236\, D^3.$

$$D = \sqrt[3]{\frac{24}{0.5236}} \text{ in.} = 3.578 \text{ in. } \textit{Ans.}$$

$$\begin{array}{rl} \log\ \ 24 = & 1.3802 \\ \text{colog } 0.5236 = & \underline{0.2810} \\ 3\,\big| & \underline{1.6612} \\ & 0.5537 \\ & = \log 3.578. \end{array}$$

$V = (32 \times 4)$ cu. in. $= 128$ cu. in.

$V = 0.5236\, D^3.$

$128 = 0.5236\, D^3.$

$$D = \sqrt[3]{\frac{128}{0.5236}} \text{ in.} = 6.253 \text{ in. } \textit{Ans.}$$

$$\begin{array}{rl} \log\ \ 128 = & 2.1072 \\ \text{colog } 0.5236 = & \underline{0.2810} \\ 3\,\big| & \underline{2.3882} \\ & 0.7961 \\ & = \log 6.253. \end{array}$$

84. At $\frac{1}{4}$ lb. to the cubic inch, what is the weight of a rectangular block of iron 17.36 in. by 8.7 in. by 1.76 in.? What would be its diameter if cast into a ball, if 11 % is allowed for waste?

$$17.36 \times 8.7 \times \overset{0.44}{\cancel{1.76}} \times \frac{1}{\cancel{4}} \text{ lb.} = 66.454 \text{ lb. } \textit{Ans.}$$

$$\begin{array}{r} 17.36 \\ \underline{8.7} \\ 12152 \\ \underline{13888} \\ 151.032 \\ \underline{0.44} \\ 604128 \\ \underline{604128} \\ 66.45408 \end{array}$$

$$\text{Diameter} = \sqrt[3]{\frac{0.89 \times 4 \times 66.454}{0.5236}} \text{ in.}$$

$$= 7.673 \text{ in. } \textit{Ans.}$$

$$\begin{array}{rl} \log\ \ 0.89 = & 9.9494 - 10 \\ \log\ \ 4 = & 0.6021 \\ \log 66.454 = & 1.8225 \\ \text{colog } 0.5236 = & 0.2810 \\ 3\,\big| & \underline{2.6550} \\ & 0.8850 \\ & = \log 7.673. \end{array}$$

85. At $\frac{1}{4}$ lb. to the cubic inch, what is the weight of a rectangular block of iron 71.4 in. by $8\frac{2}{3}$ in. by $3\frac{1}{3}$ in.? What would be its diameter if cast into a ball, if 11 % is allowed for waste?

$$71\tfrac{2}{5} \times 8\tfrac{2}{3} \times 3\tfrac{1}{3} \times \tfrac{1}{4} \text{ lb.} = \frac{\overset{119}{\cancel{357}}}{\cancel{5}} \times \frac{\overset{13}{\cancel{26}}}{\cancel{3}} \times \frac{\overset{2}{\cancel{10}}}{3} \times \frac{1}{\underset{2}{\cancel{4}}} \text{ lb.} = \frac{1547}{3} \text{ lb.} = 515\tfrac{2}{3} \text{ lb. } \textit{Ans.}$$

$$\text{Diameter} = \sqrt[3]{\frac{0.89 \times 4 \times 515\frac{2}{3}}{0.5236}} \text{ in.} = 15.19 \text{ in. } \textit{Ans.}$$

[redacted]

$$\begin{aligned} \log\ 0.89 &= 9.9494 - 10 \\ \log\ 4 &= 0.6021 \\ \log\ 515\tfrac{5}{8} &= 2.7123 \\ \text{colog}\ 0.5236 &= 0.2810 \\ 3\,&\underline{)\,3.5448} \\ &\ \ 1.1816 \quad = \log 15.19. \end{aligned}$$

86. What is the diameter of a cylinder 11 in. long that will hold 2 gallons?

$$2 \text{ gal.} = 2 \times 231 \text{ cu. in.} = 462 \text{ cu. in.}$$

$$462 = 0.7854 \times D^2 \times 11.$$

$$D = \sqrt{\frac{462}{0.7854 \times 11}} \text{ in.} = \sqrt{\frac{42}{0.7854}} \text{ in.} = 7.313 \text{ in. } \textit{Ans.}$$

$$\begin{aligned} \log\ 42 &= 1.6232 \\ \text{colog}\ 0.7854 &= 0.1049 \\ 2\,&\underline{)\,1.7281} \\ &\ \ 0.8641 \quad = \log 7.313. \end{aligned}$$

87. What is the diameter of a cylinder 9 in. long that will hold 2 gallons?

$$462 = 0.7854 \times D^2 \times 9.$$

$$D = \sqrt{\frac{462}{0.7854 \times 9}} \text{ in.} = 8.086 \text{ in. } \textit{Ans.}$$

$$\begin{aligned} \log\ 462 &= 2.6646 \\ \text{colog}\ 0.7854 &= 0.1049 \\ \text{colog}\ 9 &= 9.0458 - 10 \\ 2\,&\underline{)\,1.8153} \\ &\ \ 0.9077 \quad = \log 8.086. \end{aligned}$$

88. What is the diameter of a cylinder 30^{cm} long that will hold 10 liters?

$$10^l = 10{,}000^{ccm}.$$

$$10{,}000 = 0.7854 \times D^2 \times 30.$$

$$D = \sqrt{\frac{1000}{0.7854 \times 3}}^{\,cm} = 20.6^{cm}. \ \textit{Ans.}$$

$$\begin{aligned} \log\ 1000 &= 3.0000 \\ \text{colog}\ 0.7854 &= 0.1049 \\ \text{colog}\ 3 &= 9.5229 - 10 \\ 2\,&\underline{)\,2.6278} \\ &\ \ 1.3139 \quad = \log 20.6. \end{aligned}$$

[redacted]

89. Find the circumference of a globe, if the number of square centimeters in its surface is three times the number of cubic centimeters in its volume.

$$V = 0.5236 \times D^3;\quad S = 3.1416 \times D^2.$$

$$3 \times 0.5236 \times D^3 = 3.1416 \times D^2.$$

Divide both sides by $3 \times 0.5236 \times D^2$, $D = 2$.

Hence, the circumference is $3.1416 \times 2^{cm} = 6.2832^{cm}$. *Ans.*

90. Find the diameter of a circle, if the number of inches in its circumference is equal to the number of square feet in its area.

Area $= 0.7854 \times D^2$ sq. ft.

Circumference $= 3.1416 \times D$ ft. $= 12 \times 3.1416 \times D$ in.

$$0.7854 \times D^2 = 12 \times 3.1416 \times D.$$

Divide both sides by $0.7854 \times D$, $D = 48$. 48 ft. *Ans.*

91. How many times does a carriage wheel 3 ft. 2 in. in diameter turn in going a mile on a smooth road?

$$\frac{5280}{3.1416 \times 3\frac{1}{6}} = 530.7.\ \textit{Ans.}$$

$$\begin{array}{r} 3.1416 \\ 3\frac{1}{6} \\ \hline 5236 \\ 94248 \\ \hline 9.9484 \end{array} \qquad \begin{array}{r} 530.7 \\ 99484\overline{)52800000.} \\ 497420 \\ \hline 305800 \\ 298452 \\ \hline 734800 \\ 696388 \\ \hline 38412 \end{array}$$

92. A point in the tire moves, while the wheel turns once, just four times the diameter of the wheel. How far does a spike head in the tire travel while a wheel, 3 ft. 2 in. in diameter, travels 1 mi.?

From Example 91, the wheel turns 530.7 times while the wheel goes 1 mi.

$$530.7 \times 4 \times 3\tfrac{1}{6}\text{ ft.} = \frac{\overset{1769}{\cancel{5307}}}{\underset{5}{\cancel{10}}} \times \overset{2}{\cancel{4}} \times \frac{19}{\underset{2}{\cancel{6}}}\text{ ft.} = \frac{33611}{5}\text{ ft.} = 6722.2\text{ ft.}\ \textit{Ans.}$$

[redacted]

93. An oil can is formed of two cylinders connected by a frustum of a cone. The upper cylinder, or neck, is 6^{cm} in diameter, and 75^{mm} high; the lower cylinder is 13^{cm} in diameter, and 153^{mm} high; the total length of the can is 30^{cm}. Find the capacity of the can in liters.

A square shaft to contain the neck would contain

$$(6 \times 6 \times 7.5)^{ccm} = 270^{ccm}.$$

A square shaft to contain the body would contain

$$(13 \times 13 \times 15.3)^{ccm} = 2585.7^{ccm}.$$

The frustum of a square pyramid to enclose the remainder would contain

$$\tfrac{1}{3} \times 7.2 \times (169 + 36 + \sqrt{169 \times 36})^{ccm} = 679.2^{ccm}.$$

$$0.7854 \times (270 + 2585.7 + 679.2)^{ccm} = 2776^{ccm} = 2.776^{l}. \textit{ Ans.}$$

7.5	13	169	3 | 7.2
36	13	36	2.4
450	39	78	
225	13	283	
270.0	169	2.4	
	15.3	1132	
	507	566	
	845	679.2	
	169		
	2585.7		

270.	3534.9
2585.7	0.7854
679.2	141396
3534.9	176745
	282792
	247443
	2776.31046

94. A common tunnel is formed of a frustum of a cone terminated with a cylinder. The height of the frustum is 14^{cm}, and the diameters of the two bases are 175^{mm} and 16^{mm}, respectively. The cylinder is 8^{cm} long. Find the capacity of the tunnel in liters.

[redacted]

The volume of the cylinder

$$= 0.7854 \times (8 \times 1.6^2)^{\text{ccm}} = 0.7854 \times 20.48^{\text{ccm}}.$$

The volume of the frustum of the cone

$$= \tfrac{14}{3} \times 0.7854 \times (17.5^2 + 1.6^2 + \sqrt{17.5^2 \times 1.6^2})^{\text{ccm}}$$
$$= \tfrac{14}{3} \times 0.7854 \times (306.25 + 2.56 + 28)^{\text{ccm}}$$
$$= \tfrac{14}{3} \times 0.7854 \times 336.81^{\text{ccm}} = 0.7854 \times 1571.78^{\text{ccm}}.$$

Therefore, the tunnel holds

$$0.7854 \times 20.48^{\text{ccm}} + 0.7854 \times 1571.78^{\text{ccm}}$$
$$= 0.7854 \times 1592.26^{\text{ccm}} = 1250^{\text{ccm}} = 1.25^{\text{l}}. \textit{ Ans.}$$

```
    1592.26
     0.7854
    -------
     636904
    796130
   1273808
  1114582
 ----------
 1250.561004
```

95. A pan in the form of a frustum of a cone is 10^{cm} deep, 12^{cm} across the bottom, and 23^{cm} across the top. Find the capacity of the pan in liters.

$$\tfrac{1}{3} \times 10 \times 0.7854 \times (23^2 + 12^2 + \sqrt{23^2 \times 12^2})$$
$$= \tfrac{1}{3} \times 10 \times 0.7854 \times (529 + 144 + 276)$$
$$= \tfrac{1}{3} \times 10 \times 0.7854 \times 949 = 2484.5.$$

```
  23          23           0.7854
  23          12             9490
 ---         ---          -------
  69          46           706860
 46          23            31416
 ---         ---          70686
 529         276         3|7453.4460
                          ---------
                           2484.482
```

$$2484.5^{\text{ccm}} = 2.4845^{\text{l}}. \textit{ Ans.}$$

96. Find the number of square centimeters of sheet iron in a stovepipe 4^{m} long, 26^{cm} in diameter, and 1^{mm} thick, if the edges lap one centimeter. Find the weight of the pipe, if the specific gravity of the sheet iron is 7.8.

[redacted]

$$4^{m} = 400^{cm};\ 1^{mm} = 0.1^{cm}.$$

Surface $= 400 \times (3.1416 \times 26 + 1)^{qcm} = 33{,}072.64^{qcm}$. *Ans.*

Weight $= 7.8 \times (0.1 \times 33{,}072.64)^{g} = 25{,}797^{g} = 25.797^{kg}$. *Ans.*

3.1416	3307.264
26	7.8
188496	26458112
62832	23150848
81.6816	25796.6592
1.	
82.6816	
400	
33072.6400	

97. A steam boiler is formed of a cylinder terminated at each end by a hemispherical cap of the same diameter. The length of the cylinder is 3.4^{m}, interior diameter 0.8^{m}. Find the number of hektoliters of water required to fill the boiler half full.

Volume of the cylinder

$$= 0.7854 \times (3.4 \times 0.8^2)^{cbm} = 1.709^{cbm} = 17.09^{hl}.$$

The two caps form a sphere, whose volume

$$= 0.5236 \times (0.8^3)^{cbm} = 0.268^{cbm} = 2.68^{hl}.$$

$$\tfrac{1}{2} \times (17.09^{hl} + 2.68^{hl}) = 9.89^{hl}.\ \textit{Ans.}$$

3.4	0.7854
0.8	2.176
2.72	47124
0.8	54978
2.176	7854
	15708
	1.7090304

0.8	0.5236
0.8	0.512
0.64	10472
0.8	5236
0.512	26180
	0.2680832

98. A spherical bomb is 32^{cm} in diameter, and the sides 38^{mm} thick. If the specific gravity of the metal is 7.2, what is the weight of the bomb and its capacity?

$$\text{Inside diameter} = 32^{cm} - 2 \times 3.8^{cm} = 24.4^{cm}.$$

$$\text{Inside volume} = 0.5236 \times (24.4^3)^{ccm} = 7607^{ccm} = 7.607^{l}. \quad \textit{Ans.}$$

$$\begin{array}{rl} \log 0.5236 = & 9.7190 - 10 \\ \log \ 24.4^3 = & \underline{4.1622} \\ & 3.8812 \qquad = \log 7607. \end{array}$$

$$\text{Total volume} = 0.5236 \times (32^3)^{ccm} = 17,150^{ccm}.$$

$$\begin{array}{rl} \log 0.5236 = & 9.7190 - 10 \\ \log \ 32^3 = & \underline{4.5153} \\ & 4.2343 \qquad = \log 17,150. \end{array}$$

$$17,150^{ccm} - 7607^{ccm} = 9543^{ccm} = 9.543^{cdm}.$$

$$7.2 \times 9.543^{kg} = 68.71^{kg}. \quad \textit{Ans.}$$

$$\begin{array}{r} 9.543 \\ \underline{7.2} \\ 19086 \\ \underline{66801} \\ 68.7096 \end{array}$$

99. The diameters of a lampshade are 25^{cm} and 7^{cm}, and its slant height is 134^{mm}. Find its curved surface in square centimeters.

$$\tfrac{1}{2} \times (25^{cm} + 7^{cm}) = 16^{cm}.$$

$$134^{mm} = 13.4^{cm}.$$

$$(13.4 \times 3.1416 \times 16)^{qcm} = 673.6^{qcm}. \quad \textit{Ans.}$$

$$\begin{array}{r} 13.4 \\ \underline{16} \\ 804 \\ \underline{134} \\ 214.4 \end{array} \qquad \begin{array}{r} 3.1416 \\ \underline{214.4} \\ 125664 \\ 125664 \\ 31416 \\ \underline{62832} \\ 673.55904 \end{array}$$

[redacted]

100. A niche is formed like a half-cylinder surmounted by a quarter of a sphere. The height of the cylinder is 1.2^{m}, the diameter 0.8^{m}. Find the volume of the niche, and the area of its interior surface.

$$\begin{aligned}\text{Volume of half-cylinder} &= \tfrac{1}{2} \times (1.2 \times 0.7854 \times 0.8^2)^{cbm}\\ &= 0.30159^{cbm} = 301.59^{l}.\\ \text{Volume of quarter-sphere} &= \tfrac{1}{4} \times (0.5236 \times 0.8^3)^{cbm}\\ &= 0.06702^{cbm} = 67.02^{l}.\end{aligned}$$

$$301.59^{l} + 67.02^{l} = 368.61^{l}. \textit{ Ans.}$$

$$\begin{array}{r} 0.8\\ \underline{0.8}\\ 0.64\\ \underline{0.6}\\ 0.384 \end{array} \qquad\qquad \begin{array}{r} 0.8\\ \underline{0.8}\\ 0.64\\ \underline{0.8}\\ 0.512 \end{array}$$

$$\begin{array}{r} 0.7854\\ \underline{0.384}\\ 31416\\ 62832\\ \underline{23562}\\ 0.3015936 \end{array} \qquad\qquad \begin{array}{r} 0.5236\\ \underline{0.512}\\ 10472\\ 5236\\ \underline{26180}\\ 4\,\overline{)\,0.2680832}\\ 0.0670208 \end{array}$$

Surface of half-cylinder $= \frac{1}{2}(1.2 \times 3.1416 \times 0.8)^{qm} = 1.5080^{qm}$.

Surface of quarter-sphere $= \frac{1}{4} \times (3.1416 \times 0.8^2)^{qm} = 0.5027^{qm}$.

Surface of the floor $= \frac{1}{2} \times (0.7854 \times 0.8^2)^{qm} = 0.2513^{qm}$.

$$1.5080^{qm} + 0.5027^{qm} + 0.2513^{qm} = 2.262^{qm}.. \textit{ Ans.}$$

$$\begin{array}{r} 0.8\\ \underline{0.6}\\ 0.48 \end{array} \quad \begin{array}{r} 3.1416\\ \underline{0.48}\\ 251328\\ \underline{125664}\\ 1.507968 \end{array} \quad \begin{array}{r} 0.8\\ \underline{0.8}\\ 4\,\overline{)\,0.64}\\ 0.16 \end{array} \quad \begin{array}{r} 3.1416\\ \underline{0.16}\\ 188496\\ \underline{31416}\\ 0.502656 \end{array} \quad \begin{array}{r} 0.7854\\ \underline{0.32}\\ 15708\\ \underline{23562}\\ 0.251328 \end{array}$$

101. What is the expense, at 30 cents a square yard, of painting the walls and ceiling of a room 22 ft. 6 in. long, 13 ft. 6 in. wide, and 10 ft. high?

Perimeter $= 2 \times (22\frac{1}{2}$ ft. $+ 13\frac{1}{2}$ ft.$) = 72$ ft.

Area of walls $= (10 \times 72)$ sq. ft. $= 720$ sq. ft.

Area of ceiling $= (22\frac{1}{2} \times 13\frac{1}{2})$ sq. ft. $= 303.75$ sq. ft.

Total area $= 720$ sq. ft. $+ 303.75$ sq. ft. $= 1023.75$ sq. ft. $= 113.75$ sq. yd.

114 sq. yd. at \$0.30 a sq. yd. will cost $114 \times \$0.30 = \$34.20$. *Ans.*

[redacted]

102. In what time will an empty cistern be filled by three pipes whose diameters are $\frac{1}{2}$ in., $\frac{3}{4}$ in., and 1 in., if the largest alone would fill it in 40 min.? The rates of flow are proportional to the squares of the diameters.

The smallest alone would fill it in $(\frac{2}{1})^2$ of 40 min. = 160 min.
The other alone would fill it in $(\frac{4}{3})^2$ of 40 min. = $71\frac{1}{9}$ min.
Hence, in 1 min. the largest fills $\frac{1}{40}$ of the cistern,
the smallest fills $\frac{1}{160}$ of the cistern,
the other fills $\frac{9}{640}$ of the cistern,
and all three together fill $\frac{1}{40}+\frac{1}{160}+\frac{9}{640}=\frac{29}{640}$ of it.

Hence, it will take $\frac{640}{29}$ min. = $22\frac{2}{29}$ min. *Ans.*

103. How many gallons of water are contained in a length of 50 yd. of a canal, if its width at the top is 8 yd. and at the bottom 7 yd., and its depth 5 ft.?

The average width is $\frac{8+7}{2}$ yd. = $7\frac{1}{2}$ yd. = $22\frac{1}{2}$ ft.

50 yd. = 150 ft.

$$\frac{150\times 22\frac{1}{2}\times 5\times 1728}{231}=\frac{\overset{50}{\cancel{150}}\times 45\times 5\times \overset{864}{\cancel{1728}}}{\cancel{2}\times \underset{77}{\cancel{231}}}=\frac{9720000}{77}=126{,}233.8.$$

126,233.8 gal. *Ans.*

```
    864                 126233.8
      5              77)9720000.
   ----                 77
   4320                 --
     50                 202
   ----                 154
 216000                 ---
     45                  480
 ------                  462
1080000                  ---
864                       180
-------                   154
9720000                   ---
                           260
                           231
                           ---
                            290
                            231
                            ---
                            590
```

[redacted]

104. A man who rows 4 miles an hour in still water takes 1 hr. 12 min. to row 4 miles up a river. How long will it take him to row down again?

$$1 \text{ hr. } 12 \text{ min.} = 1.2 \text{ hr.}$$

In still water the man could row 1.2×4 mi. $= 4.8$ mi. in 1 hr. 12 min. Hence, the stream carries him down 0.8 mi. in 1.2 hr., or flows at the rate of $\frac{0.8}{1.2}$ mi. $= \frac{2}{3}$ mi. per hour. When he rows with the stream he will row $4\frac{2}{3}$ mi. per hour, and will row 4 mi. in $\frac{4}{4\frac{2}{3}}$ hr. $= \frac{6}{7}$ hr. $= 51\frac{3}{7}$ min. *Ans.*

105. How long must a ladder be to reach a window 40 ft. from the ground, if the distance of the foot of the ladder from the wall is 9 ft.?

The length of the ladder

$$= \sqrt{40^2 + 9^2} \text{ ft.} = \sqrt{1600 + 81} \text{ ft.} = \sqrt{1681} \text{ ft.} = 41 \text{ ft. } \textit{Ans.}$$

$$\begin{array}{r} 16\,81(41 \\ 16 \\ \hline 81)81 \\ 81 \\ \hline \end{array}$$

106. If 3 oz. of gold 15 carats fine are mixed with 7 oz. 12 carats fine, what will be the fineness of the compound? What must be the fineness of 11 oz. that, when added to this compound, the whole may be 14 carats fine?

$$\begin{array}{l} 3 \times 15 = 45 \\ \underline{7} \times 12 = \underline{84} \\ 10 \qquad \overline{\underline{)129}} \\ \qquad\quad 12.9 \end{array} \qquad\qquad \begin{array}{r} 10 \text{ oz.} + 11 \text{ oz.} = 21 \text{ oz.} \\ 21 \times 14 = 294. \\ 294 - 129 = 165. \\ 165 \div 11 = 15. \end{array}$$

12.9 carats. *Ans.* 15 carats. *Ans.*

107. Find the surface of each face of a cube whose volume is 14 cu. ft. 705.088 cu. in.

[redacted]

14 cu. ft. 705.088 cu. in. = 24,897.088 cu. in.

$$
\begin{array}{rr|l}
 & & 24\,897.088(29.2 \\
 & & 8 \\
3 \times 20^2 = & 1200 & 16897 \\
3 \times (20 \times 9) = & 540 & \\
9^2 = & 81 & \\
 & 1821 & 16389 \\
3 \times 290^2 = & 252300 & 508088 \\
3 \times (290 \times 2) = & 1740 & \\
2^2 = & 4 & \\
 & 254044 & 508088
\end{array}
\qquad
\begin{array}{r}
29.2 \\
29.2 \\
\hline
584 \\
2628 \\
584 \\
\hline
852.64
\end{array}
$$

852.64 sq. in. *Ans.*

108. Determine the depth of conical wineglasses $2\frac{1}{2}$ in. across the top that 60 of them may hold a gallon.

$$\text{Volume} = \tfrac{1}{60} \text{ of } 231 \text{ cu. in.} = \tfrac{77}{20} \text{ cu. in.}$$

$$\begin{aligned}\text{Volume} &= \tfrac{1}{3} \times (2.5 \times 2.5 \times 0.7854 \times h) \text{ cu. in.}\\ &= \tfrac{1}{3} \times (6.25 \times 0.7854 \times h) \text{ cu. in.}\end{aligned}$$

$$\therefore\ \tfrac{77}{20} = \tfrac{1}{3} \times 6.25 \times 0.7854 \times h.$$

$$\therefore\ h = \frac{\frac{77}{20}}{\frac{1}{3} \times 6.25 \times 0.7854} \text{ in.}$$

$$= \left(\frac{\cancel{77}}{\cancel{20}} \times \cancel{3} \times \frac{\overset{5}{\cancel{100}}}{\cancel{625}} \times \frac{\overset{8}{\overset{\cancel{16}}{\cancel{10000}}}}{\underset{17}{\underset{\cancel{34}}{\underset{\cancel{102}}{\cancel{7854}}}}}\right) \text{ in.} = \frac{40}{17} \text{ in.} = 2.353 \text{ in. } \textit{Ans.}$$

109. What must be the length of spermaceti candles $\frac{7}{8}$ of an inch in diameter that six of them may weigh a pound, if the specific gravity of spermaceti is 0.943?

$$V = 0.7854 \times \left(\frac{7^2}{8^2} \times h\right) \text{ cu. in.}$$

1 lb. is the weight of $\dfrac{1728}{0.943 \times 62.5}$ cu. in. of spermaceti.

$$\text{Hence, } 0.7854 \times \frac{7^2}{8^2} \times h = \frac{1728}{6 \times 0.943 \times 62.5}.$$

$$h = \frac{1728 \times 8^2}{0.7854 \times 7^2 \times 6 \times 0.943 \times 62.5} \text{ in.} = 8.124 \text{ in. } \textit{Ans.}$$

$$
\begin{array}{rrl}
\log & 1728 & = 3.2375 \\
\log & 8^2 & = 1.8062 \\
\text{colog} & 0.7854 & = 0.1049 \\
\text{colog} & 7^2 & = 8.3098 - 10 \\
\text{colog} & 6 & = 9.2218 - 10 \\
\text{colog} & 0.943 & = 0.0255 \\
\text{colog} & 62.5 & = \underline{8.2041} - 10 \\
 & & \ 0.9098 \qquad = \log 8.124.
\end{array}
$$

110. A cylinder 10 in. across and 10 in. high contains 0.3927 cu. ft. of water. How many shot 0.1 in. in diameter must be poured in to raise the water to the top?

Volume of cylinder $= 0.7854 \times (10^2 \times 10)$ cu. in. $= 785.4$ cu. in.

0.3927 cu. ft. $= 0.3927 \times 1728$ cu. in. $= 678.5856$ cu. in.

785.4 cu. in. $-$ 678.5856 cu. in. $= 106.8144$ cu. in.

Volume of each shot $= 0.5236 \times (0.1^3)$ cu. in. $= 0.0005236$ cu. in.

$106.8144 \div 0.0005236 = 204{,}000$. *Ans.*

$$
\begin{array}{r}
0.3927 \\
\underline{1728} \\
31416 \\
7854 \\
27489 \\
\underline{3927} \\
678.5856
\end{array}
\qquad\qquad
\begin{array}{r}
204000 \\
5236\overline{)1068144000} \\
\underline{10472} \\
20944 \\
\underline{20944} \\
000
\end{array}
$$

111. How deep must a round cistern 4 ft. in diameter be made to be lined with the same amount of lead as a cubical cistern 4 ft. on an edge? Compare their capacities.

Amount of lead to line cubical cistern

$$= 5 \times (4 \times 4) \text{ sq. ft.} = 80 \text{ sq. ft.}$$

Area of bottom of round cistern

$$= 0.7854 \times 16 \text{ sq. ft.} = 12.5664 \text{ sq. ft.}$$

$$80 \text{ sq. ft.} - 12.5664 \text{ sq. ft.} = 67.4336 \text{ sq. ft.}$$

$$\therefore \text{depth} = \frac{67.4336}{4 \times 3.1416} \text{ ft.} = 5.366 \text{ ft. } \textit{Ans.}$$

[redacted]

$$\begin{array}{r} 3.1416 \\ \underline{4} \\ 12.5664 \end{array}
\qquad
\begin{array}{r} 5.366 \\ 125664\overline{)674336.} \\ \underline{628320} \\ 460160 \\ \underline{376992} \\ 831680 \\ \underline{753984} \\ 776960 \\ \underline{753984} \\ 22976 \end{array}$$

Cubical cistern : round cistern

$= (4^3)$ cu. ft. : $(5.366 \times 0.7854 \times 16)$ cu. ft. $= 64 : 67.43$. *Ans.*

$$\begin{array}{r} 0.7854 \\ \underline{16} \\ 47124 \\ \underline{7854} \\ 12.5664 \end{array}
\qquad
\begin{array}{r} 12.5664 \\ \underline{5.366} \\ 753984 \\ 753984 \\ 376992 \\ \underline{628320} \\ 67.4313024 \end{array}$$

112. The material for lining a cubical cistern cost \$10. Find the cost of the material for lining two similar cisterns which shall each hold one half as much.

The cost is proportional to

$(\sqrt[3]{1})^2 : 2 \times (\sqrt[3]{\tfrac{1}{2}})^2 = 1^2 : 2 \times 0.7937^2 = 1 : 2 \times 0.63 = 1 : 1.26.$

$1 : 1.26 :: \$10 : ?. \qquad 1.26 \times \$10 = \$12.60.$ *Ans.*

113. If 5 excavators sink a circular shaft 8 ft. in diameter and 125 fathoms deep in 100 days of 10 hr. each, how many nights of 7 hr. each will 4 excavators be in sinking a shaft 6 ft. in diameter and 75 fathoms deep, if the difficulty of working by night is one seventh greater than by day, and the hardness of the ground in the smaller shaft is to that in the larger shaft as 7 is to 5?

$$\left.\begin{array}{l} 4:5 \\ 8^2:6^2 \\ 125:75 \\ 7:10 \\ 7:8 \\ 5:7 \end{array}\right\} :: 100 \text{ nights} : ?.
\qquad
\frac{\cancel{5} \times \overset{9}{\cancel{36}} \times \overset{3}{\cancel{75}} \times \overset{2}{\cancel{10}} \times \cancel{8} \times \cancel{7} \times \overset{25}{\cancel{100}} \text{ nights}}{\cancel{4} \times \underset{\underset{\cancel{4}}{\cancel{8}}}{\cancel{64}} \times \underset{\cancel{5}}{\cancel{125}} \times 7 \times \cancel{7} \times \cancel{5}}$$

$= \frac{675}{7}$ nights $= 96\frac{3}{7}$ nights. *Ans.*

[redacted]

114. Find the number of dry quarts a tub will hold that is 22 in. across the top, 20 in. across the bottom, and 18 in. deep.

Area of upper base $= 3.1416 \times 11^2 = 380.1336$.

Area of lower base $= 3.1416 \times 10^2 = 314.16$.

$$\sqrt{380.1336 \times 314.16} = \sqrt{3.1416^2 \times 11^2 \times 10^2} = 3.1416 \times 11 \times 10 = 345.576.$$

$$V = \tfrac{1}{3} \times 18 \times (380.1336 + 314.16 + 345.576)$$
$$= 6 \times 1039.8696 = 6239.2176.$$

$$6239.2176 \text{ cu. in.} = \frac{6239.2176}{67.2} \text{ dry qt.} = 92.8455 \text{ dry qt. } \textit{Ans.}$$

115. Find the number of dry quarts a cylinder will hold that is 28 in. long and has a diameter of 18 in.

$$V = \frac{28 \times 3.1416 \times 9^2}{67.2} \text{dry qt.}$$
$$= 106 \text{ dry qt. } \textit{Ans.}$$

$$\begin{aligned} \log\ 28 &= 1.4472 \\ \log 3.1416 &= 0.4971 \\ \log\ 81 &= 1.9085 \\ \text{colog}\ 67.2 &= 8.1726 - 10 \\ &\overline{\ 2.0254\ } = \log 106.0. \end{aligned}$$

116. How high will 2 quarts of milk stand in a cylindrical pail 7 in. in diameter? How high will 2 quarts of oats stand in the same pail?

2 liquid qt. $= 2 \times 57\tfrac{3}{4}$ cu. in.
$= 115\tfrac{1}{2}$ cu. in.

$V = 3.1416 \times 3.5^2 \times h.$

$115.5 = 3.1416 \times 3.5^2 \times h.$

$$\therefore h = \frac{115.5}{3.1416 \times 3.5^2}.$$

$$\begin{aligned} \log\ 115.5 &= 2.0626 \\ \text{colog}\ 3.1416 &= 9.5029 - 10 \\ \text{colog}\ 3.5^2 &= 8.9118 - 10 \\ &\overline{\ 0.4773\ } \\ &= \log 3.001. \end{aligned}$$

3.001 in. *Ans.*

2 dry qt. $= 2 \times 67\tfrac{1}{5}$ cu. in.
$= 134\tfrac{2}{5}$ cu. in.

$V = 3.1416 \times 3.5^2 \times h.$

$134.4 = 3.1416 \times 3.5^2 \times h.$

$$\therefore h = \frac{134.4}{3.1416 \times 3.5^2}.$$

$$\begin{aligned} \log\ 134.4 &= 2.1284 \\ \text{colog}\ 3.1416 &= 9.5029 - 10 \\ \text{colog}\ 3.5^2 &= 8.9118 - 10 \\ &\overline{\ 0.5431\ } \\ &= \log 3.492. \end{aligned}$$

3.492 in. *Ans.*

117. Find the capacity in gallons of a cylindrical boiler 1 ft. in diameter and 4 ft. 10 in. long; of a cylindrical boiler 1 ft. 6 in. in diameter and 3 ft. 6 in. long; of a cylindrical boiler 2 ft. 8 in. in diameter and 5 ft. 6 in. long.

1 ft. = 12 in.; 4 ft. 10 in. = 58 in.

$$V = \frac{3.1416 \times 6^2 \times 58}{231}.$$

$$\begin{aligned} \log 3.1416 &= 0.4971 \\ \log 36 &= 1.5563 \\ \log 58 &= 1.7634 \\ \text{colog } 231 &= \underline{7.6364} - 10 \\ & 1.4532 \\ &= \log 28.39. \end{aligned}$$

28.39 gal. *Ans.*

1 ft. 6 in. = 18 in.; 3 ft. 6 in. = 42 in.

$$V = \frac{3.1416 \times 9^2 \times 42}{231}.$$

$$\begin{aligned} \log 3.1416 &= 0.4971 \\ \log 81 &= 1.9085 \\ \log 42 &= 1.6232 \\ \text{colog } 231 &= \underline{7.6364} - 10 \\ & 1.6652 \\ &= \log 46.26. \end{aligned}$$

46.26 gal. *Ans.*

2 ft. 8 in. = 32 in.; 5 ft. 6 in. = 66 in.

$$V = \frac{3.1416 \times 16^2 \times 66}{231}.$$

$$\begin{aligned} \log 3.1416 &= 0.4971 \\ \log 16^2 &= 2.4082 \\ \log 66 &= 1.8195 \\ \text{colog } 231 &= \underline{7.6364} - 10 \\ & 2.3612 = \log 229.7. \end{aligned}$$

229.7 gal. *Ans.*

118. Find the capacity of a tumbler $3\frac{1}{4}$ in. across the bottom, $3\frac{1}{2}$ in. across the top, and $3\frac{1}{2}$ in. deep; of a cylindrical tumbler $3\frac{1}{2}$ in. in diameter and $3\frac{1}{2}$ in. deep.

Area of upper base $= 0.7854 \times (3.5^2)$ sq. in. $= 9.62115$ sq. in.

Area of lower base $= 0.7854 \times (3.25^2)$ sq. in. $= 8.29579$ sq. in.

$$\begin{aligned} \sqrt{9.62115 \times 8.29579} &= \sqrt{3.5^2 \times 3.25^2 \times 0.7854^2} \\ &= 3.5 \times 3.25 \times 0.7854 = 8.93392. \end{aligned}$$

$$\begin{aligned} \text{Volume} &= \tfrac{1}{3} \times 3.5 \times (9.62115 + 8.29579 + 8.93392) \text{ cu. in.} \\ &= \tfrac{1}{3} \times 3.5 \times 26.85086 \text{ cu. in.} = 31.326 \text{ cu. in.} \quad \textit{Ans.} \end{aligned}$$

3.5	0.7854	3.25	10.5625
3.5	12.25	3.25	0.7854
175	39270	1625	422500
105	15708	650	528125
12.25	15708	975	845000
	7854	10.5625	730375
	9.621150		8.29578750

[redacted] [redacted]

$$\begin{array}{r} 3.25 \\ 3.5 \\ \hline 1625 \\ 975 \\ \hline 11.375 \end{array} \qquad \begin{array}{r} 11.375 \\ 0.7854 \\ \hline 45500 \\ 56875 \\ 91000 \\ 79625 \\ \hline 8.9339250 \end{array} \qquad \begin{array}{r} 26.85086 \\ 3.5 \\ \hline 13425430 \\ 8055258 \\ 3\,\underline{)93.978010} \\ 31.326 \end{array}$$

$$\text{Volume} = \left(\overset{0.3927}{\cancel{0.7854}} \times \frac{7}{\cancel{2}} \times \frac{7}{2} \times \frac{7}{2}\right) \text{cu. in.} = 33.674 \text{ cu. in.} \quad \textit{Ans.}$$

$$\begin{array}{r} 7 \\ 7 \\ \hline 49 \\ 7 \\ \hline 343 \end{array} \qquad \begin{array}{r} 0.3927 \\ 343 \\ \hline 11781 \\ 15708 \\ 11781 \\ \hline 134.6961 \end{array} \qquad \begin{array}{r} 4\,\underline{)134.6961} \\ 33.674 \end{array}$$

119. Find the area of an ellipse whose longest and shortest diameters are 11 in. and 8 in., respectively.

$$0.7854 \times (11 \times 8) \text{ sq. in.} = 69.115 \text{ sq. in.} \quad \textit{Ans.}$$

$$\begin{array}{r} 0.7854 \\ 88 \\ \hline 62832 \\ 62832 \\ \hline 69.1152 \end{array}$$

120. The ends of a rope 100 ft. long are fastened to stakes placed 80 ft. apart on level ground. A ring, to which a kid is tied, plays freely on the rope. How far from a straight line joining the stakes can the ring be pulled?

$$\sqrt{50^2 - 40^2} \text{ ft.} = \sqrt{2500 - 1600} \text{ ft.} = \sqrt{900} \text{ ft.} = 30 \text{ ft.} \quad \textit{Ans.}$$

121. If the stakes of Ex. 120 are placed 25 ft. apart, by how many per cent is the kid's pasturage increased, provided he can graze 18 in. beyond the rope when stretched?

$\sqrt{50^2 - 12.5^2}$ ft. $= \sqrt{2500 - 156.25}$ ft. $= \sqrt{2343.75}$ ft. $= 48.4$ ft.

$$\begin{array}{r|l} & 23\,43.75(48.4 \\ & 16 \\ 88) & 743 \\ & 704 \\ 964) & 3975 \\ & 3856 \end{array}$$

The diameters of the ellipse are 100 ft. + 2 × $1\frac{1}{2}$ ft. and 2 × 48.4 ft. + 2 × $1\frac{1}{2}$ ft.; that is, 103 ft. and 99.8 ft.

Area = 0.7854 × (103 × 99.8) sq. ft.

Diameters of ellipse of Ex. 120 are 103 ft. and 63 ft.

Area = 0.7854 × (103 × 63) sq. ft.

$$\frac{0.7854 \times 103 \times 99.8}{0.7854 \times 103 \times 63} = \frac{99.8}{63} = 1.584.$$

$$\begin{array}{r|l} & 1.584 \\ 63) & 99.8 \\ & 63 \\ & 368 \\ & 315 \\ & 530 \\ & 504 \\ & 260 \\ & 252 \end{array}$$

Hence, the increase is 58.4 %. *Ans.*

122. A cylindrical log, 11 in. in diameter, is sawed off at such a slant that the pieces are 8 in. longer on the longest than on the shortest side. Find the diameters of the ellipse thus made, and its area.

The shortest diameter is evidently the diameter of the log, or 11 in. The longest diameter is

$$\sqrt{11^2 + 8^2} \text{ in.} = \sqrt{121 + 64} \text{ in.} = \sqrt{185} \text{ in.} = 13.6 \text{ in.}$$

Area = (13.6 × 11 × 0.7854) sq. in. = 117.5 sq. in. *Ans.*

$$\begin{aligned} \log 13.6 &= 1.1335 \\ \log 11 &= 1.0414 \\ \log 0.7854 &= 9.8951 - 10 \\ & \overline{2.0700} \quad = \log 117.5. \end{aligned}$$

[redacted]

123. Find the area of an ellipse, if its longest diameter is 12 in. and its shortest diameter 9 in.

$$\text{Area} = 0.7854 \times (12 \times 9) \text{ sq. in.} = 84.8232 \text{ sq. in. } \textit{Ans.}$$

$$\begin{array}{r} 0.7854 \\ \underline{108} \\ 62832 \\ \underline{7854} \\ 84.8232 \end{array}$$

124. Find the number of quarts a conical vessel will hold if it is 9 in. across the top and 8 in. deep.

$$V = \frac{\frac{1}{3} \times 0.7854 \times 9^2 \times 8}{57.75} \text{ qt.} = 2.938 \text{ qt. } \textit{Ans.}$$

$$\begin{array}{lrl} \text{colog} & 3 & = 9.5229 - 10 \\ \log & 0.7854 & = 9.8951 - 10 \\ \log & 81 & = 1.9085 \\ \log & 8 & = 0.9031 \\ \text{colog} & 57.75 & = \underline{8.2384} - 10 \\ & & \ 0.4680 \qquad = \log 2.938. \end{array}$$

125. Find the number of pints a spherical bowl will hold if it is 5 in. across the top and $2\frac{1}{4}$ in. deep.

$$V = \frac{\frac{2}{3} \times 2.25 \times 5^2 \times 0.7854}{\frac{1}{2} \times 57.75} \text{ pt.} = \frac{1.5 \times 5^2 \times 0.7854}{28.875} \text{ pt.} = 1.02 \text{ pt. } \textit{Ans.}$$

$$\begin{array}{lrl} \log & 1.5 & = 0.1761 \\ \log & 25 & = 1.3979 \\ \log & 0.7854 & = 9.8951 - 10 \\ \text{colog} & 28.875 & = \underline{8.5395} - 10 \\ & & \ 0.0086 \qquad = \log 1.02. \end{array}$$

126. Find the number of pints a spherical bowl will hold if it is 4 in. across the top and $3\frac{1}{2}$ in. deep.

$$V = \frac{\frac{2}{3} \times 3\frac{1}{2} \times 4^2 \times 0.7854}{28.875} \text{ pt.} = 1.016 \text{ pt. } \textit{Ans.}$$

$$\begin{array}{lrl} \log & 2 & = 0.3010 \\ \text{colog} & 3 & = 9.5229 - 10 \\ \log & 3.5 & = 0.5441 \\ \log & 16 & = 1.2041 \\ \log & 0.7854 & = 9.8951 - 10 \\ \text{colog} & 28.875 & = \underline{8.5395} - 10 \\ & & \ 0.0067 \qquad = \log 1.016. \end{array}$$

127. Find the capacity in pints of a coffee cup 3 in. across the top and 3 in. deep.

$$V = \frac{\frac{1}{2} \times 3 \times 3^2 \times 0.7854}{\frac{1}{2} \times 57.75}\text{ pt.} = \frac{27 \times 0.7854}{57.75}\text{ pt.} = 0.3672\text{ pt. } \textit{Ans.}$$

$$\begin{aligned} \log\ 27 &= 1.4314 \\ \log 0.7854 &= 9.8951 - 10 \\ \operatorname{colog}\ 57.75 &= \underline{8.2384 - 10} \\ & \ 9.5649 - 10 = \log 0.3672. \end{aligned}$$

128. Find the capacity in liters of a spherical wash bowl 30^{cm} in diameter and 5^{cm} deep.

$$V = \tfrac{2}{3} \times (5 \times 3.1416 \times 15^2)^{ccm} = \left(\frac{2}{\cancel{3}} \times 5 \times 3.1416 \times \overset{75}{\cancel{225}}\right)^{ccm}$$

$$= 2356.2^{ccm} = 2.356^{l}.\ \textit{Ans.}$$

$$\begin{array}{r} 3.1416 \\ \underline{750} \\ 1570800 \\ \underline{219912} \\ 2356.2000 \end{array}$$

129. Find the capacity in liters of the basin of a fountain 89^{cm} in diameter and 31^{cm} deep.

$$V = \tfrac{2}{3} \times (31 \times 0.7854 \times 89^2)^{ccm} = 128{,}600^{ccm} = 128.6^{l}.\ \textit{Ans.}$$

$$\begin{aligned} \log\ 2 &= 0.3010 \\ \operatorname{colog}\ 3 &= 9.5229 - 10 \\ \log\ 31 &= 1.4914 \\ \log 0.7854 &= 9.8951 - 10 \\ \log\ 89^2 &= \underline{3.8988} \\ & \ 5.1092 \qquad = \log 128{,}600. \end{aligned}$$

130. Find the capacity in quarts of a bowl 10 in. in diameter and 4 in. deep.

$$V = \frac{\frac{2}{3} \times 4 \times 0.7854 \times 10^2}{57.75}\text{ qt.} = 3.627\text{ qt. } \textit{Ans.}$$

$$\begin{aligned} \log\ 2 &= 0.3010 \\ \operatorname{colog}\ 3 &= 9.5229 - 10 \\ \log\ 4 &= 0.6021 \\ \log 0.7854 &= 9.8951 - 10 \\ \log\ 100 &= 2.0000 \\ \operatorname{colog}\ 57.75 &= \underline{8.2384} - 10 \\ & \ 0.5595 \qquad = \log 3.627. \end{aligned}$$

$$\text{colog } 28.875 = \underline{8.5395 - 1}$$
$$9.9909 - 1$$

$$V = \frac{\frac{2}{3} \times 3 \times 0.7854 \times 7^2}{28.875} \text{ pt.} = \frac{2 \times 0.7854}{28.875}$$

$$\begin{aligned} \log \quad 2 &= 0.3010 \\ \log 0.7854 &= 9.8951 - 10 \\ \log \quad 49 &= 1.6902 \\ \text{colog } 28.875 &= \underline{8.5395} - 10 \\ & 0.4258 \end{aligned}$$

132. How many gallons will a spherical b
2 ft. deep hold?

$$V = \frac{\frac{2}{3} \times 2 \times 0.7854 \times 5^2 \times 1728}{231} \text{ gal.}$$

$$\begin{aligned} \log \quad 2 &= 0.3010 \\ \text{colog} \quad 3 &= 9.5229 - 10 \\ \log \quad 2 &= 0.3010 \\ \log 0.7854 &= 9.8951 - 10 \\ \log \quad 25 &= 1.3979 \\ \log \quad 1728 &= 3.2375 \\ \text{colog} \quad 231 &= \underline{7.6364} - 10 \\ & 2.2918 \quad = \end{aligned}$$

133. How many gallons will a spherical bow
1 ft. deep hold?

[redacted]

134. Find the capacity in pints of a saucer 5 in. across and 2 in. deep.

$$V=\frac{\frac{2}{3}\times 2\times 0.7854\times 5^2}{28.875}\text{ pt.}=\frac{100\times 0.7854}{3\times 28.875}\text{ pt.}=0.9068\text{ pt. } \textit{Ans.}$$

$$\begin{aligned}
\log\ 100 &= 2.0000\\
\log 0.7854 &= 9.8951-10\\
\text{colog}\ 3 &= 9.5229-10\\
\text{colog}\ 28.875 &= \underline{8.5395-10}\\
& \ 9.9575-10=\log 0.9068.
\end{aligned}$$

135. Find the capacity in gallons of a paraboloid (shaped like a coffee cup) boiler 25 in. across and 14 in. deep.

$$V=\frac{\frac{1}{2}\times 14\times 0.7854\times 25^2}{231}\text{ gal.}=\frac{7\times 0.7854\times 625}{231}\text{ gal.}=14.88\text{ gal. } \textit{Ans.}$$

$$\begin{aligned}
\log\ 7 &= 0.8451\\
\log 0.7854 &= 9.8951-10\\
\log\ 625 &= 2.7959\\
\text{colog}\ 231 &= \underline{7.6364}-10\\
& \ 1.1725 \qquad =\log 14.88.
\end{aligned}$$

136. Find the capacity in quarts of a conical vessel 9 in. across and 7 in. deep.

$$V=\frac{\frac{1}{3}\times 7\times 0.7854\times 9^2}{57.75}\text{qt.}=\frac{7\times 0.7854\times 27}{57.75}\text{qt.}=2.571\text{ qt.}$$

$$\begin{aligned}
\log\ 7 &= 0.8451\\
\log 0.7854 &= 9.8951-10\\
\log\ 27 &= 1.4314\\
\text{colog}\ 57.75 &= \underline{8.2384}-10\\
& \ 0.4100 \qquad =\log 2.571.
\end{aligned}$$

137. Find the number of gallons contained in a full cask whose bung diameter is 24 inches, head diameter 22 inches, and length 30 inches.

$$24\text{ in.}-22\text{ in.}=2\text{ in.}$$

Mean diameter $= 22\text{ in.} + 0.65\times 2\text{ in.} = 22\text{ in.} + 1.3\text{ in.} = 23.3\text{ in.}$

$$V=\frac{30\times 23.3^2}{294}\text{ gal.}=55.41\text{ gal. } \textit{Ans.}$$

$$\begin{aligned}
\log\ 30 &= 1.4771\\
\log 23.3^2 &= 2.7348\\
\text{colog}\ 294 &= \underline{7.5317}-10\\
& \ 1.7436 \qquad =\log 55.41.
\end{aligned}$$

[redacted]

138. Find the number of gallons contained in a full cask whose bung diameter is 22 inches, head diameter 20 inches, and length 28 inches.

$$22 \text{ in.} - 20 \text{ in.} = 2 \text{ in.}$$

$$\text{Mean diameter} = 20 \text{ in.} + 0.65 \times 2 \text{ in.} = 20 \text{ in.} + 1.3 \text{ in.} = 21.3 \text{ in.}$$

$$V = \frac{28 \times 21.3^2}{294} \text{ gal.} = 43.22 \text{ gal. } \textit{Ans.}$$

$$\begin{aligned} \log\ 28 &= 1.4472 \\ \log 21.3^2 &= 2.6568 \\ \text{colog}\ 294 &= \underline{7.5317} - 10 \\ &\ 1.6357 \qquad = \log 43.22. \end{aligned}$$

139. Find the number of gallons contained in a full cask whose bung diameter is 20 inches, head diameter 18 inches, and length 28 inches.

$$20 \text{ in.} - 18 \text{ in.} = 2 \text{ in.}$$

$$\text{Mean diameter} = 18 \text{ in.} + 0.65 \times 2 \text{ in.} = 18 \text{ in.} + 1.3 \text{ in.} = 19.3 \text{ in.}$$

$$V = \frac{28 \times 19.3^2}{294} \text{ gal.} = 35.49 \text{ gal. } \textit{Ans.}$$

$$\begin{aligned} \log\ 28 &= 1.4472 \\ \log 19.3^2 &= 2.5712 \\ \text{colog}\ 294 &= \underline{7.5317} - 10 \\ &\ 1.5501 \qquad = \log 35.49. \end{aligned}$$

140. The flash of a gun is seen $7\frac{1}{2}$ sec. before the report of the gun is heard; there is no wind, and the temperature is 73° F. How far off is the gun?

$$73^\circ - 32^\circ = 51^\circ. \qquad 51 \times 1.1 \text{ ft.} = 56.1 \text{ ft.}$$

$$1090 \text{ ft.} + 56.1 \text{ ft.} = 1146.1 \text{ ft.}$$

$$\begin{array}{r} 1146.1 \text{ ft.} \\ \underline{7.5} \\ 57305 \\ \underline{80227} \\ 8595.75 \text{ ft. } \textit{Ans.} \end{array}$$

141. A meteor was seen to burst; the report followed in 4 min. 17 sec. What was its distance, if the average temperature of the intervening air was 50° F.?

$$4 \text{ min. } 17 \text{ sec.} = 257 \text{ sec.} \qquad 50^\circ - 32^\circ = 18^\circ.$$

$$18 \times 1.1 \text{ ft.} = 19.8 \text{ ft.}$$

[redacted]

1090 ft. + 19.8 ft. = 1109.8 ft.

$$\begin{array}{r} 1109.8 \\ \underline{257} \\ 77686 \\ 55490 \\ \underline{22196} \\ 285218.6 \end{array} \qquad \begin{array}{r} 54.01 \\ 5280)\overline{285218.6} \\ \underline{26400} \\ 21218 \\ \underline{21120} \\ 9860 \\ \underline{5280} \\ 4580 \end{array}$$

54.02 mi. *Ans.*

142. How long will it take for an explosion at the equator to be heard at the antipodes of the place, if the circumference of the earth at the equator is reckoned at $40,000^{km}$, and the average temperature at the equator at 23° C.?

$23 \times 0.609^{m} = 14.007^{m}$.

$332^{m} + 14.007^{m} = 346.007^{m}$.

$20,000^{km} = 20,000,000^{m}$.

$20,000,000 \div 346.007 = 57,802$.

57,802 sec. = 16 hr. 3 min. 22 sec. *Ans.*

$$\begin{array}{r} 0.609 \\ \underline{23} \\ 1827 \\ \underline{1218} \\ 14.007 \\ \underline{332.} \\ 346.007 \end{array} \qquad \begin{array}{r} 57802 \\ 346007)\overline{20000000000} \\ \underline{1730035} \\ 2699650 \\ \underline{2422049} \\ 2776010 \\ \underline{2768056} \\ 795400 \\ \underline{692014} \\ 103386 \end{array}$$

143. If an explosion at the equator occurs at sunset and the average temperature east of the spot is 22° C., and that to the west 24° C., how far from the antipodes will the sound waves meet?

[redacted] [redacted]

$22 \times 0.609^{m} = 13.398^{m}$.
$332^{m} + 13.398^{m} = 345.398^{m}$.
$24 \times 0.609^{m} = 14.616^{m}$.
$332^{m} + 14.616^{m} = 346.616^{m}$.
$345.398^{m} + 346.616^{m} = 692.014^{m}$,

the velocity per second with which the two sound waves are approaching each other.

$692.014 : 346.616 :: 40{,}000^{km} : ?$.

$$\frac{346616 \times 40000^{km}}{692014} = 20{,}035.2^{km}.$$

$$20{,}035.2^{km} - 20{,}000^{km} = 35.2^{km}.$$ *Ans.*

```
  0.609
     22
   1218
  1218
 13.398

  0.609
     24
   2436
  1218
 14.616

      346616
       40000
 13864640000

                 20035.2
 692014)13864640000.
        1384028
          2436000
          2076042
           3599580
           3460070
            1395100
            1384028
              11072
```

144. How far off is the lightning when the thunder follows in 13 sec., the temperature being 76° F.?

$76^{\circ} - 32^{\circ} = 44^{\circ}$.
44×1.1 ft. $= 48.4$ ft.
1090 ft. $+ 48.4$ ft. $= 1138.4$ ft.

```
 1138.4 ft.
     13
  34152
 11384
 14799.2 ft.

           2.8
 5280)14799.2
      10560
       42392
       42240
         152
```

2.8 mi. *Ans.*

145. How long would it take sound to go through a whispering tube 3 mi. long, temperature 61° F.?

$61° - 32° = 29°.$
29×1.1 ft. $= 31.9$ ft.
1090 ft. $+ 31.9$ ft. $= 1121.9$ ft.
3 mi. $= 3 \times 5280$ ft. $= 15{,}840$ ft.
$15{,}840 \div 1121.9 = 14.1.$

14.1 sec. *Ans.*

$$\begin{array}{r} 14.1 \\ 11219\overline{)158400.} \\ \underline{11219} \\ 46210 \\ \underline{44876} \\ 13340 \\ \underline{11219} \\ 2121 \end{array}$$

146. Sound travels in iron about $10\frac{1}{2}$ times as fast as in air. How long, then, after seeing the blow of a sledge hammer given on the other end of an iron pipe $1\frac{1}{2}$ mi. long, may I expect to hear the sound by the iron; and how long after, to hear the sound through the air in the pipe; thermometer 63° F.?

$63° - 32° = 31°.$
31×1.1 ft. $= 34.1$ ft.
1090 ft. $+ 34.1$ ft. $= 1124.1$ ft.
$10\frac{1}{2} \times 1124.1$ ft. $= 11{,}803.05$ ft.
$1\frac{1}{2}$ mi. $= \frac{3}{\cancel{2}} \times \overset{2640}{\cancel{5280}}$ ft. $= 7920$ ft.
$7920 \div 11{,}803.05 = 0.671.$
$9\frac{1}{2} \times 0.671$ sec. $= 6.375$ sec.

$$\begin{array}{r} 0.671 \\ 1180305\overline{)792000.} \\ \underline{7081830} \\ 8381700 \\ \underline{8262135} \\ 1195650 \\ \underline{1180305} \\ 15345 \end{array} \qquad \begin{array}{r} 0.671 \\ \underline{9.5} \\ 3355 \\ \underline{6039} \\ 6.3745 \end{array}$$

By the iron in 0.671 sec.; through the air 6.375 sec. after. *Ans.*

[redacted]

147. Two gunners fire at each other simultaneously from forts 1½ mi. apart; the wind at 70° F., blows steadily from one fort to the other, at 11 mi. an hour. How soon will each hear the report of the other's gun? Suppose one ball flies on the average 987 ft. a second, the other 818 ft. a second; when will each receive the other's shot?

$$70^\circ - 32^\circ = 38^\circ.$$

$$38 \times 1.1 \text{ ft.} = 41.8 \text{ ft.}$$

$$1090 \text{ ft.} + 41.8 \text{ ft.} = 1131.8 \text{ ft.}$$

$$11 \text{ mi. per hour} = 16.1 \text{ ft. per second.}$$

The velocity of the sound with the wind = 1131.8 ft. + 16.1 ft. = 1147.9 ft. per second.

The velocity of the sound against the wind = 1131.8 ft. − 16.1 ft. = 1115.7 ft. per second.

```
           6.89                      7.09
11479)79200.              11157)79200.
      68874                     78099
      103260                     110100
       91832                     100413
       114280                      9687
       103311
        10969
```

Therefore, it will take the first sound 6.9 sec. and the second 7.1 sec. *Ans.*

```
         8.02                  9.68
   987)7920.            818)7920.
       7896                 7362
        2400                 5580
        1974                 4908
         426                  6720
                              6544
                               176
```

Therefore, it will take the first ball 8.02 sec., and the second 9.68 sec. *Ans.*

148. Sound travels in water about 4.26 times as fast as in air. How many seconds sooner would the sound of a torpedo exploded under water 2 mi. off reach you by water than by air, at 68° F.?

[redacted]

$$68^\circ - 32^\circ = 36^\circ.$$

$$36 \times 1.1 \text{ ft.} = 39.6 \text{ ft.}$$

$$1090 \text{ ft.} + 39.6 \text{ ft.} = 1129.6 \text{ ft.}$$

The velocity by water is 4.26×1129.6 ft. $= 4812.096$.

$$\begin{array}{r} 1129.6 \\ 4.26 \\ \hline 67776 \\ 22592 \\ 45184 \\ \hline 4812.096 \end{array} \qquad \begin{array}{r} 5280 \\ 2 \\ \hline 10560 \end{array}$$

$$\begin{array}{r} 9.35 \\ 11296\overline{)105600.} \\ 101664 \\ \hline 39360 \\ 33888 \\ \hline 5472 \end{array} \qquad \begin{array}{r} 2.19 \\ 4812096\overline{)10560000.} \\ 9624192 \\ \hline 9358080 \\ 4812096 \\ \hline 45459840 \\ 43308864 \\ \hline 2150976 \end{array}$$

9.35 sec. $- 2.19$ sec. $= 7.16$ sec. *Ans.*

149. A hill 482 ft. high is 8 mi. from the shore. How many miles out at sea is it visible?

$$\begin{array}{r} \tfrac{1}{2}\log 482 = 1.3415 \\ 0.1215 \\ \hline 1.4630 = \log 29.04. \end{array}$$

29.04 mi. $- 8$ mi. $= 21.04$ mi. *Ans.*

150. A sailor at the topmast 80 ft. above the sea can just see a sailor at the topmast of a similar ship. How many miles apart are the vessels?

$$\begin{array}{r} \tfrac{1}{2}\log 80 = 0.9516 \\ 0.1215 \\ \hline 1.0731 = \log 11.83. \end{array}$$

2×11.83 mi. $= 23.66$ mi. *Ans.*

[redacted]

151. How far is a mountain 1000^{m} high visible? a mountain 2000^{m} high?

$$\begin{array}{rl} \frac{1}{2}\log 1000 = & 1.5000 \\ & \underline{0.5880} \\ & 2.0880 = \log 122.5. \end{array}$$

$$\begin{array}{rl} \frac{1}{2}\log 2000 = & 1.6505 \\ & \underline{0.5880} \\ & 2.2385 = \log 173.2. \end{array}$$

122.5^{km}; 173.2^{km}. *Ans.*

152. If a man stands on a bluff that raises his eyes 11^{m} above the sea, how far can he see from the shore?

$$\begin{array}{rl} \frac{1}{2}\log 11 = & 0.5207 \\ & \underline{0.5880} \\ & 1.1087 = \log 12.84. \end{array}$$

12.84^{km}. *Ans.*

153. A sailor at sea is at a distance of 171^{km} from a mountain when the top of the mountain is just visible. How high is the mountain?

$$H = \left(\frac{171^2}{15}\right)^{m} = 1950^{m}. \textit{ Ans.}$$

$$\begin{array}{rl} \log 171^2 = & 4.4660 \\ \text{colog}\quad 15 = & \underline{8.8239} - 10 \\ & 3.2899 \qquad = \log 1950. \end{array}$$

154. A vessel approaching Valparaiso at daybreak just makes out the peak of Aconcagua, 22,427 ft. high and 140 mi. back from the coast. How far is the vessel from land if the eye of the observer is 30 ft. above the water?

$$\begin{array}{rl} \frac{1}{2}\log 30 = & 0.7386 \\ & \underline{0.1215} \\ & 0.8601 = \log 7.247. \end{array}$$

$$\begin{array}{rl} \frac{1}{2}\log 22427 = & 2.1754 \\ & \underline{0.1215} \\ & 2.2969 = \log 198.1. \end{array}$$

198.1 mi. − 140 mi. = 58.1 mi.

58.1 mi. + 7.25 mi. = 65.35 mi. *Ans.*

155. If Mount Washington is 6293 ft. high and 76 mi. in an air line from Cape Elizabeth, how far out from the Cape will its peak be visible in the ordinary state of the atmosphere?

$$\begin{array}{rl} \frac{1}{2}\log 6293 = & 1.8994 \\ & \underline{0.1215} \\ & 2.0209 = \log 104.9. \end{array}$$

104.9 mi. − 76 mi. = 28.9 mi. *Ans.*

156. How many acres of water can a man see if he stands on a raft with his eyes just 6 ft. above the water, and no land is in sight?

$$\begin{array}{rl} \tfrac{1}{2}\log 6 = & 0.3891 \\ & \underline{0.1215} \\ & 0.5106 \\ & \underline{\quad\quad 2} \\ & 1.0212 \\ \log 3.1416 = & 0.4971 \\ \log \quad 640 = & \underline{2.8062} \\ & 4.3245 = \log 21{,}110. \qquad 21{,}110 \text{ A. } \textit{Ans.} \end{array}$$

157. How far would a mountain 29,000 ft. high be visible? one of 5000 ft. high? one of 1000 ft. high?

$$\begin{array}{rl} \tfrac{1}{2}\log 29000 = & 2.2312 \\ & \underline{0.1215} \\ & 2.3527 = \log 225.3. \qquad 225.3 \text{ mi. } \textit{Ans.} \\ \tfrac{1}{2}\log \quad 5000 = & 1.8495 \\ & \underline{0.1215} \\ & 1.9710 = \log 93.54. \qquad 93.54 \text{ mi. } \textit{Ans.} \\ \tfrac{1}{2}\log \quad 1000 = & 1.5000 \\ & \underline{0.1215} \\ & 1.6215 = \log 41.83. \qquad 41.83 \text{ mi. } \textit{Ans.} \end{array}$$

158. How high must a mountain be in order to be visible at sea level 50 miles? 100 miles? 150 miles?

$$\tfrac{4}{7} \text{ of } 50^2 = \tfrac{4}{7} \times 2500 = 1429.$$
$$\tfrac{4}{7} \text{ of } 100^2 = \tfrac{4}{7} \times 10{,}000 = 5714.$$
$$\tfrac{4}{7} \text{ of } 150^2 = \tfrac{4}{7} \times 22{,}500 = 12{,}857.$$

1429 ft.; 5714 ft.; 12,857 ft. *Ans.*

159. What distance can be seen from the top of a mountain 4 miles high?

$$4 \text{ mi.} = 21{,}120 \text{ ft.}$$

$$\begin{array}{rl} \tfrac{1}{2}\log 21120 = & 2.1624 \\ & \underline{0.1215} \\ & 2.2839 = \log 192.3. \qquad 192.3 \text{ mi. } \textit{Ans.} \end{array}$$

160. Find the length of a pendulum that beats half-seconds; of a pendulum that beats quarter-seconds.

$2^2 : 1^2 :: 39.138 \text{ in.} : ?.$ $\qquad$ $4^2 : 1^2 :: 39.138 \text{ in.} : ?.$

$4 : 1 :: 39.138 \text{ in.} : ?.$ $\qquad$ $16 : 1 :: 39.138 \text{ in.} : ?.$

$\tfrac{1}{4} \times 39.138 \text{ in.} = 9.785 \text{ in.}$ *Ans.* $\qquad$ $\tfrac{1}{16} \times 39.138 \text{ in.} = 2.446 \text{ in.}$ *Ans.*

[redacted]

161. How many centimeters long is a pendulum that swings 80 times a minute? a pendulum that swings 30 times a minute?

$$1 \text{ in.} = 2.53998^{\text{cm}}.$$

$$80^2 : 60^2 :: 39.138 \times 2.53998^{\text{cm}} : ?.$$

$$\frac{3600 \times 39.138 \times 2.53998^{\text{cm}}}{6400} = 55.91^{\text{cm}}. \textit{ Ans.}$$

$$30^2 : 60^2 :: 39.138 \times 2.53998^{\text{cm}} : ?.$$

$$\frac{3600 \times 39.138 \times 2.53998^{\text{cm}}}{900} = 397.6^{\text{cm}}. \textit{ Ans.}$$

$$\begin{array}{lrl} \log & 3600 = & 3.5563 \\ \log & 39.138 = & 1.5926 \\ \log & 2.53998 = & 0.4048 \\ \text{colog} & 6400 = & \underline{6.1938} - 10 \\ & & 1.7475 \\ & & \quad = \log 55.91. \end{array} \qquad \begin{array}{lrl} \log & 3600 = & 3.5563 \\ \log & 39.138 = & 1.5926 \\ \log & 2.53998 = & 0.4048 \\ \text{colog} & 900 = & \underline{7.0458} - 10 \\ & & 2.5995 \\ & & \quad = \log 397.6. \end{array}$$

162. If a cannon ball is suspended by a fine wire 176 ft. long in the central well of the Bunker Hill Monument, how many times a minute will it swing?

$$176 \text{ ft.} = 2112 \text{ in.}$$

$$\sqrt{2112} : \sqrt{39.138} = 60 : ?.$$

$$\frac{\sqrt{39.138} \times 60}{\sqrt{2112}} = 8.17. \textit{ Ans.}$$

$$\begin{array}{lrl} \log & \sqrt{39.138} = & 0.7963 \\ \log & 60 = & 1.7782 \\ \text{colog} & \sqrt{2112} = & \underline{8.3376} - 10 \\ & & 0.9121 = \log 8.168. \end{array}$$

163. How long is a pendulum that swings three times in two seconds? that swings five times in two seconds?

$$3^2 : 2^2 = 39.138 \text{ in.} : ?.$$

$$9 : 4 = 39.138 \text{ in.} : ?.$$

$$\frac{4 \times \overset{13.046}{\cancel{39.138}} \text{ in.}}{\underset{3}{\cancel{9}}} = \frac{52.184}{3} \text{ in.}$$

$$= 14.061 \text{ in. } \textit{Ans.}$$

$$5^2 : 2^2 = 39.138 \text{ in.} : ?.$$

$$25 : 4 = 39.138 \text{ in.} : ?.$$

$$\frac{4 \times 39.138 \text{ in.}}{25} = 6.262 \text{ in. } \textit{Ans.}$$

$$\begin{array}{r} 39.138 \\ \underline{0.16} \\ 234828 \\ \underline{39138} \\ 6.26208 \end{array}$$

[redacted]

164. What velocity in meters a second will a cannon ball acquire in falling three quarters of a second? in falling three and a quarter seconds?

$$\begin{array}{r} 9.806^{m} \\ 0.75 \\ \hline 49030 \\ 68642 \\ \hline 7.35450^{m} \end{array} \quad \textit{Ans.} \qquad\qquad \begin{array}{r} 9.806^{m} \\ 3.25 \\ \hline 49030 \\ 19612 \\ 29418 \\ \hline 31.86950^{m} \end{array} \quad \textit{Ans.}$$

165. How long will it take a leaden ball, rolling off a table 29 in. high, to reach the floor?

$16\frac{1}{12}$ ft. $= 193$ in.

$193 : 29 :: 1^2 : (?)^2$.

$\sqrt{\frac{29}{193}}$ sec. $= 0.3876$ sec. *Ans.*

$$\begin{array}{rl} \log\ 29 = & 1.4624 \\ \text{colog}\ 193 = & 7.7144 - 10 \\ \hline & 9.1768 - 10 \\ & 10. - 10 \\ 2\,\big) & 19.1768 - 20 \\ & 9.5884 - 10 \\ & = \log 0.3876. \end{array}$$

166. What velocity will a crowbar attain in falling endwise from a balloon 2000^{m} high? How long will it be in coming down?

$4.903 : 2000 :: 1^2 : (?)^2$.

$\sqrt{\frac{2000}{4.903}}$ sec. $= 20.2$ sec. *Ans.*

$$\begin{array}{rl} \log\ 2000 = & 3.3010 \\ \text{colog}\ 4.903 = & 9.3095 - 10 \\ 2\,\big) & 2.6105 \\ & 1.3053 = \log 20.2. \end{array}$$

$$\begin{array}{r} 9.806^{m} \\ 20.2 \\ \hline 19612 \\ 19612 \\ \hline 198.0812^{m} \end{array} \quad \textit{Ans.}$$

167. What velocity will a crowbar attain in falling endwise from a balloon one mile and a quarter high? How long will it be in coming down?

[redacted]

[illegible] $= \frac{5}{4} \times$ [illegible] ft. $=$ [illegible] ft. [illegible]

$\sqrt{\frac{[illegible]}{[illegible]}}$ sec. $= \sqrt{\frac{12 \times [illegible]}{193}}$ sec. $=$ [illegible] sec. *Ans.*

log 12 = 1.0792	30.36
log [illegible] = [illegible]	32[illegible]
colog 193 = 7.7144 − 10	[illegible]
2 | [illegible]	[illegible]
[illegible] = log 30.36.	[illegible]
	651.68[illegible] 651.7 ft. *Ans.*

168. How long will it take a ball, rolling off a table, to drop 1^{cm}? 1 in.? 10^{cm}? 6 in.?

$4.903^m = 490.3^{cm}$.

$490.3 : 1 = 1^2 : (?)^2$.

$\sqrt{\frac{1}{490.3}}$ sec. $= 0.04517$ sec. *Ans.*

$$\begin{aligned}
\log\ 1 &= 0.0000\\
\text{colog}\ 490.3 &= 7.3095 - 10\\
&\overline{\ 7.3095 - 10\ }\\
&\ \ 10. \qquad - 10\\
2\ &\underline{|\,17.3095 - 20}\\
&\ \ 8.6548 - 10\\
&= \log 0.04517.
\end{aligned}$$

$490.3 : 10 = 1^2 : (?)^2$.

$\sqrt{\frac{10}{490.3}}$ sec. $= 0.1428$ sec. *Ans.*

$$\begin{aligned}
\log\ 10 &= 1.0000\\
\text{colog}\ 490.3 &= 7.3095 - 10\\
&\overline{\ 8.3095 - 10\ }\\
&\ \ 10. \qquad - 10\\
2\ &\underline{|\,18.3095 - 20}\\
&\ \ 9.1548 - 10\\
&= \log 0.1428.
\end{aligned}$$

$16\frac{1}{12}$ ft. $= 193$ in.

$193 : 1 = 1^2 : (?)^2$.

$\sqrt{\frac{1}{193}}$ sec. $= 0.07198$ sec. *Ans.*

$$\begin{aligned}
\log\ 1 &= 0.0000\\
\text{colog}\ 193 &= 7.7144 - 10\\
&\overline{\ 7.7144 - 10\ }\\
&\ \ 10. \qquad - 10\\
2\ &\underline{|\,17.7144 - 20}\\
&\ \ 8.8572 - 10\\
&= \log 0.07198.
\end{aligned}$$

$193 : 6 = 1^2 : (?)^2$.

$\sqrt{\frac{6}{193}}$ sec. $= 0.1763$ sec. *Ans.*

$$\begin{aligned}
\log\ 6 &= 0.7782\\
\text{colog}\ 193 &= 7.7144 - 10\\
&\overline{\ 8.4926 - 10\ }\\
&\ \ 10. \qquad - 10\\
2\ &\underline{|\,18.4926 - 20}\\
&\ \ 9.2463 - 10\\
&= \log 0.1763.
\end{aligned}$$

[redacted]

169. If Carisbrook Well is 210 ft. deep, how long after a pebble is dropped will it be heard to strike the bottom, if the velocity of sound is 1120 ft. a second?

$16\frac{1}{12} : 210 = 1^2 : (?)^2. \qquad 193 : 2520 = 1^2 : (?)^2.$

$\sqrt{\frac{2520}{193}}$ sec. $= 3.613$ sec.

$$\begin{aligned} \log 2520 &= 3.4014 \\ \text{colog } 193 &= 7.7144 - 10 \\ 2\,&\underline{|1.1158} \\ &0.5579 \qquad = \log 3.613. \end{aligned}$$

The sound requires $\frac{210}{1120}$ sec. $= \frac{3}{16}$ sec. $= 0.188$ sec.

3.613 sec. + 0.188 sec. = 3.801 sec. *Ans.*

170. How long after a pebble is dropped will it be heard to strike the bottom of a ventilating shaft 1600 ft. deep, if the temperature is 68° F.?

$16\frac{1}{12} : 1600 = 1^2 : (?)^2. \qquad 193 : 19{,}200 = 1^2 : (?)^2.$

$\sqrt{\frac{19200}{193}}$ sec. $= 9.975$ sec.

$$\begin{aligned} \log 19200 &= 4.2833 \\ \text{colog } 193 &= 7.7144 - 10 \\ 2\,&\underline{|1.9977} \\ &0.9989 \qquad = \log 9.975. \end{aligned}$$

68° − 32° = 36°. 36 × 1.1 ft. = 39.6 ft. 1090 ft. + 39.6 ft. = 1129.6 ft.

The sound requires $\frac{1600}{1129.6}$ sec. $= 1.416$ sec.

$$\begin{aligned} \log 1600 &= 3.2041 \\ \text{colog } 1129.6 &= 6.9471 - 10 \\ &\overline{0.1512} \qquad = \log 1.416. \end{aligned}$$

9.975 sec. + 1.416 sec. = 11.391 sec. *Ans.*

171. If a rock dropped over a precipice strikes the bottom in $7\frac{1}{2}$ sec., how high is the precipice?

$1^2 : (7\frac{1}{2})^2 = 16\frac{1}{12}$ ft. : ?. $\qquad 1 : \frac{225}{4} = 16\frac{1}{12}$ ft. : ?.

$$\frac{225}{4} \times 16\tfrac{1}{12}\text{ ft.} = \frac{\overset{75}{\cancel{225}}}{4} \times \frac{193}{\underset{4}{\cancel{12}}}\text{ ft.} = \frac{14475}{16}\text{ ft.} = 904.7\text{ ft. } \textit{Ans.}$$

[redacted]

172. How long after a pebble is dropped down a shaft 193 ft. deep will it be heard to strike the bottom, if the temperature is 50° F.?

[illegible] $193 : 1546 = 1^2 : [illegible]$

[illegible] sec. = 2.875 sec.

[illegible]

$3750 - 3584 = [illegible]$ [illegible] ft. = 29.7 ft. [illegible] ft. − 29.7 ft. = 1119.7 ft.

The sound requires [illegible] sec. = 0.1188 sec.

[illegible]

$9.0748 - 10 = \log 0.1188.$

2.875 sec. − 0.119 sec. = 2.994 sec. *Ans.*

173. Find the lifting power of a hydraulic press, the plunger being [illegible] in diameter and driven with a force of 100kg, if the lifting piston is [illegible] in diameter.

$1^m = 100^{cm}.$

[illegible] = 1,000,000kg = 1000t. *Ans.*

174. If the plunger is [illegible] in. in diameter, and is driven with a force of 1000 lb., how much can it lift with a lifting piston 4 ft. in diameter?

4 ft. = 48 in.; 1000 lb. = ½ t.

$\frac{(48)^2}{[illegible]} \times \frac{1}{2}$ t. = 48 × 48 × 2 × 2 × $\frac{1}{2}$ t. = 4608 t. *Ans.*

175. If the plunger is 2 in. in diameter, and is driven with a force of 1000 lb., how much can it lift with a lifting piston 2 ft. in diameter?

2 ft. = 24 in.; 1000 lb. = ½ t.

$[illegible] \times \frac{1}{2}$ t. = $12^2 \times \frac{1}{2}$ t. = 144 × ½ t. = 72 t. *Ans.*

176. The water stands in a fissure in a rock 10m high and 12m long. What pressure is exerted to split the rock on the lowest meter's width? on the highest meter's width? in the whole fissure?

$(1 \times 12 \times 9.5)^{cbm} = 114^{cbm}.$ $(10 \times 12 \times 5)^{cbm} = 600^{cbm}.$

$(1 \times 12 \times 0.5)^{cbm} = 6^{cbm}.$ 114t.; 6t.; 600t. *Ans.*

[redacted]

177. A dam is 100 ft. long and 10 ft. deep, and the water is just flowing over it. What pressure is exerted on the lowest two feet of the dam?

$$(100 \times 9 \times 2) \text{ cu. ft.} = 1800 \text{ cu. ft.}$$

$$1800 \times 62\tfrac{1}{2} \text{ lb.} = \overset{900}{\cancel{1800}} \times \frac{125}{\cancel{2}} \text{ lb.} = 112{,}500 \text{ lb.} = 56\tfrac{1}{4} \text{ t. } \textit{Ans.}$$

178. Water is running 2 ft. over a dam that is 180 ft. long and 12 ft. deep. Find the pressure on the dam.

$$(180 \times 12 \times 7) \text{ cu. ft.} = 15{,}120 \text{ cu. ft.}$$

$$15{,}120 \times 62.5 \text{ lb.} = 945{,}000 \text{ lb.} = 472.5 \text{ t. } \textit{Ans.}$$

12	15120
7	62.5
84	75600
180	30240
6720	90720
84	945000.0
15120	

179. Water is running 9 in. deep over a dam that is 78 ft. long and 8 ft. deep. Find the pressure on the dam.

$$(78 \times 8 \times 4\tfrac{3}{8}) \text{ cu. ft.} = 2730 \text{ cu. ft.}$$

$$2730 \times 62.5 \text{ lb.} = 170{,}625 \text{ lb.} = 85 \text{ t. } 625 \text{ lb. } \textit{Ans.}$$

$4\frac{3}{8}$	62.5
8	2730
35	18750
78	4375
280	1250
245	170625.0
2730	

180. With what velocity will water flow through a hole 9 ft. below the surface?

$\sqrt{9} : \sqrt{16} = 3 : 4.$ $\frac{3}{4}$ of 32 ft. = 24 ft. *Ans.*

181. With what velocity will water leave a fountain having free play, and a head of 25 ft.? a head of 100 ft.?

$\sqrt{25} : \sqrt{16} = 5 : 4.$ $\qquad$ $\sqrt{100} : \sqrt{16} = 10 : 4 = 5 : 2.$

$\frac{5}{4}$ of 32 ft. = 40 ft. *Ans.* $\qquad$ $\frac{5}{2}$ of 32 ft. = 80 ft. *Ans.*

182. If a hole in the side of a cistern 4 ft. below the surface of the water is delivering 10 gal. an hour, how many gallons would it deliver with 5 ft. more head?

$\sqrt{4} : \sqrt{9} = 10$ gal. : ?.

$2 : 3 = 10$ gal : ?.

$$\frac{3 \times \overset{5}{\cancel{10}} \text{ gal.}}{\cancel{2}} = 15 \text{ gal. } \textit{Ans.}$$

183. If a pipe 2 in. in diameter, and 1 ft. long, inserted in a dam, the head of water being kept constant, delivers 4 gallons of water a minute, how many gallons a minute may be expected when another pipe of the same length, but $2\frac{1}{2}$ in. in diameter, is substituted for the two-inch pipe?

$2^2 : (2\frac{1}{2})^2 = 4$ gal. : ?.

$4 : 6\frac{1}{4} = 4$ gal. : ?.

$$\frac{6\frac{1}{4} \times 4 \text{ gal.}}{4} = 6\frac{1}{4} \text{ gal. } \textit{Ans.}$$

184. If a one-inch pipe, 20 in. long, is substituted for the two-inch pipe, 1 ft. long, in Example 183, and the flow is found to be 5 pints a minute, what part of the decrease of flow is due to the smaller area of the orifice, and what part to the increased friction on the sides of the longer pipe?

$2^2 : 1^2 = 4$ gal. : ?. $\qquad$ $4 : 1 = 4$ gal. : ?. $\qquad$ $\frac{1 \times 4 \text{ gal.}}{4} = 1$ gal.

4 gal. − 1 gal. = 3 gal. *Ans.*

1 gal. − 5 pt. = 8 pt. − 5 pt. = 3 pt. *Ans.*

185. A miller is using water flowing through the gateway under 4 ft. head. How much more work could he do if the head was raised to 9 ft.? how much more if the head was raised to 25 ft.?

$\sqrt{4^3} : \sqrt{9^3} = 1 : ?.$ $\qquad$ $\sqrt{64} : \sqrt{729} = 1 : ?.$ $\qquad$ $8 : 27 = 1 : ?.$

$$\frac{27 \times 1}{8} = \frac{27}{8} = 3\frac{3}{8}. \textit{ Ans.}$$

$\sqrt{4^3} : \sqrt{25^3} = 1 : ?.$ $\qquad$ $\sqrt{64} : \sqrt{15625} = 1 : ?.$ $\qquad$ $8 : 125 = 1 : ?.$

$$\frac{125 \times 1}{8} = \frac{125}{8} = 15\frac{5}{8}. \textit{ Ans.}$$

[redacted]

186. A cross section of a stream of water is a rectangle 6 ft. by $2\frac{1}{2}$ ft.; the velocity is 40 ft. per minute. There is a fall of 10 ft. where a water wheel is erected that utilizes 70% of the work. Find the horse power of the wheel.

The volume of water going over the fall per minute is

$$(6 \times 2\tfrac{1}{2} \times 40) \text{ cu. ft.} = 600 \text{ cu. ft.}$$

The weight of the water per minute $= 600 \times 62.5$ lb.

The work of the wheel per minute $= (10 \times 600 \times 62.5)$ ft.-lb.

The horse power of the wheel $= \dfrac{10 \times 600 \times 62.5}{33000}$.

The utilized horse power of the wheel $= \dfrac{0.70 \times 10 \times 600 \times 62.5}{33000}$

$= 7.955$. *Ans.*

187. Find the horse power of the wheel of Ex. 186, if the fall of the water is 14 ft.

The horse power $= \dfrac{0.70 \times 14 \times 600 \times 62.5}{33000} = 11.136$. *Ans.*

188. A cross section of a stream of water is a rectangle 5 ft. by 4 ft.; the velocity is 50 ft. per minute. There is a fall of 12 ft. where a water wheel is erected that utilizes 65% of the work. Find the horse power of the wheel.

The volume of water per minute

$$= (5 \times 4 \times 50) \text{ cu. ft.} = 1000 \text{ cu. ft.}$$

The horse power of the wheel

$$= \frac{0.65 \times 12 \times 1000 \times 62.5}{33000} = 14.773. \textit{ Ans.}$$

189. Find the horse power of the wheel of Ex. 188, if the fall of the water is 16 ft.

The horse power $= \dfrac{0.65 \times 16 \times 1000 \times 62.5}{33000} = 19.697$. *Ans.*

190. A cross section of a stream of water is a trapezoid whose altitude is $3\frac{1}{2}$ ft., and parallel sides 6 ft. and 5 ft., respectively; the velocity is 150 ft. per minute. There is a fall of 9 ft. where a water wheel is erected that utilizes 75% of the work. Find the horse power of the wheel.

Area of cross section $= [3\frac{1}{2} \times \frac{1}{2}(6 + 5)]$ sq. ft. $= 19\frac{1}{4}$ sq. ft.

Volume of water per minute $= (150 \times 19\frac{1}{4})$ cu. ft. $= 2887.5$ cu. ft.

The horse power $= \dfrac{0.75 \times 9 \times 2887.5 \times 62.5}{33000} = 36.914$. *Ans.*

[redacted]

191. [illegible] diameter is making [illegible] revolutions a second. With what force does the [illegible] pull away from the centre?

[illegible]

[illegible]

1.227 [illegible] 6135.000 [illegible]

[illegible] times the weight of the material. *Ans.*

192. If a [illegible] in. long contains a stone that weighs ½ lb., and is whirled round [illegible] times a minute, what is the force pulling on the string?

Radius = 30 in. = $2\frac{1}{2}$ ft. 80 times a minute = $\frac{4}{3}$ times a second.

$$1.227 \times 2\tfrac{1}{2} \times \left(\tfrac{4}{3}\right)^2 \times \tfrac{1}{2} \text{ lb.} = \overset{0.409}{\cancel{1.227}} \times \frac{5}{\cancel{2}} \times \frac{\overset{2}{\cancel{4}}}{\cancel{3}} \times \frac{4}{3} \times \frac{1}{\cancel{2}} \text{ lb.}$$

$$= \frac{8.18}{3} \text{ lb.} = 2.727 \text{ lb. } \textit{Ans.}$$

193. With what force does a locomotive that weighs 60 tons running 30 mi. an hour, on a curve of 800 ft. radius, bear against the outer rail? If the locomotive is running 60 mi. an hour, with what force does it bear on the outer rail?

30 mi. per hr. = ½ mi. per min. = 2640 ft. per min. = 44 ft. per sec.

The circumference of the curve = 2 × 3.1416 × 800 ft. = 5026.56 ft.

Hence, the locomotive makes $\frac{44}{5026.56}$ revolutions per second.

$$\text{Force} = 1.227 \times 800 \times \left(\frac{44}{5026.56}\right)^2 \times 120{,}000 \text{ lb.} = 9028 \text{ lb. } \textit{Ans.}$$

$$\begin{aligned} \log\ 1.227 &= 0.0889 \\ \log\ 800 &= 2.9031 \\ \log\ 44^2 &= 3.2870 \\ \text{colog}\ 5026.56^2 &= 2.5974 - 10 \\ \log\ 120{,}000 &= \underline{5.0792} \\ &\ 3.9556 \quad = \log 9028. \end{aligned}$$

If the locomotive is running 60 mi. an hour, it makes twice as many revolutions a second. Hence, since the force contains as a factor the square of the number of revolutions a second, the force is four times as great as at 30 mi. per hour.

$$4 \times 9028 \text{ lb.} = 36{,}112 \text{ lb. } \textit{Ans.}$$

[redacted]

194. If washed wool is put wet into a wire basket 1.2^m in diameter, and the basket is set to spinning at the rate of 180 revolutions a second, with what force is water wrung out of the wool?

$$\text{Radius} = \tfrac{1}{2} \text{ of } 1.2^m = 0.6^m.$$

$$4.025 \times 0.6 \times 180^2 = 78,246.$$

180	4.025
180	0.6
14400	2.4150
180	32400
32400	966000
	4830
	7245
	78246.000

78,246 times its weight. *Ans.*

195. If steel pens are revolved in a basket 32^{cm} in diameter, 17 revolutions a second, with what force is the oil drained from them?

Radius $= \tfrac{1}{2}$ of $32^{cm} = 16^{cm} = 0.16^m$.

$$4.025 \times 0.16 \times 17^2 = 186.116.$$

4.025	0.644	10.948
0.16	17	17
24150	4508	76636
4025	644	10948
0.64400	10.948	186.116

186.116 times its weight. *Ans.*

196. The top of a wheel is at each instant moving with twice the velocity of the carriage, and is moving in a curve whose centre, at the instant, is as far below ground as the point is above ground. What, then, is the force exerted to separate the mud from the top of a wheel 3 ft. 2 in. in diameter, when the carriage is moving at the rate of 10 miles an hour?

When the carriage is going at the rate of 10 mi. an hour, the top of the wheel is going at the rate of 20 mi. an hour, or $29\tfrac{1}{3}$ ft. a second. The radius of the curve $= 2 \times 3\tfrac{1}{6}$ ft. $= 6\tfrac{1}{3}$ ft. The circumference of the curve is $2 \times 3.1416 \times 6\tfrac{1}{3}$ ft. $= 39.7936$ ft. The force $= 1.227 \times 6\tfrac{1}{3} \times \left(\dfrac{29.3333}{39.7936}\right)^2 = 4.224$ times the weight of the mud. *Ans.*

$\log$ [illegible] $=$ [illegible]
$\log$ [illegible] $=$ [illegible]
$\log$ [illegible] $=$ [illegible]
[illegible] $=$ [illegible] $- 10$
[illegible] $= \log 4.224$

197. [illegible] a chain, weighing half a [illegible] make the lowest part curve with an [illegible]?

[illegible] $=$ [illegible] $\qquad$ 6 ft. $= 2$ yd.
[illegible] $\qquad$ 2 yd. weighs 1 lb.
[illegible] *Ans.* $\qquad$ ∴ tension $= 1$ lb. *Ans.*

198. A [illegible] rope, weighing [illegible] lb. to the yard, is fastened at one [illegible] the same level, runs over [illegible]. What is the radius of its [illegible]?

[illegible] represents 100 yd. of rope.
[illegible] *Ans.*

199. A [illegible] the rope of Example 198, and increases its [illegible] become?

[illegible] $=$ [illegible] of what it was, it takes only [illegible] *Ans.*

200. A [illegible] move a ship, straightened the [illegible] point was 1980 ft. The rope was [illegible] the yard. With what force was it stretched?

$$1980 \text{ ft.} = 660 \text{ yd.}$$

$$660 \times \text{[illegible] lb.} = 2145 \text{ lb. } \textit{Ans.}$$

201. A [illegible] hangs between points on a level, and sags 4 ft. What is the radius at the lowest point?

$$\text{Radius} = \frac{(\tfrac{1}{2}\text{ chain} - \text{sag}) \times (\tfrac{1}{2}\text{ chain} - \text{sag})}{2 \times \text{sag}} \text{ ft.}$$

$$= \frac{(23.5-4) \times (15.5-4)}{2 \times 4} \text{ ft.} = \frac{19.5 \times 11.5}{8} \text{ ft.} = 28.031 \text{ ft. } \textit{Ans.}$$

$$\begin{array}{r} 19.5 \\ 11.5 \\ \hline 975 \\ 195 \\ 195 \\ \hline 8\,)\,224.25 \\ \hline 28.031 \end{array}$$

202. The whole chain, in Example 201, weighs 18 lb. What is the horizontal tension? What is the distance between the points? What is the slant, or batter, of the end of the chain?

$$\begin{aligned}\text{Tension} &= \text{weight of radius}\\ &= 28.031 \times \tfrac{18}{31} \text{ lb.}\\ &= 16.28 \text{ lb. } \textit{Ans.}\end{aligned}$$

$$\begin{array}{lrl} \log & 28.031 = & 1.4477 \\ \log & 18 = & 1.2553 \\ \text{colog} & 31 = & \underline{8.5086 - 10} \\ & & 1.2116 \\ & & \quad = \log 16.28. \end{array}$$

$$\log \tfrac{1}{2}\text{ span} = \log 19.5 + \log 11.5 + \log(1.2900 - 1.0607) + \text{colog}\, 4 + 0.0612.$$

$$1.2900 - 1.0607 = 0.2293.$$

$$\begin{array}{lrl} \log & 19.5 = & 1.2900 \\ \log & 11.5 = & 1.0607 \\ \log & 0.2293 = & 9.3604 - 10 \\ \text{colog} & 4 = & 9.3979 - 10 \\ & & \underline{0.0612} \\ & & 1.1702 \\ & & \quad = \log 14.8. \end{array}$$

$$\text{Span} = 2 \times 14.8 \text{ ft.} = 29.6 \text{ ft. } \textit{Ans.}$$

$$\text{Batter} = \frac{\text{Radius}}{\tfrac{1}{2}\text{chain}} = \frac{28.031}{15.5} = 1.809. \ \textit{Ans.}$$

$$\begin{array}{lrl} \log & 28.031 = & 1.4477 \\ \text{colog} & 15.5 = & \underline{8.8097 - 10} \\ & & 0.2574 \quad = \log 1.809. \end{array}$$

203. A chain weighing 1^{kg} to the meter is suspended from points on a level; the length of chain is 31^m, and it sags 1.3^m. Find all the conditions, and find how much it falls below a level at 10^{cm} from each end.

$$\begin{aligned}\text{Radius} &= \frac{(15.5+1.3)\times(15.5-1.3)^m}{2\times 1.3}\\ &= \frac{16.8 \times 14.2^m}{2.6}\\ &= 91.75^m. \ \textit{Ans.}\end{aligned}$$

$$\begin{array}{lrl} \log & 16.8 = & 1.2253 \\ \log & 14.2 = & 1.1523 \\ \text{colog} & 2.6 = & \underline{9.5850 - 10} \\ & & 1.9626 \\ & & \quad = \log 91.75. \end{array}$$

$$\text{Tension} = 91.75^{kg}. \ \textit{Ans.}$$

$$\tfrac{1}{2}\text{ span} = 15.42^m.$$

$$\text{Span} = 2 \times 15.42^m = 30.84^m. \ \textit{Ans.}$$

$$\begin{array}{lrl} \log & 16.8 = & 1.2253 \\ \log & 14.2 = & 1.1523 \\ \log & 0.0730 = & 8.8633 - 10 \\ \text{colog} & 1.3 = & 9.8861 - 10 \\ & & \underline{0.0612} \\ & & 1.1882 \\ & & \quad = \log 15.42. \end{array}$$

$$\text{Batter} = \frac{91.75}{15.5} = 5.92. \ \textit{Ans.}$$

[redacted]

215. ... 204 ... [illegible] ...

... [illegible] ... $= 38.75$ ... Ans.

$\log 45.1 = \ldots - 10$

1.5883

$= \log 38.75.$

Tension $= 38.75 \times 3.28 \times 0.875$ lb.

$= 111.2$ lb. Ans.

$\log 38.75 = 1.5883$

$\log 3.28 = 0.5159$

$\log 0.875 = 9.9420 - 10$

2.0462

$= \log 111.2.$

206. How heavy a rock placed 6 in. from the fulcrum can a man, who weighs 180 lb., raise with a crowbar 5 ft. 6 in. long?

$$180 \text{ lb.} : W = 6 \text{ in.} : 5 \text{ ft.}$$
$$180 \text{ lb.} : W = 1 : 10.$$
$$\therefore\ W = 10 \times 180 \text{ lb.} = 1800 \text{ lb.}\ \textit{Ans.}$$

207. Two weights of 30 lb. and 20 lb., respectively, at the ends of a horizontal lever 5 ft. long, balance. Find how far and in which direction the fulcrum must be moved for the weights to balance when each is increased by 5 lb.

$$30 : 20 = 20\text{-lb. arm} : 30\text{-lb. arm}.$$
$$\therefore\ 20\text{-lb. arm} = \tfrac{3}{5} \text{ of } 5 \text{ ft.} = 3 \text{ ft.}$$
$$35 : 25 = 25\text{-lb. arm} : 35\text{-lb. arm}.$$
$$\therefore\ 25\text{-lb. arm} = \tfrac{7}{12} \text{ of } 5 \text{ ft.} = 2\tfrac{11}{12} \text{ ft.} = 2 \text{ ft. } 11 \text{ in.}$$

Therefore, the fulcrum must be moved 1 in. toward the lighter weight. *Ans.*

208. A man who weighs 160 lb., wishing to raise a rock, leans with his whole weight on a horizontal crowbar 5 ft. long, which is propped at the distance of 4 in. from the end in contact with the rock. Find the force he exerts on the rock, and the pressure the prop has to sustain, if the weight of the crowbar is not reckoned.

$$4 : 56 = 160 \text{ lb.} : ?.$$
$$\frac{56 \times \overset{40}{\cancel{160}} \text{ lb.}}{\cancel{4}} = 2240 \text{ lb.}\ \textit{Ans.}$$
$$2240 \text{ lb.} + 160 \text{ lb.} = 2400 \text{ lb.}\ \textit{Ans.}$$

209. A child weighing 56 lb. is seated at one end of a plank 16 ft. long, and a child weighing 72 lb. is at the other end. Find the distance of each child from the fulcrum when the plank is used for a seesaw.

$$56 : 72 = 7 : 9.$$

Therefore, the 56-lb. child is 9 ft. from the fulcrum, and the 72-lb. child is 7 ft. *Ans.*

210. In a pair of nutcrackers if the nut is placed at a distance of 1 in. from the hinge, and the hand presses at a distance of 8 in. from the hinge, find the pressure upon the nut for every ounce of pressure exerted by the hand.

$$1 : 8 = 1 \text{ oz.} : ?. \qquad 8 \times 1 \text{ oz.} = 8 \text{ oz.}\ \textit{Ans.}$$

[redacted]

211. A body is weighed in both arms of a false balance, and its apparent weights are 2.56 lb. and 2.25 lb. Find its true weight.

$$2.56 : \text{true weight} = \text{true weight} : 2.25.$$

$$\text{True weight} = \sqrt{2.56 \times 2.25}\text{ lb.} = (1.6 \times 1.5)\text{ lb.} = 2.4\text{ lb. } \textit{Ans.}$$

212. In a steelyard the weight of the beam is 15 lb., and the distance of its centre of gravity from the fulcrum is 3 in. Find the distance from the fulcrum a weight of 6 lb. must be placed to balance the beam.

$$6 : 15 = 3\text{ in.} : ?.$$

$$\frac{15 \times 3\text{ in.}}{6} = 7\tfrac{1}{2}\text{ in. } \textit{Ans.}$$

213. A cask weighing 160^{kg} is attached to a rope wound on an axle 19^{cm} in diameter; at one end of the axle is a wheel 175^{cm} in diameter. With what force must a man pull down on a rope passing over the wheel to raise the cask?

$$175 : 19 = 160^{kg} : ?.$$

$$\frac{19 \times \overset{32}{\cancel{160^{kg}}}}{\underset{35}{\cancel{175}}} = 17\tfrac{13}{35}{}^{kg} = 17.37^{kg}.\ \textit{Ans.}$$

214. A rope passes over a single pulley. How much force is required to raise 180 lb. attached to one end of a rope if 1 % of the force is required to overcome friction?

$$\frac{100}{\underset{11}{\cancel{99}}} \text{ of } \overset{20}{\cancel{180}}\text{ lb.} = \frac{2000}{11}\text{ lb.} = 181.82\text{ lb. } \textit{Ans.}$$

215. If the radius of the wheel is four times that of the axle, and the string round the wheel can support a weight of 50 lb. only, find the greatest weight that can be lifted.

$$1 : 4 = 50\text{ lb.} : ?.$$

$$4 \times 50\text{ lb.} = 200\text{ lb. } \textit{Ans.}$$

216. Find the ratio of the radii of a wheel and axle that a force of 100 lb. may just support a weight of 1 ton.

$$1\text{ t.} = 2000\text{ lb.}$$

$$\text{Radius of wheel} : \text{radius of axle} = 2000 : 100 = 20 : 1.\ \textit{Ans.}$$

[redacted]

217. The radius of a wheel is 80^{cm} and the radius of the axle is 12^{cm}. What weight can be supported by a force of 30^{kg}? Find the work done if the weight is raised 60^{cm}.

$$12 : 80 = 30^{kg} : ?. \qquad \frac{\overset{40}{\cancel{80}} \times \overset{5}{\cancel{30}}^{kg}}{\underset{2}{\cancel{12}}} = 200^{kg}. \ \textit{Ans.}$$

$$60^{cm} = 0.6^{m}.$$

Work = (200 × 0.6) kilogram-meters = 120 kilogram-meters. *Ans.*

218. The power arm of a screw is 16 in. long, and by one turn of the screw the head advances one eighth of an inch. If the power is 3 lb., find the weight lifted.

The circumference described by the end of the power arm is

$$(2 \times 16 \times 3.1416) \text{ in.} = 100.5312 \text{ in.}$$

$$\tfrac{1}{8} : 100.5312 = 3 \text{ lb.} : ?.$$

$$8 \times 100.5312 \times 3 \text{ lb.} = 2412.75 \text{ lb. } \textit{Ans.}$$

219. In a screw used to raise a load of 10 tons, the power is 50 lb., acting by an arm 4 ft. long. Find the distance between two consecutive threads.

The circumference described by the end of the power arm is

$$(2 \times 48 \times 3.1416) \text{ in.} = 301.5936 \text{ in.}$$

$$20{,}000 : 50 = 301.6 \text{ in.} : ?.$$

$$\frac{\cancel{50} \times 301.6 \text{ in.}}{\underset{400}{\cancel{20000}}} = 0.754 \text{ in. } \textit{Ans.}$$

220. The lever of a screw is 1 ft. 9 in. long, and the power applied at the end is 100 lb. What must be the distance between the threads that a pressure of 5000 lb. may act on the press board?

The circumference described by the end of the power arm is

$$2 \times 3.1416 \times 21 \text{ in.} = 131.9472 \text{ in.}$$

$$5000 : 100 = 131.95 \text{ in.} : ?.$$

$$\frac{\cancel{100} \times 131.95 \text{ in.}}{\underset{50}{\cancel{5000}}} = 2.639 \text{ in. } \textit{Ans.}$$

221. The lever of a screw is 3 ft. 6 in. long, and the distance between the threads is $\frac{1}{5}$ in. What power must be applied at the end of the lever to produce a pressure of 10 tons on the press board?

The circumference described by the end of the power arm is

$$2 \times 3.1416 \times 42 \text{ in.} = 263.8944 \text{ in.}$$

$$263.8944 : \tfrac{1}{5} = 20{,}000 \text{ lb.} : ?.$$

$$\frac{\frac{1}{5} \times 20000 \text{ lb.}}{263.8944} = 15.158 \text{ lb. } \textit{Ans.}$$

```
             15.157
2638944)40000000.
        2638944
        13610560
        13194720
          4158400
          2638944
          15194560
          13194720
           19998400
           18473508
            1524892
```

222. What per cent of water is oxygen? what per cent hydrogen?

$$(2 \times 1) + 16 = 2 + 16 = 18.$$

$$\tfrac{2}{18} \text{ of } 100\% = 11\tfrac{1}{9}\% \text{ H. } \textit{Ans.}$$

$$100\% - 11\tfrac{1}{9}\% = 88\tfrac{8}{9}\% \text{ O. } \textit{Ans.}$$

223. What per cent of quicklime, CaO, is oxygen?

$40 + 16 = 56.$ $\quad \frac{16}{56}$ of $100\% = 28\frac{4}{7}\%$. *Ans.*

224. What per cent of water in slacked lime, CaO_2H_2?

$$\begin{array}{lr} Ca = & 40 \\ O_2 = & 32 \\ H_2 = & 2 \\ \hline & 74 \end{array} \qquad \begin{array}{lr} H_2 = & 2 \\ O = & 16 \\ \hline & 18 \end{array} \qquad \tfrac{18}{74} \text{ of } 100\% = 24.32\%. \ \textit{Ans.}$$

225. What per cent of pure marble, $CaCO_3$, is oxygen?

$40 + 12 + 48 = 100.$ $\quad \frac{48}{100}$ of $100\% = 48\%$. *Ans.*

226. What per cent of gypsum, called plaster of Paris, $CaSO_4 + 2\,H_2O$, is sulphur?

$$40 + 32 + 64 + 2(2 + 16) = 136 + 36 = 172.$$

$$\tfrac{32}{172} \text{ of } 100\% = 18\tfrac{26}{43}\%. \ \textit{Ans.}$$

[redacted]

227. What per cent of washing soda, $Na_2CO_3 + 10\ H_2O$, is carbon?

$$46 + 12 + 48 + 10(2 + 16) = 106 + 180 = 286.$$

$$\tfrac{12}{286} \text{ of } 100\,\% = 4\tfrac{28}{143}\,\%. \quad \textit{Ans.}$$

228. In 118 lb. of Glauber salts, $Na_2SO_4 + 10\,H_2O$, how many ounces of sulphur?

$$46 + 32 + 64 + 10(2 + 16) = 142 + 180 = 322.$$

$$\tfrac{32}{322} \text{ of } 118 \times 16 \text{ oz.} = 187.6 \text{ oz. } \textit{Ans.}$$

229. How many ounces of soda, $Na_2O + H_2O$, in 7 lb. of borax, $Na_2B_4O_7 + 10\ H_2O$?

$$46 + 16 + (2 + 16) = 80.$$

$$46 + 44 + 112 + 10(2 + 16) = 202 + 180 = 382.$$

$$7 \text{ lb.} = 112 \text{ oz.}$$

$$\tfrac{80}{382} \times 112 \text{ oz.} = 23.46 \text{ oz. } \textit{Ans.}$$

230. What per cent of pure alcohol, C_2H_6O, is carbon? What per cent of pure white marble, $CaCO_3$, is carbon?

$$24 + 6 + 16 = 46. \qquad 40 + 12 + 48 = 100.$$

$$\tfrac{24}{46} \text{ of } 100\,\% = 52\tfrac{4}{23}\,\%. \ \textit{Ans.} \qquad \tfrac{12}{100} \text{ of } 100\,\% = 12\,\%. \ \textit{Ans.}$$

231. What per cent of pure acetic acid (the acid of vinegar) is carbon, the formula being $C_2H_4O_2$?

$$24 + 4 + 32 = 60. \qquad \tfrac{24}{60} \text{ of } 100\,\% = 40\,\%. \ \textit{Ans.}$$

232. How much acetic acid can be obtained from 12 lb. of alcohol, C_2H_6O, if there is no waste?

$$C_2H_4O_2 = 60, \text{ acid.} \qquad 24 + 6 + 16 = 46, \text{ alcohol.}$$

Alcohol contains $\tfrac{16}{46}$ of O, and acid $\tfrac{32}{60}$. $\therefore\ \tfrac{32}{60} : \tfrac{16}{46} :: 12 : ?$.

$$\tfrac{16}{46} \times \tfrac{60}{32} \times 12 \text{ lb.} = 7.83 \text{ lb. } \textit{Ans.}$$

233. How many grains of carbon in 1 oz. avoirdupois of oxalic acid, $C_2H_2O_4 + 2\ H_2O$?

$$24 + 2 + 64 + 2\,(2 + 16) = 90 + 36 = 126.$$

$$\frac{\overset{2}{\cancel{24}}}{\underset{\underset{3}{\cancel{42}}}{\cancel{126}}} \text{ of } \frac{\overset{\overset{250}{\cancel{500}}}{\cancel{7000}}}{\underset{\cancel{2}}{\cancel{16}}} \text{ gr.} = \frac{250}{3} \text{ gr.} = 83\tfrac{1}{3} \text{ gr. } \textit{Ans.}$$

[redacted]

234. How many milligrams of carbon in 3^g of tartaric acid, $C_4H_6O_6$?

$48 + 6 + 96 = 150.$ $\quad 3^g = 3000^{mg}.$

$$\frac{48}{\cancel{150}} \times \overset{20}{\cancel{3000^{mg}}} = 960^{mg}. \text{ Ans.}$$

235. How many kilograms of carbon in 95^{kg} of white sugar, $C_{12}H_{22}O_{11}$?

$144 + 22 + 176 = 342.$

$$\frac{\overset{8}{\cancel{144}}}{\underset{19}{\cancel{342}}} \times \overset{5}{\cancel{95^{kg}}} = 40^{kg}. \text{ Ans.}$$

236. The formula of camphor is $C_{10}H_{16}O$. How many grams of carbon in 14^{kg} of camphor?

$120 + 16 + 16 = 152.$

$$\frac{\overset{15}{\cancel{120}}}{\underset{19}{\cancel{152}}} \times 14000^g = \frac{210000^g}{19} = 11{,}052.6^g. \text{ Ans.}$$

237. In 20^{kg} of oil of vitriol, H_2SO_4, how many grams of sulphur?

$2 + 32 + 64 = 98.$

$$\frac{\overset{16}{\cancel{32}}}{\underset{49}{\cancel{98}}} \times 20000^g = \frac{320000^g}{49} = 6530.6^g. \text{ Ans.}$$

238. What per cent of oil of vitriol is water? what per cent sulphuric acid, SO_3?

$H_2SO_4 = 98.$ $\quad \frac{18}{98}$ of $100\% = 18.37\%$, water.

$H_2O \;\; = 18.$ $\quad 100\% - 18.37\% = 81.63\%$, sulphuric acid.

239. In 3.5^g of black oxide of iron, FeO, how many milligrams of iron?

$3.5^g = 3500^{mg}.$ $\quad 56 + 16 = 72.$

$$\frac{\overset{7}{\cancel{56}}}{\underset{9}{\cancel{72}}} \text{ of } 3500^{mg} = \frac{24500^{mg}}{9} = 2722\tfrac{2}{9}^{mg}. \text{ Ans.}$$

240. Red iron-rust consists of 70 % iron and 30 % oxygen. Find its formula.

Fe = 56 and O = 16. $\quad$ 56 : 16 = 7 : 2. $\quad$ 70 : 30 = 7 : 3.

First seek multiples of 56 and 16 in the ratio of 70 to 30; that is, of 7 to 3. $\therefore$ Fe : O = 2 : 3.

Formula = Fe_2O_3. *Ans.*

241. The choking vapor of burning sulphur is sulphur and oxygen in equal parts. Find its formula.

$$S = 32. \qquad O = 16. \qquad O_2 = 32.$$

Formula $= SO_2$. *Ans.*

242. Copperas is 28.9 % sulphuric acid, 25.7 % oxide of iron, 45.4 % water. Find its formula.

Water being 18, oxide of iron 72, and sulphuric acid 80, first seek multiples of 72 and 80, in the ratio of 25.7 to 28.9; that is, of 0.8893 to 1. But 72 and 80 are in almost exactly that ratio. This gives $FeSO_4$ + water; and it remains to find a multiple of 18 which is to 152 as 45.4 is to 54.6; that is, which is 0.8315 of 152, or 126.4. But $7 \times 18 = 126$; and the addition of 7 parts of water gives as the complete formula, $FeSO_4 + 7\,H_2O$. *Ans.*

243. Spirits of turpentine is 11.76 % hydrogen and 88.24 % carbon. Find its formula. What per cent of oxygen combined with spirits of turpentine are required to make camphor, $C_{10}H_{16}O$?

Hydrogen being 1 and carbon 12, and 88.24 being almost exactly 7.5 times 11.76, we seek the smallest multiple of 12 that is 7.5 times a whole number. This is evidently 5 times 12, equal to 7.5 times 8. Therefore the formula is C_5H_8, or as chemistry gives it $C_{10}H_{16}$. *Ans.*

Add O, and we have $C_{10}H_{16}O$, the formula of camphor; containing $\frac{16}{136} = 11.76\%$ of oxygen added to spirits of turpentine. *Ans.*

244. If the resistance of 1 mile of wire 2^{mm} in diameter is 4.72 ohms, what is the resistance of 3 miles of wire of the same material 3^{mm} in diameter?

$$\left.\begin{array}{c} 1 : 3 \\ 3^2 : 2^2 \end{array}\right\} :: 4.72 \text{ ohms} : \text{resistance}.$$

$$\text{Resistance} = \frac{\cancel{3} \times 2 \times 2 \times 4.72 \text{ ohms}}{1 \times \cancel{3} \times 3} = 6.29 \text{ ohms. } \textit{Ans.}$$

245. What length of copper wire 1^{mm} in diameter has the same resistance as 720^m of copper wire 4^{mm} in diameter?

$$(\tfrac{1}{4})^2 \text{ of } 720^m = \tfrac{1}{16} \text{ of } 720^m = 45^m. \ \textit{Ans.}$$

246. The conductivity of iron is $\frac{1}{7}$ that of copper. If the resistance of a copper wire 1 mile long and $\frac{1}{8}$ in. in diameter is 6.8 ohms, what is the resistance of an iron wire $\frac{1}{16}$ in. in diameter and 5 miles long?

$$\begin{array}{l} 1:7 \\ 1:5 \\ (\tfrac{1}{16})^2 : (\tfrac{1}{8})^2 \end{array} \Bigg\} :: 6.8 \text{ ohms} : \text{resistance.}$$

$$\text{Resistance} = \frac{7 \times 5 \times \frac{1}{8} \times \frac{1}{8} \times 6.8 \text{ ohms}}{\frac{1}{16} \times \frac{1}{16}} = 7 \times 5 \times 4 \times 6.8 \text{ ohms}$$

$$= 952 \text{ ohms. } \textit{Ans.}$$

247. If 50 volts force 54.8 ampères of electrical current through a lamp, what is the resistance?

$$\text{Ampères} = \frac{\text{volts}}{\text{ohms}}. \qquad 54.8 = \frac{50}{\text{ohms}}.$$

$$\text{Resistance} = \frac{50}{54.8} \text{ ohms} = 0.912 \text{ ohm. } \textit{Ans.}$$

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        0.912
548)500.
     4932
      680
      548
     1320
     1096
      224
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248. If the resistance of an electric lamp is 2.8 ohms when a current of 10 ampères is passing through it, what is the voltage?

$$10 = \frac{\text{volts}}{2.8}. \qquad \text{Voltage} = 10 \times 2.8 \text{ volts} = 28 \text{ volts. } \textit{Ans.}$$

249. Five arc lamps on a circuit have each a resistance of 2.35 ohms. The resistance of the wires is 1.2 ohms and of the dynamo is 0.75 ohm. What voltage is required to send a current of 15 ampères through the circuit?

$$5 \times 2.35 \text{ ohms} = 11.75 \text{ ohms.}$$

$$\text{Total resistance} = 11.75 \text{ ohms} + 1.2 \text{ ohms} + 0.75 \text{ ohm} = 13.7 \text{ ohms.}$$

$$\text{Voltage} = 15 \times 13.7 \text{ volts} = 205.5 \text{ volts. } \textit{Ans.}$$

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